Chapter 5

- Operational Amplifiers

Outline

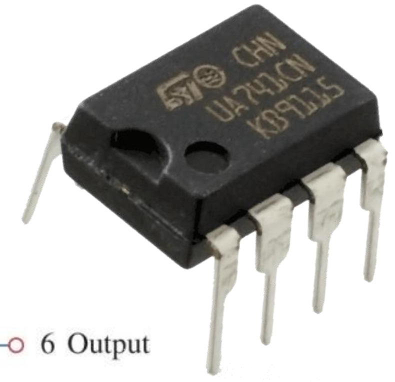
- Operational amplifier (op amp)
- Basic operations
- DAC
- Instrumentation amplifier

The Op Amp

- Behaves like a voltage-controlled voltage source with very high amplification factor

- When combined with resistors, capacitors, and inductors, can perform various functions:
  - amplification/scaling
  - sign changing
  - addition/subtraction
  - integration
  - differentiation
  - analog filtering

Op Amp Terminals



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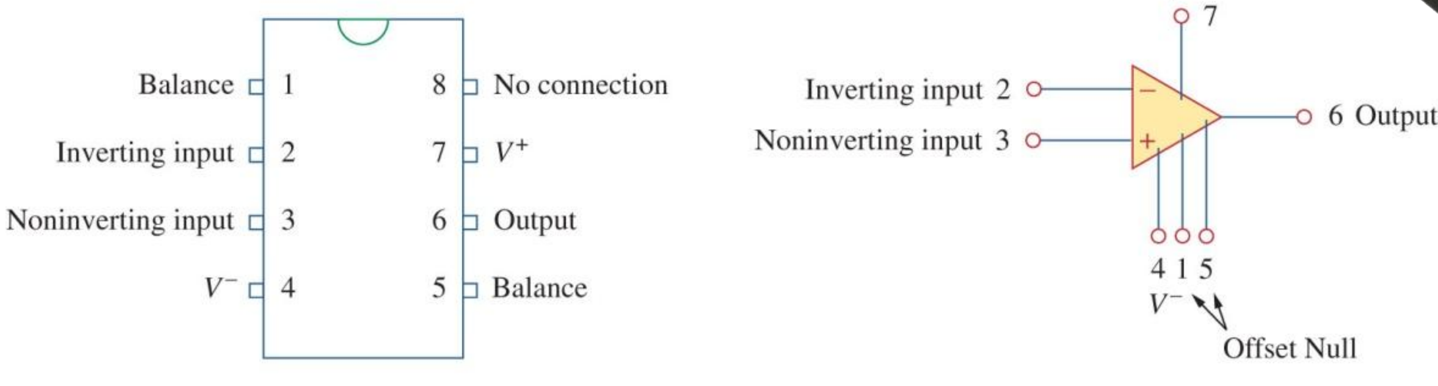

(a) (b)

- Five important terminals
  - The inverting input
  - The noninverting input
  - The output
  - The positive (+) power supply
  - The negative (-) power supply
- The rest three terminals
  - 2 Offset Null (Balance)
    - May be used to compensate for performance degradation due to aging etc.
  - 1 No Connection (NC)
    - Unused, not connected to the amplifier circuit.

Output Voltage

- The voltage output of an op-amp is proportional to the difference between the noninverting and inverting inputs

$$\boxed{v_o = Av_d = A(v_2 - v_1)}$$

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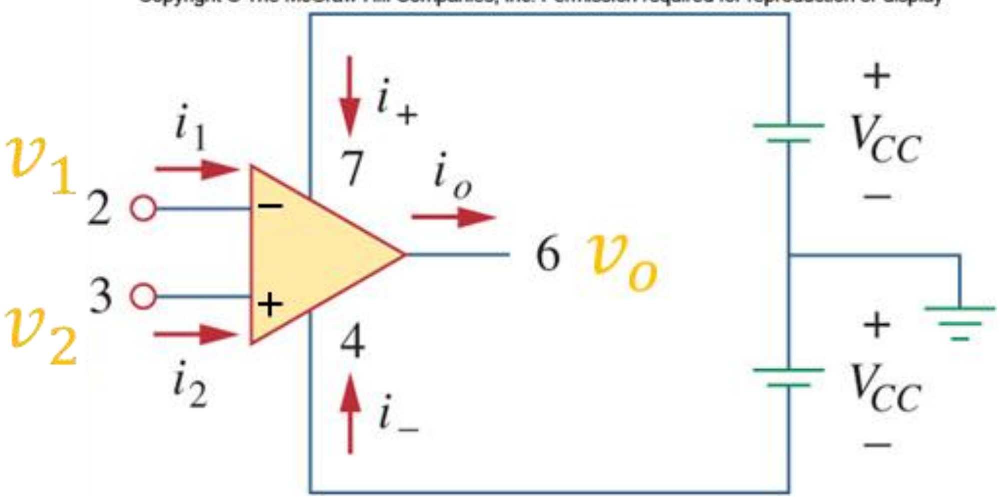


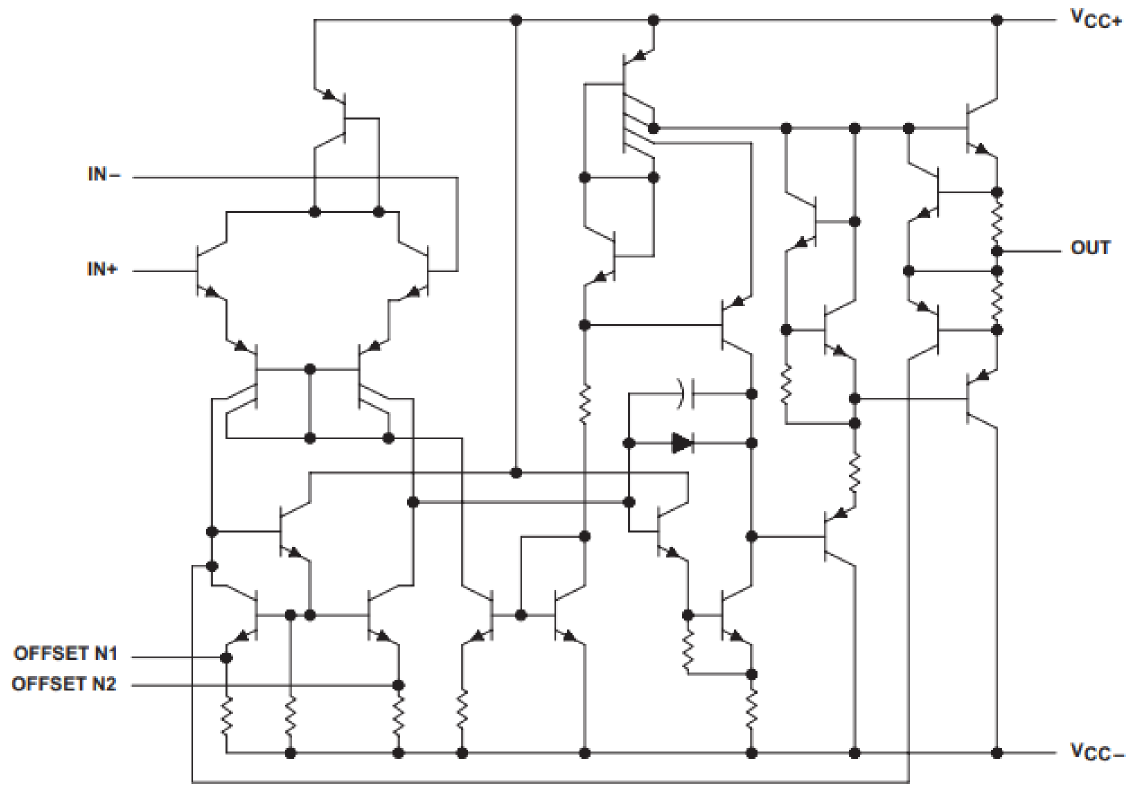
Op Amp in the real world

$$\boxed{v_o = A_\mathrm{d}(v_2 - v_1) + A_c(v_2 + v_1)/2}$$

共模抑制比：$\frac{A_d}{A_c}$ or $20\log_{10}\frac{A_d}{A_c}$ (dB)

uA741 Functional Block Diagram

7.2 Functional Block Diagram



Component Count	
Transistors	22
Resistors	11
Diode	1
Capacitor	1

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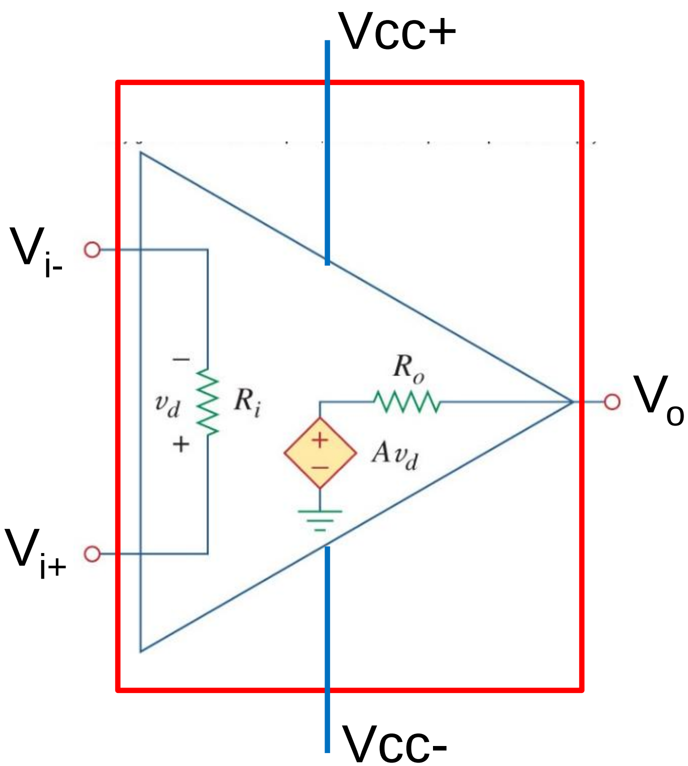
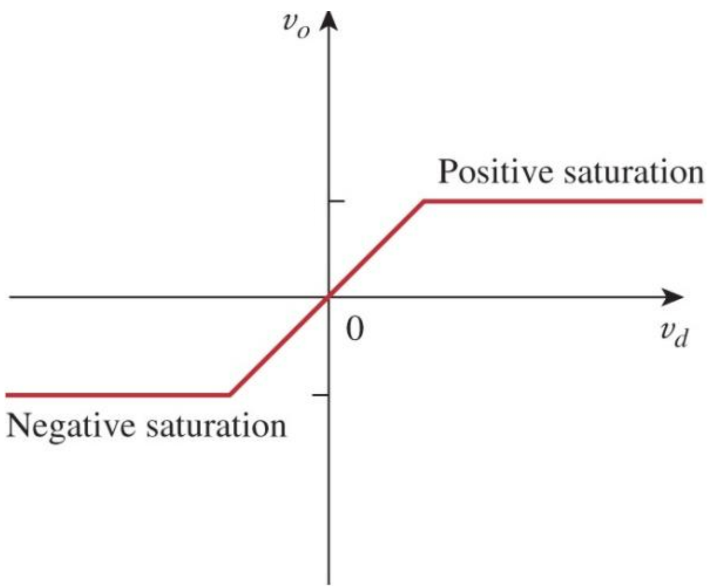
Equivalent Circuit of an Op Amp

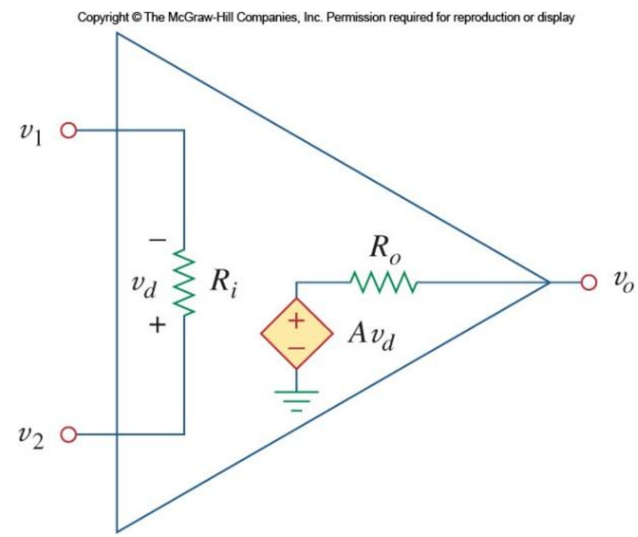


The total current into the three terminals (V_{i+}, V_{i-}, and V_o) of an amp is nonzero.
KCL can still be applied to each node, but not to the model circuit as a whole (the red box) because Vcc+ and Vcc- are not part of the model circuit.

Voltage Saturation

- Is the output voltage unlimited?





$$\boxed{v_o = Av_d = A(v_2 - v_1)}$$

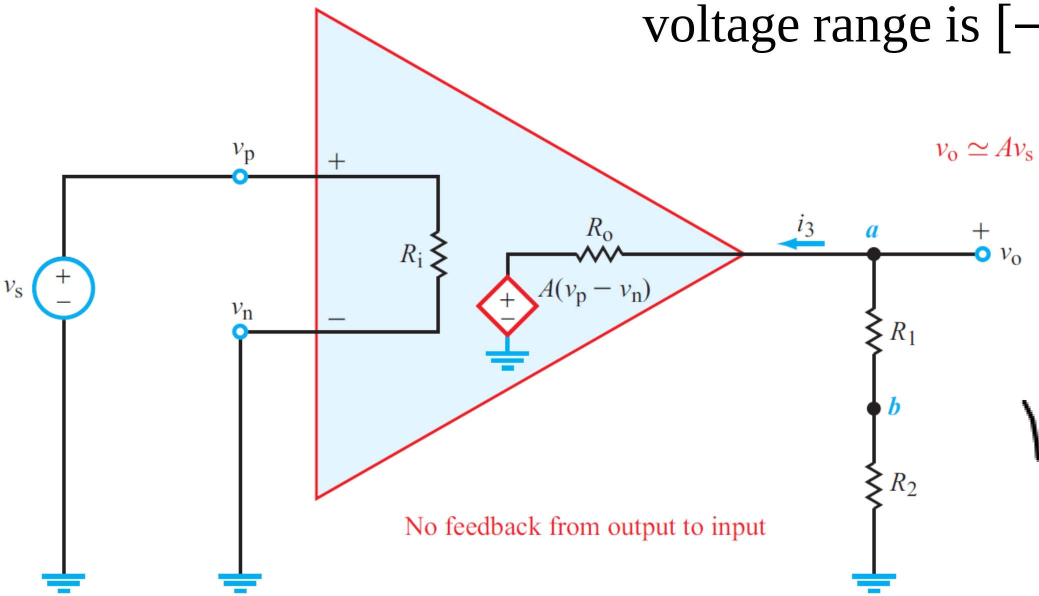
6.4 Electrical Characteristics: µA741C

at specified virtual junction temperature, $V_{CC\pm}$ = ±15 V (unless otherwise noted)

V_{OM}	Maximum peak output voltage swing	R_L = 10 kΩ	25°C	±12	±14	V
		R_L ≥ 10 kΩ	Full range	±12		
		R_L = 2 kΩ	25°C	±10		
		R_L ≥ 2 kΩ	Full range	±10		

Example 1

For $V_{cc} = 10$ V, $A = 10^6$, $R_i = 10^7$ Ω, $R_o = 10$ Ω, $R_1 = 80$ kΩ, and $R_2 = 20$ kΩ, find v_o, assuming the output voltage range is $[-V_{cc}, V_{cc}]$.



$$V_o = A V_s \cdot \frac{R_1+R_2}{R_o+R_1+R_2}$$

$$= 0.9999 A V_s$$

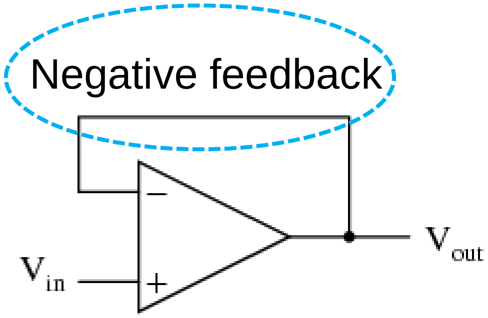
$\simeq 10^6 V_s$ $\quad V_s \le 10\mu V$

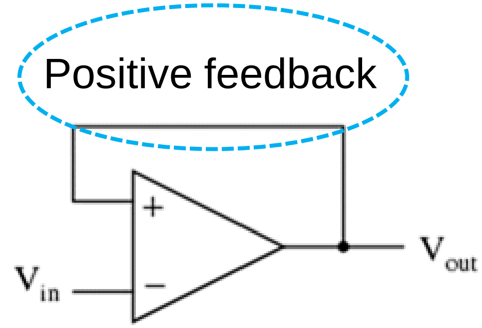
$\simeq 10V$ $\quad V_s \ge 10\mu V$

$-10V$ $\quad V_s < -10\mu V$

假设输出电压的工作范围为

Feedback





$V_{\text{out}} \uparrow \Longrightarrow$ voltage difference $\downarrow \Longrightarrow\ V_{\text{out}} \downarrow$

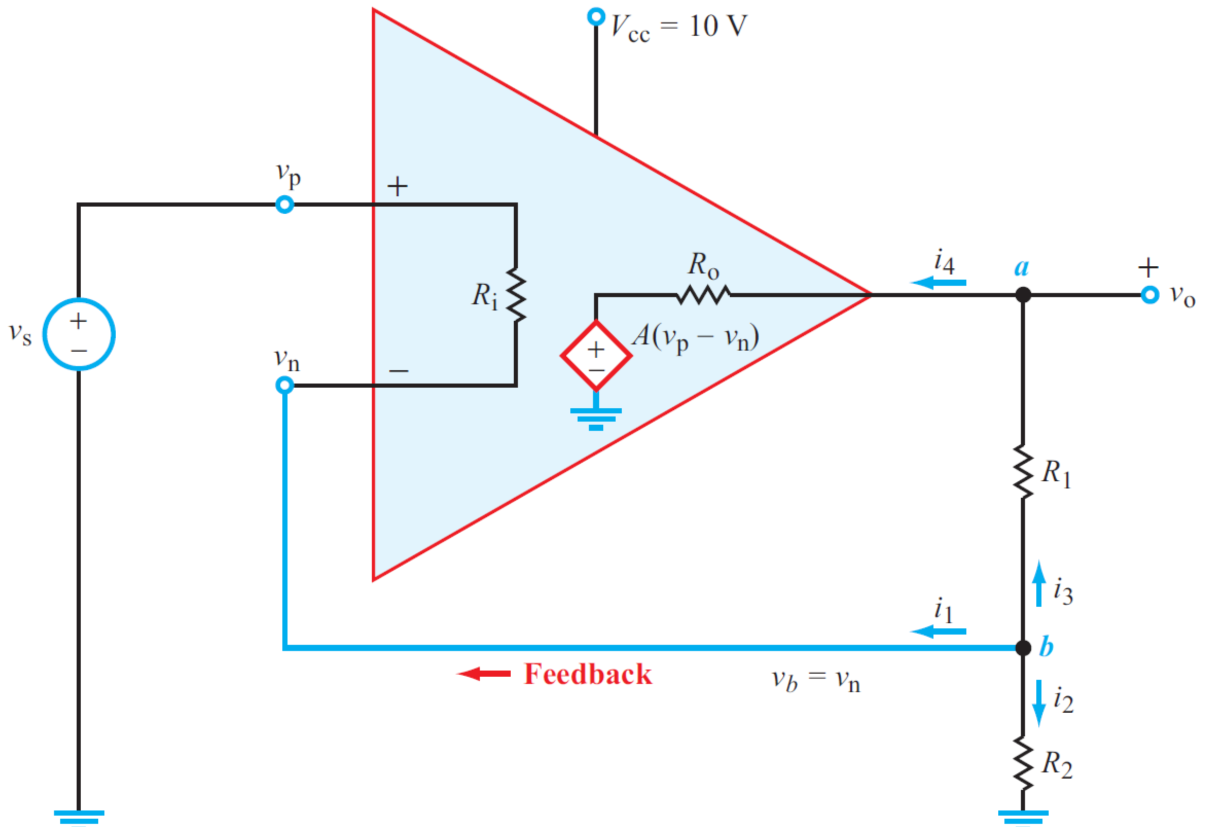
$V_{\text{out}} \uparrow \Longrightarrow$ voltage difference$\uparrow\Longrightarrow\ V_{\text{out}} \uparrow$

负反馈：当输出端电压变化通过外部电路反馈至输入端时，反馈信号引起的输出端电压变化方向与原变化方向相反且通常幅度更大,从而能抵消输出电压的变化，维持输出电压稳定。

正反馈：当输出端电压变化通过外部电路反馈至输入端时，反馈信号引起的输出端电压变化方向与原变化方向相同,从而加剧输出电压的变化。

Example 2

For $V_{cc} = 10$ V, $A = 10^6$, $R_i = 10^7$ Ω, $R_o = 10$ Ω, $R_1 = 80$ kΩ, and $R_2 = 20$ kΩ, Find v_o / v_s



Example 2 (solution)

$n=5$
$n_{vs}=2$ } $\therefore$ 2 independent voltage

$$\frac{V_b-V_0}{R_1}+\frac{V_b}{R_2}+\frac{(V_b-V_s)}{R_i}=0$$

$$\frac{V_b-V_0}{R_1}-\frac{V_0-A(V_s-V_b)}{R_0}=0$$

$$\begin{pmatrix} -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_i} \\ -\frac{1}{R_1}-\frac{1}{R_0} & \frac{1}{R_1}-\frac{A}{R_0} \end{pmatrix}\begin{pmatrix} V_0 \\ V_b \end{pmatrix}+\begin{pmatrix} -\frac{V_s}{R_i} \\ \frac{AV_s}{R_0} \end{pmatrix}=0$$

$$Mx+b=0 \qquad x=-M^{-1}\cdot b$$

$$V_0=+V_s\frac{AR_i(R_1+R_2)+R_2R_0}{R_i(AR_2+R_1+R_2+R_0)+R_1R_2+R_2R_0}$$

$$\approx V_s\frac{AR_i(R_1+R_2)}{AR_iR_2}=V_s\frac{R_1+R_2}{R_2}$$

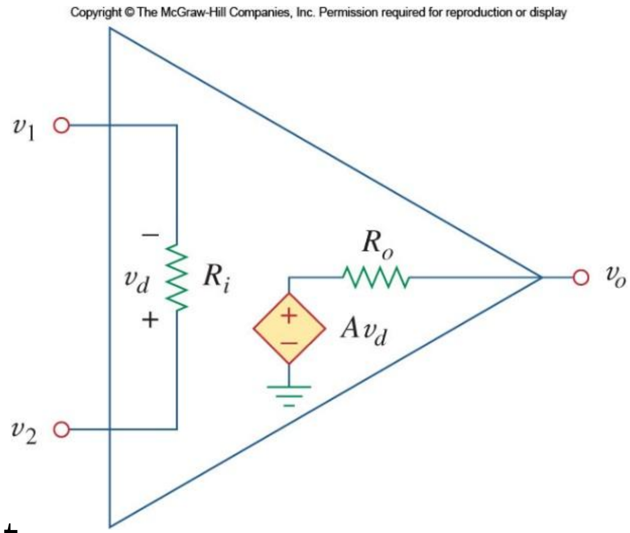
$$=5V_s$$

$$V_s=V_0\cdot\frac{R_2}{R_1+R_2}$$

注意：当$A=-10^6$时（正反馈），同样可以得到$V_s\approx V_0\frac{R_2}{R_1+R_2}$，但此时电压不稳定。

Attributes of an Ideal Op Amp

- ✓ infinite input resistance: $\mathrm{R}_i \approx \infty\ \Omega$
- ✓ infinite open-loop gain: $A \cong \infty$
- ✓ zero output impedance: $\mathrm{R}_o \approx 0\ \Omega$ (not necessary for applying the golden rules)

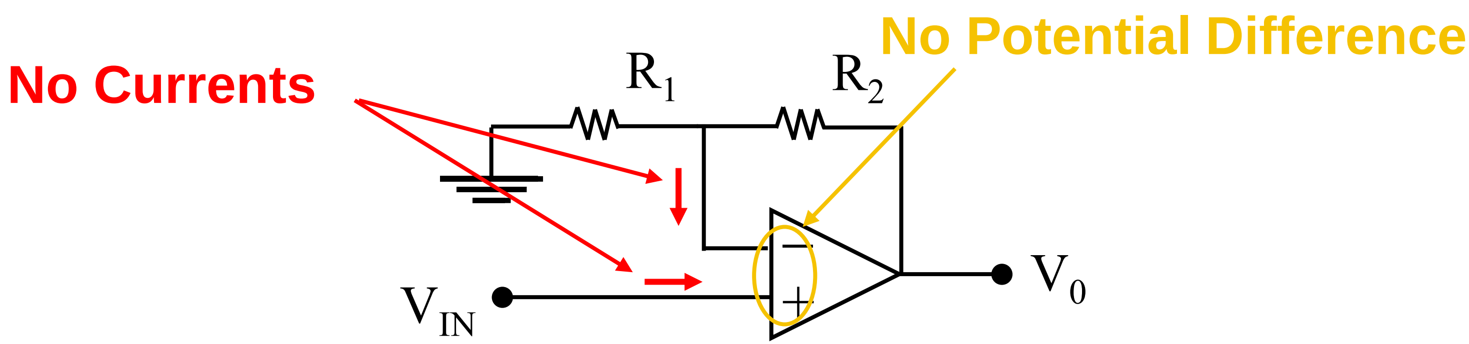


Ideal Op-Amp Analysis

When there exists negative feedback,

Property 1: Gain is infinite ⟹ The potential between the op-amp input terminals, $v_{(+)} - v_{(-)}$, equals zero.

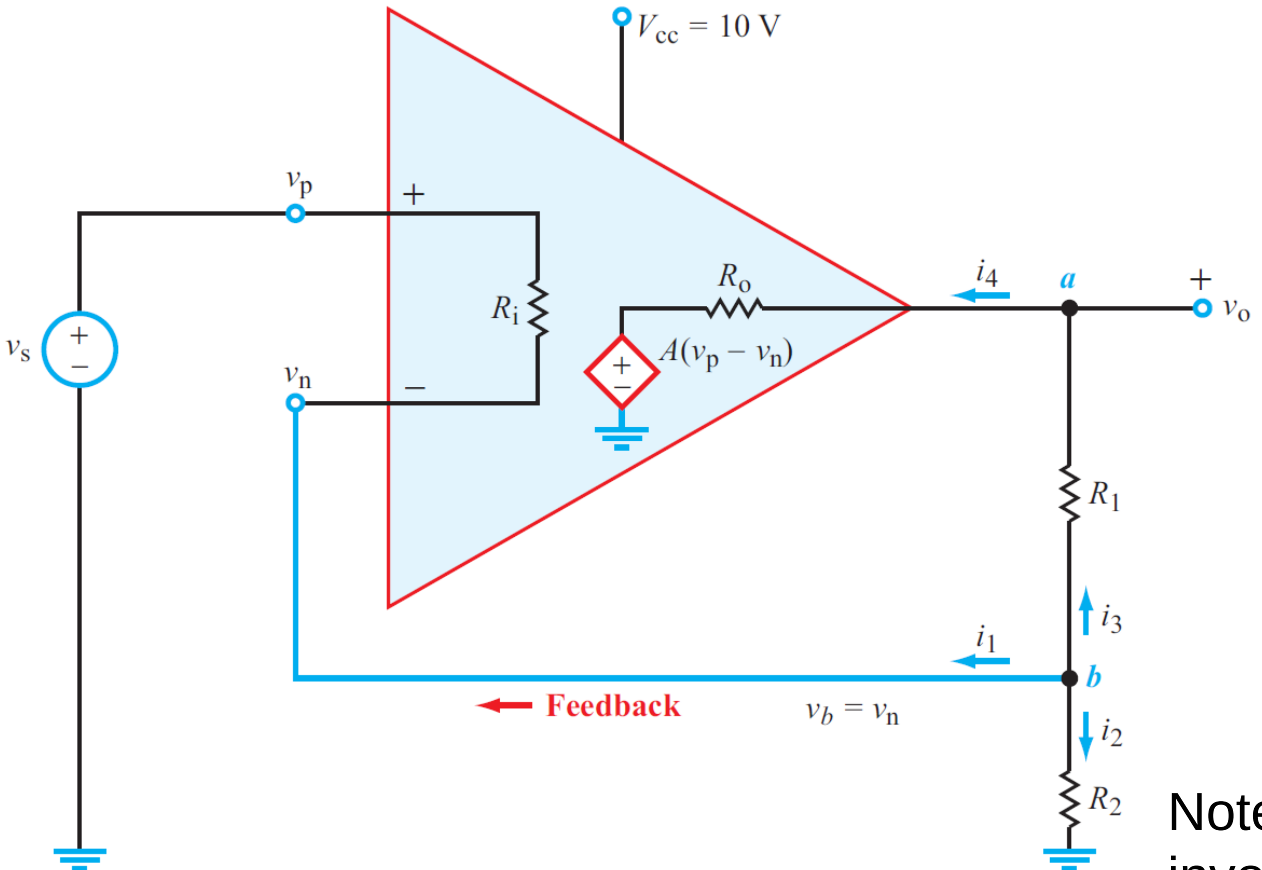
Property 2: Input resistance is infinite ⟹ The currents flowing into the op-amp’s two input terminals both equal zero.



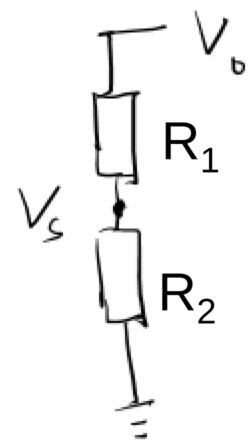
Property 1 introduces an additional equation which replaces the KCL equation at the output node.

Example 3

For $V_{cc} = 10$ V, $A = 10^6$, $R_i = 10^7$ Ω, $R_o = 10$ Ω, $R_1 = 80$ kΩ, and $R_2 = 20$ kΩ, Find v_o / v_s



Equivalent circuit



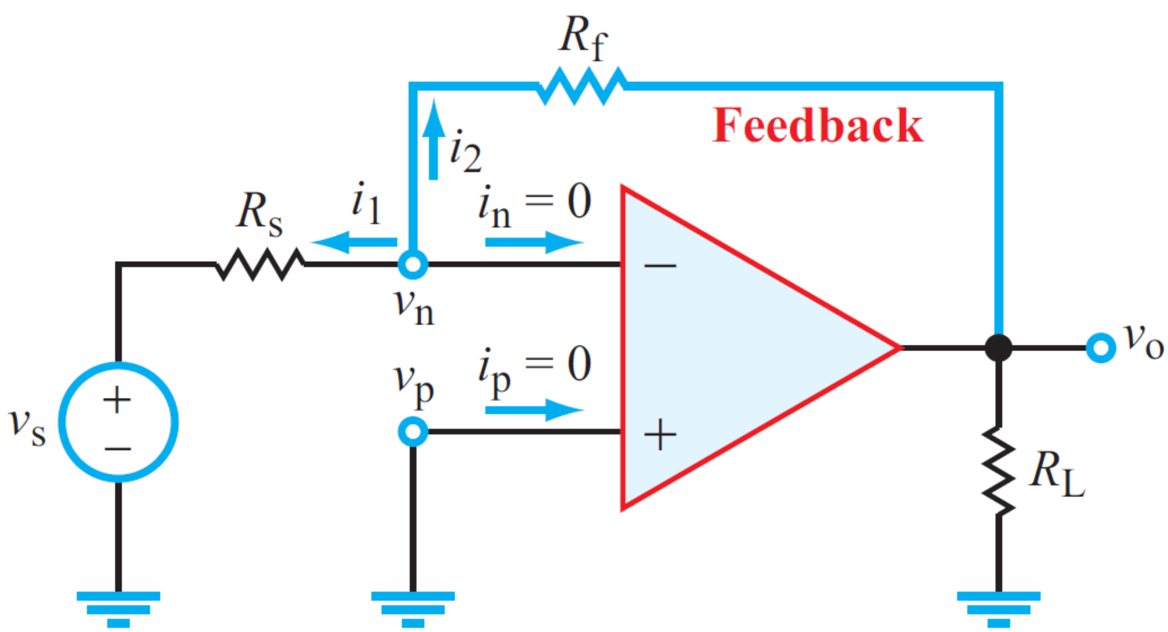
$$V_s = V_o \cdot \frac{R_2}{R_1+R_2}$$

Note： there is no involvement of R_0 in the analysis.

Outline

- Operational amplifier (op amp)
- Basic operations
  - sign changing
  - amplification/scaling
  - addition/subtraction
  - integration
  - differentiation
  - analog filtering
- DAC
- Instrumentation amplifier

Inverting Amplifier



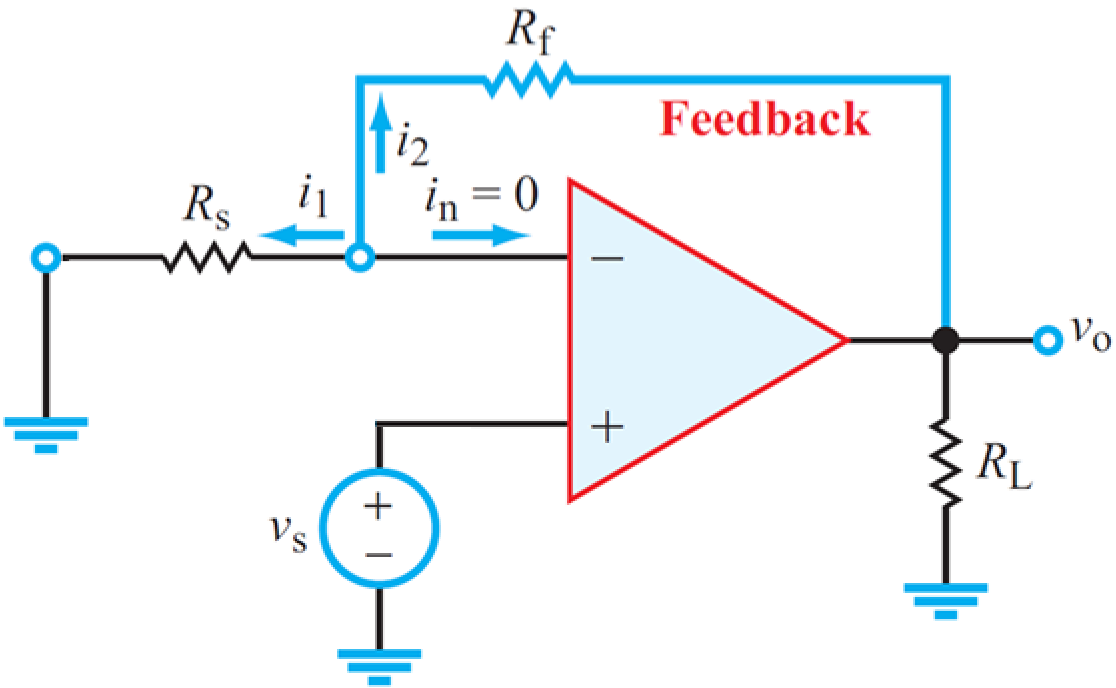
Apply KCL at node -,

we have

$$\frac{v_0}{R_f} = -\frac{v_s}{R_s}$$

$$v_0 = -\frac{R_f}{R_s} v_s$$

Non-inverting Amplifier



Apply KCL at node -,

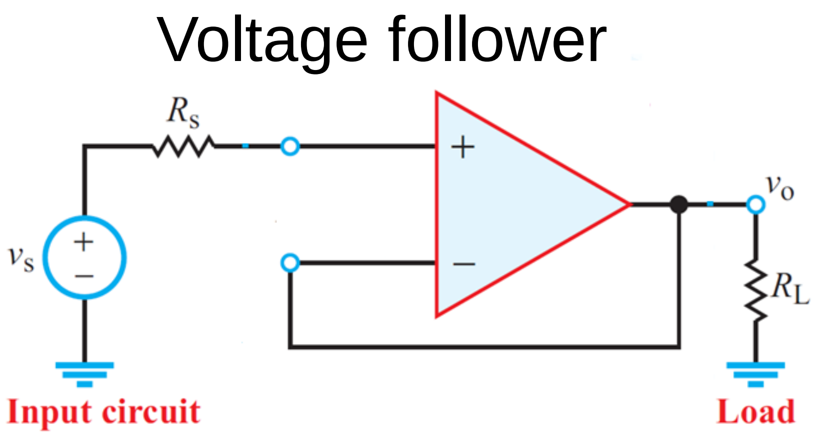
we have

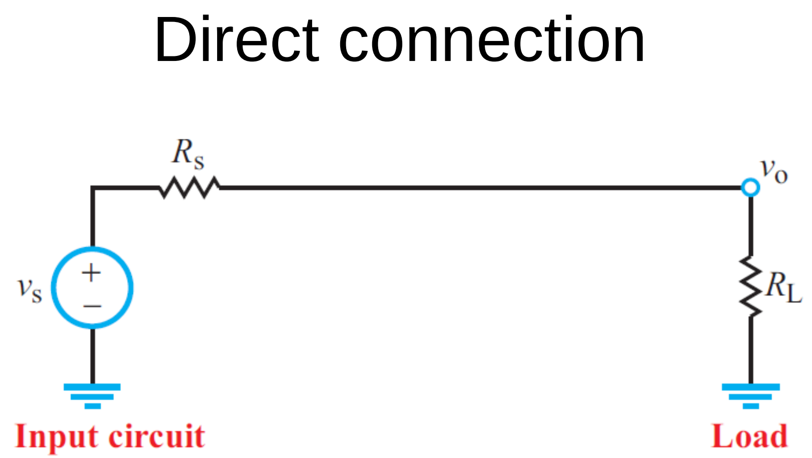
$$\frac{v_s - v_0}{R_f} + \frac{v_s}{R_s} = 0$$

$$v_0 = v_s(1 + \frac{R_f}{R_s})$$

When $R_f = 0$ and $R_s = \infty$, we get $v_0 = v_s$, i.e. a voltage follower.

Application: Voltage Follower





Equivalent Thevenin circuit:

Input side: $V_{Th}=V_s$; $R_{Th} \approx R_s$, load ∞

Output side: $V_{Th}=V_s$; $R_{Th} \approx 0$, load R_L

Equivalent Thevenin circuit:
$V_{Th}=V_s$; $R_{Th} = R_s$, load R_L

Practice 1

- Determine v_o in the circuit shown below

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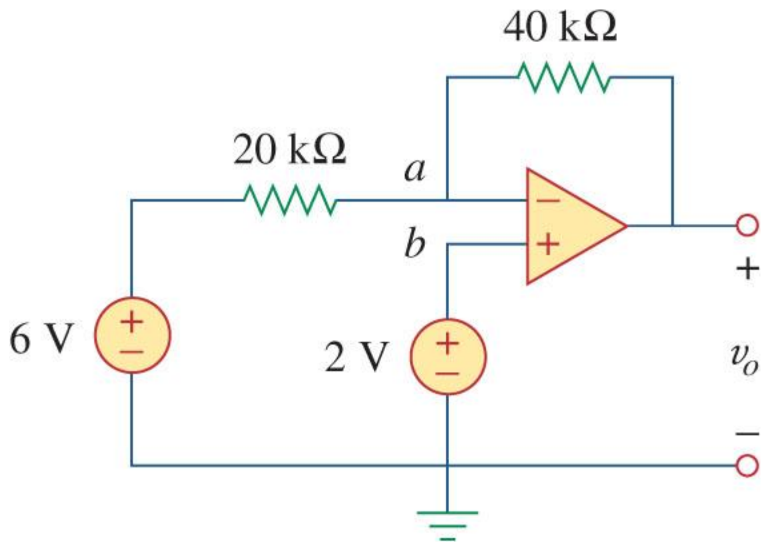
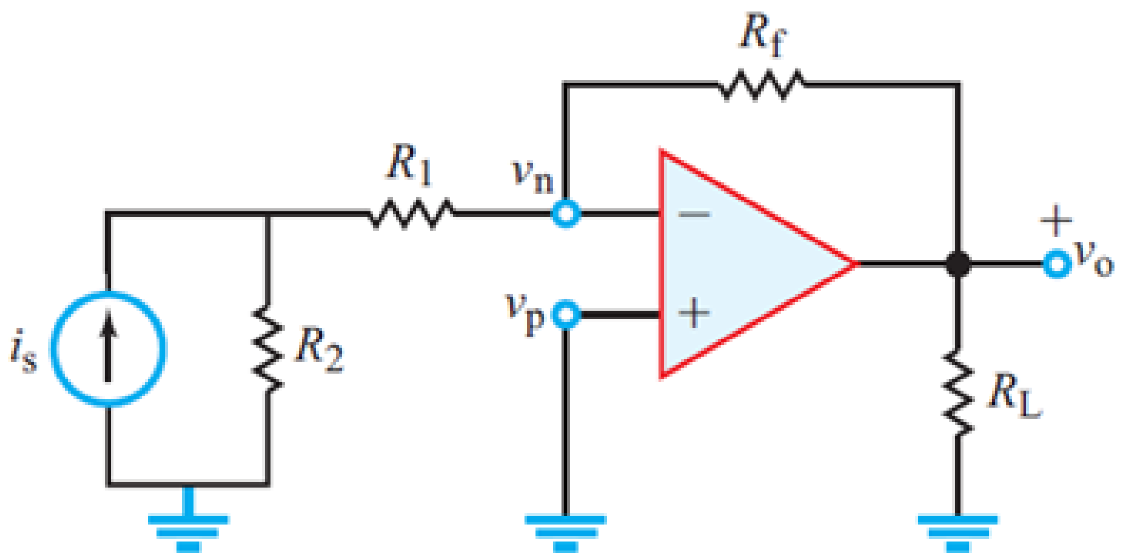


Answer: $(V_0-2)/40=-4/20$; $V_0 = -6$ V;

Practice 2

- Determine v_o in the circuit shown below.

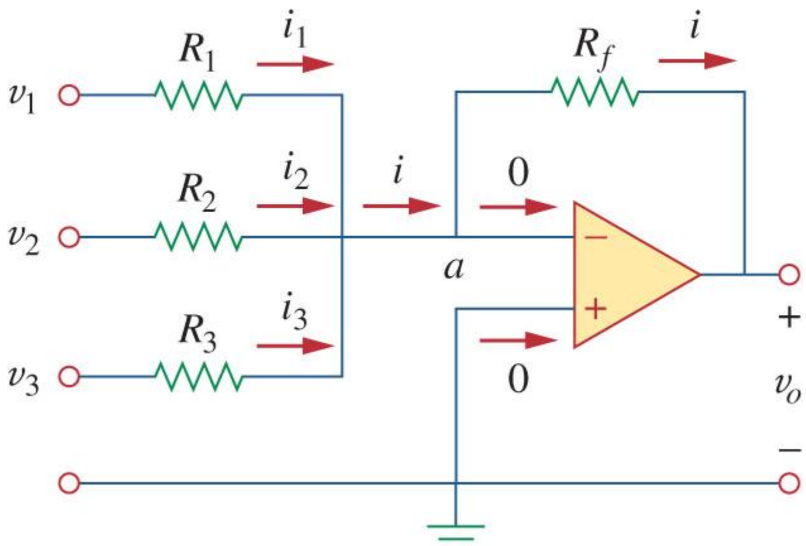


$V_0/R_f = -[(i_s * R_2)/(R_1 + R_2)]$

右边[..]内可理解为：
并联电路分流或
电源等效变换后串联电路的电流

Summing Amplifier

- Aside from amplification, the op-amp can be made to do addition readily.

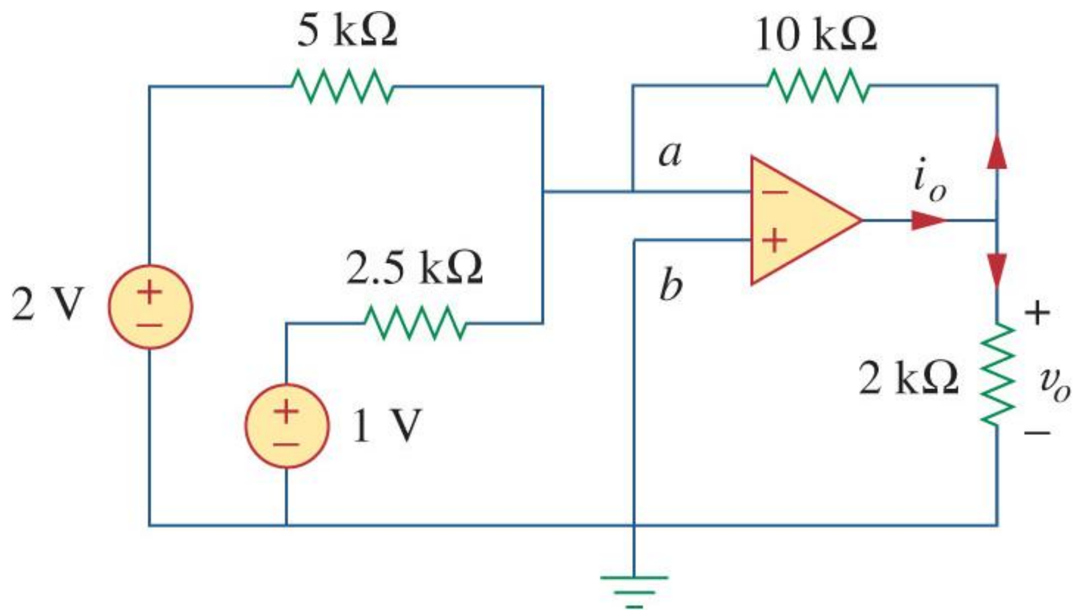


$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = \frac{-V_o}{R_f}$$

$$V_o = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$

Practice

- Find v_o and i_o in the circuit shown below

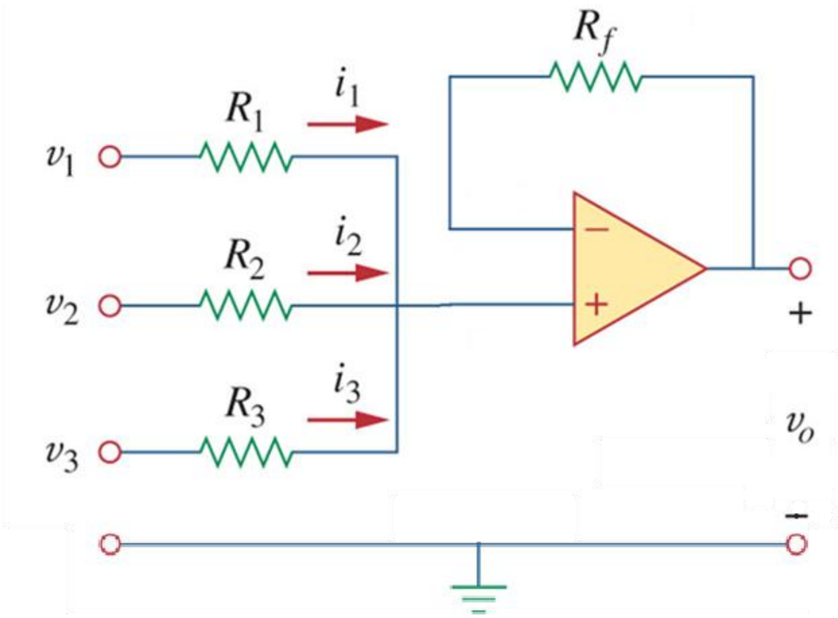


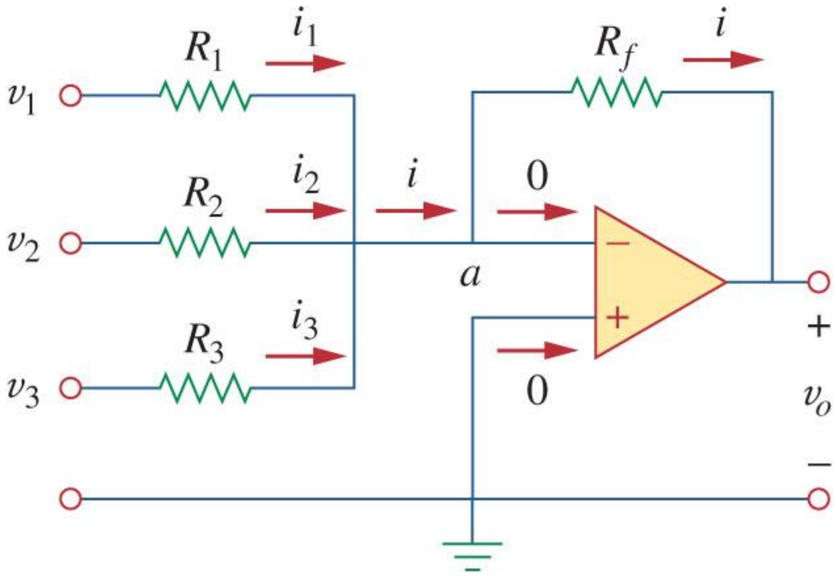
$$\frac{v_0}{10\ \text{K}} + \frac{2}{5\ \text{K}} + \frac{1}{2.5\ \text{K}} = 0$$

$$v_0 = -8\ \text{V}$$

$$i_0 = -4 - 0.8 = -4.8\ \text{mA}$$

Exercise: find the relationship between v_0 and input voltages v_1, v_2, and v_3. What is the disadvantage of such a summing amplifier compared to the one in the previous slide?





Answer:

$$V_0 = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)(R_1 \| R_2 \| R_3)$$

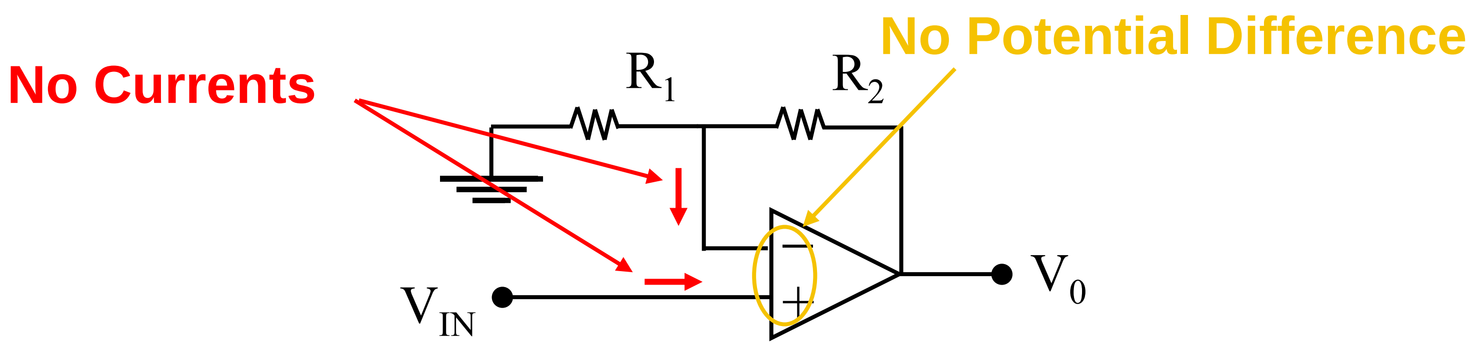
However, the current into each terminal is affected by the voltages at the other input terminals, which is not desirable and should be avoided.
Remember: for non-ideal voltage source, the output voltage varies with output current.

Ideal Op-Amp Analysis

When there exists negative feedback,

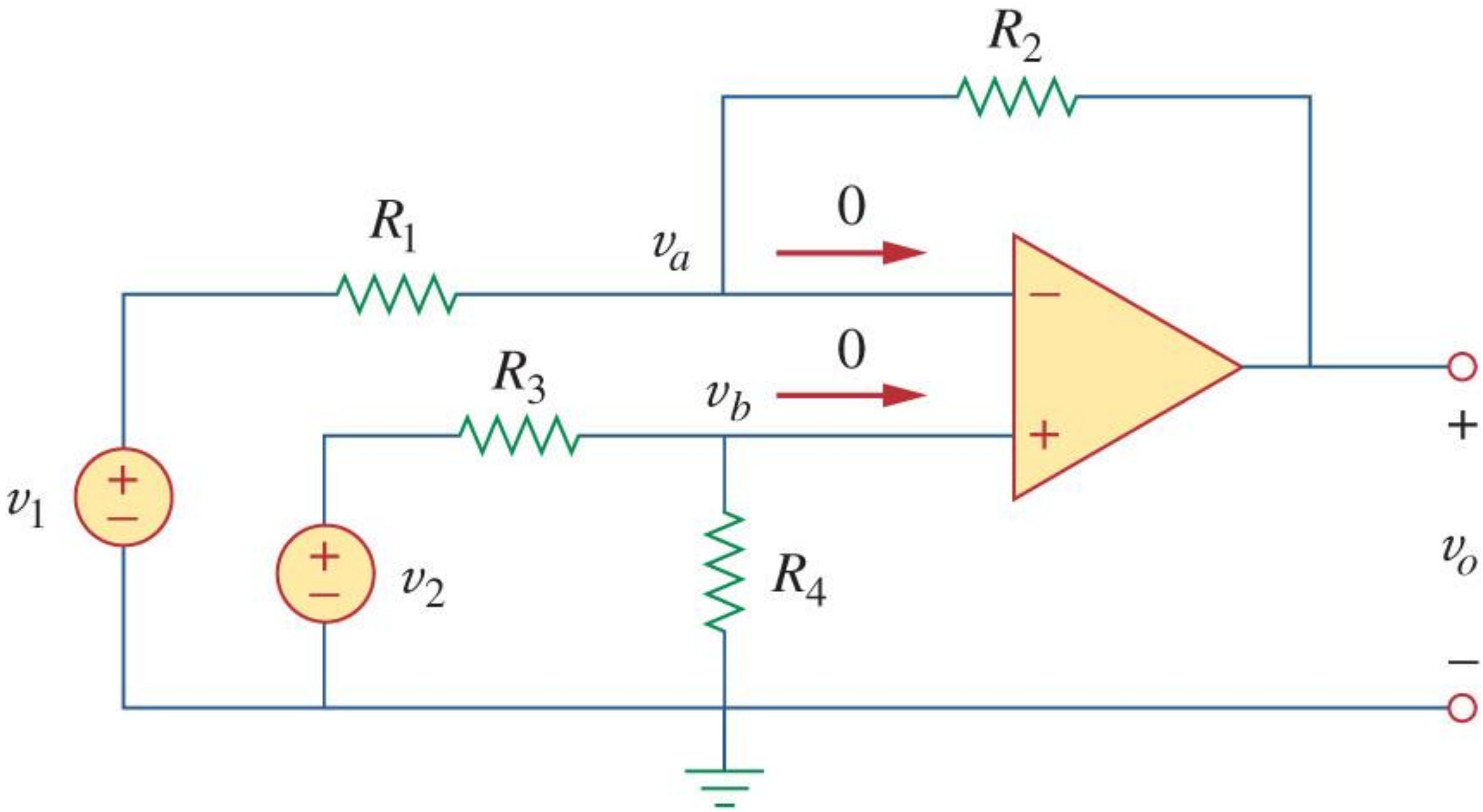
Property 1: Gain is infinite ⟹ The potential between the op-amp input terminals, $v_{(+)} - v_{(-)}$, equals zero.

Property 2: Input resistance is infinite ⟹ The currents flowing into the op-amp's two input terminals both equal zero.



Property 1 reduces the number of unknown voltages by 1 which compensates the lack of KCL equation at the output node.

Difference Amplifier



Difference Amplifier (solution)

$$V_a = V_b = V_2 \cdot \frac{R_4}{R_3+R_4}$$

$$\frac{V_1 - V_a}{R_1} = \frac{V_a - V_o}{R_2}$$

$$\frac{V_1}{R_1} - V_a\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = -\frac{V_o}{R_2}$$

$$\frac{V_1}{R_1} - V_2\frac{R_4}{R_3+R_4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = -\frac{V_o}{R_2}$$

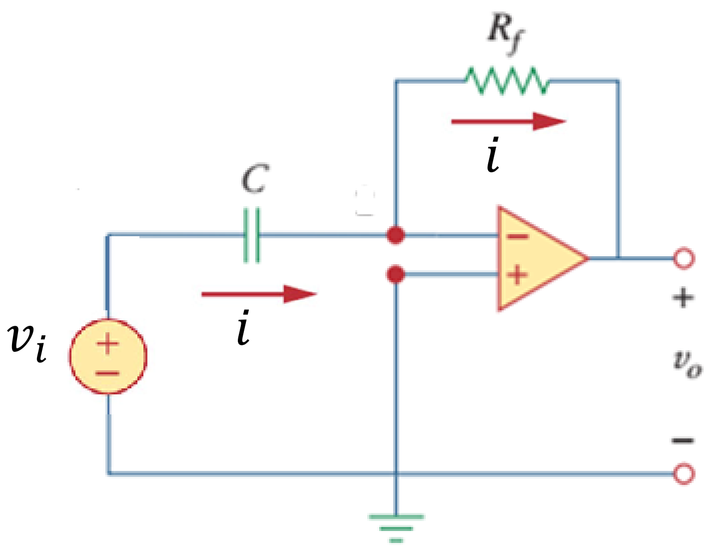
$$V_o = -\frac{R_2}{R_1}V_1 + \frac{\left(\frac{R_2}{R_1}+1\right)V_2}{\left(\frac{R_3}{R_4}+1\right)}$$

$$= \frac{R_2}{R_1}\left[\frac{\left(1+\frac{R_1}{R_2}\right)}{\left(1+\frac{R_3}{R_4}\right)}V_2 - V_1\right]$$

Note: current from voltage source 1 is affected by v_2, but not the other way around.

Differentiator

Find the relationship between v_o and v_i in the circuit shown below

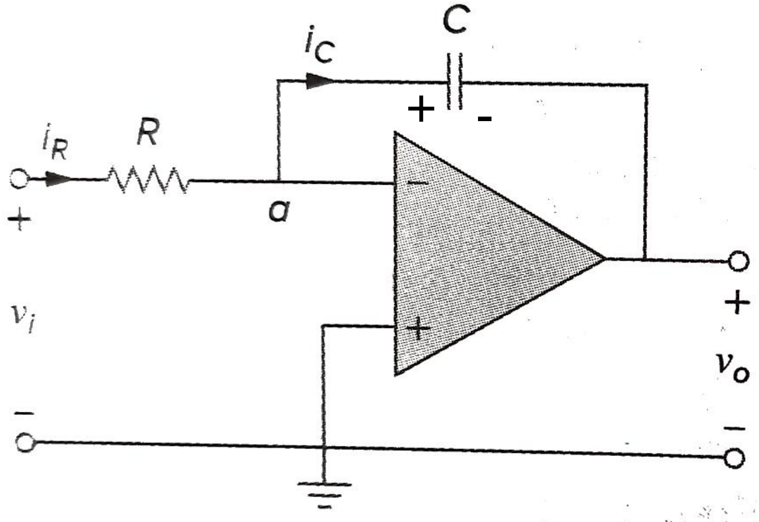


$$v_0 = -R_f i = -R_f C \frac{dv_i}{dt}$$

For this equation to be valid, the response time of the op-amp must be much shorter than $R_f C$.

Integrator

Find the relationship between v_o and v_i in the circuit shown below. What if the capacitor is replaced by a inductor?



$$v_0 = -\frac{Q}{C} = -\frac{1}{C}\int_0^t I dt + v_{0,i}$$

$$= -\frac{1}{C}\int_0^t \frac{v_i}{R} dt + v_{0,i}$$

$$= -\frac{1}{RC}\int_0^t v_i dt, \text{ if } v_{0,i} = 0.$$

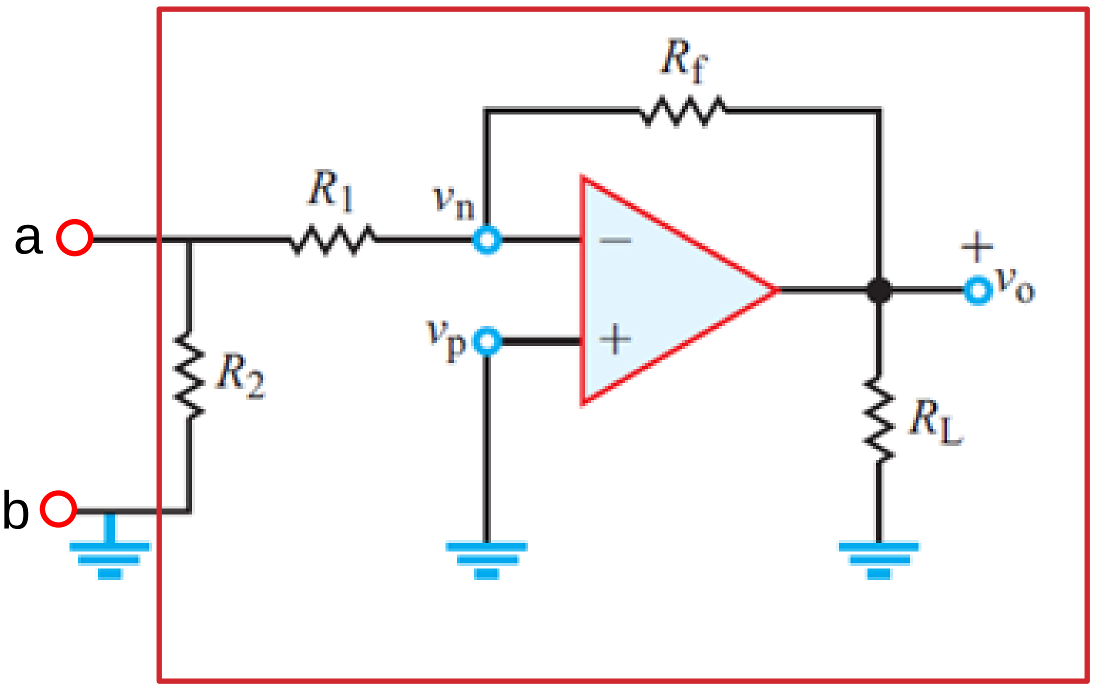
If C is replaced by an inductor,

$$v_0 = -L\frac{dI}{dt} = -\frac{L}{R}\frac{dv_i}{dt}$$

Note: both RC and $\frac{\mathrm{L}}{\mathrm{R}}$ have the dimension of time.

Practice 3

Determine the equivalent resistance at terminals a-b.



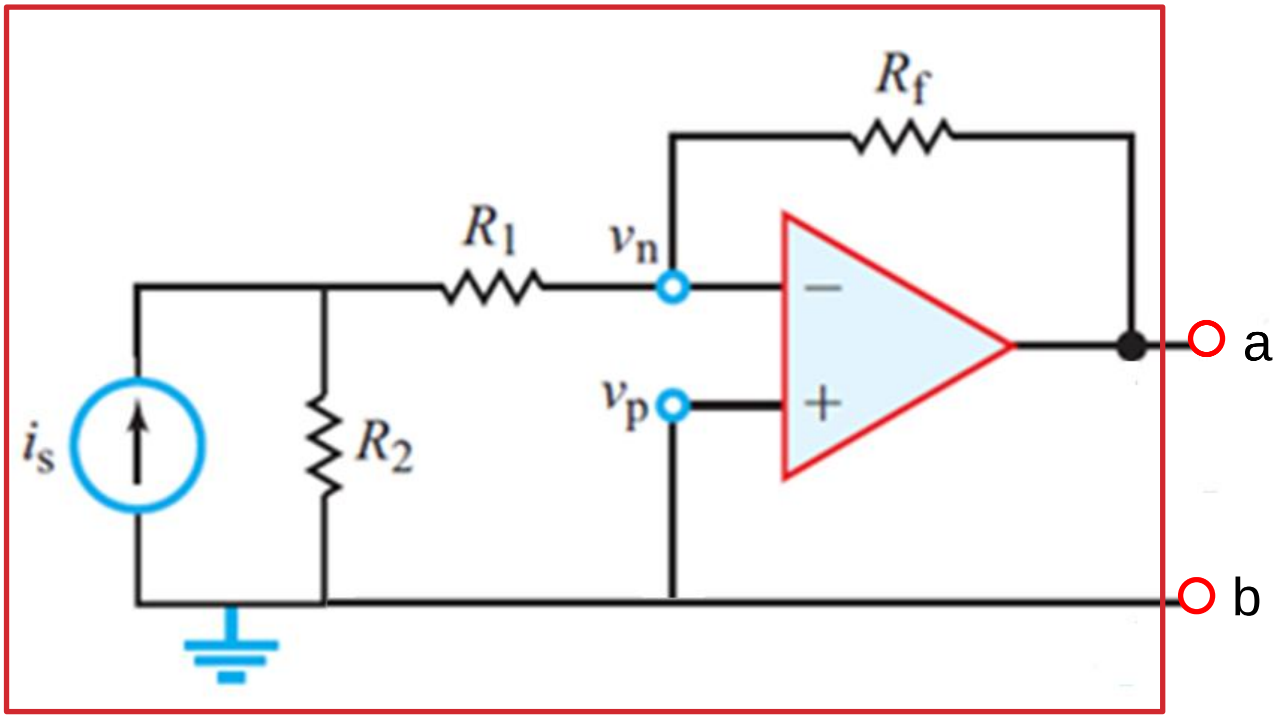
Answer:

$$I = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2}$$

$$R_{eq} = \frac{V_{ab}}{I} = \frac{R_1 R_2}{R_1 + R_2} = R_1 || R_2$$

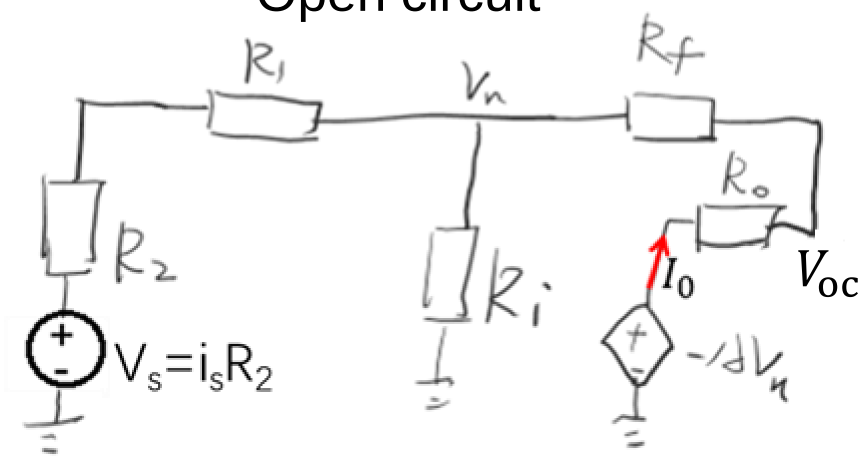
Practice 4

Determine the Thevenin’s equivalent resistance at terminals a-b.



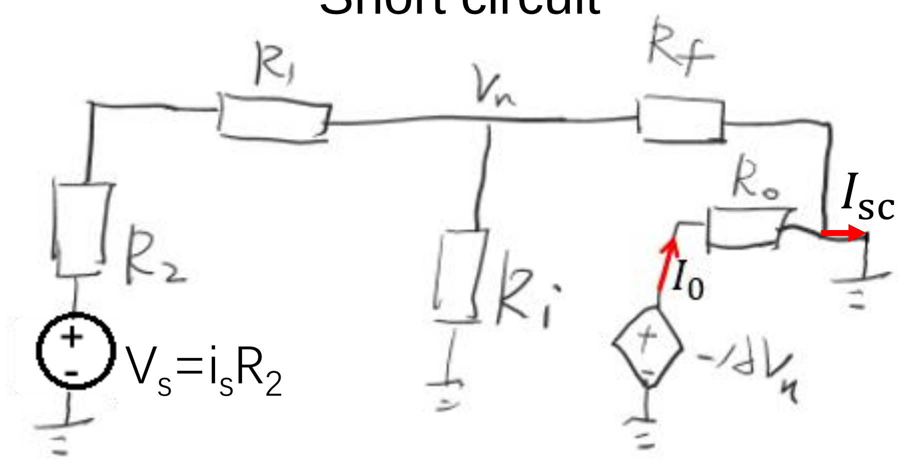
Practice 4 (solution)

Open circuit



From ideal op-amp approximation, we have $V_{\mathrm{oc}} = \frac{-i_s R_2 R_f}{R_1+R_2}$

Short circuit

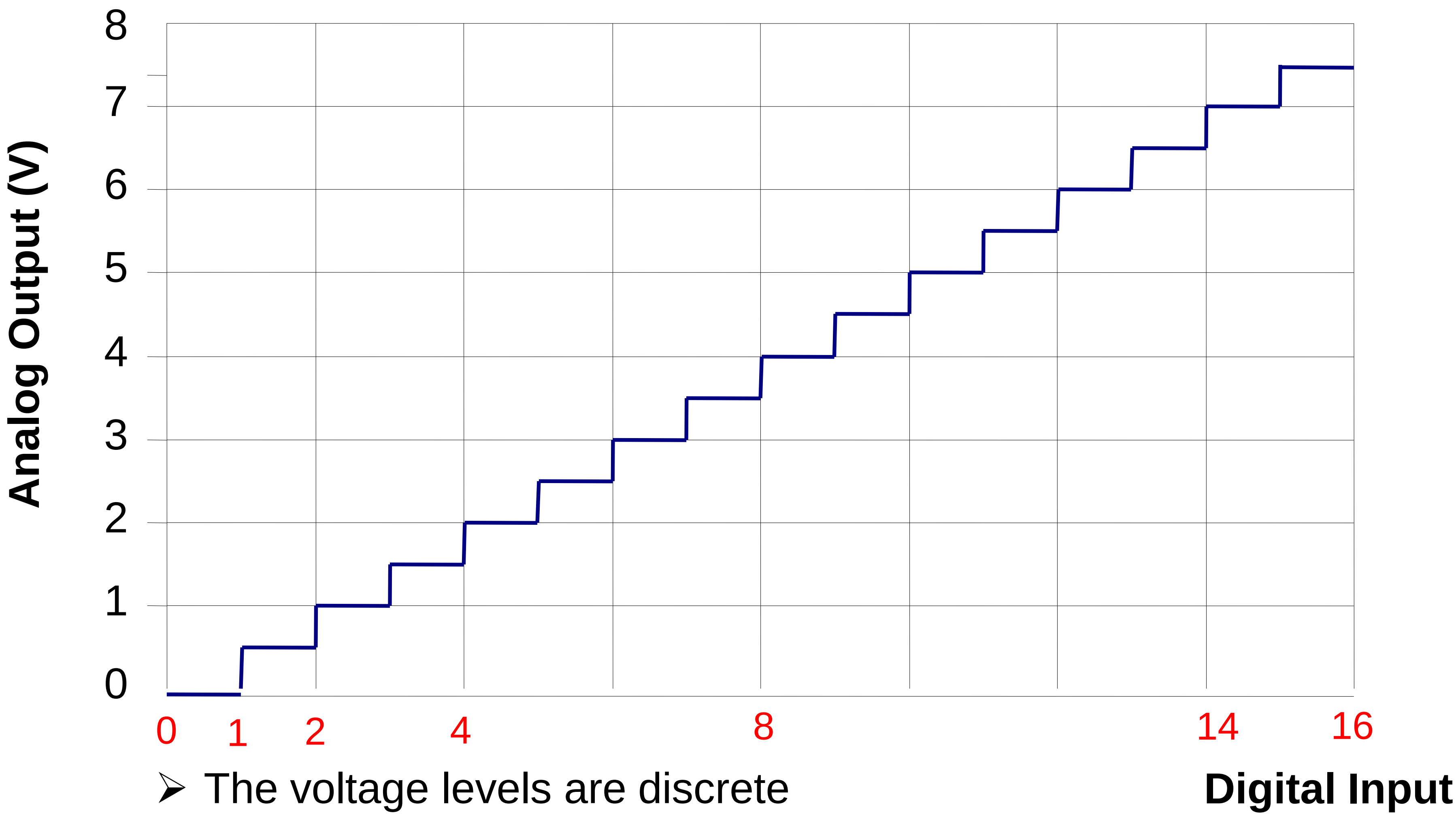


$$V_n = \frac{(i_s R_2) R_f}{R_1+R_2+R_f}, \quad (R_i \sim \infty)$$

$$I_{\mathrm{sc}} = -\frac{AV_n}{R_0} + \frac{V_n}{R_f} \approx \frac{-A(i_s R_2) R_f}{R_0(R_1+R_2+R_f)}$$

$$R_{\mathrm{Th}} = \frac{V_{\mathrm{oc}}}{I_{\mathrm{sc}}} = \frac{R_0(R_1+R_2+R_f)}{A(R_1+R_2)} \sim \frac{R_0}{A}$$

Application - DAC



- The voltage levels are discrete
- It is proportional to the digital value
- Approximately continuous if the step is small enough

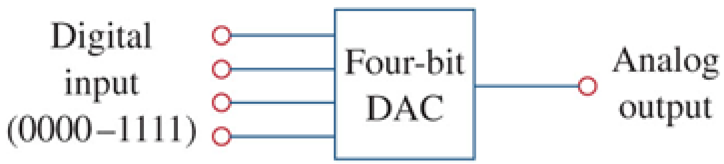
Binary representation

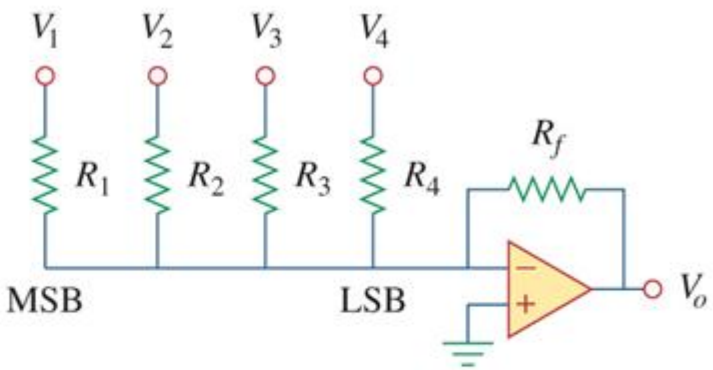
$[v_n \ldots \ldots v_3 v_2 v_1]$, v_i = 0 or 1.

MSB LSB

Digital value =$v_1 + 2v_2 + 4v_3 + \cdots 2^{n-1} v_n$

DAC



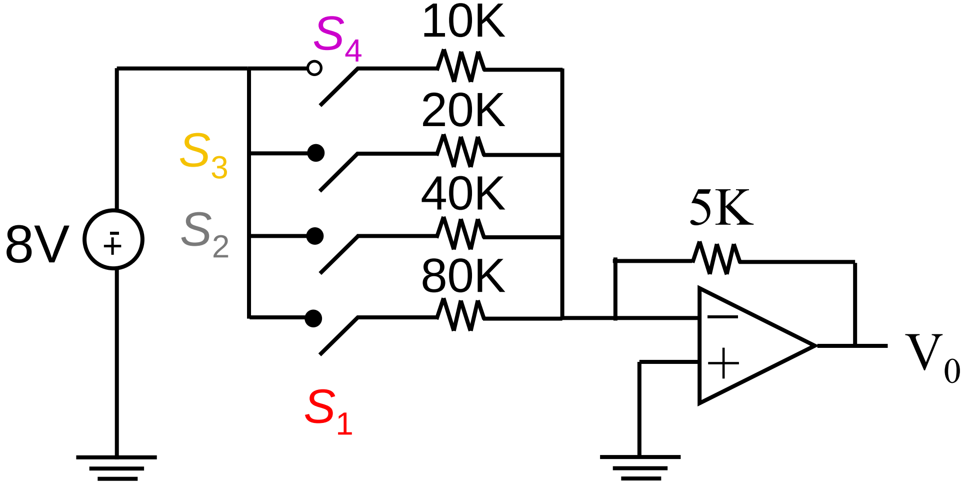


$$-V_o = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4 = \frac{R_f}{R_1}\left(V_1 + \frac{R_1}{R_2}V_2 + \frac{R_1}{R_3}V_3 + \frac{R_1}{R_4}V_4\right)$$

If $\frac{R_1}{R_2} = 2$, $\frac{R_1}{R_3} = 4$, $\frac{R_1}{R_4} = 8$, then $V_o = -\frac{R_f}{R_1}\underbrace{(V_1 + 2V_2 + 4V_3 + 8V_4)}_{\text{二进制数字的值}}$

DAC

"Weighted-adder D/A converter"



4-Bit D/A

(Transistors are used
as electronic switches)

$$V_i = -8S_i$$

$$V_o = -\frac{R_f}{R_1}(V_1 + 2V_2 + 4V_3 + 8V_4) = 0.5(S_1 + 2S_2 + 4S_3 + 8S_4)$$

Instrumentation Amplifier

1. The amplification factor controllable and not too large to allow a sufficient dynamic range. → multiple resistance options.

2. The input signal often has an unknown offset → differential amplifier.

3. The Thevinen resistance of the input signal is not negligible. → two stage design.

Instrumentation Amplifier

Ad8231

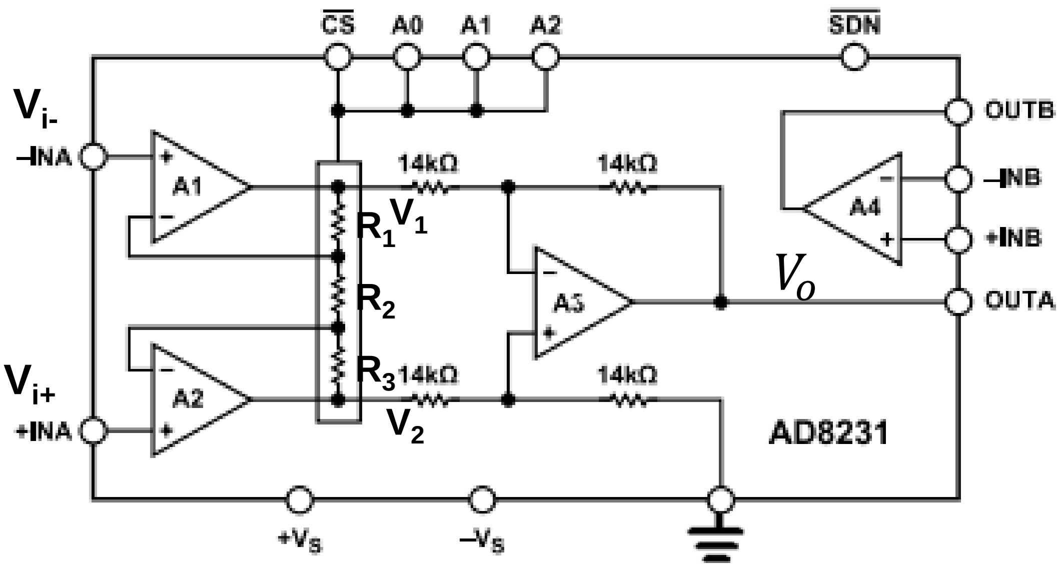


Table 7. Truth Table for AD8231 Gain Settings

CS	A2	A1	A0	Gain
Low	Low	Low	Low	1
Low	Low	Low	High	2
Low	Low	High	Low	4
Low	Low	High	High	8
Low	High	Low	Low	16
Low	High	Low	High	32
Low	High	High	Low	64
Low	High	High	High	128
High	X	X	X	No change

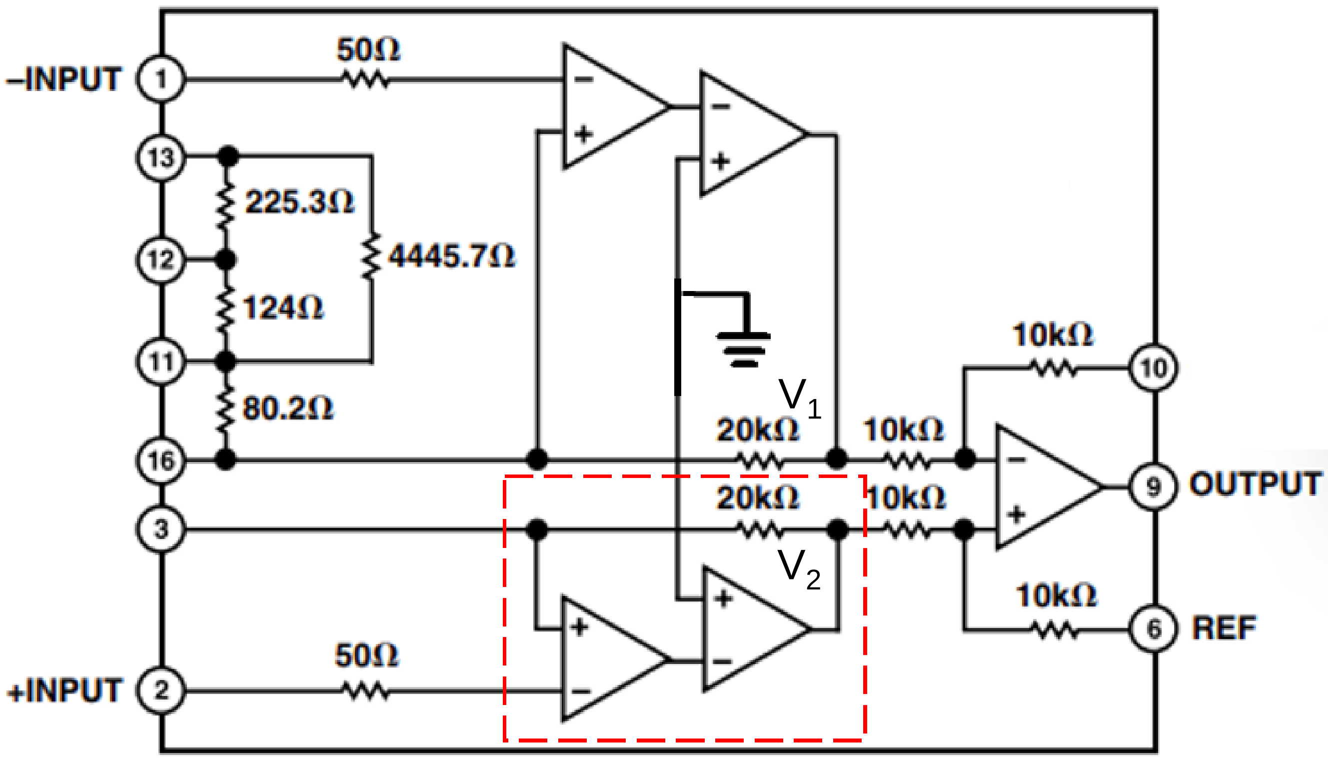
The circuit can be separated into 2 stages

Stage 1: $(V_{i+} - V_{i-}) = (V_2 - V_1)\frac{R_2}{R_1+R_2+R_3}$

Stage 2: $V_o = (V_2 - V_1)$

Instrumentation Amplifier (practice 5)

下图为某仪表放大器的内部电路图，如果将端口12与3相连，端口9与10相连，端口6接地。求 OUTPUT(端口9)的电压与“+input”(端口2)- “-input” (端口1)间电压差的关系。



practice 5 solution

V_{output}=V_2-V_1

V_{+input}-V_{-input}
$=(V_2-V_1)\frac{R_{3,16}}{40000+R_{3,16}}$

$V_{output}=(V_{+input}-V_{-input})\left(1+\frac{40000}{R_{3,16}}\right)$

$$R_{3,16}=80.2+\frac{124\cdot(225.3+4445.7)}{124+225.3+4445.7}=200.99$$

$V_{output}=200(V_{+input}-V_{-input})$