

Forward Pass

$$z_1^1 = (0,15 \cdot 0,05) + (0,25 \cdot 0,1) + 0,35 = 0,3825$$

$$a_1 = \frac{1}{1 + e^{-0,3825}} = 0,594$$

$$z_2 = (0,2 \cdot 0,05) + (0,3 \cdot 0,1) + 0,35 = 0,39$$

$$a_2 = 0,596$$

$$z_1^2 = (0,4 \cdot 0,594) + (0,5 \cdot 0,596) + 0,60 = 1,136$$

$$a_1 = 0,757$$

$$z_2^2 = (0,45 \cdot 0,594) + (0,45 \cdot 0,596) + 0,60 = 1,136$$

$$a_2 = 0,757$$

$$Loss_1 = \frac{1}{2} (0,01 - 0,757)^2 = 0,279$$

$$Loss_2 = \frac{1}{2} (0,99 - 0,757)^2 = 0,027$$

$$Loss = 0,306$$

# Backward Pass

$$\frac{\partial \ell}{\partial \theta_{11}^2} = \frac{\partial \ell}{\partial a_1^3} \cdot \frac{\partial a_1^3}{\partial z_1^2} \cdot \frac{\partial z_1^2}{\partial \theta_{11}^2}$$

$$\frac{\partial \ell}{\partial \theta_{12}^2} = \frac{\partial \ell}{\partial a_1^3} \cdot \frac{\partial a_1^3}{\partial z_1^2} \cdot \frac{\partial z_1^2}{\partial \theta_{12}^2}$$

$$\frac{\partial \ell}{\partial \theta_{21}^2} = \frac{\partial \ell}{\partial a_2^3} \cdot \frac{\partial a_2^3}{\partial z_2^2} \cdot \frac{\partial z_2^2}{\partial \theta_{21}^2}$$

$$\frac{\partial \ell}{\partial \theta_{22}^2} = \frac{\partial \ell}{\partial a_2^3} \cdot \frac{\partial a_2^3}{\partial z_2^2} \cdot \frac{\partial z_2^2}{\partial \theta_{22}^2}$$

$$\frac{\partial \ell}{\partial \theta_{11}^1} = \frac{\partial \ell_1}{\partial a_1^2} \cdot \frac{\partial a_1^2}{\partial z_1^1} \cdot \frac{\partial z_1^1}{\partial \theta_{11}^1} + \frac{\partial \ell_2}{\partial a_1^2} \cdot \frac{\partial a_1^2}{\partial z_1^1} \cdot \frac{\partial z_1^1}{\partial \theta_{11}^1}$$

$$\frac{\partial \ell}{\partial \theta_{12}^1} = \frac{\partial \ell}{\partial a_1^2} \cdot \frac{\partial a_1^2}{\partial z_1^1} \cdot \frac{\partial z_1^1}{\partial \theta_{12}^1} + \frac{\partial \ell_2}{\partial a_1^2} \cdot \frac{\partial a_1^2}{\partial z_1^1} \cdot \frac{\partial z_1^1}{\partial \theta_{12}^1}$$

$$\frac{\partial \ell}{\partial \theta_{21}^1} = \frac{\partial \ell_1}{\partial a_2^2} \cdot \frac{\partial a_2^2}{\partial z_2^1} \cdot \frac{\partial z_2^1}{\partial \theta_{21}^1} + \frac{\partial \ell_2}{\partial a_2^2} \cdot \frac{\partial a_2^2}{\partial z_2^1} \cdot \frac{\partial z_2^1}{\partial \theta_{21}^1}$$

$$\frac{\partial \ell}{\partial \theta_{22}^1} = \frac{\partial \ell_1}{\partial a_2^2} \cdot \frac{\partial a_2^2}{\partial z_2^1} \cdot \frac{\partial z_2^1}{\partial \theta_{22}^1} + \frac{\partial \ell_2}{\partial a_2^2} \cdot \frac{\partial a_2^2}{\partial z_2^1} \cdot \frac{\partial z_2^1}{\partial \theta_{22}^1}$$

a)  $\frac{\partial \ell}{\partial a_1^3} = \frac{1}{2} \cdot 2 \cdot (0.01 - a_1^3) \cdot (-1) + 0 = 0,99$

b)  $\frac{\partial a_1^3}{\partial z_1^2} = a_1^3 / (1 - a_1^3)^2 = (0,99 / (1 - 0,99)^2) = 0,124$

c)  $\frac{\partial z_1^2}{\partial \theta_{11}^2} = a_1^2 = 0,594$

d)  $\frac{\partial z_2^1}{\partial \theta_{12}^1} = a_2^2 = 0,396$

e)  $\frac{\partial \ell}{\partial a_2^3} = \frac{1}{2} \cdot 2 \cdot (0,99 - a_2^3) \cdot (-1) + 0 = -(0,99 - 0,396) = -0,594$

$$f) \frac{\partial a_2^3}{\partial z_2^2} = a_2^3(1-a_2^3) = 0,852(1-0,852) = 0,184$$

$$g) \frac{\partial z_2^2}{\partial a_2^2} = 0,594$$

$$h) \frac{\partial z_2^2}{\partial a_2^1} = 0,596$$

$$i) \frac{\partial \ell_1}{\partial a_1^2} = 0,343 \cdot 0,184 \cdot 0,4 = 0,055$$

$$j) \frac{\partial a_1^2}{\partial z_1^1} = 0,594(1-0,594) = 0,201$$

$$k) \frac{\partial z_1^1}{\partial a_1^1} = 0,05$$

$$l) \frac{\partial \ell_2}{\partial a_1^2} = (-0,233) \cdot 0,184 \cdot 0,45 = -0,019$$

$$m) \frac{\partial z_1^1}{\partial a_1^2} = 0,10$$

$$n) \frac{\partial \ell_1}{\partial a_2^2} = 0,343 \cdot 0,184 \cdot 0,5 = 0,069$$

$$o) \frac{\partial a_2^2}{\partial z_2^1} = 0,596(1-0,596) = 0,41$$

$$p) \frac{\partial z_2^1}{\partial a_2^1} = 0,05$$

$$q) \frac{\partial \ell_2}{\partial a_2^2} = (-0,233) \cdot 0,184 \cdot 0,2 = -0,019$$

$$r) \frac{\partial z_2^1}{\partial a_2^1} = 0,10$$

weights:

$$1) 0,343 \cdot 0,184 \cdot 0,594 = 0,082$$

$$2) 0,343 \cdot 0,184 \cdot 0,596 = 0,082$$

$$3) (-0,232) \cdot 0,184 \cdot 0,594 = -0,025$$

$$4) (-0,232) \cdot 0,184 \cdot 0,596 = -0,026$$

$$5) 0,055 \cdot 0,241 \cdot 0,05 + (-0,019) \cdot 0,241 \cdot 0,05 = 4,33 \cdot 10^{-4}$$

$$6) 0,055 \cdot 0,241 \cdot 0,10 + (-0,019) \cdot 0,241 \cdot 0,05 = 1,10 \cdot 10^{-3}$$

$$7) 0,069 \cdot 0,241 \cdot 0,05 + (-0,019) \cdot 0,241 \cdot 0,05 = 6,02 \cdot 10^{-4}$$

$$8) 0,069 \cdot 0,241 \cdot 0,10 + (-0,019) \cdot 0,241 \cdot 0,05 = 1,43 \cdot 10^{-3}$$