Set Theory

What is Set?

Set

- sets inside **curly brackets** like this:

  {hat, shirt, jacket, pants, ...}

- Set of whole numbers: {0, 1, 2, 3, ...}
- Set of prime numbers: {2, 3, 5, 7, 11, 13, 17, ...}

Set is just things grouped together with a certain property in common.

Set:- The collection of well-defined distinct objects is known as a **set**

Notation

A={a,b,c,d}

- We simply list each element (or "member") separated by a comma, and then put some curly brackets around the whole thing:
- a,b,c,d are elements of sets
- A is name of set

- when we say an element **a** is in a set **A**, we use the symbol $\in$ to show

it. And if something is not in a set use $\notin$

- a ϵ A, bur e $\notin$ A

Set Representation

Roster Notation

we enumerate or list all the element.

Examples :

1) A is a set of whole numbers less than 6.

A = { 0,1,2,3,4,5}

2) C is the set of letters in the word excellent .

C = { e, x, c, l, n, t }

Set Representation

Set-builder form (Rule method)

In this method , we specify the rule or property or statement.
A = { x | x has a property of p}
This is read as A is the set of elements x such that(|) x has a property p.

Examples :

1) Given : A = { 2,4,6,8,10,12}
Solution :
In set A all the elements are even natural number up to 12.So this is the rule for the set A
So set builder notation will be
A = { x | x is an even natural number, x ≤ 12}
or
A= { x | x ∈N, x is even number and x<=12}.

2) B = { 4,5,6,7}

Solution :

In set B all the elements are natural numbers between 3 and 8.This is the rule.

So set builder notation will be

B ={ x | x is a natural number, 3 < x < 8} Or

B = { x | x ∈N, 3 < x < 8}.

Natural Numbers : **N**

The whole numbers from 1 upwards. (Or from 0 upwards in some fields of mathematics).

The set is {1,2,3,...} or {0,1,2,3,...}

Integers : **Z**

The whole numbers, {1,2,3,...} negative whole numbers {..., -3,-2,-1} and zero {0}. So the set is {..., -3, -2, -1, 0, 1, 2, 3, ...}

Rational Numbers : **Q**

The numbers you can make by dividing one integer by another (but not dividing by zero). In other words fractions

Examples: 3/2 (=1.5),
-1/1000 (=-0.001)

Write following sets in set bulder form

A= { -6, -5, -4, -3, -2, ... }

B= {1,2,3,4,5,6,------}

C= {-1, 0,1, 2, 3, 4}

D= {a,e,i,o,u}

$A = \{ x \,/\, x \geq -6, x \text{ is an integer} \}$

$B = \{x \,/\, x \text{ is an natural}, x >= 1\}$

$C = \{x \,/\, x \text{ is an integer and} -1 \leq x < 5\}$

$D = \{x/x \text{ is an vowel alphbet}\}$

Set Equality

Two sets are equal if they have precisely the same members

$A = \{1, 2, 3\}$ $B = \{3, 1, 2\}$, $A=B$

$\{a, c, t\} = \{c, a, t\} = \{t, a, c\}$, but $\{a, c, t\} \neq \{a, c, t, o, r\}$

$\{b,o,o,k\}=\{b,o,k\}$

$A = \{f,o,l,l,o,w\}$

$B = \{w,o,l,f\}$

$A = B$?

Empty Set

The *empty set* is a set that has no members. It can also called void set or null set.

Notation: The symbol $\varnothing$ is used to represent the empty set, { }.

$\varnothing = \{ \}$

$\{\varnothing\}$ does ***not*** symbolize the empty set; it represents a set that contains
an empty set as an element

Singleton Set

These are those sets that have only a single element.

Ex: A= {A number which is prime and even both}

Equivalent Sets

Equivalent sets are those which have an equal number of elements irrespective of what the elements are.

A = {1, 2, 3, 4, 5}
B = {set of vowel letter}

A and B sets are equivalent sets because both these sets have 5 elements each.

Finite Sets

Any set which is empty or contains a definite and countable number of elements is called a finite set.

$A = \{a, e, i, o, u\}$ is a finite set

Infinite Sets

Any set which contains a indefinite and uncountable elements is called a Infinite set.

A = {prime numbers}

Subset

If A and B are two sets, and every element of set A is also an element of set B, then A is called a subset of B and we write it as **A ⊆ B** or **B ⊇ A**

Every set is a subset of itself, i.e.,

A = {2, 4, 6}

B = {6, 4, 8, 2}, **A ⊆ B**

A is a subset of B, all the elements of set A are contained in set B. But B is not the subset of A . Since, all the elements of set B are not contained in set A.

Set Operations

- Set Union
- Set Intersection
- Set Difference
- Complement of Set
- Cartesian Product.

Union of sets

A and B (denoted by $A \cup B$) is the set of elements that are in A, in B, or in both A and B.

Hence, $A \cup B = \{ x \mid x \in A \text{ OR } x \in B \}$.

Example − If $A = \{ 10, 11, 12, 13 \}$ and $B = \{ 13, 14, 15 \}$, then

$A \cup B = \{ 10, 11, 12, 13, 14, 15 \}$.

(The common element occurs only once)

Properties of union of sets

$A \cup B = B \cup A$

$A \cup \varnothing = A$

$A \cup A = A$

$U \cup A = U$

Set Intersection

The intersection of sets A and B (denoted by $A \cap B$) is the set of elements which are in both A and B.

Hence, $A \cap B = \{ x \mid x \in A \text{ AND } x \in B \}$.

Example − If $A = \{ 11, 12, 13 \}$ and $B = \{ 13, 14, 15 \}$,

then $A \cap B = \{ 13 \}$.

Properties of Intersection of sets

$A \cap B = B \cap A$.

$\varphi \cap A = \varphi$

$U \cap A = A$

$A \cap A = A$

Universal Set

A Universal Set is the set of all elements under consideration, denoted by capital U .
All other sets are subsets of the universal set.
Ex: U={Set of natural numbers}

Complement of a Set

The complement of a set A (denoted by A’) is the set of elements which are not in set A.

The complement of a set can be represented with several different notations.

The complement of set A can be written as:

A^c or A' or $\overline{A}$ or $\sim A$

Hence, A' = { x | x $\notin$ A }.

Example of Complement of a Set

Example: Let U = {1, 2, 3, 4, 5, 6} and A = {1, 3, 5}.

Complement of a Set A' = {2, 4, 6}.

U= {a, b, c, ..., x, y, z},

P = {a, b, c, d, e} and

Q = {x, y, z}, find P’ AND Q'.

Set Difference

The relative complement or set difference of sets A and B, denoted A – B, is the set of all elements in A that are not in B.

In set-builder notation, $A - B = \{ x \in A \text{ and } x \notin B\} = A \cap B'$.

Example: Let A = {a, b, c, d} and B = {b, d, e}. Then A – B = {a, c} and B – A = {e}.

Let G = {t, a, n} and H = {n, a, t}. Then G – H = ∅.

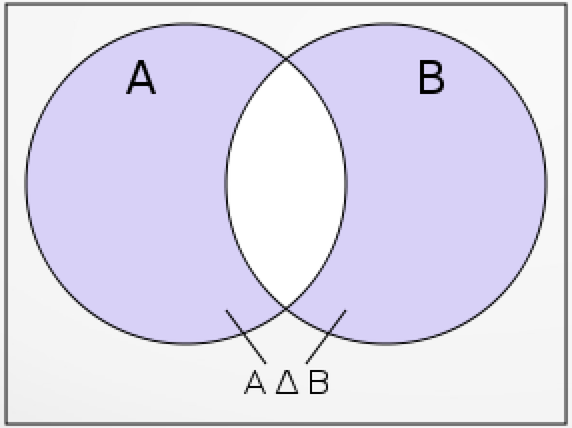
Symmetric Difference of sets

The symmetric difference of set A with respect to set B is the set of elements which are in either of the sets A and B, but not in their intersection. This is denoted as A△B

$A \triangle B = (A \cup B) - (A \cap B)$

or

$A \triangle B = (A - B) \cup (B - A)$



Properties

$$A \triangle \emptyset = A$$

$$A \triangle A = \emptyset$$

$$A \triangle B = B \triangle A$$

If $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{1, 3, 5, 6, 7, 8, 9\}$,

then $A – B = \{2, 4\}$,

$B – A = \{9\}$

$(A-B)U(B-A)=\{2, 4, 9\}$.

$A \triangle B = \{2, 4, 9\}$.

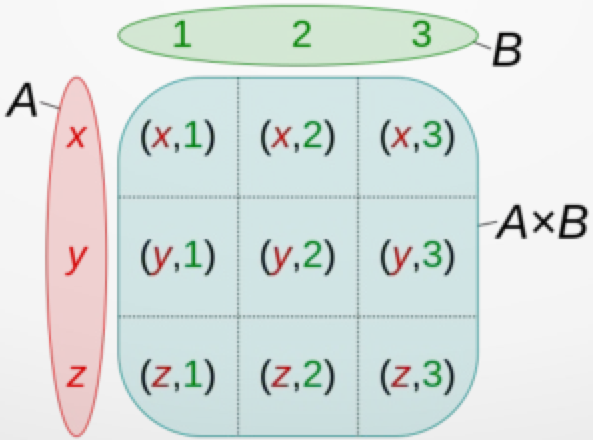
If A = {1, 2, 4, 7, 9} and
B = {2, 3, 7, 8, 9} then
A-B={1,4}
B-A={3,8}
A △ B = (A-B) U (B-A) = {1,3,4,8}

Cartesian Product

The Cartesian product of two sets A and B, denoted A × B, is the set of all possible ordered pairs where the elements of A are first and the elements of B are second. ...
A= {x,y,z} and B ={1,2,3}
A X B= {(x,1),(x,2),(x,3),

(y,1),(y,2),(y,3)

(z,1),(z,2),(z,3)}



Cartesian Product

It is interesting to know that (a1,b1) will be different from (b1,a1).

If either of the two sets is a null set,
i.e., either $A = \Phi$ or $B = \Phi$, then, $A \times B = \Phi$

If $A = \{7, 8\}$ and $B = \{2, 4, 6\}$, find $A \times B$.

If $A = \{ 1, 3, 5\}$ and $B = \{2, 3\}$, then

Find: (i) $A \times B$

(ii) $B \times A$

(iii) $A \times A$

(iv) $(B \times B)$

$A \times B = \{(7, 2); (7, 4); (7, 6); (8, 2); (8, 4); (8, 6)\}$

$A \times B = \{1, 3, 5\} \times \{2,3\} = \{\{1, 2\},\{1, 3\},\{3, 2\},\{3, 3\},\{5, 2\},\{5, 3\}\}$

$B \times A = \{2, 3\} \times \{1, 3, 5\} = [\{2, 1\},\{2, 3\},\{2, 5\},\{3, 1\},\{3, 3\},\{3, 5\}]$

$A \times A = \{1, 3, 5\} \times \{1, 3, 5\} = [\{1, 1\},\{1, 3\},\{1, 5\},\{3, 1\},\{3, 3\},\{3, 5\},\{5, 1\},\{5, 3\},\{5, 5\}]$

$B \times B = \{2, 3\} \times \{2, 3\} = [\{2, 2\},\{2, 3\},\{3, 2\},\{3, 3\}]$

Example: The shop has banana, chocolate and lemon ice cream.

What do you order?
Nothing at all: {}
Or maybe just banana: {banana}. Or just {chocolate} or just {lemon}
Or two together: {banana,chocolate} or {banana,lemon} or {chocolate,lemon}
Or all three! {banana, chocolate,lemon}

Power Set

The power set is a set which includes all the subsets including the empty set and the original set itself. Represented by P(A).

If set A = {x,y,z} is a set,
then all its subsets {x}, {y}, {z}, {x,y}, {y,z}, {x,z}, {x,y,z} and {} are the elements of powerset.

How is Power set Calculated?

If the given set has n elements, then its Power Set will contain 2^n elements. It also represents the cardinality of powerset.

Example

Let us say Set A = { a, b, c } The power set P(A) = { { } , { a }, { b }, { c }, { a, b }, { b, c }, { c, d }, { a, b, c } }

Number of elements: 3 Therefore, the subsets of the set are: $2^3 = 8$

{ } which is the null or the empty set

{ a }

{ b }

{ c }

{ a, b }

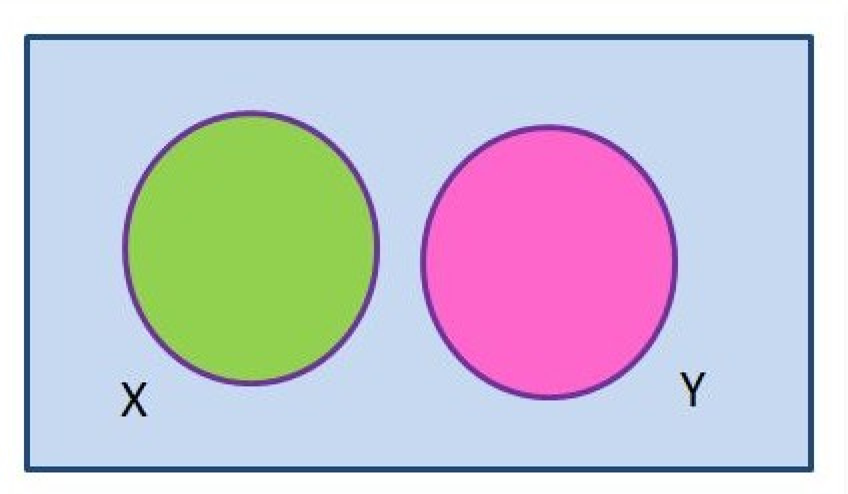
{ b, c }

{ c, a }

{ a, b, c }

Disjoint Set

When the intersection of two sets is a null or empty set,
then they are called disjoint sets.
Hence, if A and B are two disjoint sets, then;
$A \cap B = \phi$



Example: The shop has banana, chocolate and lemon ice cream.
A={chocolate, banana, lemon}
What do you order?
Nothing at all: {}
Or maybe just banana: {banana}. Or just {chocolate} or just {lemon}
Or two together: {banana,chocolate} or {banana,lemon} or {chocolate,lemon}
Or all three! {banana, chocolate,lemon}

Venn Diagram

The english mathematician John Venn began usng diagrams to represent set. That diagrams are called venn diagram