**FI7015-1 Cosmología**
**Profesor:** Domenico Sapone
**Auxiliares:** Francisco Colipí & Vicente Pedreros

# Tarea 2

**Fecha entrega**: 21 de octubre.

*Perform the full calculations with all the steps; the numerical parts should be written in python and the code uploaded to your personal Github repository.*

**P1.** Starting from the metric of the conformal Newtonian Gauge

$$ds^2 = a^2(\tau)[-(1 + 2\psi(\vec{x}, t)d\tau^2 + (1 - 2\phi(\vec{x}, t)\delta_{ij}dx^i dx^j], \tag{1}$$

evaluate everything needed for finding the perturbed Einstein equations.

**P2.** The energy momentum tensor $T^u_\nu$ for a single fluid satisfies the equation $\nabla_\mu T^\mu_\nu$. Solve the first order part of this equation for a general perfect fluid, defined by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} \tag{2}$$

**P3.** Derive the conservation equation of energy and momentum by taking moments of the collisionless Boltzmann equation. Consider the Newtonian limit, giving

$$\frac{Df}{Dt} = 0 \iff \frac{\partial f}{\partial t} + \frac{\vec{p}}{ma^2} \cdot \frac{\partial f}{\partial \vec{x}} - m\nabla\Phi \cdot \frac{\partial f}{\partial \vec{p}} = 0, \tag{3}$$

where $\Phi = \Phi(\vec{x}, t)$ is the gravitational potential and it satisfies the Poisson equation

$$\nabla^2\Phi = 4\pi G a^2 \rho. \tag{4}$$

*Hint: use the following definitions for the energy density $\rho$, momentum density $\pi^i$ and kinetic tensor $\sigma^{ij}$*

$$\rho(\vec{x}, t) = \frac{m}{a^3} \int \frac{d^3p}{(2\pi)^3} f(\vec{x}, \vec{p}) \tag{5}$$

$$\pi^i(\vec{x}, t) = \frac{1}{a^4} \int \frac{d^3p}{(2\pi)^3} p^i f(\vec{x}, \vec{p}) \tag{6}$$

$$\sigma^{ij}(\vec{x}, t) = \frac{1}{ma^5} \int \frac{d^3p}{(2\pi)^3} p^i p^j f(\vec{x}, \vec{p}) \tag{7}$$

**P4.** Numerically solve the perturbation equations for a matter component with $w = \delta p = \sigma = 0$, where $w$ is the equation of state parameter, $\delta p$ the pressure perturbation and $\sigma$ the anistropic stress. Assume a $\Lambda$CDM model with $\Omega_{\mathrm{r},0} = 10^{-4}$ and $\Omega_{\mathrm{m},0} = 0.3$. Plot your results for $10^{-4} \leq a \leq 1$ and four different scales: $k = H_0, k = 5H_0, k = 20H_0$ and $k = 200H_0$. Consider $H_0 = 67$ Km/(sMpc).

**P5.** Using `CAMB` compute the matter power spectrum at different redshift. Then, manually set the density of massive neutrinos to zero. Comment the differences and justify the results.

**P6.** Using `CAMB` compute the TT angular power spectrum. Then, explore different cosmological parameters and explain the differences in the results. Give at least one compelling example and its justification.