



第二章

电路术语与基尔霍夫定律



主要知识点

➤ 集总参数电路模型

➤ 基本物理量

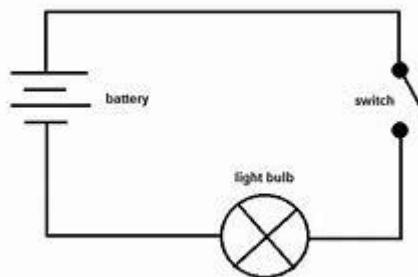
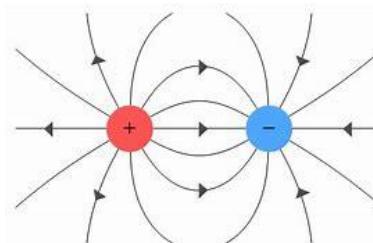
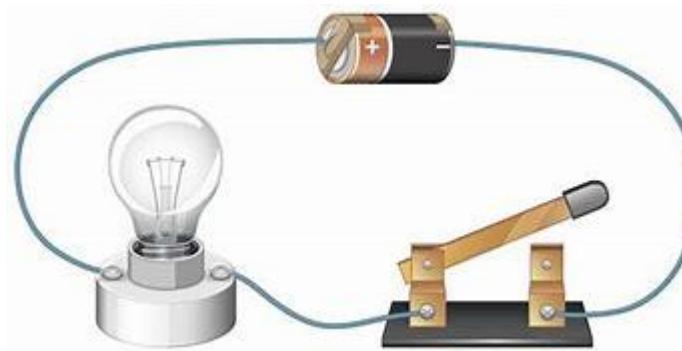
➤ 电路元器件

➤ 基尔霍夫定律

➤ 分压与分流



集总参数电路



- | | |
|-------------------|----|
| (1) 灯泡为什么会亮? | 电流 |
| (2) 为什么会有电流? | 电压 |
| (3) 电压怎么产生的? | 电场 |
| (4) 电场从何而来? | 电池 |
| (5) 电场如何从电池转移到灯泡? | 导线 |
| (6) 导线为何能转移电场? | 电荷 |



集总参数电路

*Electrical effects happen instantaneously throughout a system. A system that is small enough so that we can make this assumption is called a **lumped-parameter system**.*

- *The net charge on every component in the system is always zero.*
- *There is no magnetic coupling between the components in a system.*

- Nilsson&Riedel



集总参数电路模型：条件一

光速 \times 特征时间 \gg 电路尺寸

波长 \times 频率 \times 特征时间 \gg 电路尺寸

波长 \gg 电路尺寸



集总参数电路模型：条件二

- 元件不产生电场 → 元件之间无电耦合
- 流入任意节点的总电流等于流出同一节点的总电流 → 基尔霍夫电流定律

集总参数电路模型：条件三

- 元件之间无磁耦合

无电磁耦合

→ 元件之间的相对独立性

→ 在描述一个元器件的伏安特性时无需考虑其他元器件

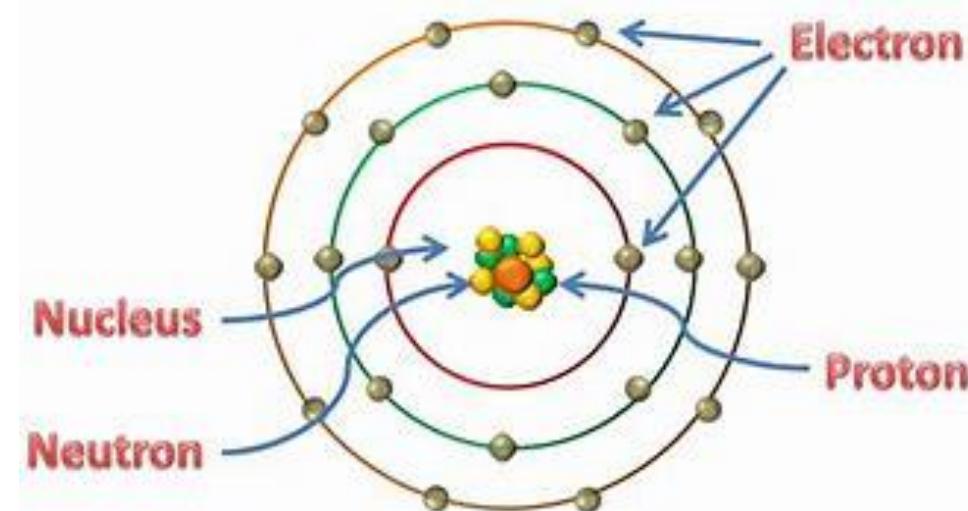


电荷

基本粒子的一个基本属性

原子结构

最小单位: 1.602×10^{-19} C





电流

定义：单位时间内通过某一横截面的总电荷数。

单位：Ampere (A) = C/s

$$I = \frac{dq}{dt} \quad q: 0 \text{ 到 } t \text{ 时段内流过的总电荷}$$

电流方向：与正电荷流动方向相同，与负电荷流动方向相反。



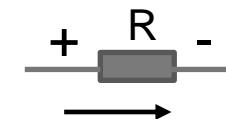


电压

定义：单位电荷流过时元器件所获得的能量

单位： $V = J/C$

方向：电压的方向与电流相同（相反）时，元器件获得（失去）能量。通常用+-表示，电压方向从+指向-。



能量的总类：电场（电容），磁场（电感），化学能（电池），热能（电阻）

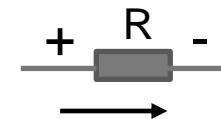


电压电流的参考方向

在复杂电路的分析中，我们往往无法事先知道电流和电压的真实方向。

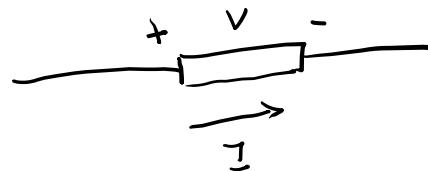
需要设定它们的参考方向（相当于参考坐标系），以便能够列出电路方程

若最终方程的解为正（负）值，则真实方向与参考方向相同（相反）。





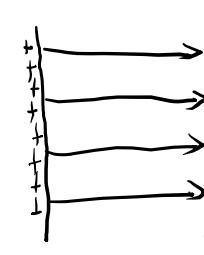
电阻器



- 功→热能
- 电压与电流的真实方向相同
- 当电流电压的参考方向相同时: $V = IR$;
- 当电流电压的参考方向相反时: $V = -IR$;
- 电阻单位 Ohm (Ω)= V/A

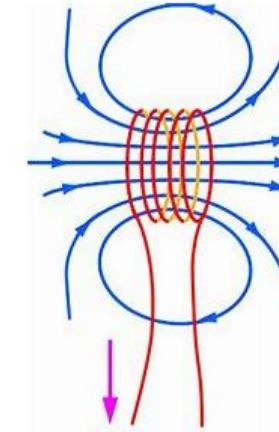


电容器



- 电荷无法在电容中通过 → 电荷在两级积聚
- 电荷产生电场，从而导致两极之间的电势差
- 电容： $q = CV$; 电容单位：Farad (F) = C/V
 - $I = \frac{dq}{dt} = C \frac{dV}{dt}$ 物理量
 - 单位
- 功 → 电场能量 (电场变强，能量增加)

电感器

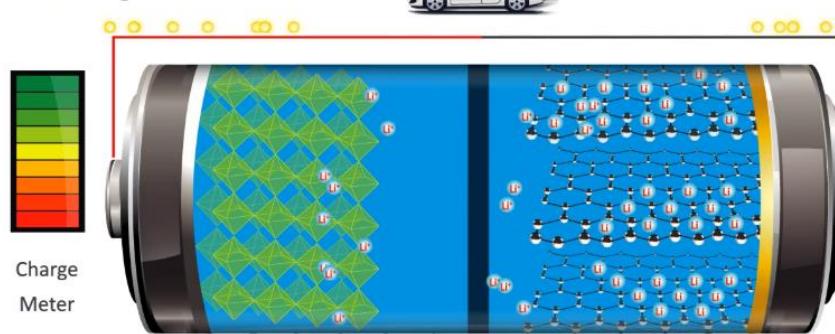


- 磁场强度与电流成正比
- 电流变化导致磁场变化，从而在线圈中感生出电场，在电感器两端出现电势差
- 电感(L): $V = L \frac{dI}{dt}$; 单位: Henry (H) = $\Omega \text{ s}$
- 感生电压起到阻碍电流增加的作用 (楞次定律) (电场施加于外部电路，并非电感导线)
- 功 → 磁场能量 (磁场变强)

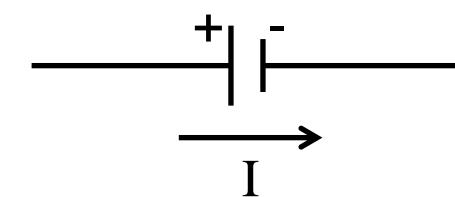
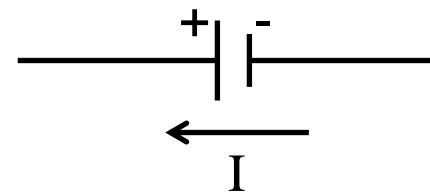
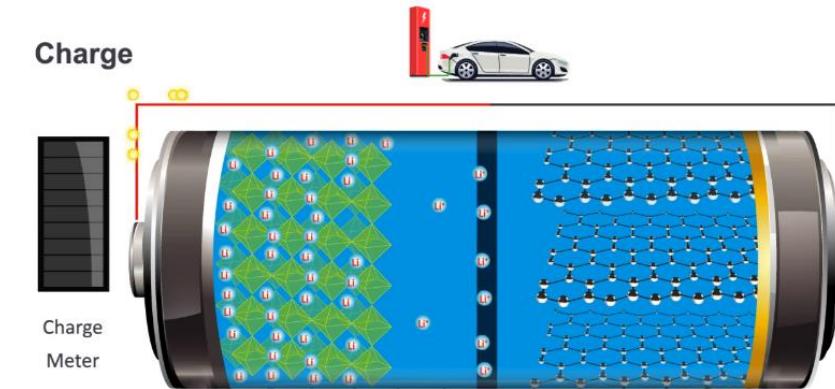


电池

Discharge

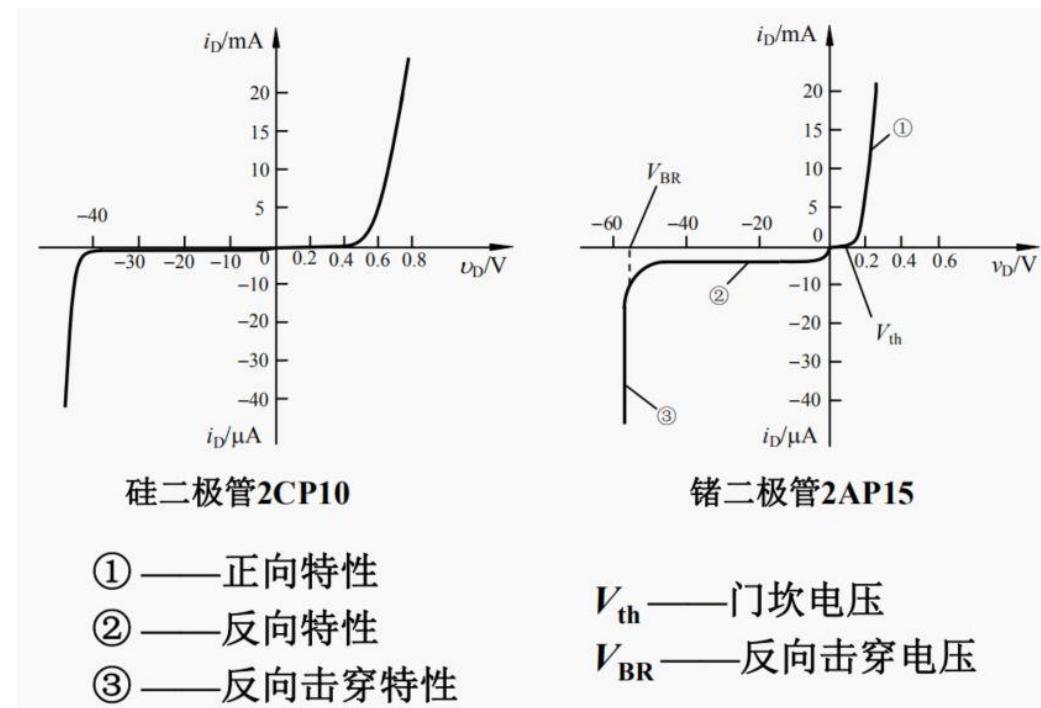
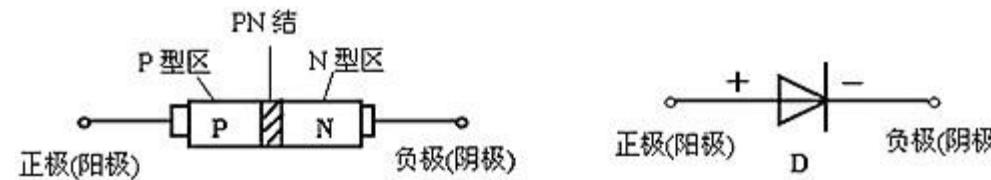


Charge

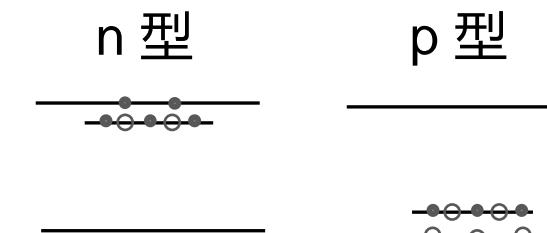
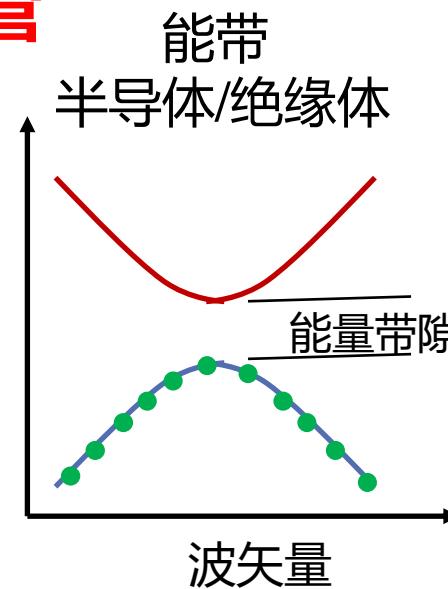




二极管



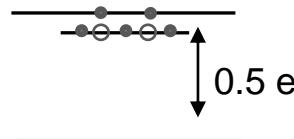
二极管



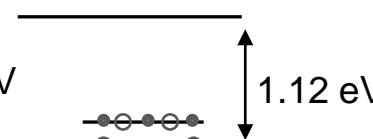


二极管

n 型



p 型



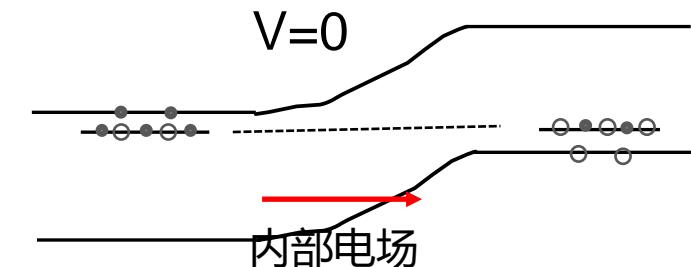
$0 < V < 0.5 \text{ V}$



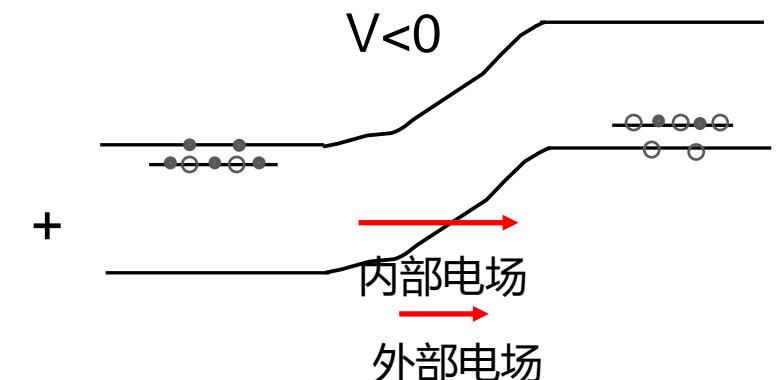
$V \geq 0.5 \text{ V}$



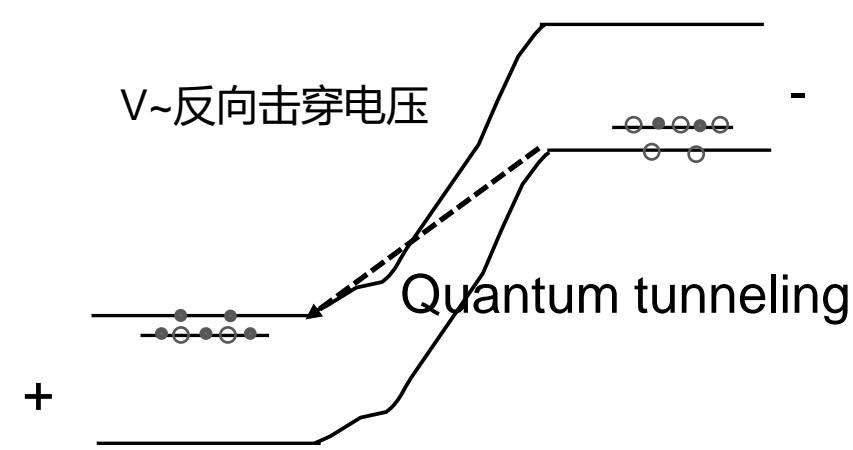
$V=0$



$V < 0$

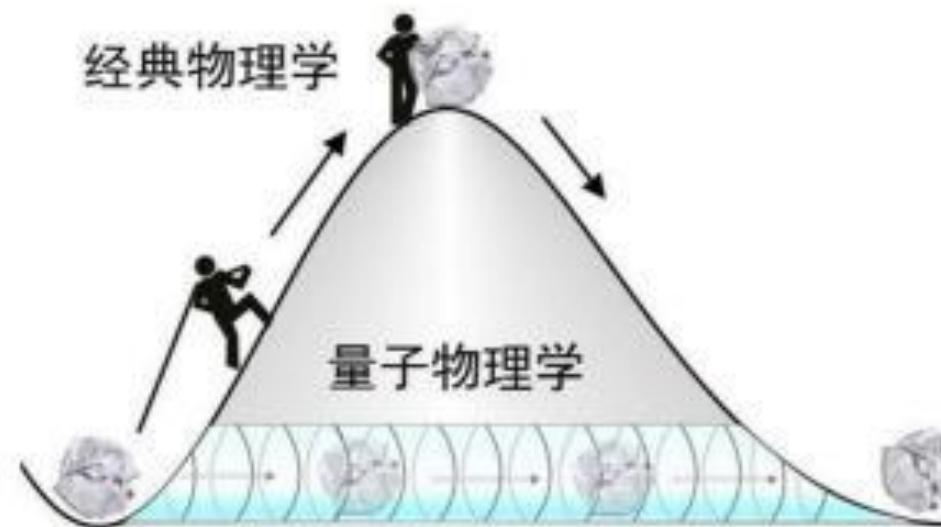


$V \sim \text{反向击穿电压}$





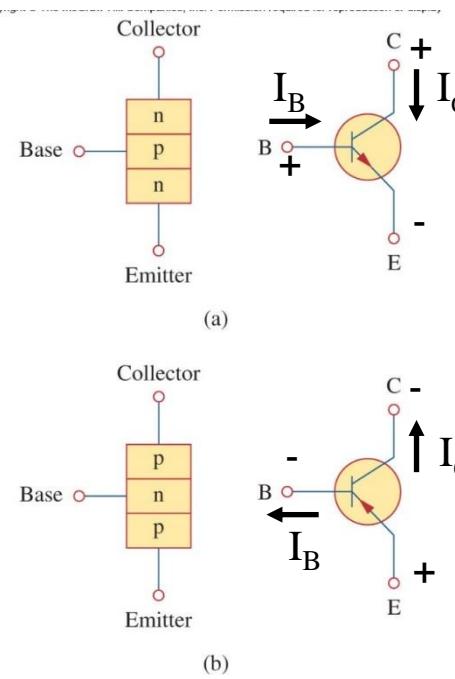
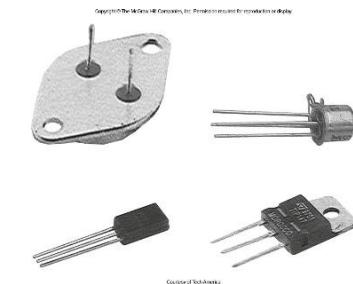
量子隧穿 (Quantum tunneling)





晶体管

- There are two types of transistors commonly used: Field Effect (FET; 场效应) and Bipolar Junction (BJT; 双极结).



A BJT is a three terminal device, where

基极 the base: typically used as to provide control current
发射极 emitter: emit electrons/holes into the base and collector
集电极 collector: collects the majority of the electrons/holes from the emitter.

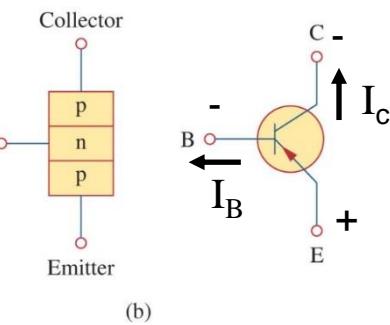
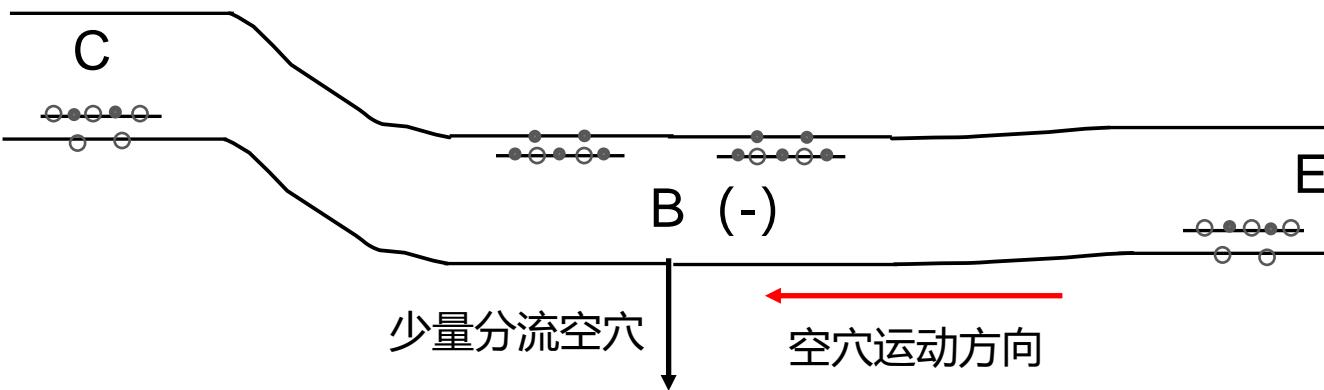
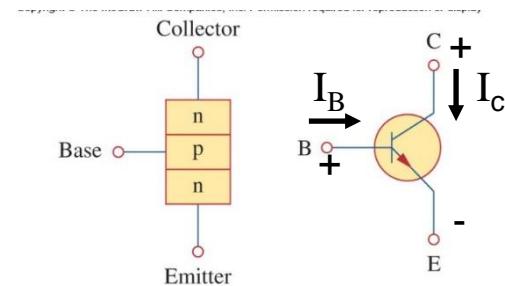
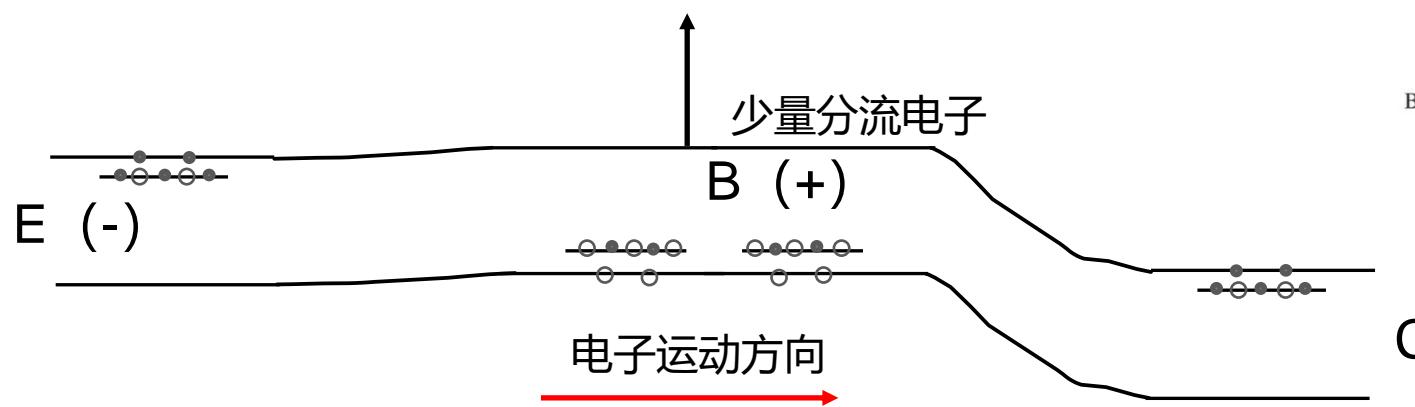
Under normal working condition, the current ratio between the collector and the base is constant $I_C/I_B=\beta$. Typical range of β : 50-1000

npn type: $V_{BE} \approx 0.7$ V; $I_{BE} > 0$; $V_{CB} > 0$

pnp type: $V_{BE} \approx -0.7$ V; $I_{BE} < 0$; $V_{CB} < 0$



晶体管

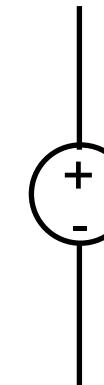




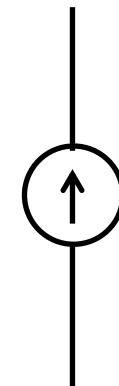
电源

- 分为独立电压和受控电源
- 独立电源包括独立电压源
 1. 前者包括发电机和蓄电池
 2. 后者包括光电池

独立电压源



独立电流源



- 受控电源包括 VCVS, VCCS, CCVS, CCCS。

1. CCCS: 三极管
2. VCVS: 运算放大器
3. CCVS: CCCS+R+VCVS
4. VCCS: VCVS+R+CCCS

受控电压源



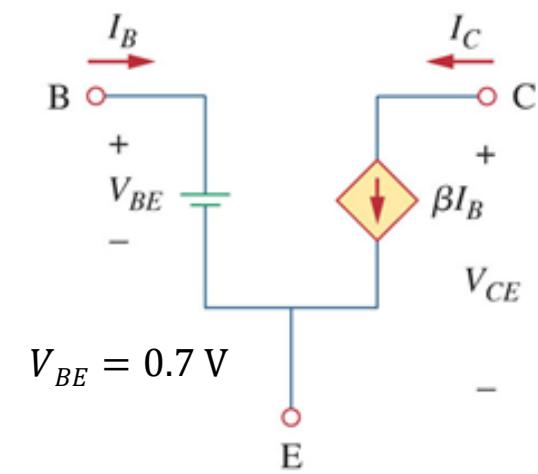
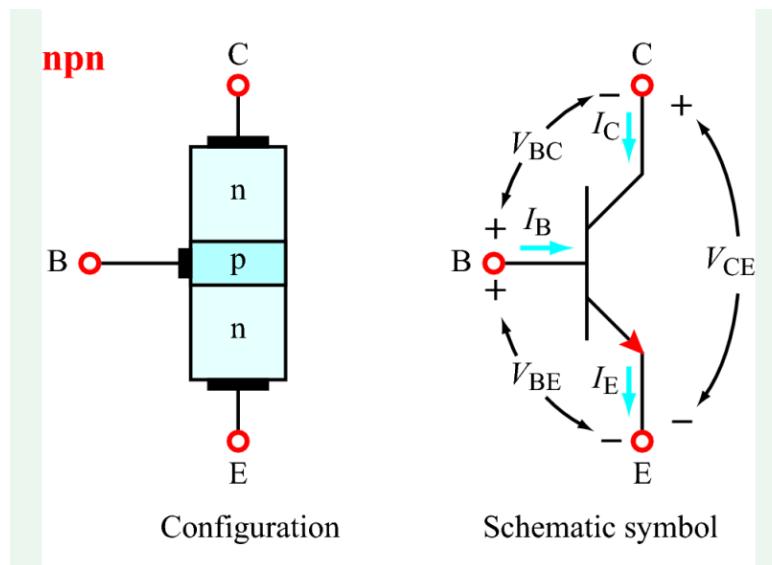
CI/CV

受控电流源



CI/CV

DC model of a npn-BJT

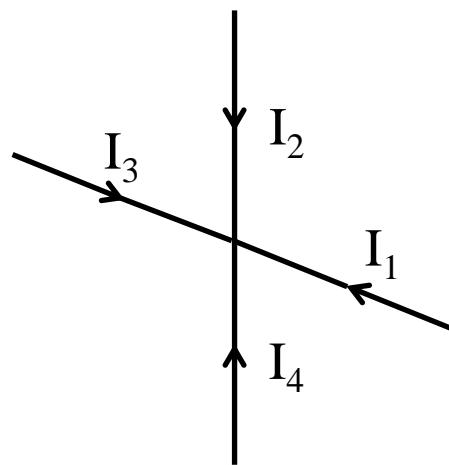




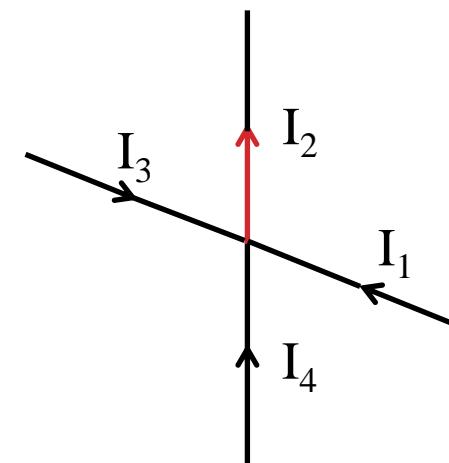
Kirchhoff's current law

- The algebraic sum of currents entering a node (or closed boundary) is zero.
- **Implication 1:** there is no accumulation of charge in any node or within any closed boundary, which is a necessary condition for the lumped circuit model.
- **Implication 2:** the currents through all components in a series circuit are equal.
- **Implication 3:** The algebraic sum of currents entering a node (or closed boundary) is equal to the sum of currents leaving the same node.

Kirchhoff's current law



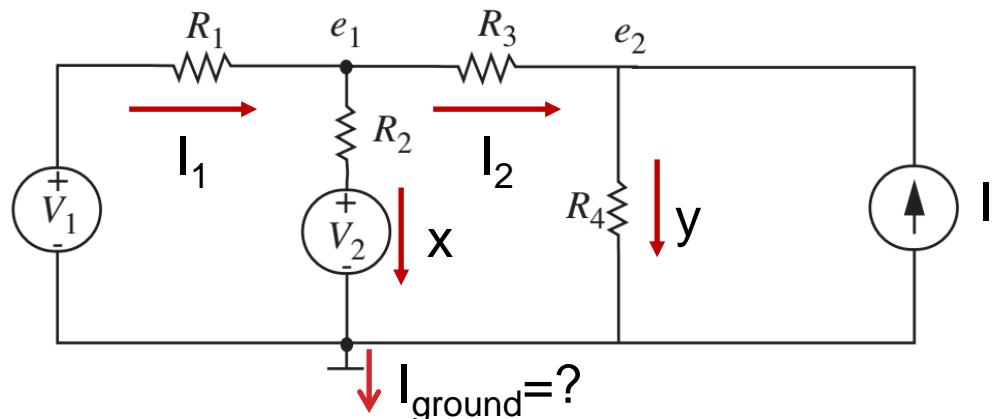
$$\sum_i I_i = 0$$



$$I_1 + I_3 + I_4 = I_2$$

$$I_1 + I_3 + I_4 - I_2 = 0$$

Exercise



1. 已知 I_1 , I_2 , 和 I_s , 求流过 R_2 与 R_4 的电流。

$$y = I_2 + I_s$$

$$x + I_2 = I_1 \rightarrow x = I_1 - I_2$$

2. 流入地线的电流是多少?

解法一: $I_{\text{ground}} = x + y - I_1 - I_s = 0$

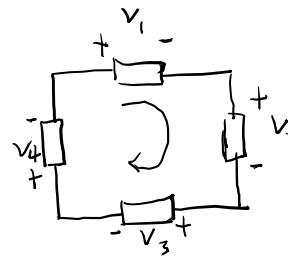
解法二: 将电路整体看作一个节点, $I_{\text{ground}} = 0$



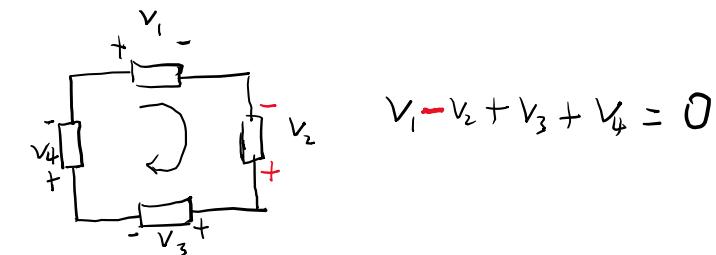
Kirchhoff's voltage law

- The algebraic sum of all voltages around a closed path (or loop) is zero.
- 其物理学基础为麦克斯韦方程 $\oint_L \vec{E} \cdot d\vec{l} = - \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$
 - 电场的回路积分包括了电阻，电容，电池，二极管等两端的电压
 - 磁场的面积分对应电感器中由磁通量变化所产生的电动势 ($V = L \frac{dI}{dt} = \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$)
 - 电路元器件之外不存在变化磁场 (集总参数电路的必要条件)

Kirchhoff's voltage law



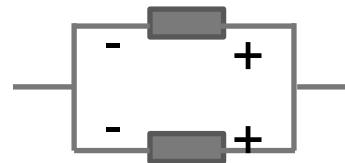
$$V_1 + V_2 + V_3 + V_4 = 0$$



$$V_1 - V_2 + V_3 + V_4 = 0$$

Implication 1: Voltages across all branches in a parallel circuit are equal.

支路1

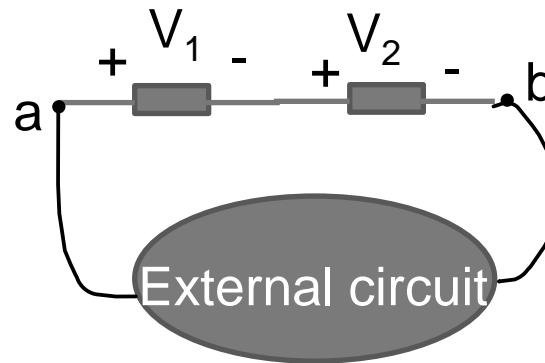


$$V_2 - V_1 = 0 \rightarrow V_1 = V_2$$

支路2

Kirchhoff's voltage law

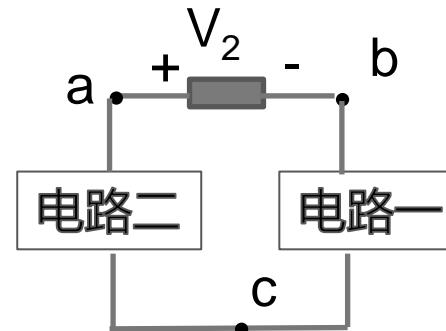
Implication 2: Voltage across two components in series is equal to the sum of voltages across each component.



$$V_1 + V_2 - V_{ab} = 0$$

$$V_{ab} = V_1 + V_2$$

Implication 3: Voltage across two nodes is equal to difference of the voltages between the two nodes and a third node.





KCL and KVL 简单实例一：串联电路

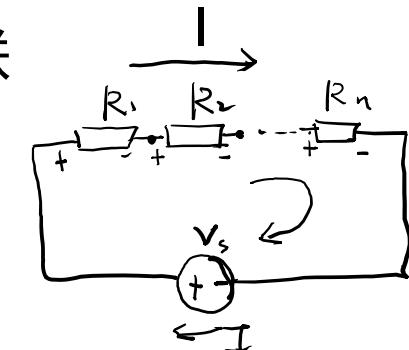
使用基尔霍夫定律，求由n个电阻和电压源 V_s 所构成的串联电路中每个电阻器的电流和电压。

$$\text{KVL: } V_1 + V_2 + \dots + V_n - V_s = 0$$

$$V_s = \sum_{i=1}^n V_i = \sum_{i=1}^n R_i I$$

$$I_s = \frac{V_s}{\sum_{i=1}^n R_i} \quad R_{\text{eff}} = \sum_{i=1}^n R_i$$

$$V_i = V_s - \frac{R_i}{R_{\text{eff}}}$$



- 串联电路的分压与阻值成正比
- 等效电阻为所有阻值之和

KCL and KVL 简单实例二：并联电路

使用基尔霍夫定律，求由n个电阻和电流源 I_s 所构成的并联电路中每个电阻器的电流和电压。

$$\text{KCL: } I_s - \sum_{i=1}^n I_i = 0$$

$$I_s = \sum_{i=1}^n I_i$$

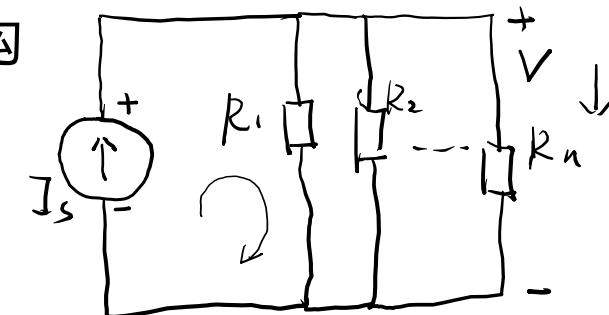
$$I_i = \frac{V}{R_i}$$

$$I_s = \sum_{i=1}^n V \left(\frac{1}{R_i} \right)$$

定义电导: $S = \frac{1}{R}$, $V = RI$, $I = SV$

$$I_s = V \sum_i S_i \rightarrow V = \frac{I_s}{\sum_i S_i}$$

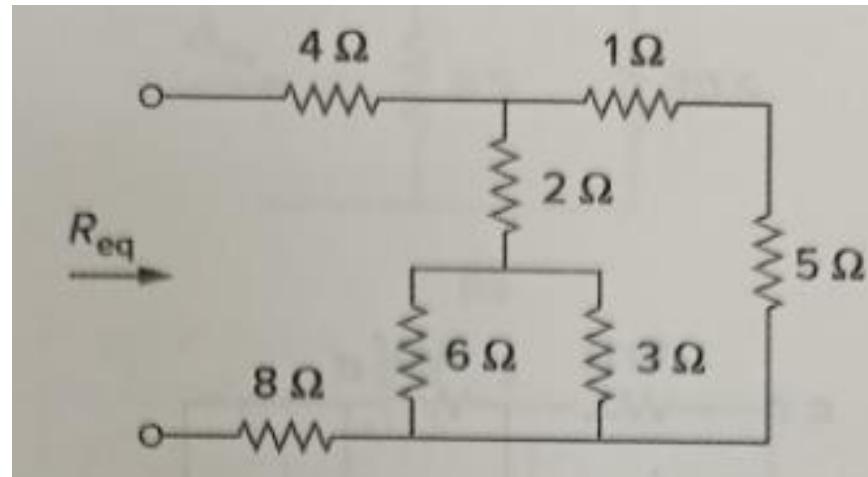
$$I_i = VS_i = \frac{S_i}{\sum_i S_i} I_s$$



- 并联电路的分流与电导成正比
- 等效电导为所有支路的电导之和
- 等效电阻为所有支路的电导之和的倒数



练习：求如下电阻网络的等效电阻



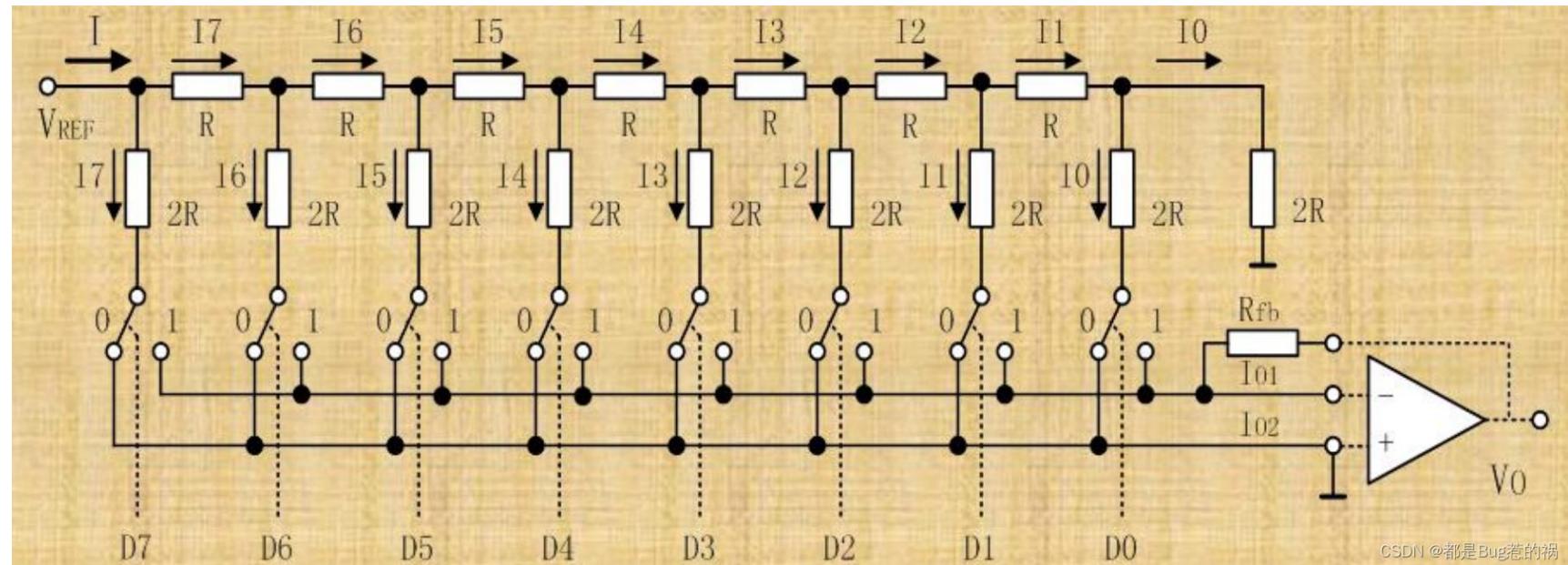
$$3\parallel 6 = 2$$

$$4\parallel 6 = 2.4$$

$$R_{eq} = 4 + 8 + 2.4 = 14.4 \Omega$$



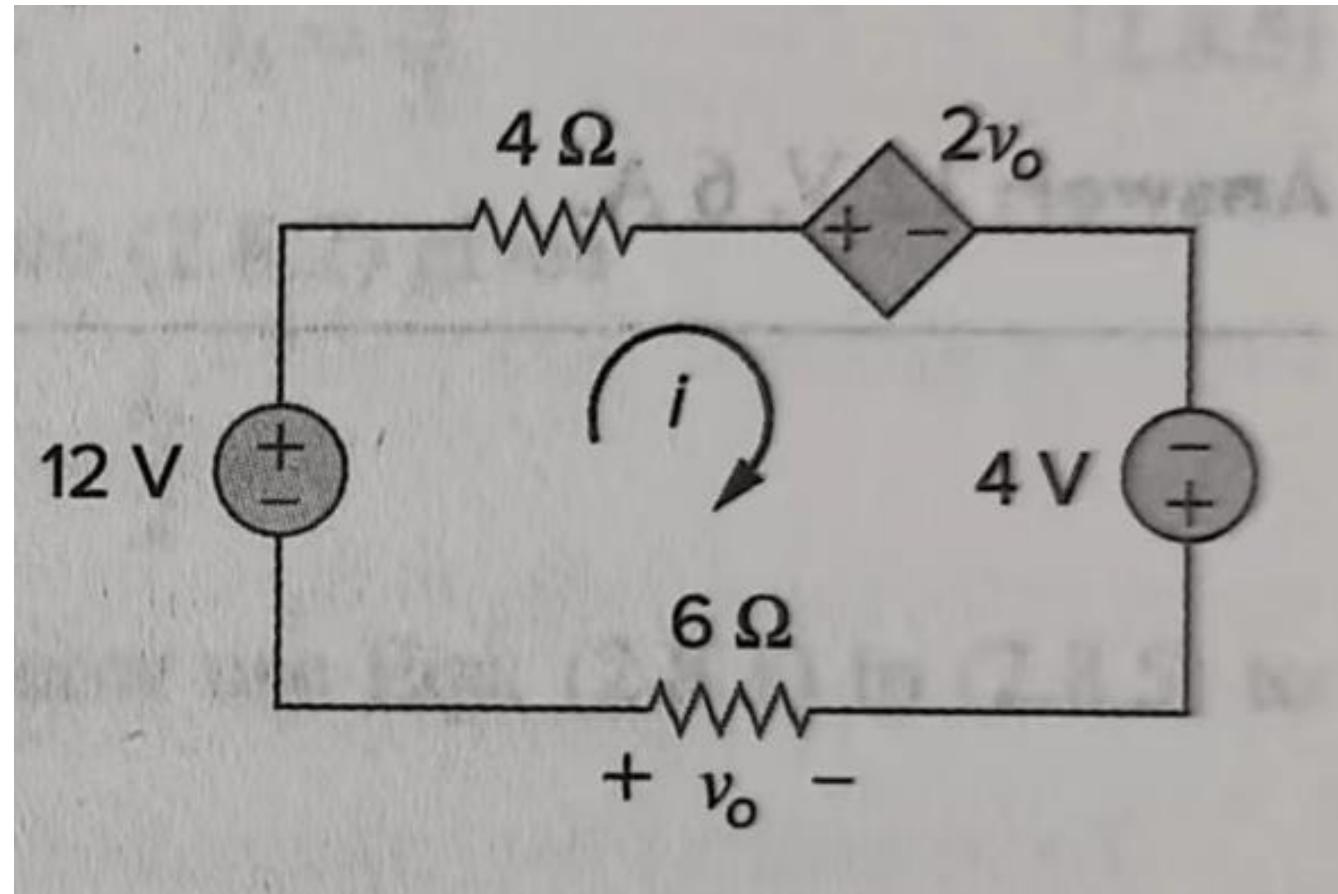
练习：求如下反T型DAC电路中 V_{REF} 与地之间的等效电阻。





KCL和KVL 练习实例三

求如下电路中的电流*i*和6 Ω电阻两端的电压*v*_o.



实例三解答

Alexander & Sadiku, Fig. 2.23

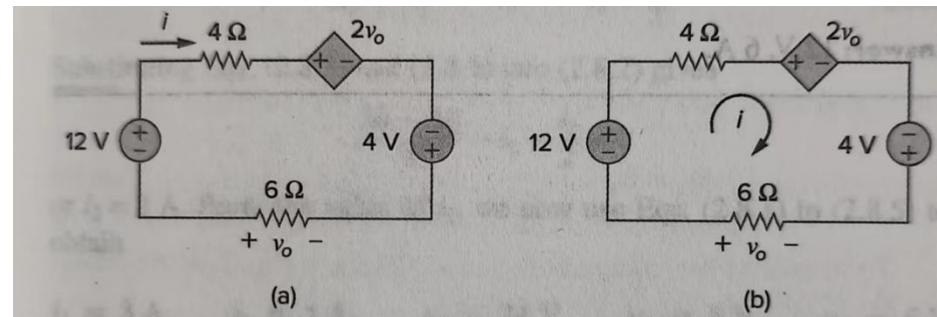


Figure 2.23
For Example 2.6.

Solution:

We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \quad (2.6.1)$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \quad (2.6.2)$$

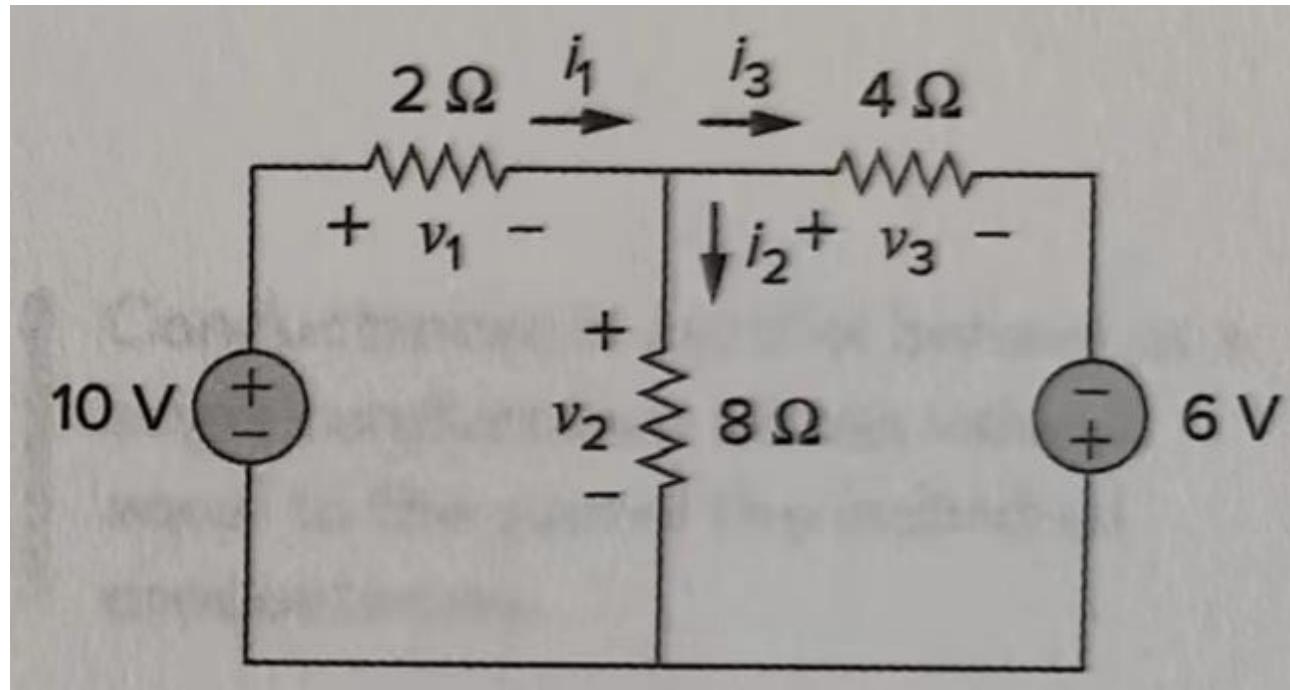
Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

$$-16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8 \text{ A}$$

and $v_o = 48 \text{ V}$.

KCL and KVL 练习实例四

求如下电路中的各电阻中的电流和两端的电压。计算电阻和电源的功率并验证其满足能量守恒。





实例四解答

$$\left\{ \begin{array}{l} -10 + 2i_1 + 8i_2 = 0 \\ -8i_2 + 4i_3 - 6 = 0 \\ i_1 - i_2 - i_3 = 0 \end{array} \right.$$

计算各个器件的功率

$$\begin{pmatrix} 2 & 8 & 0 \\ 0 & -8 & 4 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 6 \\ 0 \end{pmatrix}$$

$$P(10 \text{ V}) = -30 \text{ W}$$

$$P(6 \text{ V}) = -15 \text{ W}$$

$$P(2 \Omega) = 18 \text{ W}$$

$$P(4 \Omega) = 25 \text{ W}$$

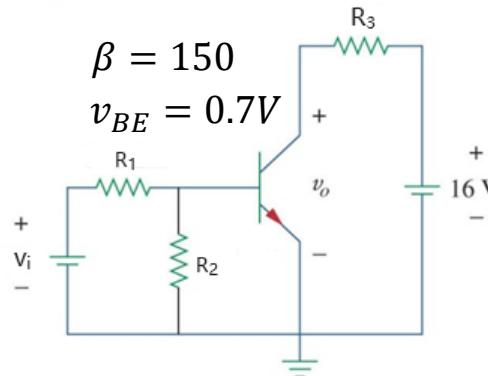
$$P(8 \Omega) = 2 \text{ W}$$

Answer: $v_1 = 6 \text{ V}$, $v_2 = 4 \text{ V}$, $v_3 = 10 \text{ V}$, $i_1 = 3 \text{ A}$, $i_2 = 500 \text{ mA}$, $i_3 = 2.5 \text{ A}$.

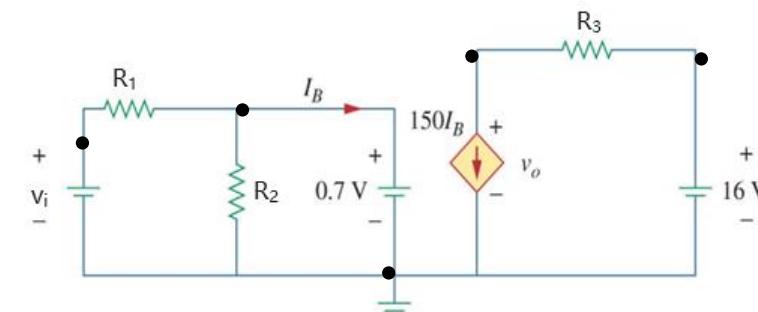
三极管电路分析

- (1) 求如下电路中输入电压(v_i)与输出电压(v_o)之间的关系。
- (2) 求 v_o 与 v_i 微小变化之间的比值。
- (3) 为了使该三极管正常工作, 求 v_i 与 v_o 的取值范围。

原电路



等效电路





Solution

Since v_i is given, the unknown variables are v_0 and I_B , which can be obtained from the following two equations:

$$I_B = \frac{(v_i - 0.7)}{R_1} - \frac{0.7}{R_2}$$

$$150I_B = \frac{16 - v_0}{R_3} \rightarrow v_0 = 16 - 150I_B R_3$$

$$v_o = -\frac{150 R_3}{R_1} v_i + 16 + \underbrace{150 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_3}_{150 \times 0.7}$$

$$A = \frac{\Delta V_o}{\Delta V_i} = -\frac{150 R_3}{R_1} = -\beta \frac{R_3}{R_1}$$

v_i与v_o的取值范围

$$I_B > 0 \rightarrow V_i > 0.7 \underbrace{\frac{(R_1 + R_2)}{R_2}}_{V_{i\min}}$$

$$V_o = 16 - 150 I_B R_3 < 16V$$

$$V_{CB} > 0 : V_o = V_{BE} + V_{CB} > 0.7V$$

$$V_i < \frac{16 + 0.7 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_3 - 0.7}{150 \frac{R_3}{R_1}} = V_{i\max}$$

