Using Cramer's Rule to Solve Two Equations with Two Unknowns

Here we will be learning how to use Cramer's Rule to solve a linear system with two equations and two unknowns. Cramer's Rule is one of many techniques that can be used to solve systems of linear equations. Cramer's Rule involves the use of determinants to find the solution and like any other technique it has its advantages and disadvantages. Cramer's Rule itself is very simple, but the notation used requires a little bit of explanation, so let's take a look at Cramer's Rule.

Cramer's Rule

$$x = \frac{D_x}{D} \text{ and } y = \frac{D_y}{D}$$

To help explain the notation, consider the following system of equations: $\begin{aligned} 3x - 2y &= 17 \\ 4x + 5y &= -8 \end{aligned}$

To find the values of x and y there are three different values that we need to calculate, they are D, D_x, and D_y. In all three cases the "D" stands for the determinant, now let's look at what they represent.

$D = \begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix}$ Here we use the x and y values from the problem to create a 2×2 matrix.

$D_x = \begin{vmatrix} 17 & -2 \\ -8 & 5 \end{vmatrix}$ Here we replace the x-values in the first column with the values after the equal sign and leave the values in the y column unchanged.

$D_y = \begin{vmatrix} 3 & 17 \\ 4 & -8 \end{vmatrix}$ Here we replace the y-values in the second column with the values after the equal sign and leave the values in the x column unchanged.

Once we have calculated the values of D, D_x, and D_y we can apply Cramer's Rule to find x and y.

To use Cramer's Rule to solve a system of two equations with two unknowns, we need to follow these steps:

Step 1:	Find the determinant, D, by using the x and y values from the problem.
Step 2:	Find the determinant, D_x, by replacing the x-values in the first column with the values after the equal sign leaving the y column unchanged.
Step 3:	Find the determinant, D_y, by replacing the y-values in the second column with the values after the equal sign leaving the x column unchanged.
Step 4:	Use Cramer's Rule to find the values of x and y.

Now we are ready to look at a couple of examples. To review how to calculate the determinant of a 2×2 matrix, click here.

Example 1: Use Cramer's Rule to solve: $\begin{array}{l} 3x - 2y = 17 \\ 4x + 5y = -8 \end{array}$

Step 1: Find the determinant, D, by using the x and y values from the problem.

–8

$$D = \begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix} = 15 - (-8) = 23$$

15

Step 2: Find the determinant, D_x, by replacing the x-values in the first column with the values after the equal sign leaving the y column unchanged.

16

$$D_x = \begin{vmatrix} 17 & -2 \\ -8 & 5 \end{vmatrix} = 85 - 16 = 69$$

85

Step 3: Find the determinant, D_y, by replacing the y-values in the second column with the values after the equal sign leaving the x column unchanged.

68

$$D_y = \begin{vmatrix} 3 & 17 \\ 4 & -8 \end{vmatrix} = -24 - 68 = -92$$

–24

Step 4: Use Cramer's Rule to find the values of x and y.

$$x = \frac{D_x}{D} = \frac{69}{23} = 3$$

$$y = \frac{D_y}{D} = \frac{-92}{23} = -4$$

The answer written as an ordered pair is (3, –4).

Example 2: Use Cramer's Rule to solve: $\begin{matrix} 5x + 4y = 11 \\ 2x + 6y = -7 \end{matrix}$

Step 1: Find the determinant, D, by using the x and y values from the problem.

8

$$D = \begin{vmatrix} 5 & 4 \\ 2 & 6 \end{vmatrix} = 30 - 8 = 22$$

30

Step 2: Find the determinant, D_x, by replacing the x-values in the first column with the values after the equal sign leaving the y column unchanged.

–28

$$D_x = \begin{vmatrix} 11 & 4 \\ -7 & 6 \end{vmatrix} = 66 - (-28) = 94$$

66

Step 3: Find the determinant, D_y, by replacing the y-values in the second column with the values after the equal sign leaving the x column unchanged.

22

$$D_y = \begin{vmatrix} 5 & 11 \\ 2 & -7 \end{vmatrix} = -35 - 22 = -57$$

–35

Step 4: Use Cramer's Rule to find the values of x and y.

$$x = \frac{D_x}{D} = \frac{94}{22} = \frac{47}{11}$$

$$y = \frac{D_y}{D} = \frac{-57}{22} = -\frac{57}{22}$$

The answer written as an ordered pair is $\left(\frac{47}{11}, -\frac{57}{22}\right)$.

Example 3: Use Cramer's Rule to solve: $\begin{array}{r} -x + 3y = 9 \\ 2x + 7y = -3 \end{array}$

Step 1: Find the determinant, D, by using the x and y values from the problem.

$$\begin{array}{c} 6 \\ D = \begin{vmatrix} -1 & 3 \\ 2 & 7 \end{vmatrix} = -7 - 6 = -13 \\ -7 \end{array}$$

Step 2: Find the determinant, D_x, by replacing the x-values in the first column with the values after the equal sign leaving the y column unchanged.

$$\begin{array}{c} -9 \\ D_x = \begin{vmatrix} 9 & 3 \\ -3 & 7 \end{vmatrix} = 63 - (-9) = 72 \\ 63 \end{array}$$

Step 3: Find the determinant, D_y, by replacing the y-values in the second column with the values after the equal sign leaving the x column unchanged.

$$\begin{array}{c} 18 \\ D_y = \begin{vmatrix} -1 & 9 \\ 2 & -3 \end{vmatrix} = 3 - 18 = -15 \\ 3 \end{array}$$

Step 4: Use Cramer's Rule to find the values of x and y.

$$x = \frac{D_x}{D} = \frac{72}{-13} = -\frac{72}{13}$$

$$y = \frac{D_y}{D} = \frac{-15}{-13} = \frac{15}{13}$$

The answer written as an ordered pair is $\left(-\frac{72}{13}, \frac{15}{13}\right)$.

Advantages and Disadvantages of Cramer's Rule

Advantages – I find that one of the advantages to Cramer's Rule is that you can find the value of x or y without having to know the other value of x or y. For example, if you needed to find just the value of x, Cramer's Rule would work well. Another thing that I like about Cramer's Rule is that if either of the values of x or y is are fractions, you do not have to plug in a fraction to find the other values. Each value can be found independently.

Disadvantages – One of the only disadvantages to using Cramer's rule is if the value of D is zero then Cramer's Rule will not work because you cannot divide by zero. However, if the value of D is zero then you know that the solution is either "No Solution" or "Infinite Solutions". You will have to use a different technique such as Addition/Elimination or Substitution to find out whether the answer is "No Solution" or "Infinite Solutions".