A(x1,y1)
B(x2,y2)
C(x3,y3)

Area of triangle = $(1/2)[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3(y_1 - y_2)]$

or

Area of triangle $=(1/2) (x_1y_2 + x_2y_3 + x_3y_1 - x_1y_3 - x_2y_1 - x_3y_2)$

x1,y1= 26,12
x2,y2 = 23,30
x3,y3=50,25

$$area = \left| \frac{26(30-25) + 23(25-12) + 50(12-30)}{2} \right| = 235.50$$
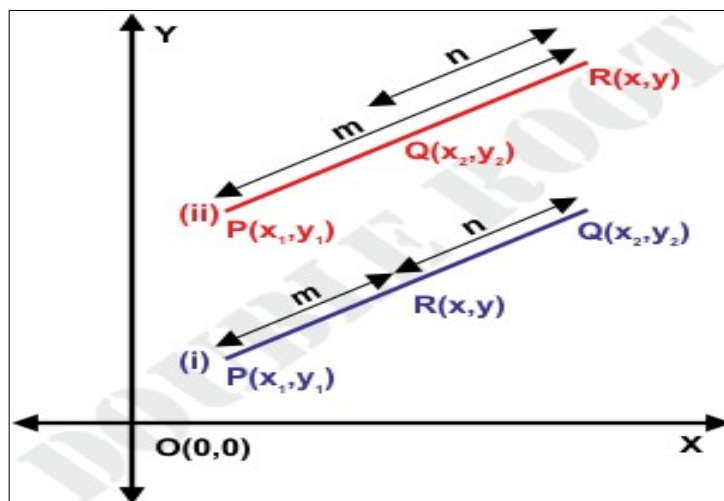
B (23,30)

C (50,25)

A (26,12)
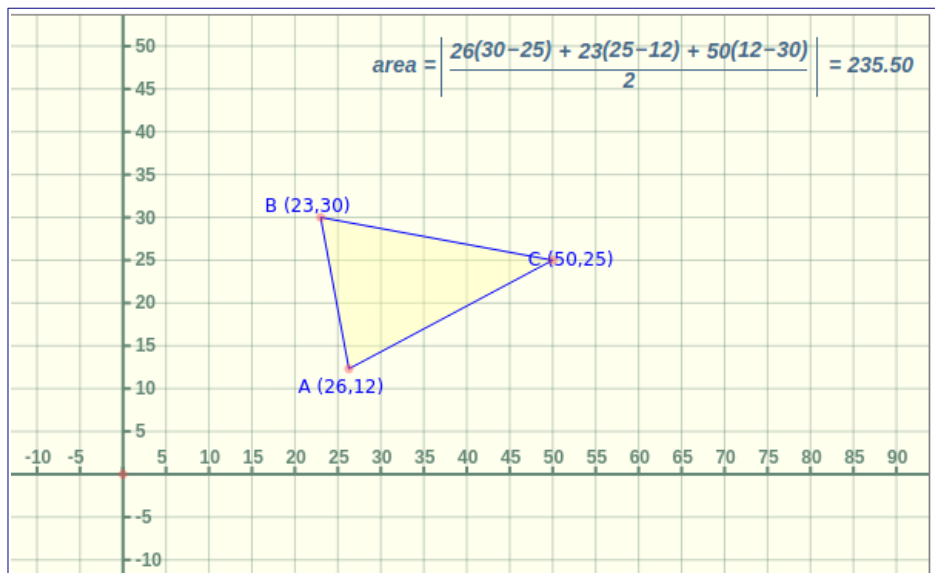
A(-3,4)
B(1,-2)
C(5,5)

Answer : 26

Section Formula

point R is divided PQ line
PR + RQ = PQ
P(x1,y1)
Q(x2,y2)
R(x3,y3) is divided PQ line in the ratio 2:3 = m:n , find co ordinates of R(x3,y3)

$x3 = (mx2+nx1) / m+n$
$y3=(my2+ny1) /m+n$

1. The coordinates of the point dividing the line segment joining (x_1, y_1) and (x_2, y_2) in the ratio m : n **internally** is given by $\left(\dfrac{mx_2+nx_1}{m+n} , \dfrac{my_2+ny_1}{m+n} \right)$

2. The coordinates of the point dividing the line segment joining (x_1, y_1) and (x_2, y_2) in the ratio m : n **externally** is given by $\left(\dfrac{mx_2-nx_1}{m-n} , \dfrac{my_2+ny_1}{m+n} \right)$

$(my2 – ny1)/m-n$

Find the coordinates of point P which divides the line joining A=(4,−5) and B=(6,3) in the ratio 2:5.

A(x1,y1) = (4,-5)
B(x2,y2) = (6,3)
m:n=2:5
P(x3,y3) =?

x3 = (mx2+nx1) / m+n
   = (2 * 6 + 5 * 4) / 2+5
=(12+20)/7
=32/7

y3=(my2+ny1) /m+n
 = (2 * 3 + 5 * -5) / 7
= (6 + -25) /7
= -19/7
p(x3,y3) = (32/7,-19/7)

Let the coordinates of P be (x, y). Then

$$P(x, y) = \left(\frac{2 \times 6 + 5 \times 4}{2 + 5}, \frac{2 \times 3 + 5 \times -5}{2 + 5} \right)$$
$$= \left(\frac{12 + 20}{7}, \frac{6 - 25}{7} \right)$$
$$= \left(\frac{32}{7}, -\frac{19}{7} \right) . \square$$

: Let P(x, y) be the point which divides the line segment joining A(4, 6) and B(-5, -4) internally in the ratio 3 : 2.

A(x1,y1) = (4,6)
B(x2,y2) = (-5,-4)
m:n = 3:2
p(x,y)=
x= mx2+nx1 / m+n
y= my2+ny1/ m+n

Solution: Let P(x, y) be the point which divides the line segment joining A(4, 6) and B(-5, -4) internally in the ratio 3 : 2.

Here,

(x_1, y_1) = (4, 6)

(x_2, y_2) = (-5, -4)

m : n = 3 : 2

Using the section formula,

$$P(x, y) = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right)$$

Coordinates of P are,

$$x = \frac{3 \times -5 + 2 \times 4}{3 + 2} \qquad y = \frac{3 \times -4 + 2 \times 6}{3 + 2}$$

$$\Rightarrow x = \frac{-15 + 8}{5} \qquad \Rightarrow y = \frac{-12 + 12}{5}$$

$$\Rightarrow x = -\frac{7}{5} \qquad \Rightarrow y = 0$$

A (6,3)
B(-4,5)
m:n = 3:2
to find point p(x,y) divides lineAB segment in externally manner
p(x,y)

x1=6, y1=3
x2=-4, y2 =5
m=3, n=2

x= (mx2-nx1) /m-n
      = 3x(-4) – 2x6 / 3-2
      =-12-12/1
=-24

y= (my2-ny1) /m-n

p(x,y) = (-24,9)

A(x1,y1) = (4,4)
B(x2,y2) = (7,7)
P(-1,-1)=(x,y)

m:n=?

p(x,y)=
x= mx2+nx1 / m+n
y= my2+ny1/ m+n

x= (mx2+nx1) /m+n

x1,y1 = 4,4
x2,y2 = 7,7
p(x,y) = - 1, -1

- 1 = (m7 + n4) / m+n

-m – n = 7m + 4n
-n -4n = 7m + m
-5n = 8m
-5=8m/n
 -5/8=m/n

- sign represent point p is diveded line AB in externally ratio.

y = my2+ny1 / m+n

6 = 9m + 5n/m+n
6m +6n = 9m + 5n
6n-5n = 9m-6m
1n=3m
m/n=1/3
m:n=1:3
+ sign represent the pont p divides the line AB in internally with ratio 1:3

for A(-2,3) and B(3,0). Find the ratio in which y-axis divides line AB.

p(x,y) = (0,y)
x1=-2,y1=3
x2=3,y2=0
x= (mx2+nx1) /m+n
0= 3m+ (-2) n /m+n

3m-2n=0
3m=2n
m/n=2/3

Line is divided by y axis. It means p point is lie on y axis. It means cordinates of x value is zero.

P(0,y)
A(-2,3) = (x1,y1)
B(3,0) = (x2,y2)

0 = (mx2 + nx1)/ m+n

= 3m + -2n/m+n
0= 3m – 2n
-3m=-2n
m/n = 2/3

find the ratio in which the line joining (2,-3) and (5,6) is divided by the X-axis.
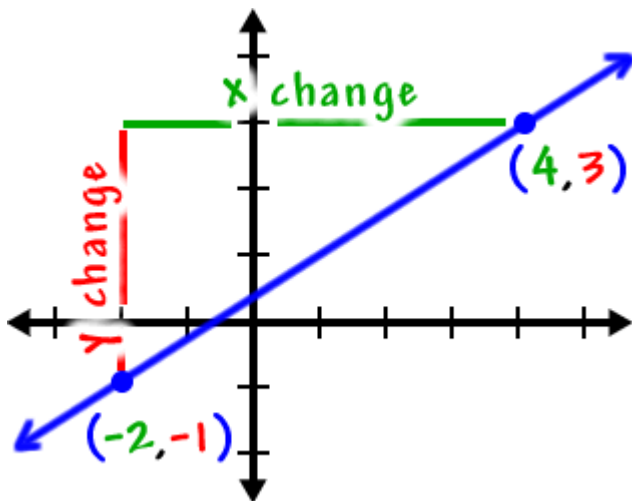
p(x,0)

$y = my_2+ny_1 / m+n$
0= 6m-3n /m+n
6m-3n=0
6m=3n
m/n=1/2

slope formula
letter m is used to represent the slope of line

If you're given two points

$$(x_1, y_1) \text{ and } (x_2, y_2)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

m represents slope

A(-2,-1) and B (4,3)
x1=-2 and y1=-1
x2=4 and y2=3

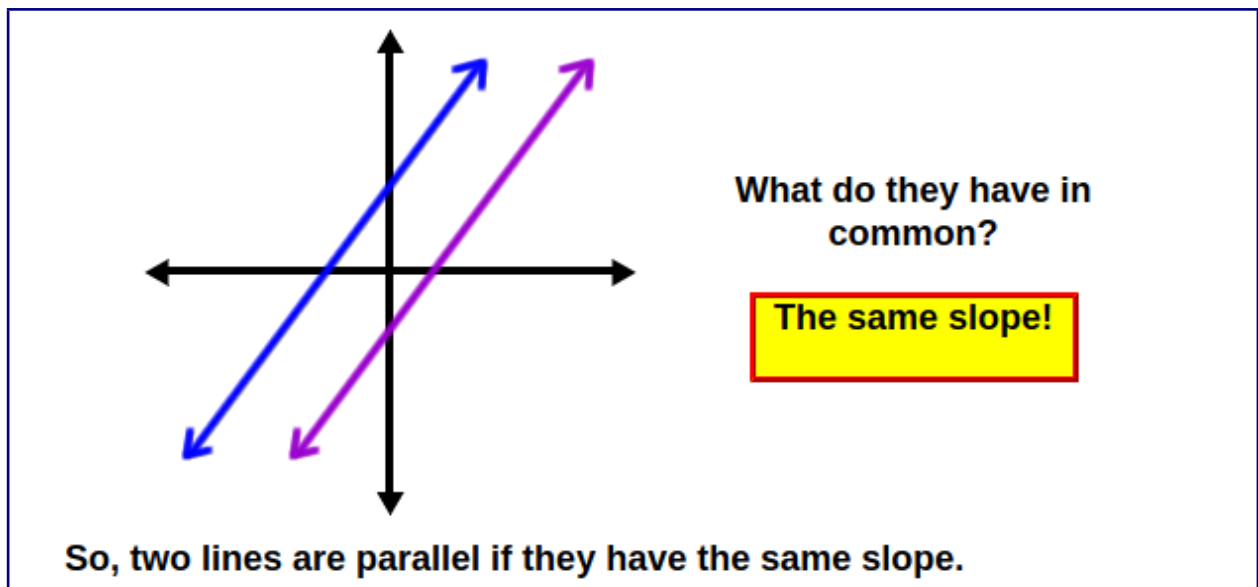$$\frac{\text{the change in the } y's}{\text{the change in the } x's}$$

$$= \frac{3-(-1)}{4-(-2)} = \frac{4}{6} = \frac{2}{3}$$

find the slope of the line that passes through the points

$$(0,-3) \text{ and } (5,4)$$

7/5

PARALLEL LINES

What do they have in common?

The same slope!

So, two lines are parallel if they have the same slope.

line l1 (blue)
line l2 (purple)
If two lines are parallel then slope of both lines are equal.

Line l1 have co -ordinates (0,- 3) and (5,4). Line l1 and l2 are parallel. So find the slope of line l2.
Slope of l1=7/5
l1 and l2 are parallel so slope of both lines are equal.
Slope of l2=7/5

line l1 has slope = 7/5

line l2 has slope = 7/5 because line l2 is parallel to line l1 and lines which are parallel to each other they have same slope.

Line l2 is parallel to line l1. To find slope of line l2.

Example:
Line l1 has co ordinates = (1,2) and (4,3). Line l1 is parallel to line l2. So find the slope of both lines.

A(x1,y1) = (1,2)
B(x2,y2) = (4,3)
 m= y2-y1/x2-x1
= 3-2 / 4-1
slope of line l1 =1 / 3
slope of line l2 = 1/3 because line l2 is parallel to line l1. So both have equal slope.

Example:
Determine x such that 3 is the slope of a line passing through points (2,5) and (x,4).
X=?
A(x1,y1) = (2,5)
B(x2,y2) =(x,4)
m=3
m= y2-y1/x2-x1
3= 4-5 / x-2
3(x-2) = 4-5
3x -6 = -1
3x= -1 +6
3x= 5
x=5/3

PERPENDICULAR LINES

These lines are perpendicular.
(They form a $90°$ angle.)

So, what's going on with the slopes?

$m_1 = 2$ and $m_2 = \frac{-1}{2}$

$y = 2x$
$y = \frac{-1}{2}x$

Notice that they are inverses but the signs are different

A typical math book would say:

$$m_1 \cdot m_2 = (2)(\tfrac{-1}{2}) = -1$$
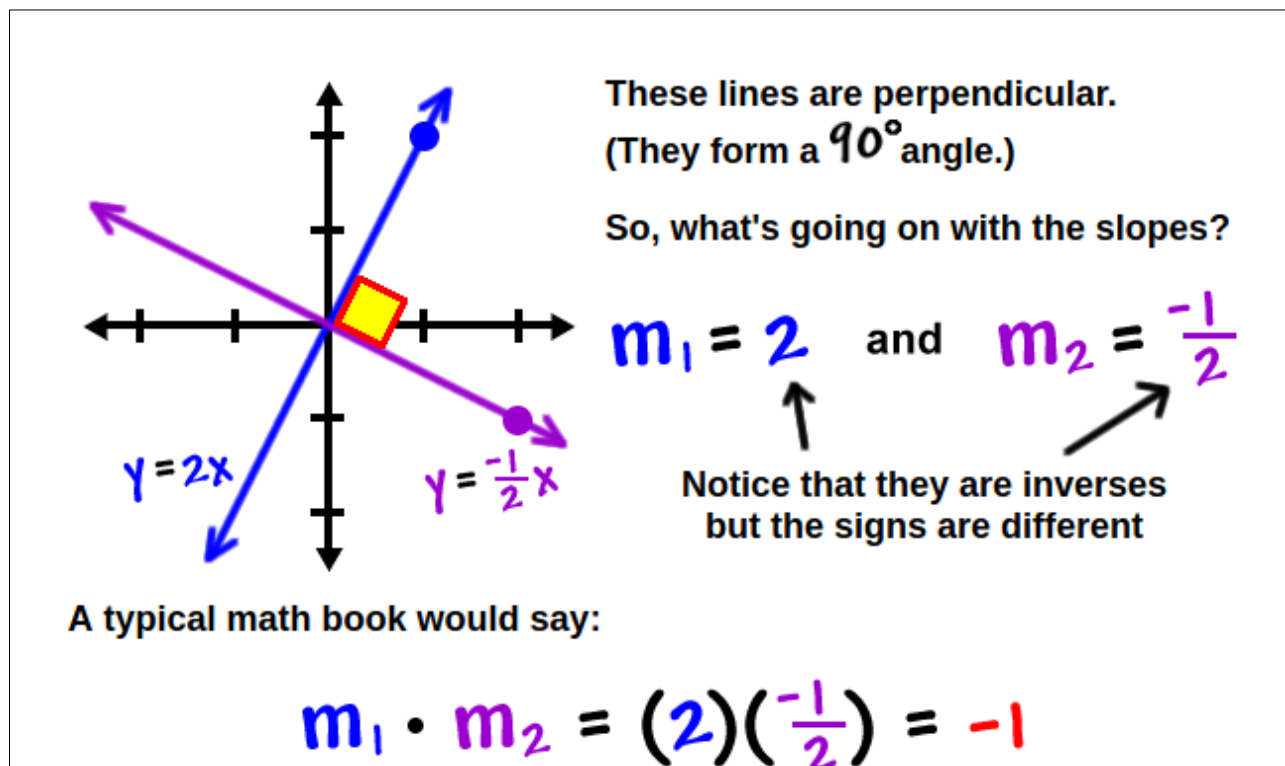
m1.m2=(-1)
line l1 has slope 4 and line l1 and l2 are perpendicular to each other. So find out slope of line l2.
m1=slope of line l1
m2=slope of line l2
l1 and l2 are perpendicular so slope is
m1.m2=-1
4 . m2= -1
m2=-1/4

Line l1 has slope m1= 2 and line l2 is perpendicular to line l1. To find slope of line l2.

M1=2
m2=?

m1.m2=-1
2.m2=-1

<mark>m2= -1/2</mark>

line can be represented in various equations.

General form of equation of line
Ax+By+C=0
Slope = -$\frac{A}{B}$
x-intercept = -$\frac{C}{A}$
y-intercept = -$\frac{C}{B}$
Ex: find intercepts and slope
  i)
        2x+3y-4=0 ---- Ax + By + C =0
        A=2, B=3, C= -4

        Slope = -$\frac{2}{3}$
        X-intercept= - C/A = - (-4)/2 =4/2= 2
        y-intercept= - C/B = - (-4) / 3 = 4/3

  ii) x=y+2
        x-y-2=0
        Slope = 1
        X-intercept= -c/A = -(-2)/1 = 2
        y-intercept= -C/B= - (-2)/-1 = 2/-1= -2
  iii) 5x+2y+3=0
slope = -5/2
x-intercept = -3/5

y- intercept = -3/2

iv) $y - 11 = 0$

slope = 0/1 =0
y-intercept= - 11/1=- 11

General form of equation of line
Ax+By+C=0
Slope = -$\frac{A}{B}$
x-intercept = -$\frac{C}{A}$
y-intercept = -$\frac{C}{B}$

Line equation
Type I : Slope intercept form
y= mx+c
where m=slope, c= y-interept

Find the equation of line where slope is
3 and y-intercept is 5.
m=3
c=5

y=mx+c
y=3x+5------
Ax+By+C=0
3x-y+5=0 (general form)

Type II: Point slope form

<mark>y-y1=m(x-x1)</mark>

m= slope
(x1,y1)= the point through which given line is passed

example:
Find the equation of line passing through the point (2,3) and slope is 5.
x1,y1 = 2,3
slope =5

y-3=5(x-2)
y-3 = 5x-10
5x-y-10+3=0
5x-y-7=0 ans is in genral form

Type III: Two Point form

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

find the equation of line passing through the points (2,3) and (5,-2)
x1,y1 = (2,3)
x2,y2 = (5,-2)

$$\frac{y-3}{-2-3} = \frac{x-2}{5-2}$$

$$\frac{y-3}{-5} = \frac{x-2}{3}$$

3(y-3) = -5 (x- 2)
3y-9 = -5x +10
5x+3y-9-10=0
5x + 3y -19=0

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

point slope form

x-y=0

-A/B = - 1/-1=1

**slope of x-y+2=0 line is 1.
Now other line is parallel to x-y+2=0 , so slpe are equal for both lines**

**m=1
x1,y1 = 1,1
y-y1=m(x-x1)
y-1 = 1(x-1)**

y-1 = x-1
y-1-x+1=0
-x+y=0

Point slope form
$y-y_1=m(x-x_1)$

m=1
$x_1,y_1= (1,1)$
x-y+2=0 --- Ax+By + C=0

slope = -A/ B = -1/- 1= 1

y- 1 = 1 (x- 1)
y-1 = x -1
x-y-1+1=0
x-y=0

Find the equation of a line passing through (1,2) and perpendicular to other line 2x-y+1=0

Point slope form
y-y1=m(x-x1)
x1,y1= (1,2)
m=?
If two lines are perpendicular to each other then product of slope is -1

2x-y+1=0 --- Ax+By + C=0
slope = -A/ B
m2= - 2/-1 = 2

m1.m2= -1 // lines are perpendicular
m1. 2 = -1
m1=-1/2

y-2 = $\frac{-1}{2}$ (x-1)

2(y-2) = -1 (x- 1)
2y-4 = -x +1
x+2y-4-1=0
x+2y-5=0

Find the equation of line passing through point (1,2) and (1,-1).

3x-3=0 or -3x+3=0
x1,y1 = 1,2
x2,y2= 1,-1

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

y-2 / -1-2 = x-1 / 1-1

y-2 / -3 = x- 1 /0

-3(x- 1) =0
-3x + 3=0
3x-3=0

Find the equation of the line passing through (-3,5) and perpendicular to the line through the points (2,5) and (-3,6).

point slope form

y-y1 = m (x-x1)

x1,y1 = (-3,5)
m1.m2=-1

m2 = y2-y1 / x2-x1
= -(1/5)

m1.m2 =-1
m1 * -1/5 = -1
m1= -1* -5=5

y-y1=m(x-x1)
y-5 = 5 (x- (-3))
y-5=5(x+3)
y-5 = 5x +15
5x-y +15 +5 =0
5x-y+20=0

Example 1

Find the acute angle between $y = 2x + 1$ and $y = -3x - 2$
(to nearest degree)

$$\therefore m_1 = 2 \text{ and } m_2 = -3$$

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\therefore \tan \theta = \left| \frac{2 + 3}{1 - 6} \right|$$

$$\therefore \tan \theta = |-1|$$

$$\therefore \tan \theta = 1 \qquad \rightarrow \theta = 45°$$

y=2x +1 ---- y=mx +C m1=2
y=-3x-2----- y =mx+C m2=-3
tan θ = $\frac{m1 - m2}{1 + m1.m2}$

$$= \frac{2 - (-3)}{1 + 2.(-3)}$$

$$= \frac{2 + 3}{1 + (-6)}$$

= 5/-5
= -1
=|-1|
tan θ= 1
θ=tan⁻¹(1)
θ=45.

$$= 2 - (-3) / 1 + 2 \cdot (-3)$$

$$= 2 + 3 / 1\text{-}6$$
$$= 5 / \text{-}5$$
$$= |\text{-}1|$$

theta=tan^{-1} 1

l1 : 3x-2y+7 =0 Ax+By+C=0
slope = -A/B = - 3/-2= 3/2
m1: 3/2

l2: 4x+2y-3=0
m2: -2

Example 2

Find the acute angle between $3x - 2y + 7 = 0$ and
(to nearest degree)

$$2y + 4x - 3 = 0$$

applying the formula $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$ $m_1 = \dfrac{3}{2}$ $m_2 = -2$

$$\therefore \tan\theta = \left|\dfrac{\dfrac{3}{2} + 2}{1 - 3}\right|$$

$$\therefore \tan\theta = \left|\dfrac{-7}{4}\right| \quad \therefore \tan\theta = \dfrac{7}{4} \quad \rightarrow \theta = 60°$$

tan θ = $\frac{3}{2}$ + 2 /-2

= $\frac{\frac{3+4}{2}}{-2}$

= $\frac{7}{2.-2}$

| $\frac{7}{-4}$ | = 7/4

tan θ = 7/4
θ=tan^{-1} (7/4)

tan θ = $\frac{m1-m2}{1+m1.m2}$

Find the measure of the angle between the lines 2x-y+1 = 0 and x + y + 7 =0. tan θ=3

Midpoint Formula

When we need to find the coordinates of a point that lies exactly at the center of any given segment we use the midpoint formula.

The midpoint formula is,

$$P = \left(\frac{x1+x2}{2}, \frac{y1+y2}{2} \text{¿} \right)$$

Find the midpoint of segment AB where A(2,3) and B(6,7).

A(2,3) = (x1,y1)

B(6,7) = (x2,y2)

P = ($\frac{x1+x2}{2}$, $\frac{y1+y2}{2}$)

p= (4 ,5)

Ans: 2x+y=0

y-y1=m (x-x1)

Find the measure of the angle between the lines 2x-y+1 = 0 and x + y + 7 =0.

Ans. tan θ=3

Find the equation of line passing through the origin and parallel to the line 3x-2y+1 =0.

Ans. 3x-2y=0

Find the equation of line passing through the intersection of the lines x+y=0 and   x-y=0. And perpendicular to the line joining (1,2) and (-1,1).

point slope form

(x1,y1) = line is passing through the intersection of two lines

m=perpendicular to the line joining (1,2) and (-1,1)

y-y1= m(x-x1)

m1= y2-y1/x2-x1

= 1-2 / -1 – 1

= - 1 / -2

=1/2

m1* m2 =-1

1/2 * m2 = -1

m2= (-1 * 2) / 1= -2

slope =m2=-2

(x1,y1)

intersection of the lines x+y=0 and x-y=0. Find the equation of line through the intersection of the lines x+y=0 and   x-y=0. And perpendicular to the line joining (1,2) and (-1,1).

point slope form

(x1,y1) = line is passing through the intersection of two lines

m=perpendicular to the line joining (1,2) and (-1,1)

y-y1= m(x-x1)

m1= y2-y1/x2-x1

= 1-2 / -1 – 1

= - 1 / -2

=1/2

m1* m2 =-1

1/2 * m2 = -1

m2= (-1 * 2) / 1= -2

slope =m2=-

Elimination method

x+y=0---1

x-y=0----2

2x=0

x=0

put the value of x into equation 1

x+y=0

0+y=0

y=0

x1,y1=(0,0)

y-y1 = m (x-x1)

y-0 = -2 (x- 0)

y=-2x-0

2x+ y = 0

General form

Ax+By+C=0

Slope intercept form

y=mx+C

x+y+1=0 ___general form

now u have to convert it in slope intercept form.