

NUXMV: Introduction*

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Thi Thieu Hoa Le, Alessandra Giordani, Patrick Trentin for FM lab 2005/15

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- NUXMV is a **new** symbolic model checker developed by **FBK-IRST**.
 - based on the NuSMV model checker
 - project url: <https://nuxmv.fbk.eu/>
 - *the binary of NUXMV is available for non-commercial or academic purposes only!*
- NUXMV allows for verifying
 - *finite-state systems* through state-of-the-art SAT-based algorithms;
 - *infinite-state systems* (e.g. systems with *real* and *integer* variables) through SMT-based techniques running on top of **MathSAT5**;
- NUXMV supports *synchronous* systems;
asynchronous systems are no longer supported!

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Interactive shell [1/3]

- `nuxmv -int` (or `NuSMV -int`) activates an interactive shell
- `help` shows the list of all commands (if a command name is given as argument, detailed information for that command will be provided).
note: option `-h` prints the command line help for each command.
- `reset` resets the whole system (in order to read in another model and to perform verification on it).
- `read_model [-i filename]` sets the input model and reads it.
- `go`, `go_bmc`, `go_msat` initialize NUXMV for verification or simulation with a specific backend engine.

- `pick_state [-v] [-a] [-r | -i]` picks a state from the set of initial states.
 - `-v` prints the chosen state.
 - `-r` picks a state from the set of the initial states randomly.
 - `-i` picks a state from the set of the initial states interactively.
 - `-a` displays all state variables (requires `-i`).
- `simulate [-p | -v] [-a] [-r | -i] -k N` generates a sequence of at most `N` transitions starting from the current state.
 - `-p` prints the changing variables in the generated trace;
 - `-v` prints changed and unchanged variables in the generated trace;
 - `-a` prints all state variables (requires `-i`);
 - `-r` at every step picks the next state randomly.
 - `-i` at every step picks the next state interactively.
- `print_current_state [-h] [-v]` prints out the current state.
 - `-v` prints all the variables.

Interacting Shell [2/3] - Output Example

```
nuXmv > reset; read_model -i example01.smv ; go ; pick_state -v; simulate -v
```

```
Trace Description: Simulation Trace
```

```
Trace Type: Simulation
```

```
-> State: 1.1 <-
```

```
  b0 = FALSE
```

```
***** Simulation Starting From State 1.1 *****
```

```
Trace Description: Simulation Trace
```

```
Trace Type: Simulation
```

```
-> State: 1.1 <-
```

```
  b0 = FALSE
```

```
-> State: 1.2 <-
```

```
  b0 = TRUE
```

```
-> State: 1.3 <-
```

```
  b0 = FALSE
```

```
-> State: 1.4 <-
```

```
  b0 = TRUE
```

```
-> State: 1.5 <-
```

```
  b0 = FALSE
```

```
-> State: 1.6 <-
```

```
  b0 = TRUE
```

```
...
```


Interacting Shell [3/3]

- `goto_state state_label` makes `state_label` the current state (it is used to navigate along traces).
- `show_traces [-t] [-v] [-a | TN[.FS[:[TS]]]` prints the trace *TN* starting from state *FS* up to state *TS*
 - `-t` prints the total number of stored traces
 - `-v` verbosely prints traces content;
 - `-a` prints all the currently stored traces
- `show_vars [-s] [-f] [-i] [-t] [-v]` prints the variables content and type
 - `-s` print state variables;
 - `-f` print frozen variables;
 - `-i` print input variables;
 - `-t` prints the number of variables;
 - `-v` prints verbosely;
- `quit` stops the program.

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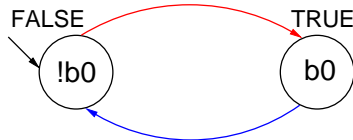
The first SMV program

- an SMV program is composed by a number of **modules**;
- each **module**, contains:
 - state variable declarations;
 - assignments defining the valid *initial states*;
 - assignments defining the *transition relation*;

Example:

```
MODULE main
VAR
    b0 : boolean;

ASSIGN
    init(b0) := FALSE;
    next(b0) := !b0;
```



Basic Types [1/2]

boolean: TRUE, FALSE, ...

`x : boolean;`

enumerative:

`s : {ready, busy, waiting, stopped};`

bounded integers (intervals):

`n : 1..8;`

integers*: -1, 0, 1, ...

`n : integer;`

rationals: 1.66, f'2/3, 2e3, 10e-1, ...

`r : real;`

words: used to model **arrays of bits** supporting bitwise logical and arithmetic operations.

- `unsigned word[3];`

- `signed word[7];`

*: integer numbers must be within C/C++ INT_MIN and INT_MAX bounds

Basic Types [2/2]

arrays:

declared with a couple of lower/upper bounds for the index and a type

VAR

```
x : array 0..10 of boolean; -- array of 11 elements
y : array -1..1 of {red, green, orange}; -- array of 3 elements
z : array 1..10 of array 1..5 of boolean; -- array of array
```

ASSIGN

```
init(x[5]) := bool(1);
init(y[0]) := {red, green}; -- any value in the set
init(z[3][2]) := TRUE;
```

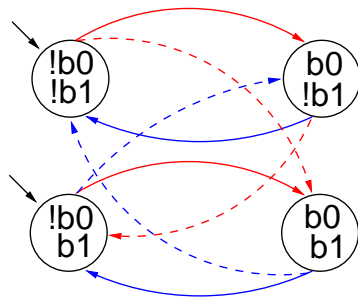
Remarks:

- Array indexes *must be constants*;

Adding a state variable

```
MODULE main
VAR
  b0 : boolean;
  b1 : boolean;

ASSIGN
  init(b0) := FALSE;
  next(b0) := !b0;
```



Remarks:

- the FSM is the result of the **synchronous composition** of the “subsystems” for b0 and b1
- the new state space is the cartesian product of the ranges of the variables.



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Initial States [1/2]

Example:

```
init(x) := FALSE; -- x must be FALSE  
init(y) := {1, 2, 3}; -- y can be either 1, 2 or 3
```

`init(<variable>) := <simple_expression>;`

- constrains the **initial value** of <variable> to satisfy the <simple_expression>;
- the **initial** value of an **unconstrained** variable can be any of those allowed by its domain;

set of initial states

is given by the set of states whose variables satisfy **all** the `init()` **constraints** in a module.

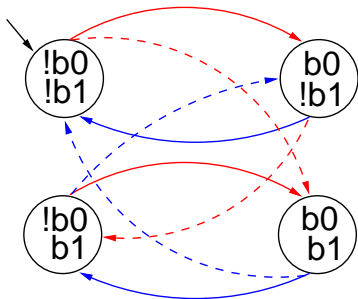
Initial States [2/2]

Example:

```
MODULE main
  VAR
    b0 : boolean;
    b1 : boolean;

  ASSIGN
    init(b0) := FALSE;
    next(b0) := !b0;

    init(b1) := FALSE;
```



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Expressions [1/3]

- arithmetic operators:

+ - * / mod - (unary)

- comparison operators:

= != > < <= >=

- logic operators:

& | xor ! (not) -> <->

- bitwise operators:

<< >>

- set operators: $\{v1, v2, \dots, vn\}$

- in: tests a value for membership in a set (*set inclusion*)
- union: takes the union of 2 sets (*set union*)



- count operator: counts number of true *boolean* expressions

count(b1 + b2 + ... + bn)



- case expression:

```
case
  c1    : e1;
  c2    : e2;
  ...
  TRUE  : en;
esac
```

C/C++ equivalent:

```
if (c1) then e1;
else if (c2) then e2;
...
else en;
```

- if-then-else expression:

```
cond_expr ? basic_expr 1 : basic_expr2
```

- **conversion operators**: `toint`, `bool`, `floor`, and
 - `swconst`, `uwconst`: convert an integer to a signed and an unsigned word respectively.
 - `word1` converts boolean to a single word bit.
 - `unsigned` and `signed` convert signed word to unsigned word and vice-versa.

- expressions in SMV do not necessarily evaluate to one value. In general, they can represent a set of possible values.

```
init(var) := {a,b,c} union {x,y,z} ;
```

- The meaning of `:=` in assignments is that the lhs can **non-deterministically** be assigned to any value in the set of values represented by the rhs.
- A constant `c` is considered as a syntactic abbreviation for `{c}` (the singleton containing `c`).

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Transition Relation [1/2]

Transition Relation

specifies a **constraint** on the values that a variable can assume in the *next state*, given the value of variables in the *current state*.

`next(<variable>) := <next_expression>;`

- `<next_expression>` can depend both on “current” and “next” variables:

```
next(a) := { a, a+1 } ;  
next(b) := b + (next(a) - a) ;
```

- `<next_expression>` must evaluate to values in the domain of `<variable>`;
- the **next** value of an **unconstrained** variable evolves **non-deterministically**;

Transition Relation [2/2]

Example:

modulo-4 counter

```
MODULE main
```

```
VAR
```

```
  b0 : boolean;
```

```
  b1 : boolean;
```

```
ASSIGN
```

```
  init(b0) := FALSE;
```

```
  next(b0) := !b0;
```

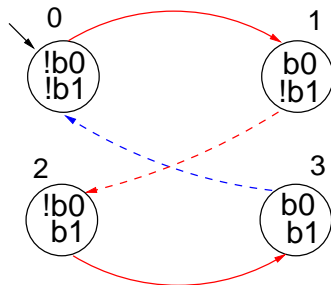
```
  init(b1) := FALSE;
```

```
  next(b1) := case
```

```
    b0   : !b1;
```

```
    TRUE : b1;
```

```
  esac;
```



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Output Variable [1/2]

output variable

is a variable whose value is deterministically depends on the value of other “current” state variables and for which no `init()` or `next()` are defined.

`<variable> := <simple_expression>;`

- `<simple_expression>` must evaluate to values in the domain of the `<variable>`.
- used to model *outputs* of a system;



Output Variable [2/2]

Example:

```
MODULE main
```

```
VAR
```

```
  b0 : boolean;
```

```
  b1 : boolean;
```

```
  out : 0..3;
```

```
ASSIGN
```

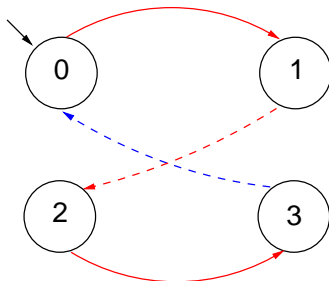
```
  init(b0) := FALSE;
```

```
  next(b0) := !b0;
```

```
  init(b1) := FALSE;
```

```
  next(b1) := ((!b0 & b1), (b0 & !b1));
```

```
  out := toint(b0) + 2*toint(b1);
```



Assignment Rules ($:=$)

- **single assignment rule** – each variable may be assigned only once;

Illegal examples:

```
init(var) := ready;
```

```
init(var) := busy;
```

```
var := ready;
```

```
var := busy;
```

```
next(var) := ready;
```

```
var := busy;
```

```
next(var) := ready;
```

```
next(var) := busy;
```

```
init(var) := ready;
```

```
var := busy;
```

Assignment Rules ($:=$)

- **single assignment rule** – each variable may be assigned only once;

Illegal examples:

<code>init(var) := ready;</code>	<code>var := ready;</code>	<code>next(var) := ready;</code>
<code>init(var) := busy;</code>	<code>var := busy;</code>	<code>var := busy;</code>
<code>next(var) := ready;</code>	<code>init(var) := ready;</code>	
<code>next(var) := busy;</code>	<code>var := busy;</code>	

- **circular dependency rule** – a set of equations must not have “cycles” in its dependency graph, unless broken by delays;

Illegal examples:

<code>next(x) := next(y);</code>	<code>x := (x + 1) mod 2;</code>	<code>next(x) := x & next(x);</code>
<code>next(y) := next(x);</code>		

Legal example:

<code>next(x) := next(y);</code>
<code>next(y) := y & x;</code>

DEFINE declarations

DEFINE <id> := <simple_expression>;

- similar to C/C++ *macro definitions*: each occurrence of the defined symbol is replaced with the body of the definition
- provide an *alternative way* of defining *output variables*;

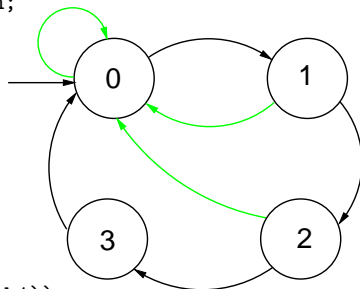
Example:

```
MODULE main
  VAR
    b0 : boolean;
    b1 : boolean;
  ASSIGN
    init(b0) := FALSE;
    next(b0) := !b0;
    init(b1) := FALSE;
    next(b1) := ((!b0 & b1) | (b0 & !b1));
  DEFINE
    out := toint(b0) + 2*toint(b1);
```

Example: modulo 4 counter with reset

The counter can be reset by an external “uncontrollable” signal.

```
MODULE main
VAR
  b0 : boolean; b1 : boolean; reset : boolean;
ASSIGN
  init(b0) := FALSE;
  init(b1) := FALSE;
  next(b0) := case
    reset = TRUE  : FALSE;
    reset = FALSE : !b0;
  esac;
  next(b1) := case
    reset : FALSE;
    TRUE  : ((!b0 & b1) | (b0 & !b1));
  esac;
DEFINE
  out := toint(b0) + 2*toint(b1);
```



Exercise 1

Exercise:

simulate the system with NUXMV and draw the FSM.

```
MODULE main
VAR
  request : boolean;
  state   : { ready, busy };

ASSIGN
  init(state) := ready;
  next(state) :=
    case
      state = ready & request : busy;
      TRUE                     : { ready, busy };
    esac;
```

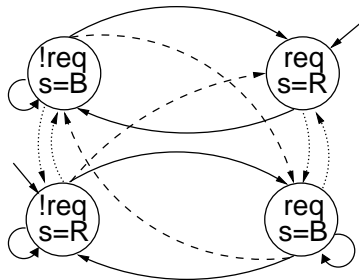

Exercise 1

Exercise:

simulate the system with `NUXMV` and draw the FSM.

```
MODULE main
VAR
  request : boolean;
  state   : { ready, busy };

ASSIGN
  init(state) := ready;
  next(state) :=
    case
      state = ready & request : busy;
      TRUE                     : { ready, busy };
    esac;
```



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Constraint Style Modeling [1/3]

```
MODULE main
VAR
  request : boolean;  state : {ready,busy};
  ASSIGN
    init(state) := ready;
    next(state) := case
      state = ready & request : busy;
      TRUE                     : {ready,busy};
    esac;
```

Every program can be alternatively defined in a *constraint style*:

```
MODULE main
VAR
  request : boolean;
  state   : {ready,busy};
  INIT
    state = ready
  TRANS
    (state = ready & request) -> next(state) = busy
```



Constraint Style Modeling [2/3]

- a model can be specified by zero or more constraints on:
 - *invariant states*:
`INVAR <simple_expression>`
 - *initial states*:
`INIT <simple_expression>`
 - *transitions*:
`TRANS <next_expression>`
- constraints can be mixed with assignments;
- any propositional formula is allowed as constraint;
- not all **constraints** can be easily rewritten in terms of assignments!

TRANS

```
next(b0) + 2*next(b1) + 4*next(b2) =  
      (b0 + 2*b1 + 4*b2 + tick) mod 8
```




- Models written in **assignment style**:
 - by construction, there is always *at least one initial state*;
 - by construction, all states have *at least one next state*;
 - *non-determinism is apparent* (unassigned variables, set assignments...).
- Models written in **constraint style**:
 - INIT constraints *can be inconsistent* \implies **no initial state!**
 - any specification (also SPEC 0) is vacuously true.
 - TRANS constraints *can be inconsistent*: \implies **deadlock state!**

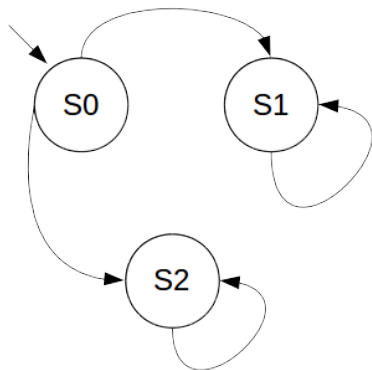
Example:

```
MODULE main
VAR b : boolean;
TRANS b -> FALSE;
```

- **tip:** use `check_fsm` to detect deadlock states
- *non-determinism is hidden:*
TRANS (state = ready & request) -> next(state) = busy


Example: Constraint Style & Case

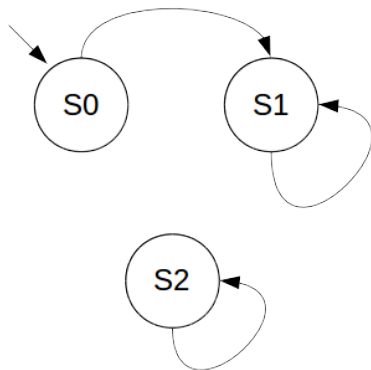
```
MODULE main()  
VAR  
  state : {S0, S1, S2};  
  
DEFINE  
  go_s1  state != S2;  
  go_s2 := state != S1;  
  
INIT  
  state = S0;  
  
TRANS  
case   
  go_  next(state) = S1;  
  go_s2 : next(state) = S2;  
esac;
```



- **Q:** does it correspond to the FSM?

Example: Constraint Style & Case

```
MODULE main()  
VAR  
  state : {S0, S1, S2};  
  
DEFINE  
  go_s1 := state != S2;  
  go_s2 := state != S1;  
  
INIT  
  state = S0;  
  
TRANS  
case  
  go_s1  next(state) = S1;  
  go_s2 : next(state) = S2;  
esac;
```



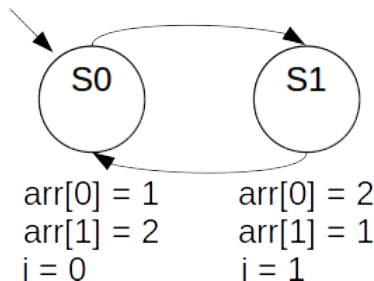
- **Q: does it correspond to the FSM?** No: cases are evaluated in order!

Example: Constraint Style & Swap

```
MODULE main()  
VAR  
  arr: array 0..1 of {1,2};  
  i : 0..1;
```

```
ASSIGN  
  init(arr[0]) := 1;  
  init(arr[1]) := 2;  
  
  init(i) := 0;  
  next(i) := 1-i;
```

```
TRANS  
  next(arr[i]) = arr[1-i] &  
  next(arr[1-i]) = arr[i];
```



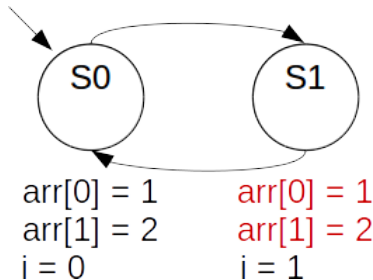
- **Q:** does it correspond to the FSM?

Example: Constraint Style & Swap

```
MODULE main()  
VAR  
  arr: array 0..1 of {1,2};  
  i : 0..1;
```

```
ASSIGN  
  init(arr[0]) := 1;  
  init(arr[1]) := 2;  
  
  init(i) := 0;  
  next(i) := 1-i;
```

```
TRANS  
  next(arr[i]) = arr[1-i] &  
  next(arr[1-i]) = arr[i];
```



- **Q: does it correspond to the FSM?** No: everything inside the **next()** operator is evaluated within the next state, indexes included!

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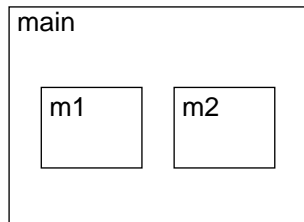
Modules [1/3]

SMV program = main module + 0 or *more* other modules

- a module can be **instantiated** as a VAR in other modules
- dot notation for accessing variables that are **local** to a module instance (e.g., m1.out, m2.out).

Example:

```
MODULE counter
  VAR out: 0..9;
  ASSIGN next(out) :=
    (out + 1) mod 10;
MODULE main
  VAR m1 : counter; m2 : counter;
    sum: 0..18;
  ASSIGN sum := m1.out + m2.out;
```



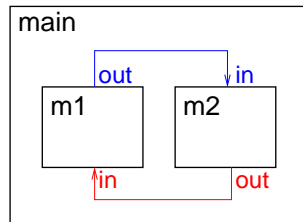
Modules [2/3]

A module declaration can be *parametric*:

- a parameter is passed by reference;
- any expression can be used as parameter;

Example:

```
MODULE counter(in)
  VAR out: 0..9;
  ...
MODULE main
  VAR m1 : counter(m2.out);
      m2 : counter(m1.out);
  ...
```



- modules can be **composed**
- modules *without parameters and assignments* can be seen as simple **records**

Example:

```
MODULE point  
VAR
```

```
  x: -10..10;  
  y: -10..10;
```

```
MODULE circle  
VAR
```

```
  center: point;  
  radius: 0..10;
```

```
MODULE main
```

```
VAR c: circle;
```

```
ASSIGN
```

```
  init(c.center.x) := 0;  
  init(c.center.y) := 0;  
  init(c.radius)   := 5;
```

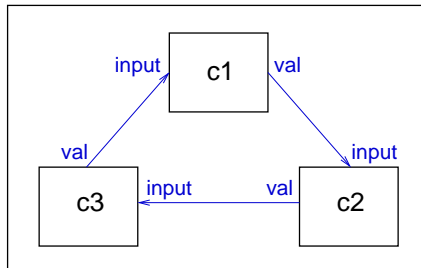
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Synchronous composition [1/2]

The composition of modules is **synchronous** by default:
all modules move at each step.

```
MODULE cell(input)
VAR
  val : {red, green, blue};
ASSIGN
  next(val) := input;

MODULE main
VAR
  c1 : cell(c3.val);
  c2 : cell(c1.val);
  c3 : cell(c2.val);
```



Synchronous composition [2/2]

A possible execution:

<i>step</i>	<i>c1.val</i>	<i>c2.val</i>	<i>c3.val</i>
0	red	green	blue
1	blue	red	green
2	green	blue	red
3	red	green	blue
4
5	red	green	blue



Asynchronous composition [1/2]

Asynchronous composition can be obtained using keyword process:
one process moves at each step.

```
MODULE cell(input)
VAR
  val : {red, green, blue};
ASSIGN next(val) := input;
FAIRNESS running
```

```
MODULE main
VAR
  c1 : process cell(c3.val);
  c2 : process cell(c1.val);
  c3 : process cell(c2.val);
```

Each process has a boolean **running** variable:

- true iff the process is selected for execution;
- can be used to guarantee a fair scheduling of processes.

Asynchronous composition [2/2]

A possible execution:

<i>step</i>	<i>running</i>	<i>c1.val</i>	<i>c2.val</i>	<i>c3.val</i>
0	-	red	green	blue
1	c2	red	red	blue
2	c1	blue	red	blue
3	c1	blue	red	blue
4	c3	blue	red	red
5	c2	blue	blue	red
6	c3	blue	blue	blue
...	...	blue	blue	blue

Warning: in NUXMV processes are deprecated!

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Exercise: Adder [1/3]

```
MODULE bit-adder(in1, in2, cin)
VAR
    sum : boolean;
    cout : boolean;
ASSIGN
    next(sum) := (in1 xor in2) xor cin;
    next(cout) := (in1 & in2) | ((in1 | in2) & cin);

MODULE adder(in1, in2)
VAR
    bit[0] : bit-adder(in1[0], in2[0], 0);
    bit[1] : bit-adder(in1[1], in2[1], bit[0].cout);
    bit[2] : bit-adder(in1[2], in2[2], bit[1].cout);
    bit[3] : bit-adder(in1[3], in2[3], bit[2].cout);
DEFINE
    sum[0] := bit[0].sum;
    sum[1] := bit[1].sum;
    sum[2] := bit[2].sum;
    sum[3] := bit[3].sum;
    overflow := bit[3].cout;
```

Exercise: Adder [2/3]

```
MODULE main
VAR
  in1 : array 0..3 of boolean;
  in2 : array 0..3 of boolean;
  a : adder(in1, in2);
ASSIGN
  next(in1[0]) := in1[0]; next(in1[1]) := in1[1];
  next(in1[2]) := in1[2]; next(in1[3]) := in1[3];
  next(in2[0]) := in2[0]; next(in2[1]) := in2[1];
  next(in2[2]) := in2[2]; next(in2[3]) := in2[3];
DEFINE
  op1 := toint(in1[0]) + 2*toint(in1[1]) + 4*toint(in1[2]) +
    8*toint(in1[3]);
  op2 := toint(in2[0]) + 2*toint(in2[1]) + 4*toint(in2[2]) +
    8*toint(in2[3]);
  sum := toint(a.sum[0]) + 2*toint(a.sum[1]) + 4*toint(a.sum[2]) +
    8*toint(a.sum[3]) + 16*toint(a.overflow);
```

Exercise: Adder [3/3]

Exercise:

- simulate a random execution of the “adder” system;
- after how many steps the adder stores the computes the final sum value?
- add a reset control which changes the values of the operands and restarts the computation of the sum

- will be uploaded on course website within a couple of days
 - send me an email if you need help or you just want to propose your own solution for a review
-
- learning programming languages requires practice: try to come up with your own solutions first!