* **Chain Rule :- Derivative of a function of a function:**

If y is a function of u and u is a function of x then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Likewise, if y is a function of v, v is a fu^n of u and u is a fu^n of x then $\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$.

Ex:- Find the derivative of the following fu^{ns}

1) $y = (2x+3)^7$

Let $u = (2x+3)$

$\therefore y = u^7$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad \text{[Chain Rule]}$$

$$= 7u^6 \cdot \frac{d}{dx}(2x+3)$$

$$= 7(2x+3)^6 \cdot (2) = 14(2x+3)^6$$

2) $(2-3x)^{1/3}$

Let $y = (2-3x)^{1/3}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(2-3x)^{1/3} \cdot \cancel{\frac{d}{dx}(2+3x)}$$

$$= \frac{1}{3}(2-3x)^{1/3-1} \frac{d}{dx}(2-3x)$$

$$= \frac{1}{3}(2-3x)^{-\frac{2}{3}} (-3)$$

$$= -(2-3x)^{-\frac{2}{3}}$$

3) $y = e^{\sqrt{x}}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} e^{\sqrt{x}} = e^{\sqrt{x}} \cdot \frac{d}{dx}\sqrt{x}$$

$$= e^{\sqrt{x}} \cdot \frac{d}{dx} x^{1/2}$$

$$= e^{\sqrt{x}} \cdot \frac{1}{2} x^{1/2-1} = e^{\sqrt{x}} \cdot \frac{1}{2} x^{-1/2}$$

4) $y = \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{1}{\sqrt{2\pi}} e^{-x^2/2}\right] = \frac{1}{\sqrt{2\pi}} \frac{d}{dx} e^{-x^2/2}$$

$$= \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2} \frac{d}{dx}\left(\frac{-x^2}{2}\right)$$

$$= \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \cdot \left(\frac{-1}{2}\right) \frac{d}{dx} x^2$$

$$= \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \cdot \left(\frac{-1}{\cancel{2}}\right) (\cancel{2}x)$$

$$= \frac{-x}{\sqrt{2\pi}} e^{-x^2/2}$$

Ex:- $y = \log(3x+5)$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left[\log(3x+5)\right]$$

$$= \frac{1}{(3x+5)} \frac{d}{dx}(3x+5)$$

$$= \frac{1}{(3x+5)} \cdot (3)$$

2nd Method:-

Let $u = (3x+5)$

$$\therefore y = \log u$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{d}{du}(\log u) \cdot \frac{d}{dx}(3x+5)$$

$$= \frac{1}{u} \cdot (3)$$

$$= \frac{3}{3x+5}$$

6) $y = 5^{\sqrt{x}}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(5^{\sqrt{x}}\right).$$

$$= 5^{\sqrt{x}} \cdot \log 5 \ \frac{d}{dx} \sqrt{x}$$

$$= 5^{\sqrt{x}} \log 5 \ \frac{d}{dx} x^{1/2}$$

$$= 5^{\sqrt{x}} \log 5 \left(\frac{1}{2} \cdot x^{1/2 - 1}\right)$$

$$= 5^{\sqrt{x}} \log 5 \left(\frac{1}{2} x^{-1/2}\right)$$

<u>2^{nd} Method</u> :-

Let $u = \sqrt{x}$

$\therefore y = 5^{u}$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{d}{du} 5^{u} \cdot \frac{d}{dx} (\sqrt{x})$$

$$= 5^{u} \cdot \log \cancel{5} \cdot \frac{d}{dx} x^{1/2}$$

$$= 5^{\sqrt{x}} \cdot \log 5 \cdot \frac{1}{2} x^{1/2 - 1}$$

$$= 5^{\sqrt{x}} \cdot \log 5 \cdot \frac{1}{2} x^{-1/2}$$

7) $y = \sqrt{\log x}$ 8) $y = \sqrt{x^2 + 5x + 6}$

9) $y = 5^{x^5}$ 10) $y = 2^{5x+2}$

11) $y = (x^2 + 5)^{10}$

* Integration:-

If the derivative of $F(x)$ w.r.t. x is $f(x)$ then $F(x)$ is called the integral of $f(x)$ w.r.t. x and is defined as

$$F(x) = \int f(x)\,dx$$

We know that $\frac{d}{dx}\{F(x)+c\}$ will also be equal to $f(x)$ where c is constant.

Hence, in general

$$\int f(x)\,dx = F(x) + c$$

where c is called the constant of integration.

* Standard Integrals:

1) $\int 1\,dx = \int dx = x + c \quad \left\{ \because \frac{d}{dx}(x+c) = 1 \right.$

2) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, \; (n \neq -1)$

$\left\{ \because \frac{d}{dx}\left(\frac{x^{n+1}}{n+1} + c\right) = x^n \right.$ for $n \neq -1$.

3) $\int \frac{1}{x}\,dx = \log x + c \quad \left\{ \because \frac{d}{dx}(\log x + c) = \frac{1}{x} \right.$

4) $\int e^x\,dx = e^x + c$

5) $\int e^{mx}\, dx = \frac{e^{mx}}{m} + c$

6) $\int a^x\, dx = \frac{a^x}{\log a} + c$

7) $\int (ax \pm b)^n\, dx = \frac{(ax \pm b)^{n+1}}{n+1} \cdot \frac{1}{a} \quad (n \neq -1)$

8) $\int \frac{1}{(ax \pm b)}\, dx = \frac{\log (ax \pm b)}{a}$

* <u>Properties</u> :-

1) $\int c\, f(x)\, dx = c \int f(x)\, dx$

2) $\int [f_1(x) \pm f_2(x) \pm \dots \pm f_n(x)]\, dx$

$= \int f_1(x)\, dx \pm \int f_2(x)\, dx \pm \int f_3(x)\, dx \dots \pm \int f_n(x)\, dx$