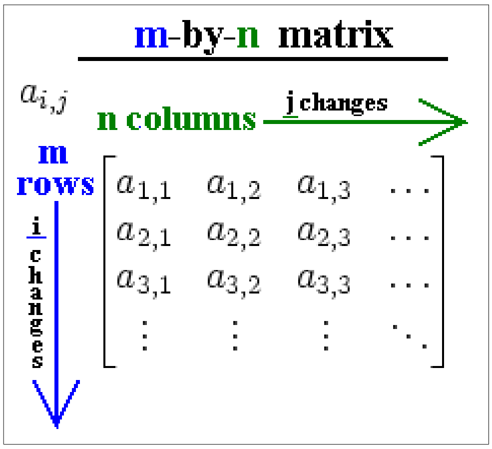
MATRIX

A matrix is a collection of numbers arranged into a fixed number of rows and columns. Usually the numbers are real numbers.

A=



A = M X N = no.of rows. X no. Of columns

Here is an example of a matrix with three rows and three columns:

$$A= \begin{array}{cc} & \text{col 1} \\ \text{row 1} & \begin{pmatrix} 1 & -2 & 3 \\ 0 & 8 & 4.6 \\ 4 & -1 & 0 \end{pmatrix} \end{array}$$

A **matrix** is a rectangular arrangement of numbers into rows and columns.
For example, matrix A has three **rows** and three **columns**.

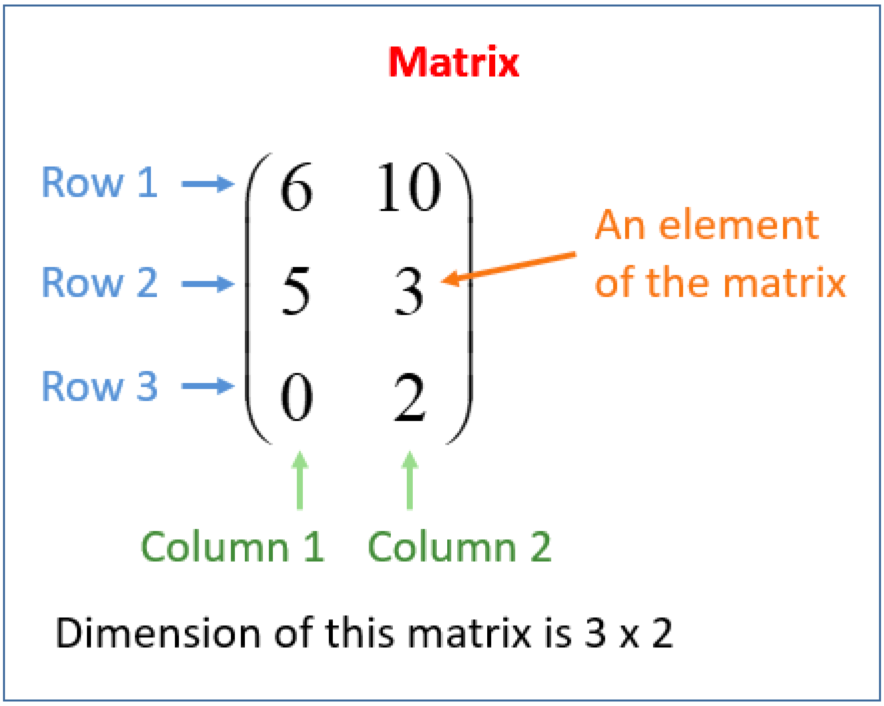
Each number that makes up a matrix is called an **element** of the matrix. The elements in a matrix have specific locations.

The upper left corner of the matrix is row 1 column 1. In the above matrix the element at row 1 col 1 is the value 1. The element at row 2 column 3 is the value 4.6.

The top row is row 1. The leftmost column is column 1. This matrix is a 3x3 matrix because it has three rows and three columns. In describing matrices, the format is:

rows X columns

The following diagram shows the rows and columns of a 3 by 2 matrix.



Dimension of matrix = order of matrix

no. of rows X no. of columns

Notation

A matrix is usually shown by a **capital letter** (such as A, or B)

Each entry (or "element") is shown by a **lower case letter** with a "subscript" of **row,column**:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}$$

Here are some sample entries: order of matrix B=2X3

A Matrix
(This one has 2 Rows and 3 Columns)

$b_{1,1} = 6$ *(the entry at row 1, column 1 is 6)*

$b_{1,3} = 24$ *(the entry at row 1, column 3 is 24)*

$b_{2,3} = 8$ *(the entry at row 2, column 3 is 8)*

$$C = \begin{pmatrix} 5 & 9 & 6 & 0 \\ 8 & 1 & 12 & 3 \end{pmatrix}$$

Rows=2

Columns= 4

Order of Matrix C= 2X 4

Elements of matrix C= 1st row 1st column: 5

1st row 2nd column : 9

12: 2nd row 3rd column

0: 1st row 4th column

6: 1st row 3rd column

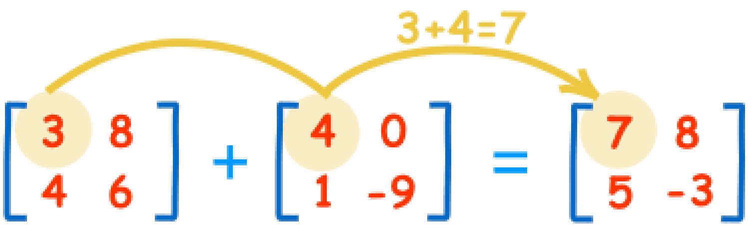
Adding

To add two matrices: add the numbers in the matching positions:

These are the calculations:

3+4=7   8+0=8

4+1=5 6−9=−3



6+ (-9)=6-9=-3

If Order of matrix A = Order of Matrix B

Then

addition of two matrices A and B can be possible

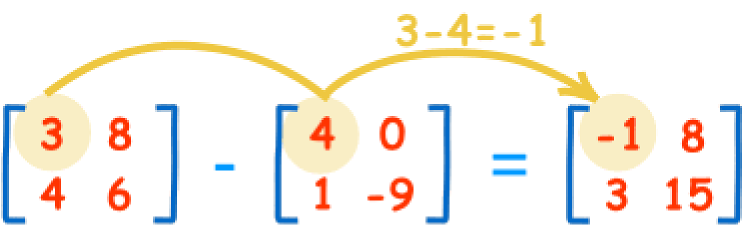
A= order 2X2

B= order 2X2

6 + (-9) = 6-9=-3

Subtracting

To subtract two matrices: subtract the numbers in the matching positions:



6-(-9)=6+9=15

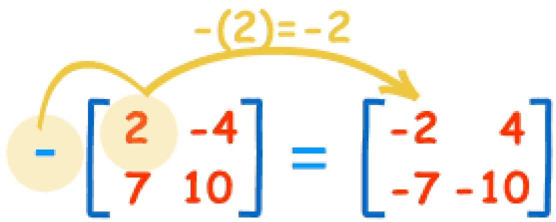
These are the calculations:

3−4=−1   8−0=8

4−1=3   6−(−9)=15

Negative

The negative of a matrix is also simple:

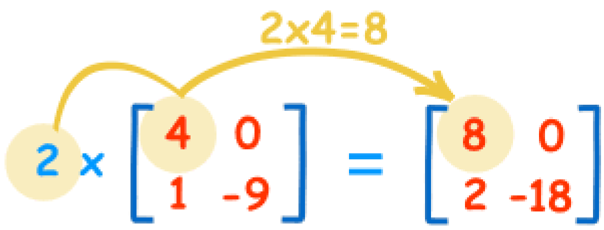


These are the calculations:

$$\begin{array}{ll} -(2)=-2 & -(-4)=+4 \\ -(7)=-7 & -(10)=-10 \end{array}$$

Multiply by a Constant

We can multiply a matrix by a **constant** *(the value 2 in this case)*:

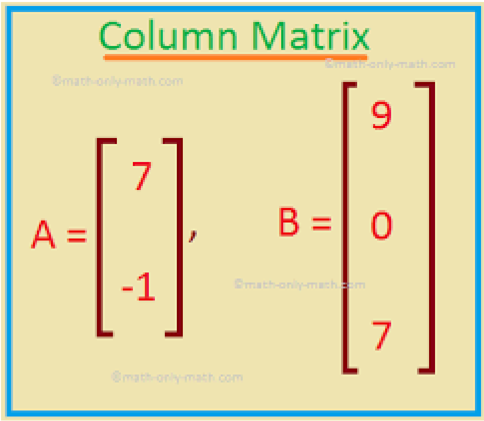


These are the calculations:

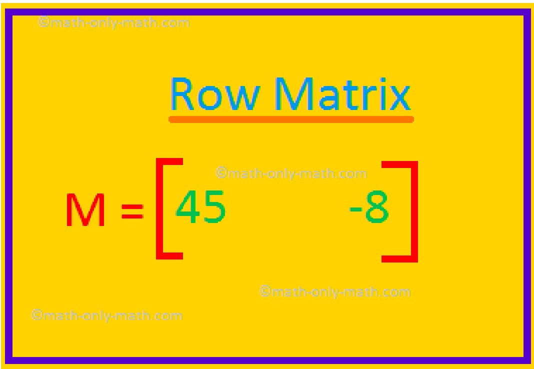
$$\begin{array}{ll} 2\times4=8 & 2\times0=0 \\ 2\times1=2 & 2\times-9=-18 \end{array}$$

Types of matrices

Column Matrix: A matrix which has exactly one column is called a column matrix.



Row Matrix: A matrix which has exactly one row is called a row matrix.



Square

A **square** matrix has the same number of rows as columns.

$$\begin{bmatrix} 2 & 0 \\ 1 & 8 \end{bmatrix}$$

above matrix is of order 2
A square matrix (2 rows, 2 columns)

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \\ 3 & 0 & 7 \end{bmatrix}$$

above matrix is of order 3
Also a square matrix (3 rows, 3 columns)

The **Main or leading Diagonal** starts at the top left and goes down to the right

$$\begin{bmatrix} 7 & 6 & 4 \\ 4 & 2 & -2 \\ 3 & 0 & 9 \end{bmatrix}$$

Transpose

A **Transpose** is where we swap entries across the main diagonal (rows become columns) like this:

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}^{T} = \begin{bmatrix} 6 & 1 \\ 4 & -9 \\ 24 & 8 \end{bmatrix}$$

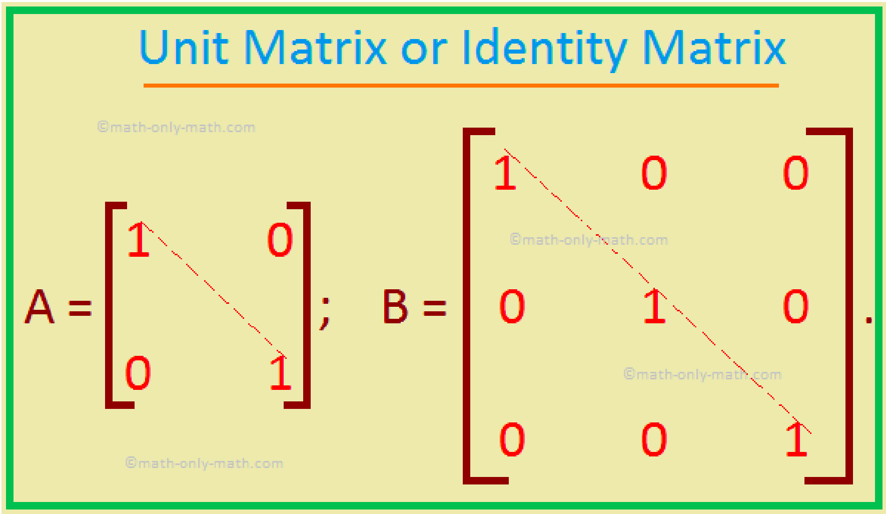
Identity Matrix

An **Identity Matrix** has **1**s on the main diagonal and **0**s everywhere else:

we can represent identity matrix using I_2=unit matrix of order 2

I_3= unit matrix of order 3

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



It is the matrix equivalent of the number "1", when we multiply with it the original is unchanged:

$$A \times I = A$$

$$I \times A = A$$

Diagonal Matrix

A square matrix is called a diagonal matrix when it has zero anywhere not on the main diagonal:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scalar Matrix

A scalar matrix has all main diagonal entries the same, with zero everywhere else:

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

A scalar matrix

Triangular Matrix

Lower triangular is when all entries above the main diagonal are zero:

$$\begin{bmatrix} 5 & 0 & 0 \\ 2 & 1 & 0 \\ 7 & 6 & -3 \end{bmatrix}$$

A lower triangular matrix

Upper triangular is when all entries below the main diagonal are zero:

$$\begin{bmatrix} 2 & -2 & 7 \\ 0 & 4 & 11 \\ 0 & 0 & 5 \end{bmatrix}$$

An upper triangular matrix

Zero Matrix (Null Matrix)

Zeros just everywhere:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Zero matrix

Zero Matrix / Null Matrix

$$A = \begin{bmatrix} 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Rules for multiplication

$$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & -4 \\ 6 & 8 & 0 \end{bmatrix}$$

$$(2 \text{ x } 2) \bullet (2 \text{ x } 3)$$

same

$$\begin{bmatrix} 5 & -3 \\ 4 & 1 \\ 7 & -2 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$

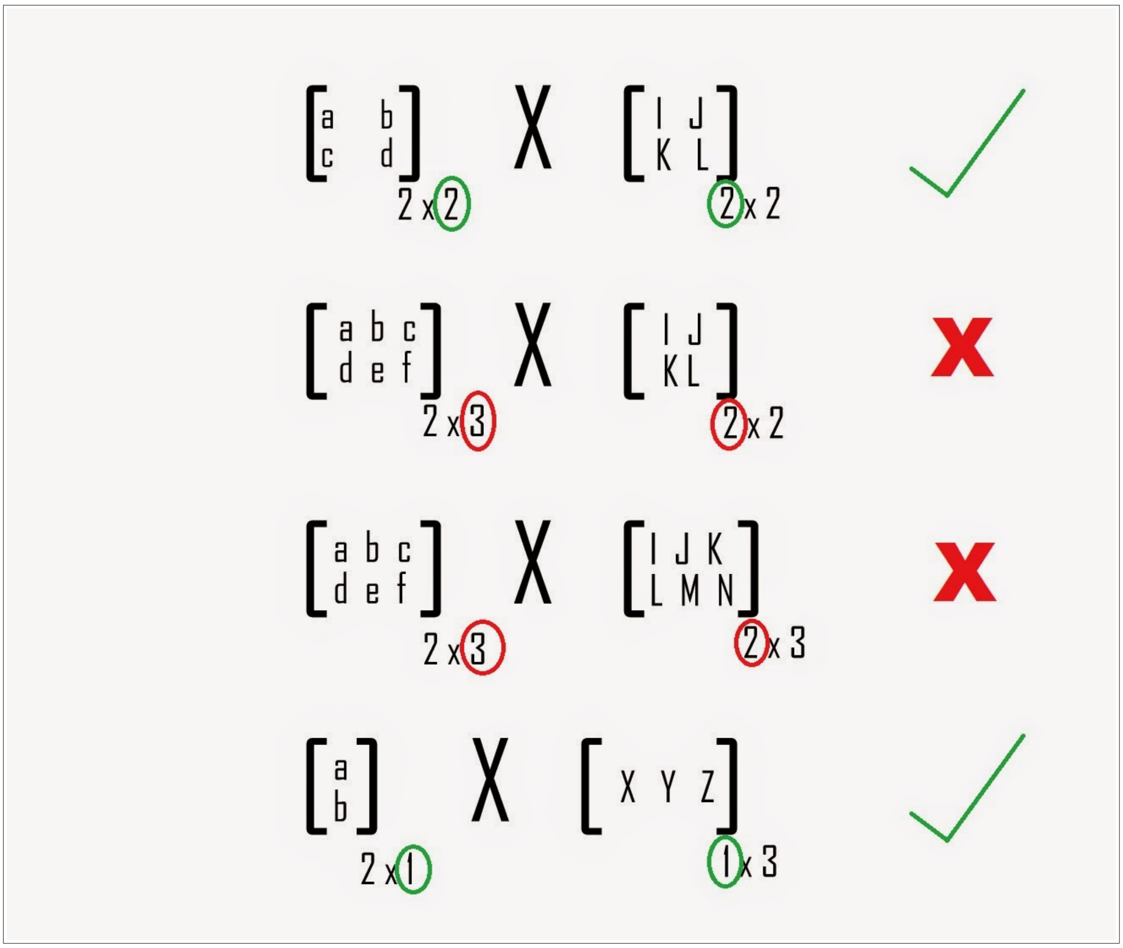
$$(3 \text{ x } 2) \bullet (2 \text{ x } 1)$$

same

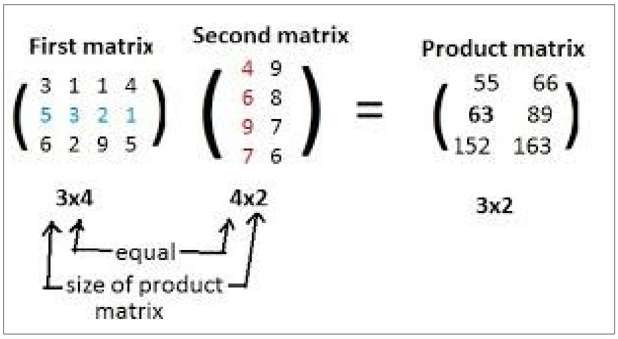
Matrix B X Matrix A

no. of columns of first matrix = no. Of rows of second matrix

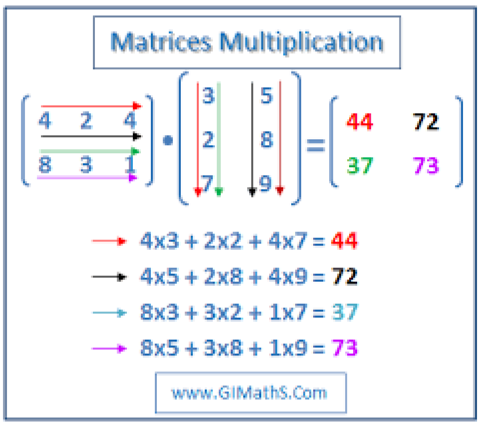
Then matrix multiplication is possible.



a b
c d
2 x 2
X
I J
K L
2 x 2
a b c
d e f
2 x 3
I J
K L
2 x 2
a b c
d e f
2 x 3
I J K
L M N
2 x 3
a
b
2 x 1
X Y Z
1 x 3



First matrix
Second matrix
Product matrix
3 1 1 4
5 3 2 1
6 2 9 5
4 9
6 8
9 7
7 6
55 66
63 89
152 163
3x4
4x2
3x2
equal
size of product matrix



Matrices Multiplication
4 2 4
8 3 1
3 5
2 8
7 9
44 72
37 73
4x3 + 2x2 + 4x7 = 44
4x5 + 2x8 + 4x9 = 72
8x3 + 3x2 + 1x7 = 37
8x5 + 3x8 + 1x9 = 73
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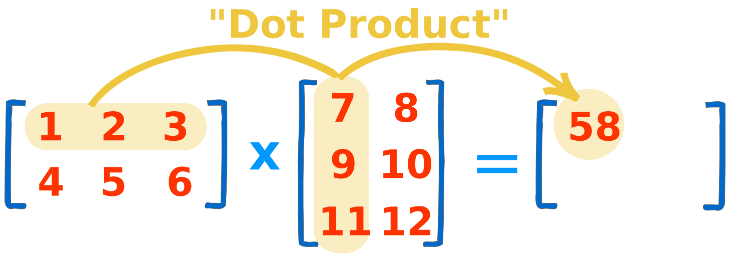
Order of first matrix = 2X3

Order of Second Matrix =3X2

Multiplying a Matrix by Another Matrix

But to multiply a matrix **by another matrix** we need to do the "dot product" of rows and columns ... what does that mean? Let us see with an example:

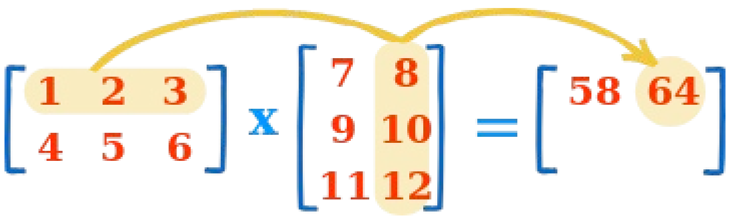
To work out the answer for the **1st row** and **1st column**:



The "Dot Product" is where we **multiply matching members**, then sum up:

$$(1, 2, 3) \bullet (7, 9, 11) = 1×7 + 2×9 + 3×11$$
$$= 58$$

We match the 1st members (1 and 7), multiply them, likewise for the 2nd members (2 and 9) and the 3rd members (3 and 11), and finally sum them up.



We can do the same thing for the **2nd row** and **1st column**:

$$(4, 5, 6) \bullet (7, 9, 11) = 4×7 + 5×9 + 6×11$$
$$= 139$$

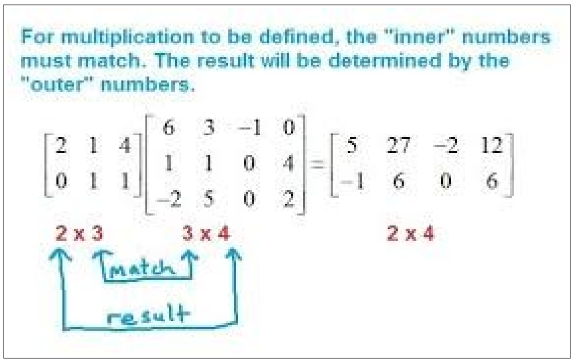
And for the **2nd row** and **2nd column**:

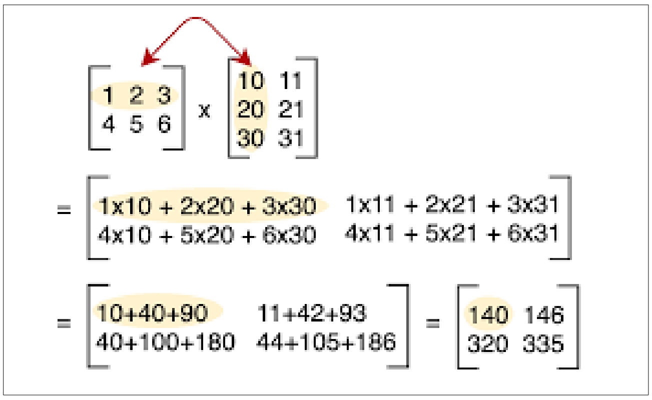
$$(4, 5, 6) \bullet (8, 10, 12) = 4×8 + 5×10 + 6×12$$
$$= 154$$

And we get:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

DONE!





$$\begin{bmatrix} 2 & -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 5 \\ -3 \end{bmatrix}$$

$$= (2)(2) + (-3)(4) + (0)(5) + (4)(-3)$$
$$= -20.$$

$$\begin{array}{ccc} A & B & A * B \\ \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} & \begin{pmatrix} 6 & 3 \\ 5 & 2 \\ 4 & 1 \end{pmatrix} & = \begin{pmatrix} 1*6+2*5+3*4 & 1*3+2*2+3*1 \\ 4*6+5*5+6*4 & 4*3+5*2+6*1 \end{pmatrix} \end{array}$$

28	**10**
73	**28**

Dividing

And what about division? Well we **don't** actually divide matrices, we do it this way:

$$A/B = A \times (1/B) = A \times B^{-1}$$

where $\mathbf{B^{-1}}$ means the "inverse" of B.