and output variables, and primed state variables (denoting the state at the end of a round).

Let us revisit our first reactive component, `Delay`, of figure 2.1. The initialization formula φ_I for the component `Delay` is $\mathrm{x} = 0$. Note that if we want to specify that the initial value of x may be either 0 or 1, the corresponding initial assignment $\mathrm{x} :=$ `choose` $\{0, 1\}$ is captured by the formula 1 (every state satisfies the constant 1). Indeed, the constraint-based or declarative style can be more convenient to specify nondeterminism.

For the `Delay` component, the reaction formula φ_R is

$$(\mathit{out} = \mathrm{x}) \;\wedge\; (\mathrm{x}' = \mathit{in})$$

that captures the relationship among old state, input, output, and updated state. In general, the reaction formula φ_R for a component C with state variables S, input variables I, and output variables O is a formula over the variables $S \cup I \cup O \cup S'$. For states s, t, input i, and output o of C, $s \xrightarrow{i/o} t$ is a reaction of C precisely when the formula φ_R is satisfied when we use state s to assign values to variables in S, input i to assign values to variables in I, output o to assign values to variables in O, and state t to assign values to the corresponding primed variables in S'.

Given a symbolic description of a synchronous reactive component C, the symbolic description of the corresponding transition system T can be obtained easily. The initialization formula for T is the same as the initialization formula for C. Recall that, for states s and t, (s, t) is a transition of T precisely when there *exists some* input i and output o such that $s \xrightarrow{i/o} t$ is a reaction of C. This relationship between transitions of T and reactions of C can be naturally expressed using the operation of *existential quantification* for logical formulas.

If f is a Boolean formula over a set V of variables and x is a variable in V, then $\exists\, x.\, f$ is a Boolean formula over $V \setminus \{x\}$. A valuation q over $V \setminus \{x\}$ satisfies the quantified formula $\exists\, x.\, f$ if q can be extended by assigning some value to x to satisfy f, that is, there exists a valuation s over V such that $s(f) = 1$ and $s(y) = q(y)$ for each variable y in $V \setminus \{x\}$. For example, if x and y are Boolean variables, then the formula $\exists\, x\, .(x \wedge y)$ expresses a constraint only over the variable y and is equivalent to the formula y (that is, $\exists\, x\, .(x \wedge y)$ evaluates to 1 exactly when y is assigned 1). Similarly, the formula $\exists\, x\, .(x \vee y)$ is equivalent to 1 (that is, this formula evaluates to 1 independent of the value of y).

The transition formula for the transition system corresponding to `Delay` is

$$\exists\, \mathit{in}.\, \exists\, \mathit{out}.\; [\,(\mathit{out} = \mathrm{x}) \;\wedge\; (\mathrm{x}' = \mathit{in}\,]).$$

This formula simplifies to the logical constant 1 since the formula is satisfied no matter what the values of x and x' are. This corresponds to the fact that for this transition system, there is a transition between every pair of states.

More generally, if φ_R is the reaction formula for the component C, then the transition formula φ_T for the corresponding transition system is obtained by existentially quantifying all the input and output variables: $\exists\, I.\, \exists\, O.\, \varphi_R$.

As another example, consider the component `TriggeredCopy` of figure 2.5. The only state variable is x of type `nat`, the initialization formula is $x = 0$, and the reaction formula is

$$(in? \wedge out = in \wedge x' = x + 1) \ \vee \ (\neg in? \wedge out = \perp \wedge x' = x).$$

The assumption that when the input event is absent, the component leaves the state unchanged and the output is absent, is captured explicitly in the reaction formula as a separate case. The corresponding transition formula is obtained by existentially quantifying the input and output variable:

$$\exists\, in, out.\ [\,(in? \wedge out = in \wedge x' = x + 1) \ \vee \ (\neg in? \wedge out = \perp \wedge x' = x)\,].$$

This transition formula can be simplified to a logically equivalent formula

$$(x' = x + 1) \ \vee \ (x' = x).$$

Composing Symbolic Representations

In section 2.3, we studied how complex components can be combined using the operations of input/output variable renaming, parallel composition, and output hiding. The symbolic description of the resulting component can be obtained naturally from the symbolic descriptions of the original components. We will use the block diagram for `DoubleDelay` from figure 2.15 to illustrate this.

Renaming of variables is useful to create instances of components so that there are no name conflicts among state variables of different components, and common names for input/output variables indicate input/output connections. Given the initialization and reaction formulas for the original component, the corresponding formulas for the instantiated component can be obtained by textual substitution. For instance, the component `Delay1` is obtained from the component `Delay` by renaming the state variable x to x_1 and the output *out* to *temp*. The initialization formula for `Delay1` then is $x_1 = 0$, and the reaction formula is $(temp = x_1) \wedge (x_1' = in)$. Similarly, the initialization formula for `Delay2` is $x_2 = 0$, and the reaction formula is $(out = x_2) \wedge (x_2' = temp)$.

Consider two compatible components C_1 and C_2. If φ_I^1 and φ_I^2 are the respective initialization formulas for C_1 and C_2, then the initialization formula for the product $C_1 \| C_2$ is simply the conjunction $\varphi_I^1 \wedge \varphi_I^2$. This captures the fact that the formula φ_I^1 constrains the initial values of the state variables of C_1, and the formula φ_I^2 constrains the initial values of the state variables of C_2. Similarly, if φ_R^1 and φ_R^2 are the respective reaction formulas for C_1 and C_2, then the reaction formula for the product $C_1 \| C_2$ is the conjunction $\varphi_R^1 \wedge \varphi_R^2$. This again captures the intuition that in synchronous composition, state s of the composite,