Limits *(An Introduction)*

Approaching ...

Sometimes we can't work something out directly ... but we **can** see what it should be as we get closer and closer!

Example:

$$\frac{(x^2 - 1)}{(x - 1)}$$

Let's work it out for x=1:

$$\frac{(1^2 - 1)}{(1 - 1)} = \frac{(1 - 1)}{(1 - 1)} = \frac{0}{0}$$

Now 0/0 is a difficulty! We don't really know the value of 0/0 (it is "indeterminate"), so we need another way of answering this.

So instead of trying to work it out for x=1 let's try **approaching** it closer and closer:

Example Continued:

x		$\frac{(x^2 - 1)}{(x - 1)}$
0.5		1.50000
0.9		1.90000
0.99		1.99000
0.999		1.99900
0.9999		1.99990
0.99999		1.99999
...		...

Now we see that as x gets close to 1, then $\frac{(x^2-1)}{(x-1)}$ gets **close to 2**

We are now faced with an interesting situation:

- When x=1 we don't know the answer (it is **indeterminate**)
- But we can see that it is **going to be 2**

We want to give the answer "2" but can't, so instead mathematicians say exactly what is going on by using the special word "limit"

The **limit** of *(x^2−1)* **(x−1)** as x approaches 1 is **2**

$$\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$$

Rules of Limit

	Limit Law in symbols	Limit Law in words
1	$\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$	The limit of a sum is equal to the sum of the limits.
2	$\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$	The limit of a difference is equal to the difference of the limits.
3	$\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$	The limit of a constant times a function is equal to the constant times the limit of the function.
4	$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)]$	The limit of a product is equal to the product of the limits.
5	$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ $\left(if \lim_{x\to a} g(x) \neq 0\right)$	The limit of a quotient is equal to the quotient of the limits.
6	$\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$	where n is a positive integer
7	$\lim_{x\to a} c = c$	The limit of a constant function is equal to the constant.
8	$\lim_{x\to a} x = a$	The limit of a linear function is equal to the number x is approaching.
9	$\lim_{x\to a} x^n = a^n$	where n is a positive integer

Methods to find limt

- Direct Substitution
- Factorization
- Rationalization

Direct Substitution

Find the limit of following functions.

lim x->2   5x+3 =?

5(2)+3= 10+3 = 13

Answer is 0/0 then u can use factorization method lim x->1

$$\frac{x^2-1}{x-1}$$

= 1-1/ *1-1* =*0*/0 indeteminate form

if u get indeterminate form then use factorization method

lim x->1 (x-1)(x+1)/ (x-1)

lim x->1 (x+1)

= 1+1 = 2

$$\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$$

$$\frac{\lim_{x \to 2} x^2-5x+6}{x^2-4}$$

= lim x->2 $(2)^2$ -5 (2) + 6 / $(2)^2$ -4

=lim x->2 4 -10 +6 / 4 – 4

= 10-10 / 0

0/0

Factorization method

x^2-5x+6 / x^2-4

x^2-5x+6

= x^2 – 3x-2x +6

= x(x-3) – 2(x-3)

= (x-3) (x-2)

x^2-4= (x-2)(x+2)

$$\frac{\lim_{x \to 2} x^2-5x+6}{x^2-4}$$

lim x->2 (x-3) (x-2) / (x-2) (x+2)

lim x->2 (x-3) / (x+2)

= 2-3 / 2+2

= -1 /4

lim x-> 1/2 $4x^2 – 1 / 2x – 1$

lim x->0 $2x^2 + 1 / x^3 + 2$

Rationalization

Example 1

Evaluate: $\lim\limits_{x\to 3} \dfrac{x-3}{\sqrt{x+22}-5}$

Step 1

Confirm that the limit has an indeterminate.

$$\lim_{x\to 3} \frac{x-3}{\sqrt{x+22}-5} = \frac{3-3}{\sqrt{3+22}-5} = \frac{0}{\sqrt{25}-5} = \frac{0}{0} \qquad \text{Indeterminate}$$

Step 2

Rationalize the denominator , then divide out the common factors.

$$\begin{aligned}
\lim_{x\to 3} \frac{x-3}{\sqrt{x+22}-5} &= \lim_{x\to 3} \frac{x-3}{\sqrt{x+22}-5} \cdot \frac{\sqrt{x+22}+5}{\sqrt{x+22}+5} && \text{Multiply by the conjugate} \\
&= \lim_{x\to 3} \frac{(x-3)(\sqrt{x+22}+5)}{(x+22)-25} \\
&= \lim_{x\to 3} \frac{(x-3)(\sqrt{x+22}+5)}{x-3} && \text{Divide out common factors} \\
&= \lim_{x\to 3} \frac{\sqrt{x+22}+5}{1} \\
&= \lim_{x\to 3} (\sqrt{x+22}+5)
\end{aligned}$$

Step 3

Evaluate the simpler limit.

$$\lim_{x\to 3} (\sqrt{x+22}+5) = \sqrt{25}+5 = 5+5 = 10$$

Answer

$$\lim_{x\to 3} \frac{x-3}{\sqrt{x+22}-5} = 10$$

X^2-4 = (x-2) (x+2)

x^2-2 = (x - $\sqrt{2}$)(x + $\sqrt{2}$)

[root(x+22) – 5] . [root(x+22) + 5]

$(root(x+22))^2$ + 5.root(x+22) – 5.root(x+22) – 25

$(root(x+22))^2$ – 25

(a+b) (a-b)

a^2-ab+ab-b^2

$(\sqrt{2})^2 = 2$

$(\sqrt{x+22})^2$ = x+22

Example 2

Evaluate: $\lim\limits_{x\to 13} \dfrac{\sqrt{x-4}-3}{x-13}$

Step 1

Confirm that the limit laws will give you an indeterminate form.

$$\lim_{x\to 13} \frac{\sqrt{x-4}-3}{x-13} = \frac{\sqrt{13-4}-3}{13-13} = \frac{\sqrt{9}-3}{0} = \frac{0}{0} \qquad \text{Indeterminate}$$

Step 2

Rationalize the numerator. Then divide out the common factors.

$$\begin{aligned}
\lim_{x\to 13} \frac{\sqrt{x-4}-3}{x-13} &= \lim_{x\to 13} \frac{\sqrt{x-4}-3}{x-13} \cdot \frac{\sqrt{x-4}+3}{\sqrt{x-4}+3} && \text{Multiply by the conjugate} \\
&= \lim_{x\to 13} \frac{(x-4)-9}{(x-13)(\sqrt{x-4}+3)} \\
&= \lim_{x\to 13} \frac{x-13}{(x-13)(\sqrt{x-4}+3)} && \text{Divide out common factors} \\
&= \lim_{x\to 13} \frac{1}{\sqrt{x-4}+3}
\end{aligned}$$

Step 3

Evaluate the simpler limit.

$$\lim_{x\to 13} \frac{1}{\sqrt{x-4}+3} = \frac{1}{\sqrt{13-4}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

Answer

$$\lim_{x\to 13} \frac{\sqrt{x-4}-3}{x-13} = \frac{1}{6}$$

Problem 1

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x+5}-3}$$

Step 1

Confirm that the limit laws give you an indeterminate form.

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x+5}-3}=\frac{4-4}{\sqrt{4+5}-3}=\frac{0}{\sqrt{9}-3}=\frac{0}{0}\qquad\text{Indeterminate!}$$

Step 2

Rationalize the denominator . Then divide out the common factors.

$$\begin{aligned}\lim_{x\to 4}\frac{x-4}{\sqrt{x+5}-3}&=\lim_{x\to 4}\frac{x-4}{\sqrt{x+5}-3}\cdot\frac{\sqrt{x+5}+3}{\sqrt{x+5}+3} &&\text{Multiply by the conjugate}\\ &=\lim_{x\to 4}\frac{(x-4)(\sqrt{x+5}+3)}{(x+5)-9}\\ &=\lim_{x\to 4}\frac{(x-4)(\sqrt{x+5}+3)}{x-4} &&\text{Divide out common factors}\\ &=\lim_{x\to 4}\frac{\sqrt{x+5}+3}{1}\\ &=\lim_{x\to 4}\left(\sqrt{x+5}+3\right)\end{aligned}$$

Step 3

Evaluate the simpler limit.

$$\lim_{x\to 4}\left(\sqrt{x+5}+3\right)=\sqrt{4+5}+3=\sqrt{9}+3=6$$

$$\lim_{x\to 5} \frac{\sqrt{x-1}-2}{x-5}$$

SHOW ANSWER

Step 1

Confirm the limit laws give you an indeterminate form.

$$\lim_{x\to 5} \frac{\sqrt{x-1}-2}{x-5} = \frac{\sqrt{5-1}-2}{5-5} = \frac{\sqrt{4}-2}{0} = \frac{0}{0} \qquad \text{Indeterminate!}$$

Step 2

Rationalize the numerator. Then divide out the common factors.

$$\begin{aligned}
\lim_{x\to 5} \frac{\sqrt{x-1}-2}{x-5} &= \lim_{x\to 5} \frac{\sqrt{x-1}-2}{x-5} \cdot \frac{\sqrt{x-1}+2}{\sqrt{x-1}+2} && \text{Multiply by the conjugate} \\
&= \lim_{x\to 5} \frac{(x-1)-4}{(x-5)(\sqrt{x-1}+2)} \\
&= \lim_{x\to 5} \frac{x-5}{(x-5)(\sqrt{x-1}+2)} && \text{Divide out the common factor} \\
&= \lim_{x\to 5} \frac{1}{\sqrt{x-1}+2}
\end{aligned}$$

Step 3

Evaluate the simpler limit.

$$\lim_{x\to 5} \frac{1}{\sqrt{x-1}+2} = \frac{1}{\sqrt{5-1}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

Answer

Find limit of following functions.

lim x->1 (2x-3) (x-1) / $2x^2$+x-3

lim x->5 x^2-9x+20 / x^2-6x+5

lim x->3 $\frac{\sqrt{x+2}-\sqrt{5}}{x-3}$

lim x->3 $\frac{x^3-27}{\sqrt{x+1}-2}$

x^3-27 = (x-3) (x2+3x+9)

lim x->3 (x-3) (x^2+3x+9) / root(x+1) – 2

lim x-> 3 $\frac{(x-3)(x^2+3x+9)}{\sqrt{x+1}-2}$ X $\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$

lim x->3 $\frac{(x-3)(x^2+3x+9).\sqrt{x+1}+2}{x+1-4}$

(3^2+3.3 + 9) . root(3+1) +4

(9 + 9 +9) X (2+2)

27 X 4

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lim x->0 $\frac{\sqrt{1+x}-1}{x}$

lim x->3 $\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}$

lim x->3 $\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}$ X $\frac{\sqrt{x-2}+\sqrt{4-x}}{\sqrt{x-2}+\sqrt{4-x}}$

lim x->3 $\frac{(x-3)X(\sqrt{x-2}+\sqrt{4-x})}{(x-2)-(4-x)}$

lim x->3 $\frac{(x-3)\sqrt{x-2}+\sqrt{4-x}}{x-2-4+x}$

lim x->3 $\frac{(x-3)\sqrt{x-2}+\sqrt{4-x}}{2x-6}$

lim x->3 $\frac{(x-3)\sqrt{x-2}+\sqrt{4-x}}{2(x-3)}$

root(3-2) + root(4-3) / 2

root (1) + root(1) /2

(1+1) / 2

2/2 = 1

Rules to remeber:

$\sqrt{5}$ + $\sqrt{5}$ = 2 $\sqrt{5}$

$\sqrt{5}$ X $\sqrt{5}$ = 5

$\sqrt{5}$ - $\sqrt{5}$ =0

$\sqrt{5}$ / $\sqrt{5}$ =1

continuity

i need to check continuity at given point x=a=2

If $f(x)$ is continuous at $x = a$ then,

$$\lim_{x \to a} f(x) = f(a) \qquad \lim_{x \to a^-} f(x) = f(a) \qquad \lim_{x \to a^+} f(x) = f(a)$$

If $f(x)$ is continuous at $x = a$ then,

$$\lim_{x \to a} f(x) = f(a) \qquad \lim_{x \to a^-} f(x) = f(a) \qquad \lim_{x \to a^+} f(x) = f(a)$$

Question 1: Let a function be defined as f(x) =

5 – 2x for x < 1
3 for x = 1
x + 2 for x > 1

Is this function continuous for all x?

Answer : Since for x < 1 and x > 1, the function f(x) is defined by straight lines (that can be drawn continuously on a graph), the function will be continuous for all x ≠ 1. Now for x = 1, let us check all the three conditions:

–> f(1) = 3 (given)

$= Lim_{x \to 1^-} f(x)$
$= Lim_{x \to 1^-} (5 - 2x)$
$= 5 - 2 \times 1$
$= 3$

–> Right-Hand Limit:
$= Lim_{x \to 1^+} f(x)$
$= Lim_{x \to 1^+} (x + 2)$
$= 1 + 2$
$= 3$

–> $Lim_{x \to 1^-} f(x) = Lim_{x \to 1^+} f(x) = 3 = f(1)$

Thus all the three conditions are satisfied and the function f(x) is found out to be continuous at x = 1. Therefore, f(x) is continuous for all x.

find the continuity of a function at x=1

f(x)= 5-2x , x<1

=3, x=1

=x+2, x>1

to find continuity at x=1

- x<1

f(x) = 5-2x = 5- 2.1 = 5-2 = 3------------1

- x=1

f(x)=3___________________ 2

- x>1

f(x)= x+2 = 1+2=3__________________3

from equation 1,2, and 3

lim x->1 f(x)= lim x->1^- f(x) = lim x->1^+ f(x) = 3

so we can say f(x) is continous at point x=1.

Find continuity of following fuction at x=0 and x=2

$$g(x) = \begin{cases} x^2 + 3x, & x \le 0 \\ x, & 0 < x \le 2 \\ 3x^2, & x > 2 \end{cases}$$

x>0,x<2, x=2----- f(x) =x

Is continuous at *x* = 0? At *x* = 2?

if i want to find continuity at x=0,

x<0 ____ x^2+3x, x=0 ____ x^2+3x, x>0 ____ x

lim x->0 = f(x)= x^2+3x = 0+0=0 ------1

lim x->0^- = f(x)= x^2 + 3x= 0+0 = 0___2

lim x->0^+= f(x) = x= 0 _____ 3

from equation 1,2, and 3

lim x->0 f(x)= lim x->0^- f(x) = lim x->0^+ f(x) = 0

so we can say f(x) is continous at point x=0.

$$g(x) = \begin{cases} x^2 + 3x, & x \le 0 \\ x, & 0 < x \le 2 \\ 3x^2, & x > 2 \end{cases}$$

X=2, x<2, x>2

We have to find out continuity at x=2.

X<2 -> f(x) = x = 2______1

x=2 -> f(x) = x = 2______2

x>2 -> f(x) = $3x^2$ = $3(2)^2$ = 3 X 4= 12_____3

from equation 1,2, and 3

lim x->2 f(x) = lim x->2^- f(x) ≠ lim x->2^+ f(x)

so we can say f(x) is dis continous at point x=2 or it is not continous.

Find continuity at x=1.

$f(x) = 1-x$, $0<= x<= 1$

$f(x) = x-1$, $x>1$

$X=1, x<1, x>1$

Find continuity at x=0 and x=1

$f(x) = -x$, $x<0$

$f(x) = x$, $0<=x<=1$

$f(x) = 2-x$, $x>1$

$0<=x$

$0<x$ or $0=x$ / $x>0$

$0<x$ means 0 is leass than x or $x>0$ means x is greater than 0.

continuity at 0

$x<0$ --- $f(x)= -x = 0$ ----1

$x>0$ --- $f(x) =x =0$----2

$x=0$--- $f(x)= x=0$-----3

$x<1$ ---$f(x)=x$

$x=1$---$f(x)=x$

$x>1$--- $f(x)= 2-x$

find the continuity of a given function at x=1.

$f(x) = 5x+2$, $0<=x<1$

$= 4x^2 + 3x$, $1<=x<2$

check continuity for $x=1$

$x=1, x<1, x>1$

$x=1$

$f(x)= 4x^2 + 3x = 7$----1

$x<1$

$f(x) = 5x+2 = 7$----2

$x>1$

$f(x)= 4x^2 + 3x =7$----3

from equation 1,2, and 3

$\lim x\to1\ f(x) = \lim x\to1^-\ f(x) = \lim x\to1^+\ f(x)$

so we can say f(x) is continous at point x=1