Chapter 9

- Sinusoidal Steady-State Analysis

Outline

- Equivalent impedance
  - Series impedance
  - Parallel impedance
- General AC phasor analysis
- AC Power

Impedance and Admittance

Resistor	$\mathbf{Z} = R$	$\mathbf{Y} = 1/R$
Inductor	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = 1/j\omega L$
Capacitor	$\mathbf{Z} = 1/j\omega C$	$\mathbf{Y} = j\omega C$

Impedance is voltage/current

$$\mathbf{Z} = R + jX$$

R = resistance = Re(Z)

X = reactance = Im(Z)

阻抗 （电阻+电抗）

Admittance is current/voltage

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = G + jB$$

G = conductance = Re(Y)

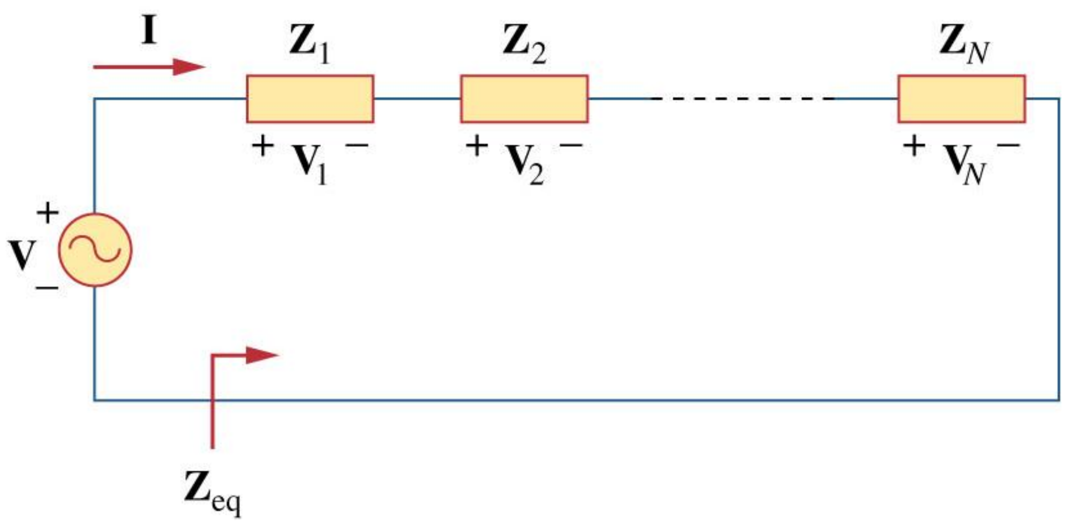
B = susceptance = Im(Y)

导纳（电导+电纳）

Equivalent Impedance

- The phasors also follow the KCL and KVL
- The phasors of I and V of R/L/C follow the same relationship as resistors in the time domain.
- The impedance can be combined in the same way as resistors to obtain equivalent impedance
- The voltage distribution rule for the series RLC connections still hold.
- The current distribution rule for the parallel RLC connections still hold.

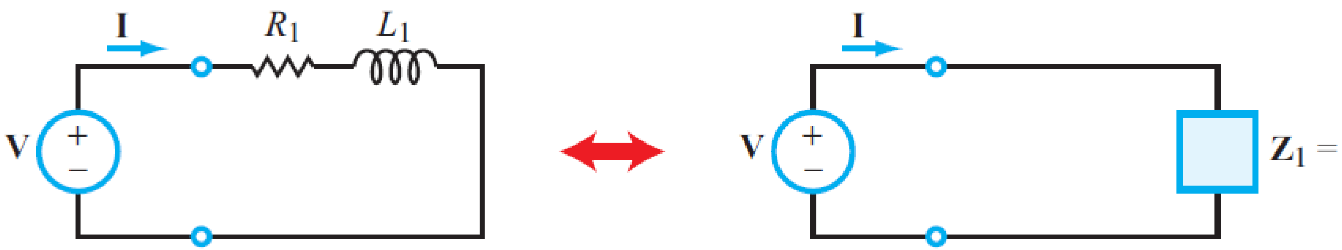
Equivalent Series Impedance



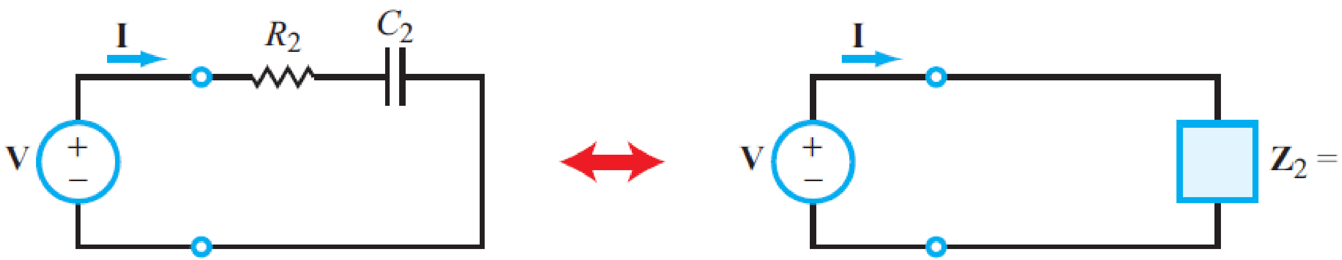
$$Z_{eq} = Z_1 + Z_2 + Z_3 + \cdots + Z_N$$

$$\mathrm{V}_i = \frac{Z_i}{Z_{eq}} V$$

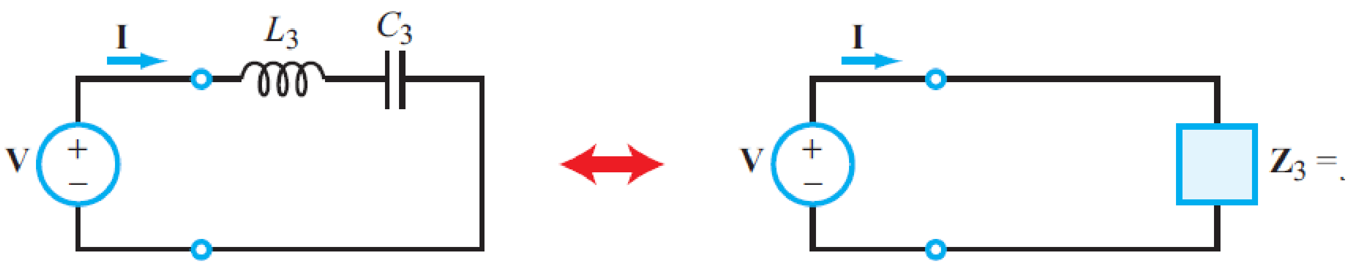
Impedance combination for RLC Circuit



(a) RL

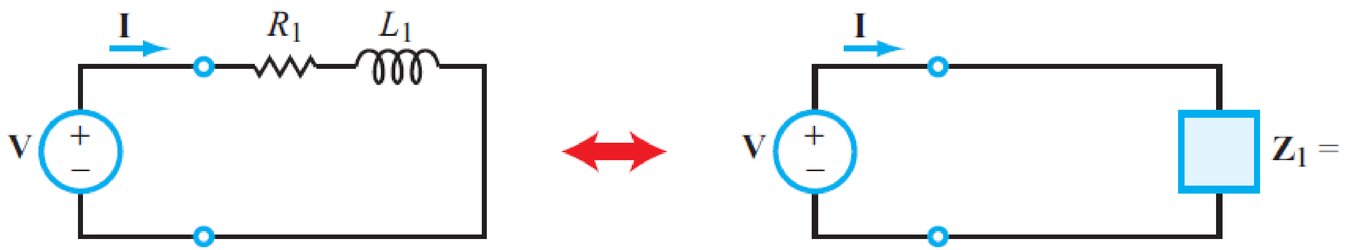


(b) RC

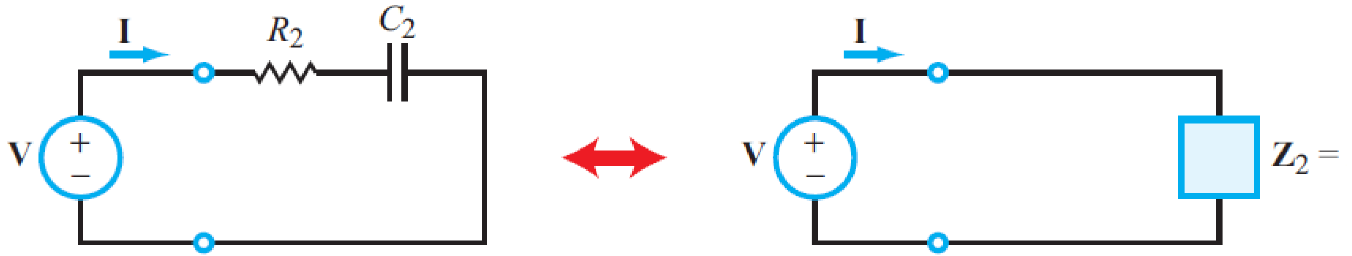


(c) LC

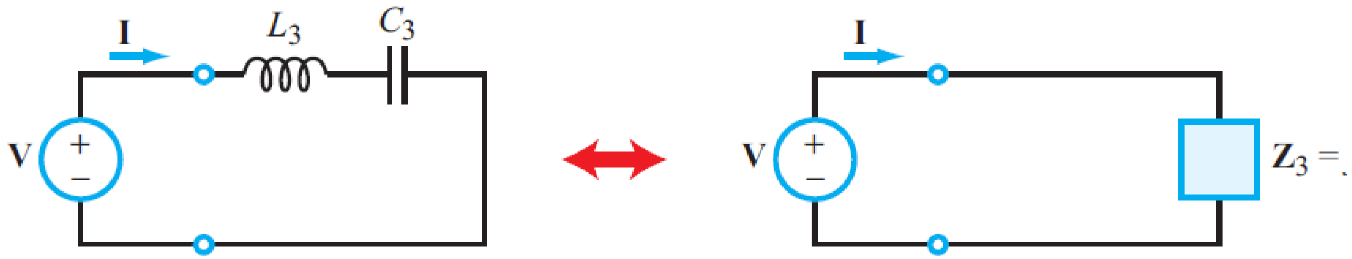
Impedance combination for RLC Circuit



(a) RL



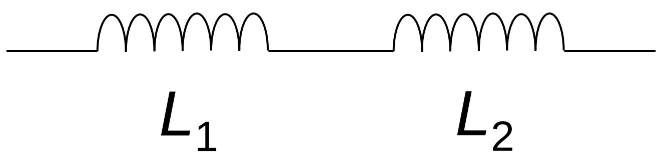
(b) RC

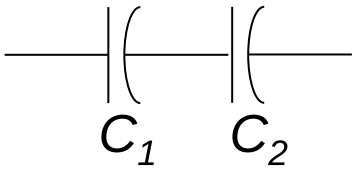


(c) LC

Series L and Series C

Find the total impedance of the following circuits.
What are the equivalent inductance and capacitance?





$$\boldsymbol{Z}_{eq} = j\omega(L_1 + L_2) = j\omega L_{eq}$$

$$L_{eq} = (L_1 + L_2)$$

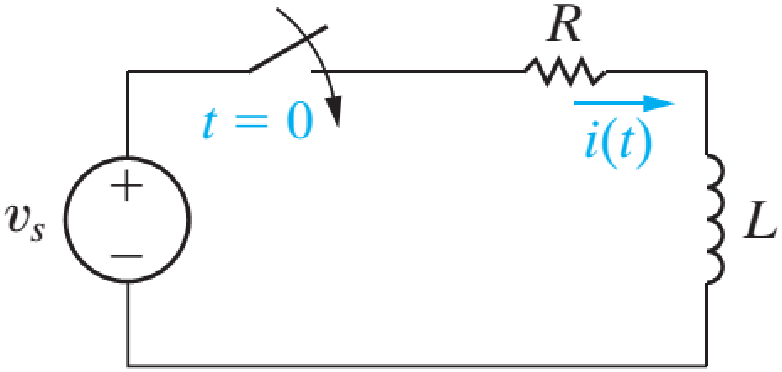
$$\boldsymbol{Z}_{eq} = \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} = \frac{1}{j\omega C_{eq}}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Exercise I

Find the current in the following circuit using phasor analysis.

$$v_s = V_m \cos(\omega t + \phi)$$



Solution Using Phasor

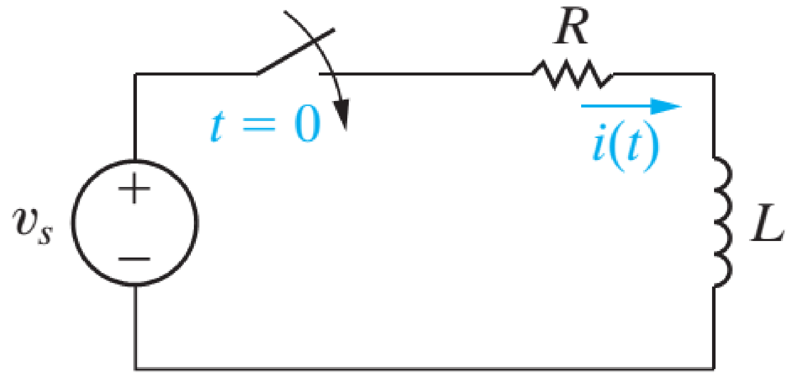
$$\boldsymbol{I} = \frac{\boldsymbol{V_m e^{j\phi}}}{\boldsymbol{R + j\omega L}} = \frac{\boldsymbol{V_m e^{j\phi}}}{\sqrt{\boldsymbol{R^2 + (\omega L)^2}}\boldsymbol{e^{j\theta}}} = \frac{\boldsymbol{V_m}}{\sqrt{\boldsymbol{R^2 + (\omega L)^2}}}\boldsymbol{e^{j(\phi-\theta)}}$$, where $\theta = \mathrm{atan}(\frac{L\omega}{R})$.

Therefore, the steady state solution is

$$i = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$$

Solution from time domain analysis

$$v_s = V_m \cos(\omega t + \phi)$$



$$i = \boxed{\frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \theta) e^{-(R/L)t}} + \boxed{\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)}$$

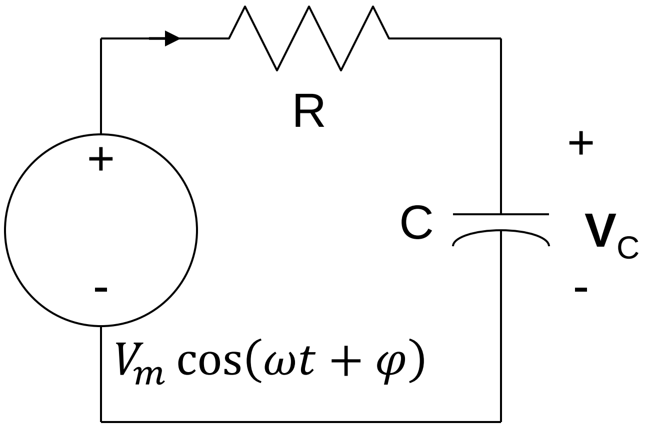
Transient response

Steady-state response

$$\theta = \mathrm{atan}\left(\frac{L\omega}{R}\right)$$

Exercise II

Find V_C in the following circuit using phasor analysis.



Solution: In phasor domain,

$$V_c = V_m e^{i\varphi} \frac{\frac{1}{i\omega C}}{R + \frac{1}{i\omega C}} = V_m e^{i\varphi} \frac{1}{i\omega RC + 1} = \frac{V_m e^{i\varphi}}{\sqrt{1 + (\omega RC)^2} e^{i\theta}}$$

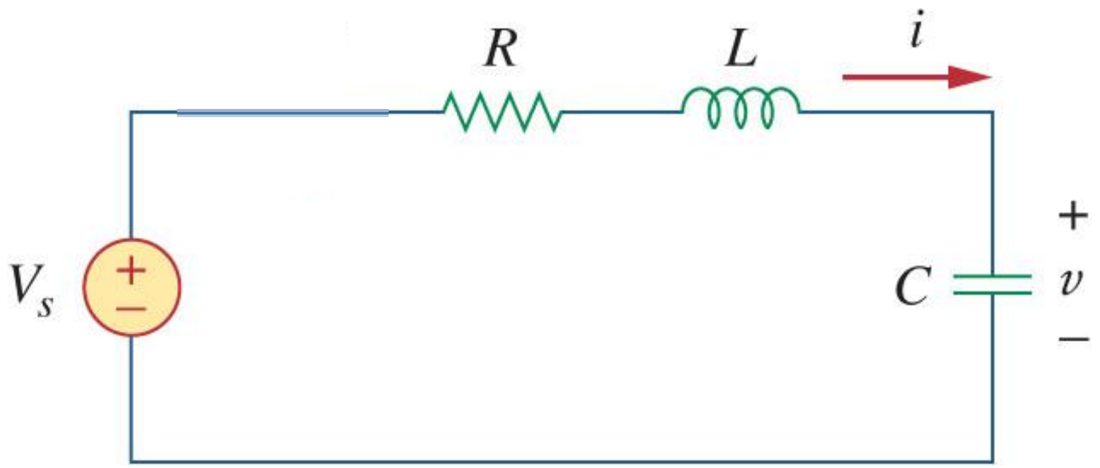
$$= \frac{V_m}{\sqrt{1 + (\omega RC)^2}} e^{i(\varphi - \theta)}, \text{where } \theta = \text{atan}(\omega RC)$$

Transforming V_c back to the time domain, we have

$$V_c(t) = \frac{V_m}{\sqrt{1 + (\omega RC)^2}} \cos(\omega t + \varphi - \theta)$$

Exercise III

- Using the phasor approach, determine the voltage $v(t)$ in the series RLC circuit.



Solution: Using the voltage divider formula,

$$v_p = \frac{\frac{1}{j\omega C}}{R + jL\omega + \frac{1}{j\omega C}} V_{s,p} = \frac{V_{s,p}}{1 + jRC\omega - LC\omega^2}.$$

Solution from the second order equation

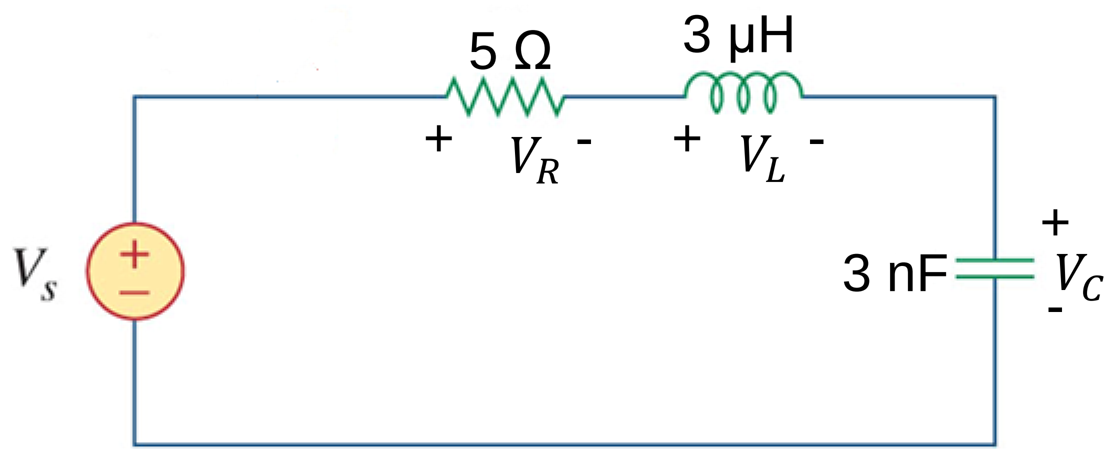
$$\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{v}{LC}=\frac{V_s}{LC}$$

$$\left(-\omega^2+\frac{R}{L}j\omega+\frac{1}{LC}\right)v_p=\frac{V_{s,p}}{LC}$$

$v_p=\frac{V_{s,p}}{1+jRC\omega-LC\omega^2}$, same as the result obtained using the voltage divider formula.

Exercise IV

- Calculate the voltages across R/L/C, if $V_s = 50\cos 2\pi f t$ V where f=1.68 MHz.



Using the voltage divider formula,

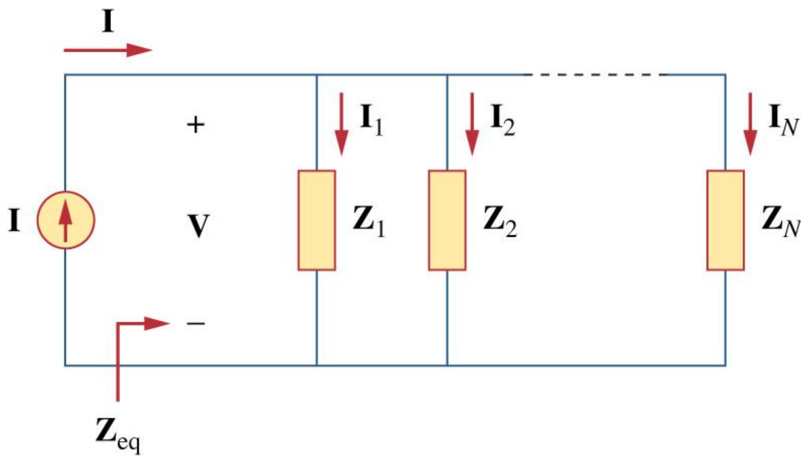
$$V_R = \frac{R}{R+jL\omega+\frac{1}{j\omega C}}50 = 50.0\angle-1^o\ \mathrm{V},$$

$$V_C = \frac{\frac{1}{j\omega C}}{R+jL\omega+\frac{1}{j\omega C}}50 = 315.7\angle 89^o\ \mathrm{V},$$

$$V_L = \frac{jL\omega}{R+jL\omega+\frac{1}{j\omega C}}50 = 316.6\angle-91^o\ \mathrm{V}$$

Parallel Combination

- Likewise, elements combined in parallel will combine in the same fashion as resistors in parallel:



$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_N}$$

$$Y_{eq} = Y_1 + Y_2 + Y_3 + \cdots + Y_N$$

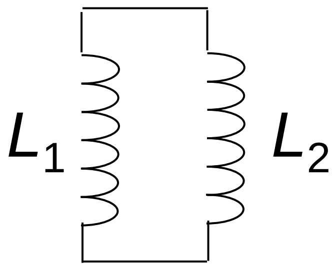
$$I_1 = \frac{Y_1}{Y_1 + \cdots + Y_N} I$$

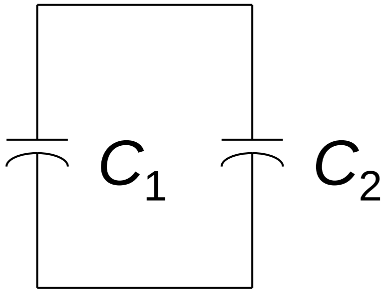
$$I_2 = \frac{Y_2}{Y_1 + \cdots + Y_N} I$$

……

Parallel Inductor and Parallel Capacitor

Find the total impedance of the following circuits.
What are the equivalent inductance and capacitance?





$$\mathbf{Z}_{eq} = j\omega \frac{L_1 L_2}{(L_1 + L_2)} = j\omega L_{eq}$$

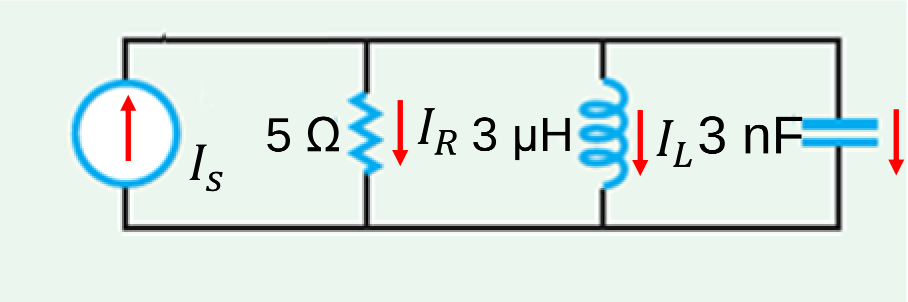
$$L_{eq} = \frac{L_1 L_2}{(L_1 + L_2)}$$

$$\mathbf{Z}_{eq} = \frac{1}{j\omega(C_1 + C_2)} = \frac{1}{j\omega C_{eq}}$$

$$C_{eq} = C_1 + C_2$$

Exercise V

- Calculate the currents into the R/L/C elements in the circuit shown below, if $I_s = 5\cos 2\pi f t$ mA and $f = 1.68$ MHz.



Using the current divider formula,

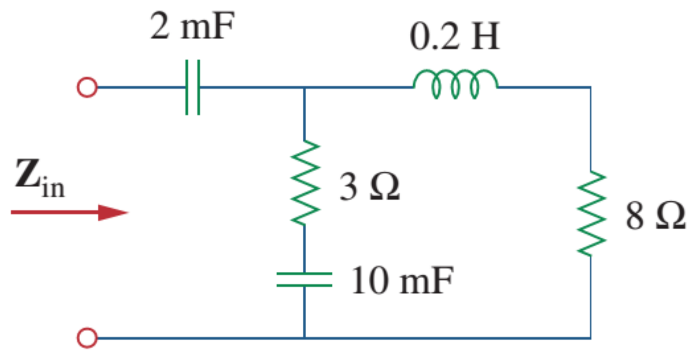
$$I_R = \frac{\frac{1}{R}}{\frac{1}{R}+j\omega C+\frac{1}{j\omega L}}5 = 5.0\angle 0^o \text{ mA}$$

$$I_C = \frac{j\omega C}{\frac{1}{R}+j\omega C+\frac{1}{j\omega L}}5 = 90.0\angle 90^o \text{ mA}$$

$$I_L = \frac{\frac{1}{j\omega L}}{\frac{1}{R}+j\omega C+\frac{1}{j\omega L}}5 = 90.0\angle -90^o \text{ mA}$$

Exercise

- Find the input impedance of the circuit below. $\omega = 50$ rad/s.



Solution:

$$z = \frac{1}{j0.002 \cdot 50} + \frac{(j0.2 \cdot 50 + 8)(3 + \frac{1}{j0.01 \cdot 50})}{(j0.2 \cdot 50 + 8) + (3 + \frac{1}{j0.01 \cdot 50})}$$

$$= -j10 + \frac{(j10 + 8)(3 - j2)}{11 + j8}$$

$$= 3.22 - j11.1 = 11.5\angle -73.8^{o}\ \Omega$$

AC Phasor Analysis
General Procedure

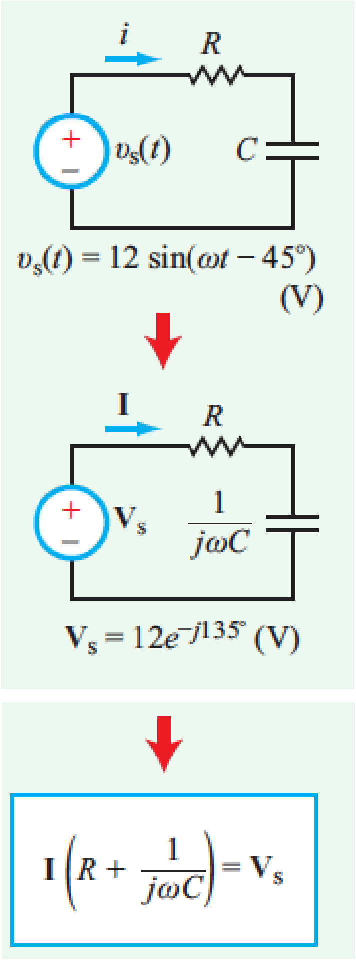
Step1: Transform power sources into phasor domain

$$\upsilon_s(t) = 12\sin(\omega t - 45^\circ)$$
$$= 12\cos(\omega t - 45^\circ - 90^\circ) = 12\cos(\omega t - 135^\circ) \text{ V}.$$

$$\mathbf{V}_s = 12e^{-j135^\circ} \text{ V}.$$

Step 2: Calculate R/L/C impedance

Step 3: Derive circuit equations for I and V phasors using KCL/KVL.



AC Phasor Analysis General Procedure

Step 4: Solve for the I/V phasors.

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + \frac{1}{j\omega C}}$$

$$\mathbf{I} = \frac{12e^{-j135^\circ}}{R + \frac{1}{j\omega C}} = \frac{j12\omega C e^{-j135^\circ}}{1 + j\omega RC}.$$

Using the specified values, namely $R = \sqrt{3}$ kΩ, $C = 1$ μF, and $\omega = 10^3$ rad/s,

$$\mathbf{I} = \frac{j12 \times 10^3 \times 10^{-6} e^{-j135^\circ}}{1 + j10^3 \times \sqrt{3} \times 10^3 \times 10^{-6}} = \frac{j12e^{-j135^\circ}}{1 + j\sqrt{3}} \text{ mA.}$$

$$\mathbf{I} = \frac{12e^{-j135^\circ} \cdot e^{j90^\circ}}{2e^{j60^\circ}} = 6e^{j(-135^\circ + 90^\circ - 60^\circ)} = 6e^{-j105^\circ} \text{ mA.}$$

Step 5: Transform the solutions back to the time domain.

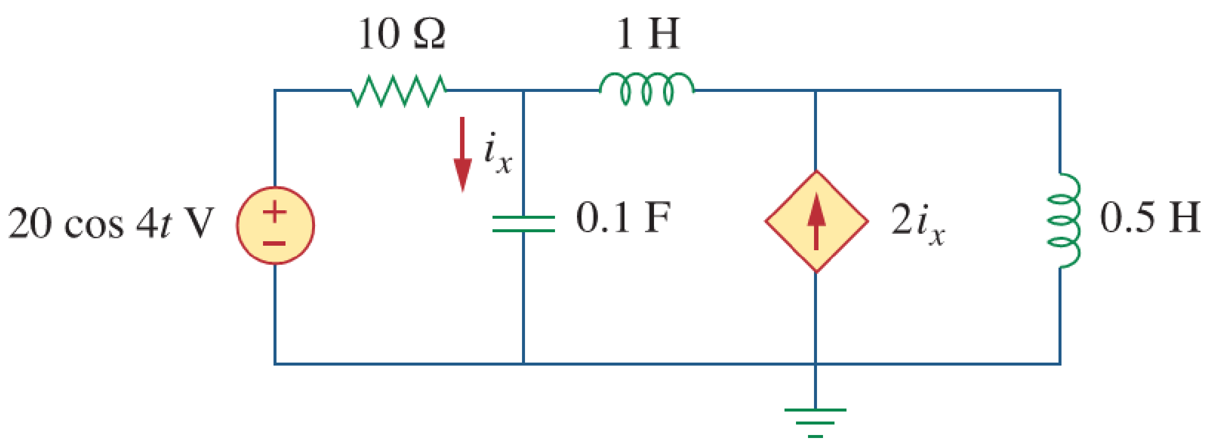
$$i(t) = \mathfrak{Re}[\mathbf{I}e^{j\omega t}] = 6\cos(\omega t - 105^\circ) \text{ (mA)}$$

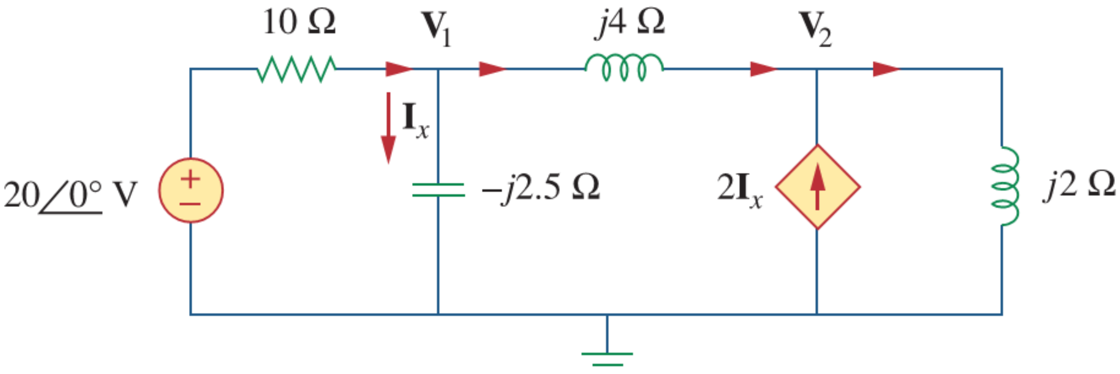
To return to the time domain, we apply the fundamental relation between a sinusoidal function and its phasor counterpart, namely

$$i(t) = \mathfrak{Re}[\mathbf{I}e^{j\omega t}] = \mathfrak{Re}[6e^{-j105^\circ} e^{j\omega t}] = 6\cos(\omega t - 105^\circ) \text{ mA.}$$

Nodal Analysis

- Example---Find i_x





$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

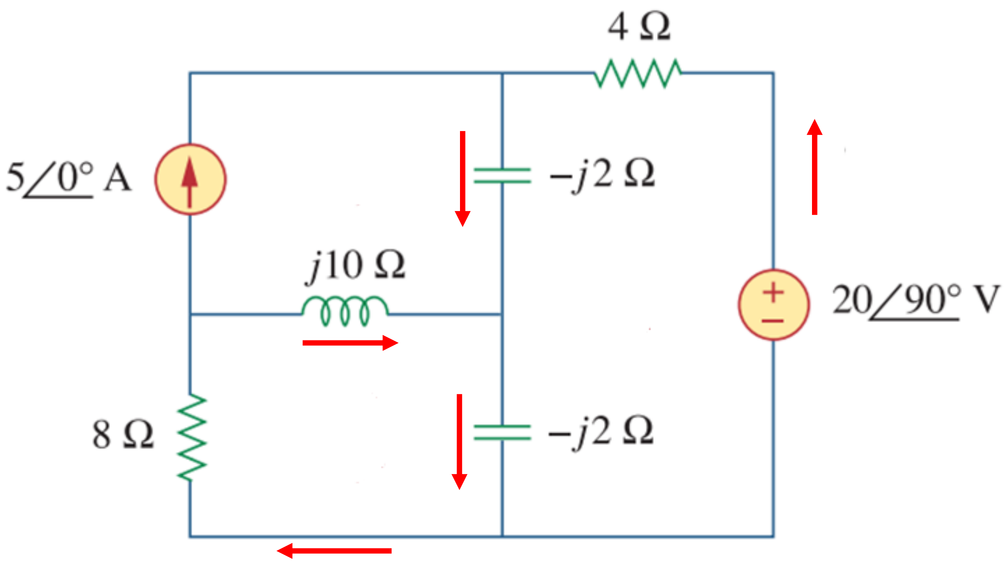
$$2\frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$\begin{bmatrix} \frac{1}{10} + j\frac{2}{5} - j\frac{1}{4} & j\frac{1}{4} \\ j\frac{4}{5} - j\frac{1}{4} & j\frac{1}{4} + j\frac{1}{2} \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

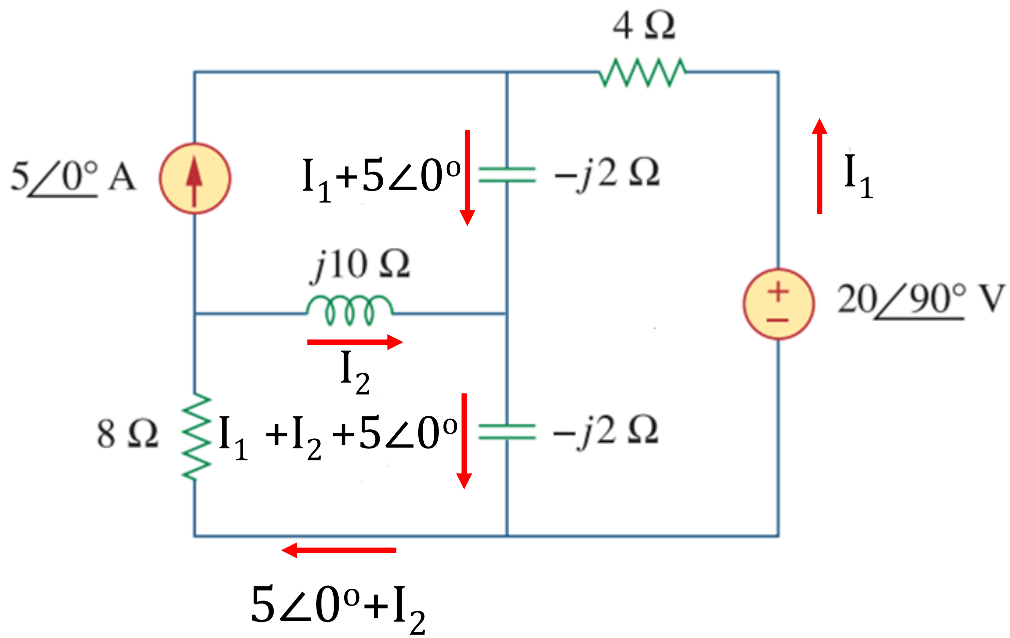
$$i_x = V_1/(-j2.5)$$

$$i_x = 7.59 \cos(4t + 108.4^\circ) \text{ A}$$

Branch Current Analysis



Branch Current Analysis



Using KVL, we have

$$-j20 + 4I_1 - j2(I_1 + 5) - j2(I_1 + I_2 + 5) = 0$$

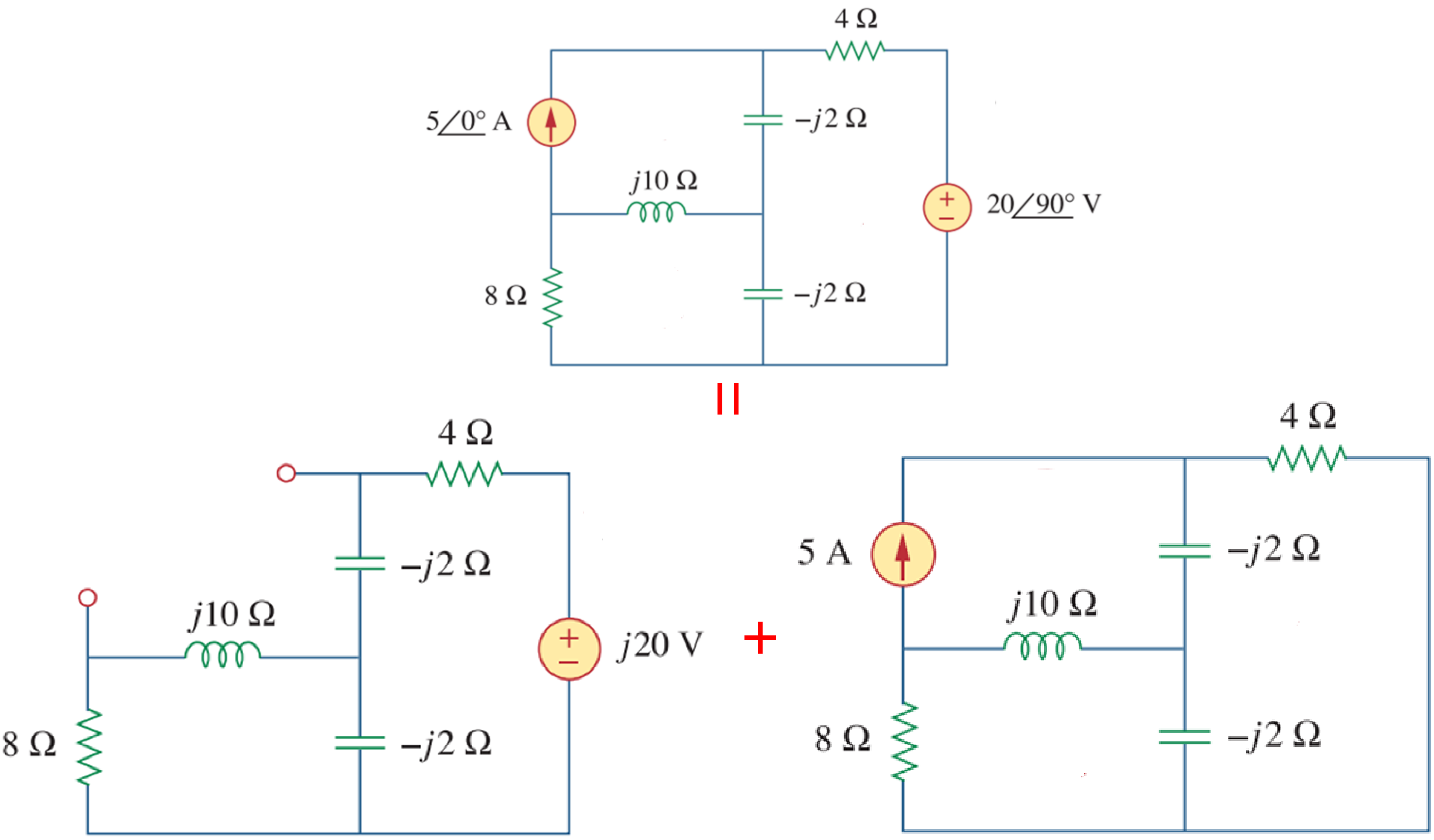
$$j10I_2 - j2(I_1 + I_2 + 5) + 8(5 + I_2) = 0$$

$\Rightarrow$

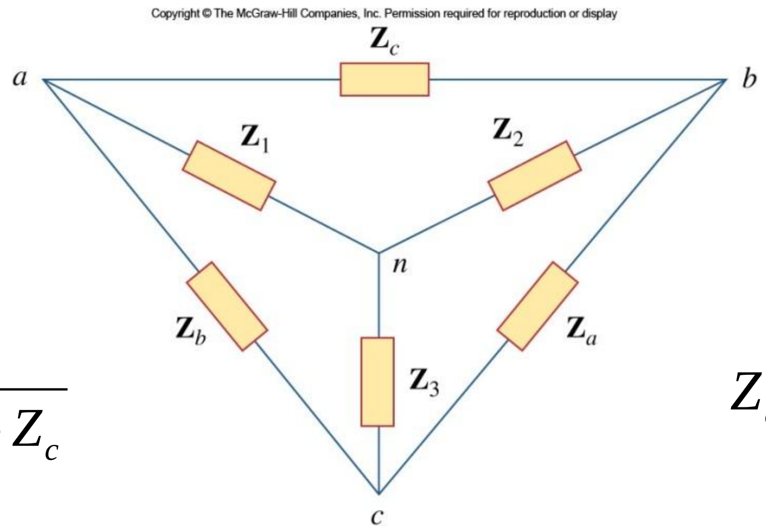
$I_1 = 6.12\angle 145^{o}$ A

$I_2 = 4.16\angle 135^{o}$ A

Superposition-Example



Delta-Wye Transformation in Phasor Domain



$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}$$

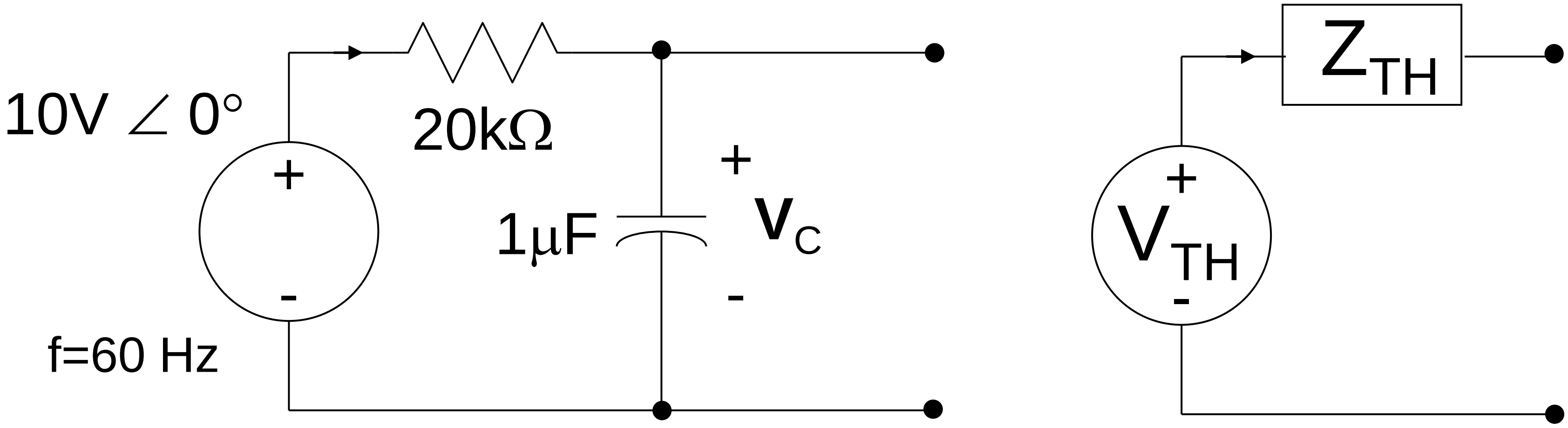
$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

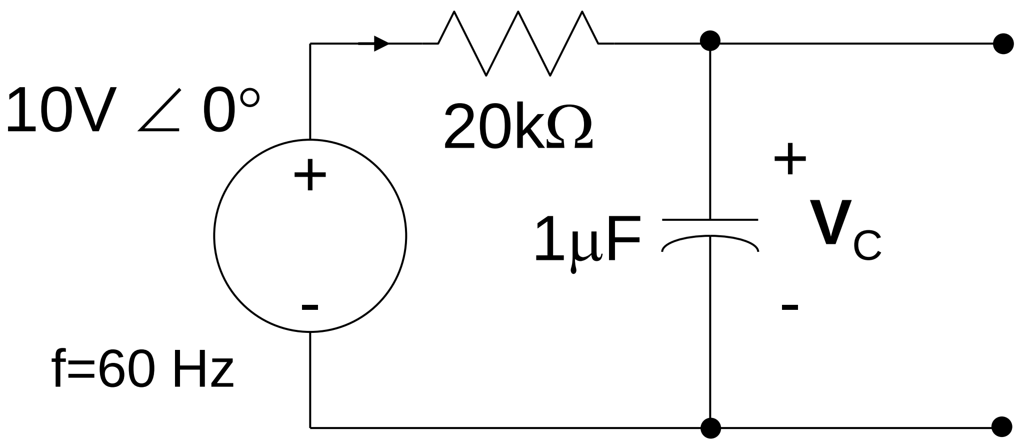
$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

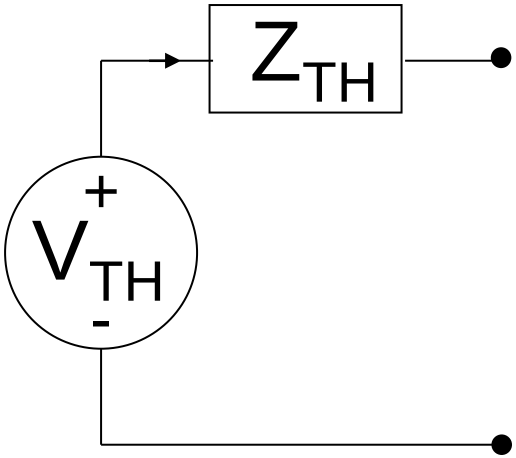
$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

Thevenin Equivalent



Thevenin Equivalent Solution



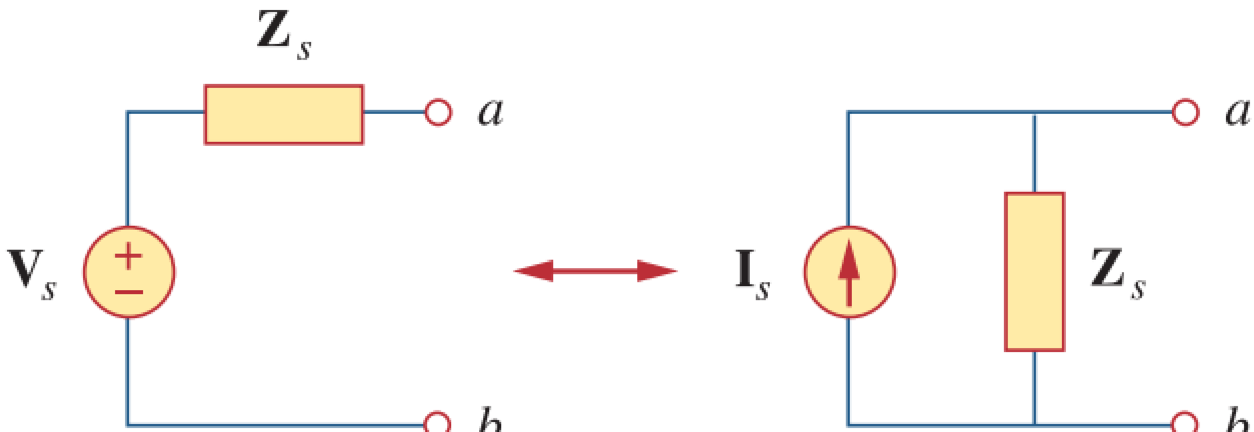


$\mathbf{Z}_R$ = R= 20kΩ = 20kΩ ∠ 0°
$\mathbf{Z}_C$ = 1/*j* (2πf x 1μF) = 2.65kΩ ∠ -90°

$$\mathbf{V}_{TH} = \mathbf{V}_{OC} = 10\text{V}\,\angle 0^\circ \left(\frac{2.65\text{k}\Omega\angle{-90^\circ}}{2.65\text{k}\Omega\angle{-90^\circ} + 20\text{k}\Omega\angle 0^\circ} \right) = 1.31\angle - 82.4^o\ \text{V}$$

$$\mathbf{Z}_{TH} = \mathbf{Z}_R || \mathbf{Z}_C = \left(\frac{20\text{k}\Omega\angle 0^\circ \cdot 2.65\text{k}\Omega\angle{-90^\circ}}{2.65\text{k}\Omega\angle{-90^\circ} + 20\text{k}\Omega\angle 0^\circ} \right) = 2.62\angle - 82.4^o\ \text{k}\Omega$$

Source transformation/Norton



$\mathbf{V}_s = \mathbf{Z}_s\mathbf{I}_s$

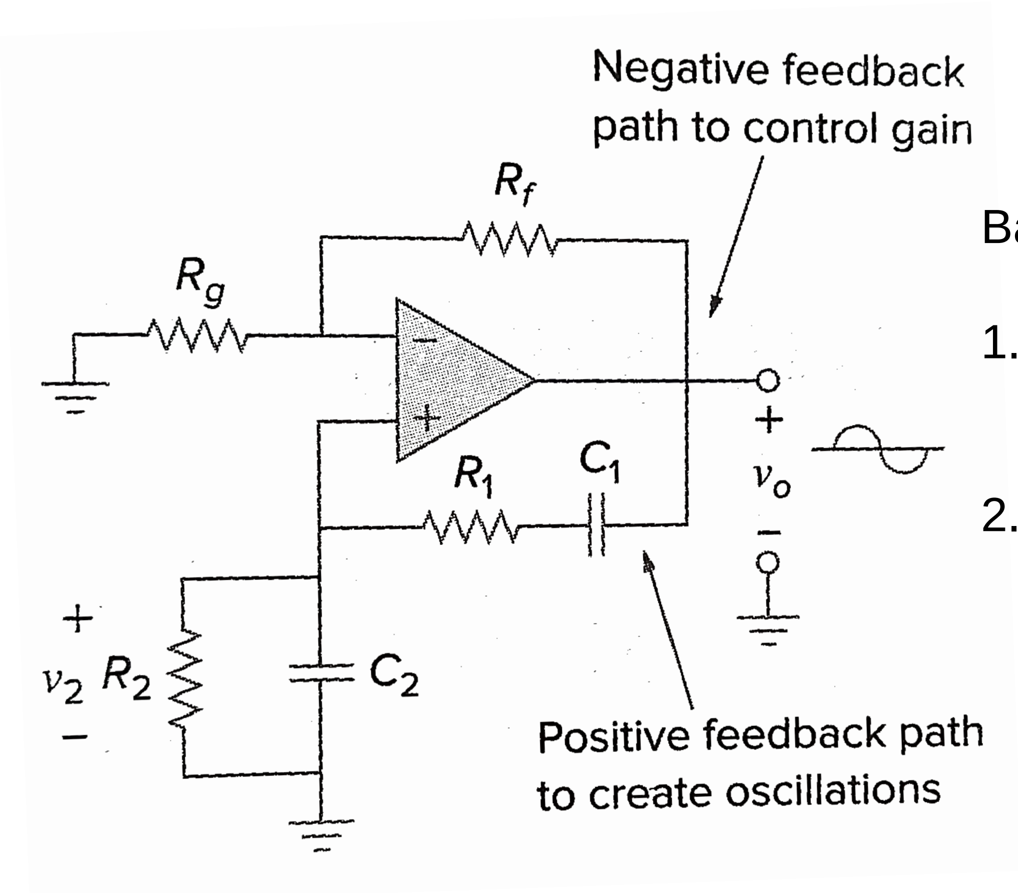
$\mathbf{I}_s = \dfrac{\mathbf{V}_s}{\mathbf{Z}_s}$

$$\boxed{\mathbf{V}_s = \mathbf{Z}_s\mathbf{I}_s \quad \Leftrightarrow \quad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}}$$

AC Op Amp Circuits

- Op amps are also applicable in AC circuits.
- The ideal op-amp model assumes very large (>10000) open-loop gain which is true at dc and low frequencies.
- The range of frequencies over which the gain is very large depends on the specific op-amp design.

Wien-bridge Oscillator



Barkhausen criteria:

1. The overall gain must be unity or greater.

2. The overall phase shift (from input to output and back to the input) must be zero.

From: Alexander & Sadiku, Fundamentals of Electric Circuits

Wien-bridge Oscillator

$$R_2 || \frac{1}{j\omega C_2} = \frac{R_2 \frac{1}{j\omega C_2}}{\frac{1}{j\omega C_2} + R_2} = \frac{R_2}{1 + j\omega C_2 R_2}$$

$$\frac{V_2}{V_0} = \frac{\frac{R_2}{1 + j\omega C_2 R_2}}{\frac{R_2}{1 + j\omega C_2 R_2} + R_1 - j\frac{1}{\omega C_1}} = \frac{\omega C_1 R_2}{\omega(C_1 R_2 + C_1 R_1 + C_2 R_2) + j(\omega^2 C_1 R_1 C_2 R_2 - 1)}$$

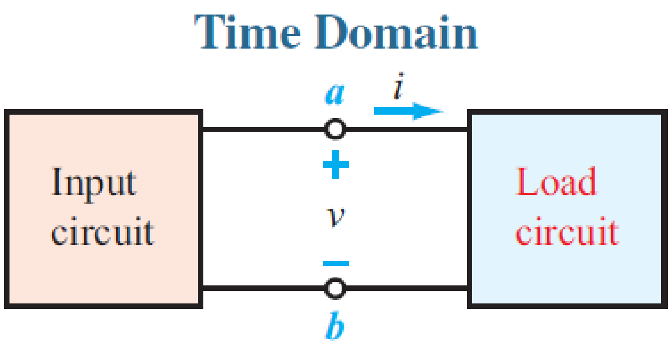
Condition 2: $\omega^2 C_1 R_1 C_2 R_2 - 1 = 0$

If R_1=R_2=R and C_1=C_2=C, then $\omega = \frac{1}{CR}$, $\frac{V_2}{V_0} = \frac{1}{3}$.

Condition 1: $\mathrm{A}(V_+ - V_-) \geq V_0 \rightarrow \frac{V_+}{V_o} - \frac{V_-}{V_o} \geq \frac{1}{A} \sim 0$

$$\frac{1}{3} - \frac{R_g}{R_g + R_f} \geq 0 \rightarrow 1 + \frac{R_f}{R_g} \geq 3, \frac{R_f}{R_g} \geq 2$$

AC Power in Time Domain: Instantaneous



$$v(t) = V_m \cos(\omega t + \theta_v) \quad i(t) = I_m \cos(\omega t + \theta_i)$$

<u>Instantaneous</u> power:
power at any instant of time.

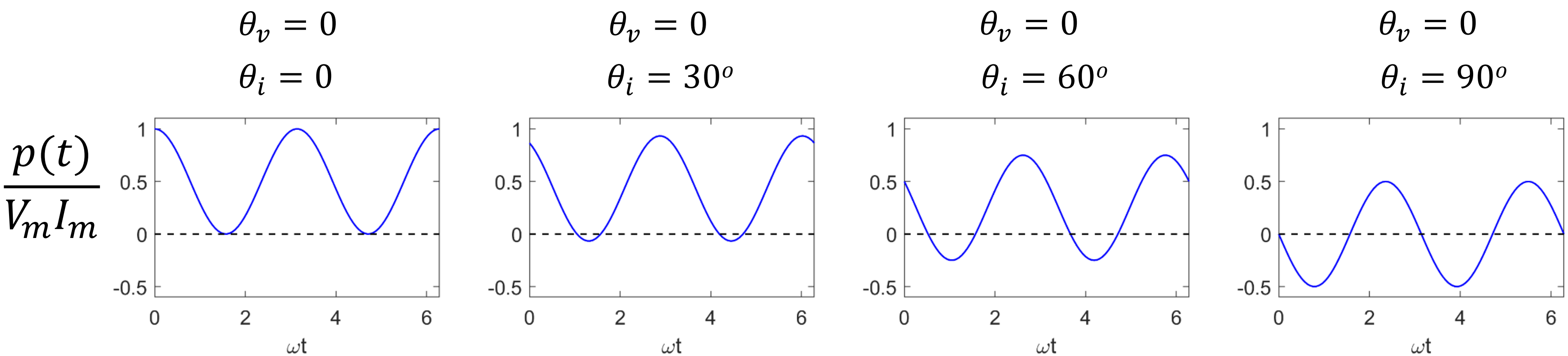
$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

AC power

- Average power formula
- Power formula in terms of RMS values of current and voltage
- Maximum power transfer

AC Power in Time Domain: Instantaneous



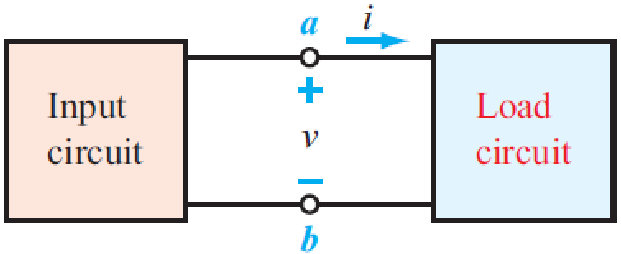
$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$$= \frac{1}{2}V_m I_m \left[\cos(\theta_v - \theta_i) + \cos(2\omega t + 2\theta_i)\cos(\theta_v - \theta_i) - \sin(2\omega t + 2\theta_i)\sin(\theta_v - \theta_i)\right]$$

$$= \underbrace{\frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\left[1 + \cos(2\omega t + 2\theta_i)\right]}_{\geq 0;\text{电源向负载输出}} - \underbrace{\frac{1}{2}V_m I_m \sin(\theta_v - \theta_i)\sin(2\omega t + 2\theta_i)}_{\text{时正时负; 负载与电源之间来回交换}}$$

Average Power

$$v(t) = V_m \cos(\omega t + \theta_v) \quad i(t) = I_m \cos(\omega t + \theta_i)$$



$$\boxed{p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)}$$

Average (or real) power (unit: watts)

The **average power**, in watts, is the average of the instantaneous power over one period.

$$P_{\text{real}} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} Re\{VI^*\}$$

$$= \frac{1}{2} Re\left\{\frac{VV^*}{Z^*}\right\} = \frac{V_m^2}{2} Re\left\{\frac{1}{Z^*}\right\} = \frac{V_m^2}{2|Z|^2} Re\{Z\}$$

$$= \frac{1}{2} Re\{II^*Z\} = \frac{I_m^2}{2} Re\{Z\}$$

$$\boxed{I = \frac{V}{Z}}$$

Complex Power

Real power: $P_{\text{real}} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} Re\{VI^*\}$

Reactive power: $P_{\text{reactive}} = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i) = \frac{1}{2} Im\{VI^*\}$

Complex power: $P_{\text{complex}} = \frac{1}{2} VI^* = P_{\text{real}} + jP_{\text{reactive}}$

Two special cases for average power *P*

- For a purely resistive load R：

$$P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = I_{\text{eff}}^2 R$$

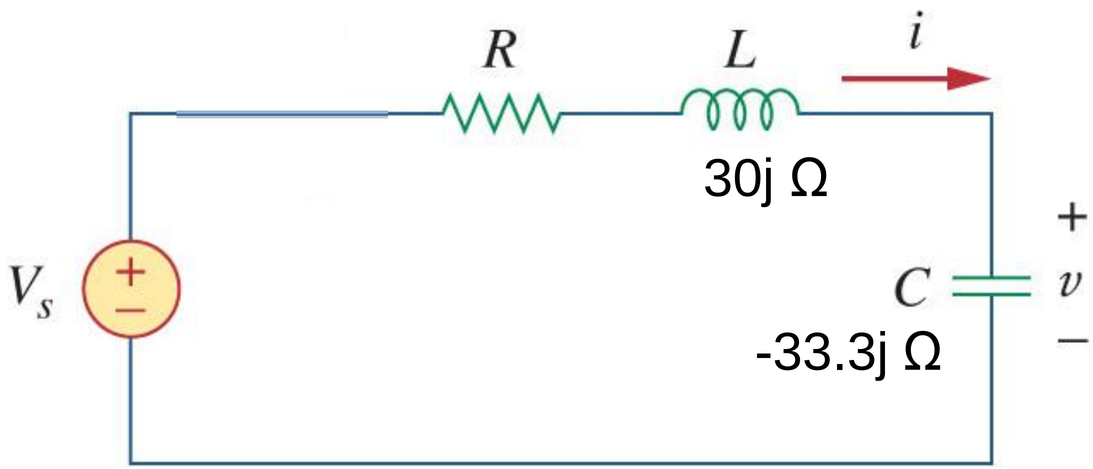
- For a purely reactive load:

$$P = \frac{1}{2}V_m I_m \cos 90^\circ = 0$$

A resistive load (*R*) absorbs power at all times, while a reactive load (*L* or *C*) absorbs zero average power.

Exercise

- Determine the average power dissipated in the circuit below, in which R=10 Ω, L=3 μH, C=3 nF, and $V_s = 50\cos(1.0 \times 10^7 t + 70^o)$ V.



$Z = 10 - 3.3j\ \Omega.$

$$P = \frac{1}{2}\frac{V_{s,m}^2}{|Z|^2}Re\{Z\} = \frac{1}{2}\frac{50^2}{(100+3.3^2)}10 = 112.5\text{ W}$$

The power dissipated by the resistor: $P_R = \overline{i(t)^2}R = \frac{V_{s,m}^2}{|Z|2}\overline{\cos^2(10^7 t + 70^o)}\,R = P$

Note: $\overline{\cos^2(10^7 t + 70^o)} = \frac{1}{2}$

Power with Multiple Frequency Components

$$v(t) = V_{m,1}\cos\left(\omega_1 t + \theta_{v,1}\right) + V_{m,2}\cos\left(\omega_2 t + \theta_{v,2}\right);$$

$$i(t) = I_{m,1}\cos\left(\omega_1 t + \theta_{i,1}\right) + I_{m,2}\cos\left(\omega_2 t + \theta_{i,2}\right);$$

$$\begin{aligned} p(t) = v(t)i(t) = {} & V_{m,1} I_{m,1}\cos\left(\omega_1 t + \theta_{v,1}\right)\cos\left(\omega_1 t + \theta_{i,1}\right) \\ & + V_{m,2} I_{m,2}\cos\left(\omega_2 t + \theta_{v,2}\right)\cos\left(\omega_2 t + \theta_{i,2}\right) \\ & + V_{m,1} I_{m,2}\cos\left(\omega_1 t + \theta_{v,1}\right)\cos\left(\omega_2 t + \theta_{i,2}\right) \\ & + V_{m,2} I_{m,1}\cos\left(\omega_2 t + \theta_{v,2}\right)\cos\left(\omega_1 t + \theta_{i,1}\right) \end{aligned}$$

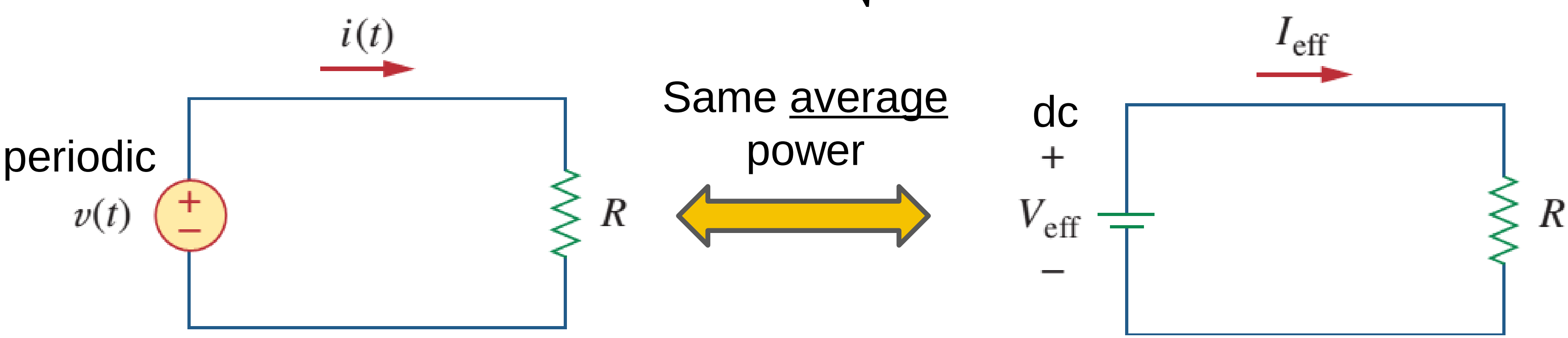
The third and fourth terms are equal to zero after time average.

The time averages of the first and second terms are $\frac{1}{2} V_{m,1} I_{m,1}\cos\left(\theta_{v,1} - \theta_{i,1}\right)$ and $\frac{1}{2} V_{m,2} I_{m,2}\cos\left(\theta_{v,2} - \theta_{i,2}\right)$.
Therefore, the total average power is equal to the sum of average powers generated by each component alone.

Effective Value (RMS)

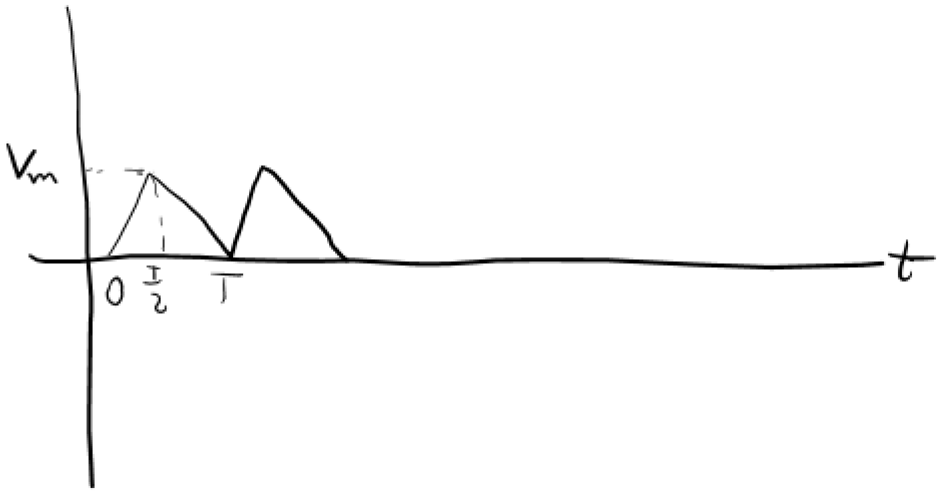
- For any periodic function $x(t)$ in general, its rms value is

$$X_{\mathrm{eff}} = X_{\mathrm{rms}} = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt}$$



$$I_{\mathrm{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} \qquad V_{\mathrm{eff}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

Example: 三角波的有效电压

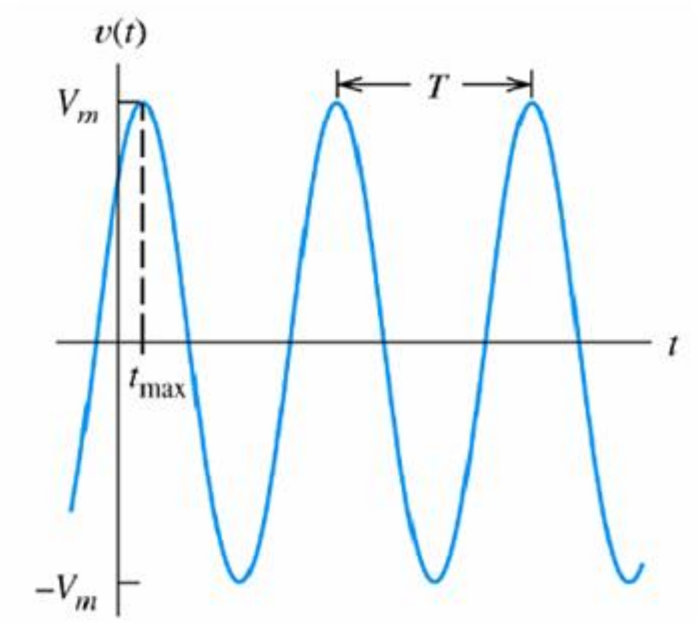


$$\frac{1}{\frac{T}{2}}\int_0^{\frac{T}{2}}\left[\frac{V_m}{(\frac{T}{2})}t\right]^2 dt = \frac{V_m^2}{(\frac{T}{2})^3}\frac{1}{3}\left(\frac{T}{2}\right)^3 = \frac{V_m^2}{3}$$

$$V_{rms} = \frac{V_m}{\sqrt{3}}$$

Example: RMS of a Sinusoidal

- The RMS value of $v(t) = V_m \cos(\omega t + \phi)$

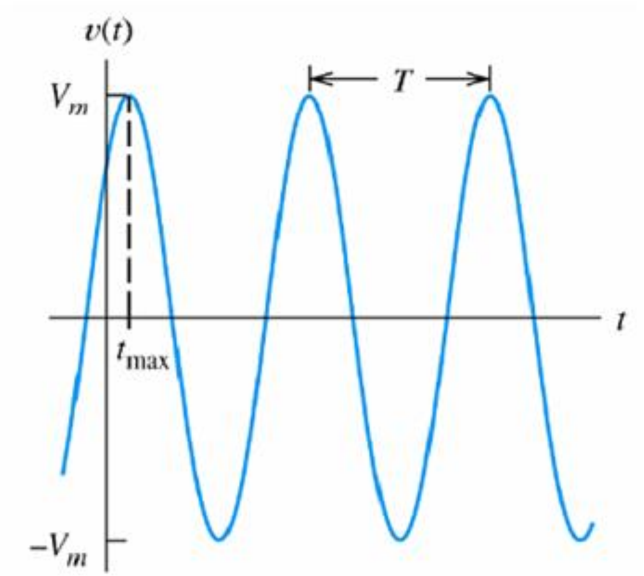


$$V_{\mathrm{rms}} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)dt}$$

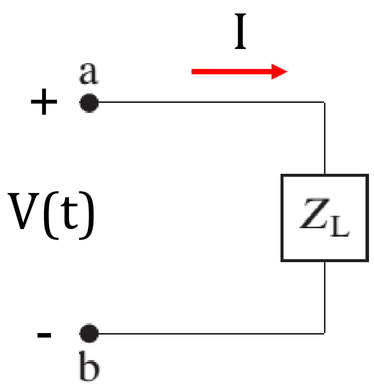
$$= \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cdot \cos^2(\omega t + \phi)dt}$$

$$= \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cdot \frac{1}{2}[1 + \cos 2(\omega t + \phi)]\, dt}$$

$$= \frac{V_m}{\sqrt{2}}$$



V_{RMS} vs I_{RMS} of a Sinusoidal



For the phasor **V** of V(t), we have $|\boldsymbol{V}| = \sqrt{2}V_{\mathrm{rms}}$.

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{Z_L} \Rightarrow I_{\mathrm{rms}} = \frac{V_{rms}}{|Z_L|}$$

Average Power

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i)$$
$$= \boxed{V_{\text{rms}} I_{\text{rms}}} \cos(\theta_v - \theta_i)$$

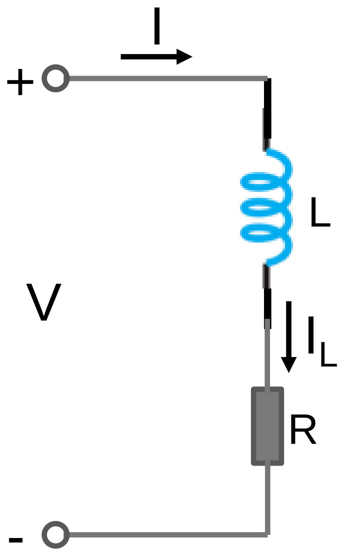
视在功率: $S = V_{rms} I_{rms}$

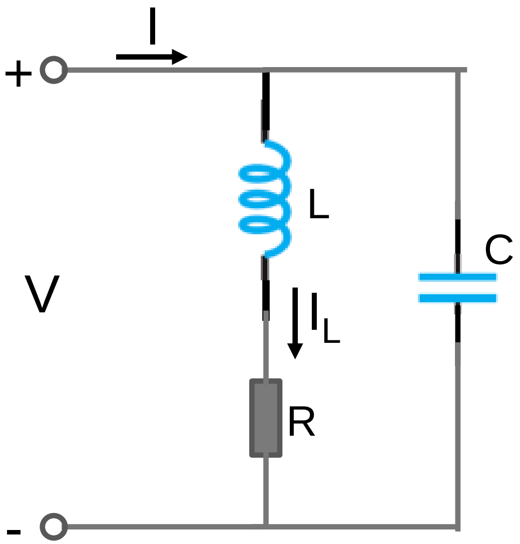
- The power factor

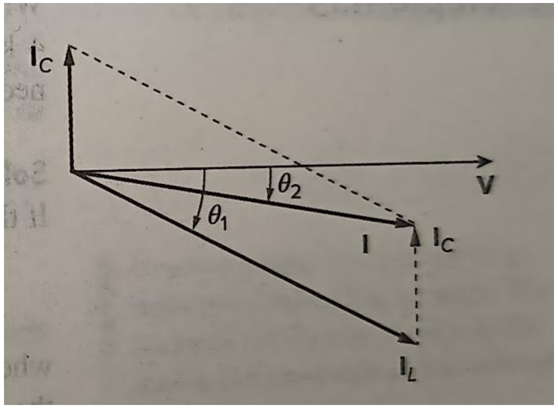
$$\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

  - $(\theta_v - \theta_i)$ is called power factor angle.
    - >0 means a *lagging* pf (current lags voltage)
    - <0 means a *leading* pf (current leads voltage)
  - pf ranges from 0 to 1.

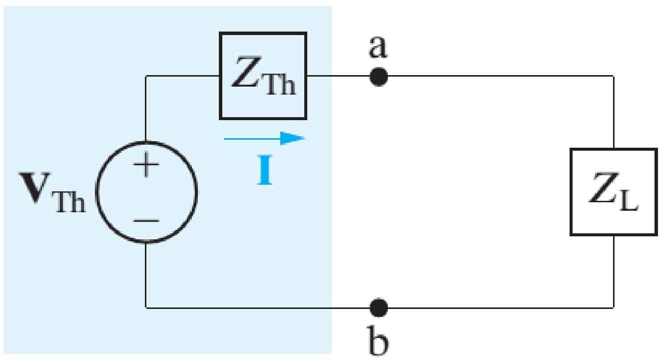
Power Factor Correction







Maximum Power Transfer



$$\boldsymbol{V}_{ab} = \frac{\boldsymbol{V}_{Th}}{Z_{Th} + Z_L} Z_L$$

$$V_{ab,\mathrm{rms}} = \frac{V_{Th,rms}}{|Z_{Th} + Z_L|} |Z_L|$$

$$I_{ab,\mathrm{rms}} = \frac{V_{Th,rms}}{|Z_{Th}+Z_L|}$$

Load power $P = V_{ab,\mathrm{rms}} I_{ab,\mathrm{rms}} Re(\frac{Z_L}{|Z_L|}) = \frac{V^2{}_{Th,rms}}{|Z_{Th}+Z_L|^2} Re(Z_L)$

$$= \frac{V^2{}_{Th,rms}}{[Re(Z_{Th}) + Re(Z_L)]^2 + [Im(Z_{Th}) + Im(Z_L)]^2} Re(Z_L)$$

It reaches maximum when $Im(Z_{Th}) = -Im(Z_L)$ and $Re(Z_{Th}) = Re(Z_L)$, i.e. $Z_L = Z_{Th}^*$. Note we have $Re(Z_{Th}) > 0$ and $Re(Z_L) > 0$.