1. Read Chapter 1 of our ROB 101 Booklet, `Notes for Computational Linear Algebra`; you will find a copy on our Canvas site, in the `file` folder. Based on your reading of the Chapter, summarize in your own words:

   (a) the purpose of the Chapter;

   (b) two things you found the most interesting.

   There are no "right" or" wrong" answers, but no answer means no points. The goal is to reflect a bit on what you are learning and why.

a) The purpose was to figure out where the elements of a matrix are coming from.

b) • The fact that 99.9% of all engineers mispronounce the Greek alphabet.

• The part 1.6 (where is Computational Linear Algebra used?) was interesting and promising.

2. Lines in the plane

(a) On a single graph, with horizontal axis x and vertical axis y, sketch the two lines $y = 0.5x + 1$ and $y = -x + 4$. Find a point (x^*, y^*) where they cross one another (read the values as closely as you can from the graph, say to an accuracy of ± 0.5). Is there more than one point on the plane where the two lines cross?

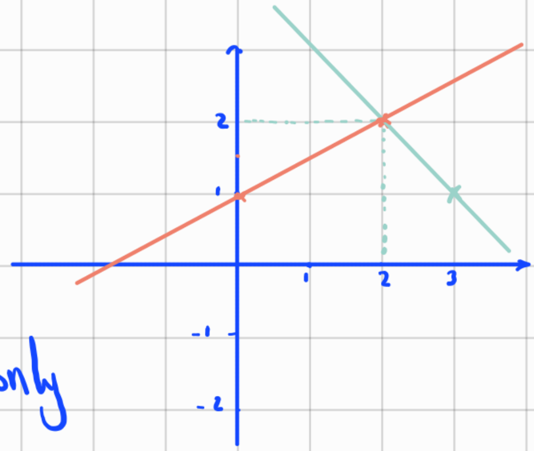
(b) Optional (means you are not required to work this part): Can you relate the point you found to the following system of linear equations?

$$x - 2y = -2$$
$$x + y = 4$$

(c) **Remark:** Sketching the graphs by hand is perfectly fine and is what we expect most of you to do. It is also OK to make the plots using Julia or any other software package. Hand sketched plots will receive the same marks as a computer generated plot.

a)

$$\begin{cases} y = 0.5x + 1 \longrightarrow \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \\ y = -x + 4 \longrightarrow \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix} \end{cases}$$

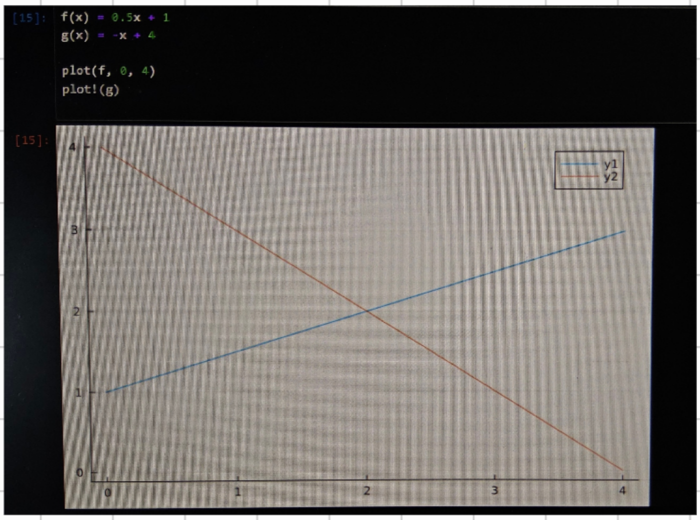


$$\Rightarrow \begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$ $\longrightarrow$ this is the only intersection point of the lines.

b)

$$\begin{cases} x - 2y = -2 \longrightarrow 2y = x + 2 \longrightarrow y = 0.5x + 1 \\ x + y = 4 \longrightarrow y = -x + 4 \end{cases}$$

$\Rightarrow$ In fact these two lines are exactly the previous ones.



3. Lines in the plane

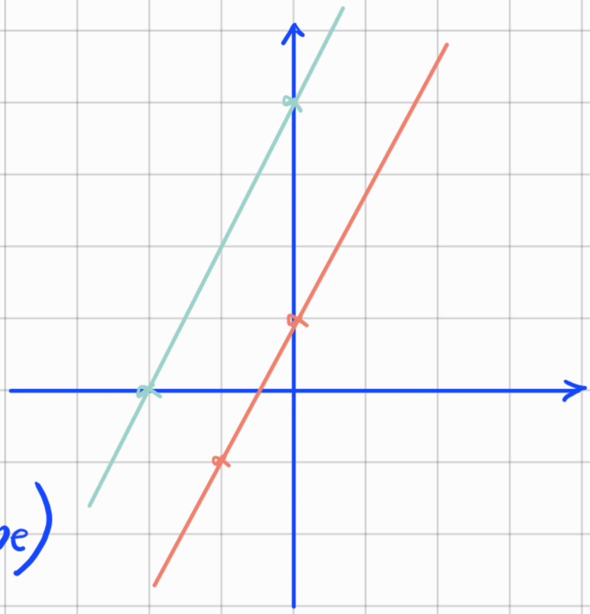
(a) On a single graph, with horizontal axis x and vertical axis y, sketch the two lines $y = 2x + 1$ and $y = 2x + 4$. Can you find a point (x^*, y^*) where they cross one another (read the values as closely as you can from the plot, say to an accuracy of ± 0.5)? If you cannot find a point where the two lines intersect, explain why not?

(b) Optional: Can you relate the results of your graph to the following system of linear equations?

$$2x - y = -1$$
$$-x + \frac{1}{2}y = 2$$

a)

$$\begin{cases} y = 2x+1 \longrightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix} \\ y = 2x+4 \longrightarrow \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \end{bmatrix} \end{cases}$$



The lines are parallel (Same slope) $m = 2$

so the do not have intersection.

$\Rightarrow$ The linear equation system does not have solution.

b)

$$\begin{cases} 2x - y = -1 \longrightarrow y = 2x+1 \\ -x + \frac{1}{2}y = 2 \longrightarrow y = 2x+4 \end{cases}$$

Again these lines are the same as those of section a.

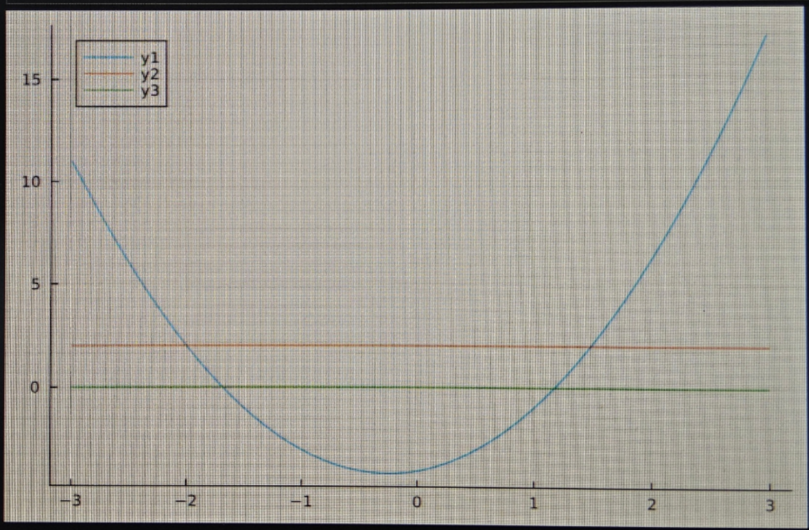
So this system of linear equations does not have solution too.

4. Plot the graph of the quadratic equation $y = 2x^2 + x - 4$, for $-3 \leq x \leq 3$. From the graph, approximately find the two values of x where $y = 0$. Repeat for $y = 2$. (read the values as closely as you can from the plot, say to an accuracy of ± 0.25)

```
[113]: p(x) = 2x^2 + x -4
       k(x) =  2
       l(x) = 0

       plot(p, -3, 3)
       plot!(k)
       plot!(l)

[113]:
```




```
[159]: p(1.2)

[159]: 0.08000000000000007

[153]: p(-1.7)

[153]: 0.07999999999999918

[129]: p(1.5)

[129]: 2.0

[155]: p(-2)

[155]: 2
```

$y = 0 \Rightarrow \begin{cases} x_1 \approx 1.2 \\ x_2 \approx -1.7 \end{cases}$

$y = 2 \Rightarrow \begin{cases} x_1 \approx 1.5 \\ x_2 \approx -2 \end{cases}$

5. Consider the equation $2x^2 + 3x - 2 = 0$

(a) Evaluate its discriminant (see Chapter 1 or the Hints)

(b) You should have found that the discriminant is positive and hence the equation has two real solutions. Compute them using the quadratic formula (see Chapter 1 or the Hints).

a)

$$2x^2 + 3x - 2 = 0 \implies \Delta = b^2 - 4ac$$

$$= 9 + 16 = 25$$

b)

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-3 \pm 5}{4}$$

$$\implies \begin{cases} x_1 = \frac{1}{2} \\ x_2 = -2 \end{cases}$$

6. Consider a system of linear equations with two unknowns

$$y - 4z = 2$$
$$2y - 10z = -2$$

(a) Solve the system of equations. Show all the steps when determining your solution. You do not need to generate a plot.

(b) Write the system in the form $Ax = b$, where you clearly identify A, x, and b. This will require some material from Chapter 2; see also the Hints.

a)

$$\begin{cases} y - 4z = 2 \\ 2y - 10z = -2 \end{cases} \xrightarrow{\times(-2)} \begin{cases} -2y + 8z = -4 \\ 2y - 10z = -2 \end{cases}$$

$$-2z = -6 \longrightarrow \boxed{z = 3}$$

$$y - 4z = 2 \Longrightarrow y = 4z + 2 \xrightarrow{z=3} \boxed{y = 14}$$

b)

$$\underbrace{\begin{bmatrix} 1 & -4 \\ 2 & -10 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y \\ z \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 2 \\ -2 \end{bmatrix}}_{b}$$

* Finito *