Definition of Mutually Exclusive Events:

If two events are such that they cannot occur simultaneously for any random experiment are said to be mutually exclusive events.

If X and Y are two mutually exclusive events, then $X \cap Y = \varnothing$

If **X** and **Y** are two mutually exclusive events, then the probability of 'X union Y' is the sum of the probability of X and the probability of Y and represented as,

$$P(X \cup Y) = P(X) + P(Y)$$

Proof: Let E be a random experiment and N(X) be the number of frequency of the event X in E. Since **X** and **Y** are two mutually exclusive events then;

$N(X \cup Y) = N(X) + N(Y)$

or, $N(X \cup Y)/N = N(X)/N + N(Y)/N$; Dividing both the sides by N.

Now taking limit N g ∞, we get probability of

P(X U Y) = P(X) + P(Y)

Worked-out problems on probability of Mutually Exclusive Events:

1. One card is drawn from a well-shuffled deck of 52 cards. What is the probability of getting a king or an ace?

Solution:

Let X be the event of 'getting a king' and,

Y be the event of 'getting an ace'

We know that, in a **well-shuffled deck of 52 cards there are 4 kings and 4 aces.**

Therefore, probability of getting a king from **well-shuffled deck of 52 cards = P(X) = 4/52 = 1/13**

Similarly, probability of getting an ace from **well-shuffled deck of 52 cards = P(Y) = 4/52 = 1/13**

According to the definition of mutually exclusive we know that, drawing of a **well-shuffled deck of 52 cards '**getting a king' and 'getting an ace' are known as mutually exclusive events.

We have to find out P(King or ace).

So according to the addition theorem for mutually exclusive events, we get;

$$P(X \cup Y) = P(X) + P(Y)$$

$$= 1/13 + 1/13$$

Therefore, $P(X \cup Y) = (1 + 1)/13$

$$= 2/13$$

Hence, probability of getting **a king or an ace** from a **well-shuffled deck of 52 cards** = 2/13

2. A bag contains 8 black pens and 2 red pens and if a pen is drawn at random. What is the probability that it is black pen or red pen?

Solution:

Let X be the event of 'getting a **black pen'** and,

Y be the event of 'getting a **red pen'.**

We know that, **there are 8 black pens and 2 red pens.**

Therefore, probability of getting a **black pen = P(X) = 8/10 = 4/5**

Similarly, probability of getting a **red pen** = P(Y) = 2/10 = 1/5

According to the definition of mutually exclusive we know that, the event of 'getting a **black pen'** and 'getting a **red pen'** from a bag are known as mutually exclusive event.

We have to find out P(getting **a black pen** or getting **a red pen**).

So according to the addition theorem for mutually exclusive events, we get;

P(X U Y) = P(X) + P(Y)

= 4/5 + 1/5

Therefore, P(X U Y) = 5/5

= 1

Hence, probability of getting '**a black pen'** or 'a **red pen'** = 1

Addition Theorem Based on Mutually Non-Exclusive Events:

If X and Y are two mutually Non- Exclusive Events, then the probability of 'X union Y' is the difference between the sum of the probability of X and the probability of Y and the probability of 'X intersection Y' and represented as,

$$\mathbf{P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)}$$

1. What is the probability of getting a diamond or a queen from a well-shuffled deck of 52 cards?

Solution:

Let X be the event of 'getting a diamond' and,

Y be the event of 'getting a queen'

We know that, in a well-shuffled deck of 52 cards there are 13 diamonds and 4 queens.

Therefore, probability of getting a diamond from well-shuffled deck of 52 cards = P(X) = 13/52 = 1/4

The probability of getting a queen from well-shuffled deck of 52 cards = P(Y) = 4/52 = 1/13

Similarly, the probability of getting a diamond queen from well-shuffled deck of 52 cards = P(X ∩ Y) = 1/52

According to the definition of mutually non-exclusive we know that, drawing of a well-shuffled deck of 52 cards 'getting a diamond' and 'getting a queen' are known as mutually non-exclusive events.

We have to find out Probability of X union Y.

So according to the addition theorem for mutually non- exclusive events, we get;

P(X ∪ Y) = P(X) + P(Y) - P(X ∩ Y)

Therefore, P(X U Y)
= 1/4 + 1/13 - 1/52

= (13 + 4 - 1)/52

= 16/52

= 4/13

Hence, probability of getting a diamond or a queen from a well-shuffled deck of 52 cards = 4/13

2. A lottery box contains 50 lottery tickets numbered 1 to 50. If a lottery ticket is drawn at random, what is the probability that the number drawn is a multiple of 3 or 5?

Solution:

Let X be the event of 'getting a multiple of 3' and,

Y be the event of 'getting a multiple of 5'

The events of getting a multiple of 3 (X) = {3,6,9,12,**15**,18,21,24,27,**30**, 33,36,39,42,**45**,48}

Total number of multiple of 3 = 16

P(X) = 16/50 = 8/25

The events of getting a multiple of 5 (Y) = {5, 10, **15**, 20, 25, **30**, 35, 40, **45**, 50}

Total number of multiple of 3 = 16

P(X) = 10/50 = 1/5

Between the events X and Y the favorable outcomes are 15, 30 and 45.

Total number of common multiple of both the number 3 and 5 = 3

The probability of getting a 'multiple of 3' and a 'multiple of 5' from the numbered 1 to 50 = P(X ∩ Y) = 3/50

Therefore, X and Y are non mutually exclusive events.

We have to find out Probability of X union Y.

So according to the addition theorem for mutually non- exclusive events, we get;

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

$= 8/25 + 1/5 - 3/50$

Therefore, $P(X \cup Y)$ $= (16 + 10 - 3)/50$

$= 23/50$

Hence, probability of getting multiple of 3 or 5 = 23/50