Question 1: A die is rolled, find the probability that an even number is obtained.
Solution to Question 1:

- Let us first write the sample space S of the experiment.
  S = {1,2,3,4,5,6}
- Let E be the event "an even number is obtained" and write it down.
  E = {2,4,6}
- We now use the formula of the classical probability.
  P(E) = n(E) / n(S) = 3 / 6 = 1 / 2

Question 2: Two coins are tossed, find the probability that two heads are obtained.
Note: Each coin has two possible outcomes H (heads) and T (Tails).
Solution to Question 2:

- The sample space S is given by.
  S = {(H,T),(H,H),(T,H),(T,T)}
- Let E be the event "two heads are obtained".
  E = {(H,H)}
- We use the formula of the classical probability.
  P(E) = n(E) / n(S) = 1 / 4

Question 4: Two dice are rolled, find the probability that the sum is
a) equal to 1
b) equal to 4
c) less than 13
Solution to Question 4:

- a) The sample space S of two dice is shown below.
  S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
  (2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
  (3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
  (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
  (5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
  (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) }
- Let E be the event "sum equal to 1". There are no outcomes which correspond to a sum equal to 1, hence
  P(E) = n(E) / n(S) = 0 / 36 = 0
- b) Three possible outcomes give a sum equal to 4: E = {(1,3),(2,2),(3,1)}, hence.
  P(E) = n(E) / n(S) = 3 / 36 = 1 / 12
- c) All possible outcomes, E = S, give a sum less than 13, hence.
  P(E) = n(E) / n(S) = 36 / 36 = 1

Question 5: A die is rolled and a coin is tossed, find the probability that the die shows an odd number and the coin shows a head.
Solution to Question 5:

- The sample space S of the experiment described in question 5 is as follows

  S = { (1,H),(2,H),(3,H),(4,H),(5,H),(6,H)
  (1,T),(2,T),(3,T),(4,T),(5,T),(6,T)}
- Let E be the event "the die shows an odd number and the coin shows a head". Event E may be described as follows

  E={(1,H),(3,H),(5,H)}
- The probability P(E) is given by

  P(E) = n(E) / n(S) = 3 / 12 = 1 / 4

<u>Question 6:</u> A card is drawn at random from a deck of cards. Find the probability of getting the 3 of diamond.
Let E be the event "getting the 3 of diamond". An examination of the sample space shows that there is one "3 of diamond" so that n(E) = 1 and n(S) = 52. Hence the probability of event E occurring is given by
P(E) = 1 / 52
<u>Question 7:</u> A card is drawn at random from a deck of cards. Find the probability of getting a queen.
<u>Solution to Question 7:</u>

- Let E be the event "getting a Queen". An examination of the sample space shows that there are 4 "Queens" so that n(E) = 4 and n(S) = 52. Hence the probability of event E occurring is given by

  P(E) = 4 / 52 = 1 / 13

<u>Question 8:</u> A jar contains 3 red marbles, 7 green marbles and 10 white marbles. If a marble is drawn from the jar at random, what is the probability that this marble is white?
<u>Solution to Question 8:</u>

- We first construct a table of frequencies that gives the marbles color distributions as follows

color	frequency
red	3
green	7
white	10

- We now use the <u>empirical</u> formula of the probability

$$P(E) = \frac{\textbf{Frequency for white color}}{\textbf{Total frequencies in the above table}}$$

= 10 / 20 = 1 / 2

a) A die is rolled, find the probability that the number obtained is greater than 4.
b) Two coins are tossed, find the probability that one <u>head</u> only is obtained.
c) Two dice are rolled, find the probability that the sum is equal to 5.
d) A card is drawn at random from a deck of cards. Find the probability of getting the King of heart.

<u>Answers to above exercises:</u>

a) 2 / 6 = 1 / 3
b) 2 / 4 = 1 / 2
c) 4 / 36 = 1 / 9
d) 1 / 52

1. A card is drawn from a well shuffled pack of 52 cards. Find the probability of:

(i) ‘2’ of spades

(ii) a jack

(iii) a king of red colour

(iv) a card of diamond

(v) a king or a queen

(vi) a non-face card

(vii) a black face card

(viii) a black card

(ix) a non-ace

(x) non-face card of black colour

(xi) neither a spade nor a jack

(xii) neither a heart nor a red king

Solution:

In a playing card there are 52 cards.

Therefore the total number of possible outcomes = 52

(i) ‘2’ of spades:

Number of favourable outcomes i.e. ‘2’ of spades is 1 out of 52 cards.

Therefore, probability of getting ‘2’ of spade

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}}$$

= 1/52

(ii) a jack

Number of favourable outcomes i.e. ‘a jack’ is 4 out of 52 cards.

Therefore, probability of getting ‘a jack’

$$P(B) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}}$$

= 4/52
= 1/13

(iii) a king of red colour

Number of favourable outcomes i.e. ‘a king of red colour’ is 2 out of 52 cards.

Therefore, probability of getting ‘a king of red colour’

$$P(C) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 2/52
= 1/26

(iv) a card of diamond

Number of favourable outcomes i.e. ‘a card of diamond’ is 13 out of 52 cards.

Therefore, probability of getting ‘a card of diamond’

$$P(D) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 13/52
= 1/4

(v) a king or a queen

Total number of king is 4 out of 52 cards.

Total number of queen is 4 out of 52 cards

Number of favourable outcomes i.e. ‘a king or a queen’ is 4 + 4 = 8 out of 52 cards.

Therefore, probability of getting ‘a king or a queen’

$$P(E) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 8/52
= 2/13

(vi) a non-face card

Total number of face card out of 52 cards = 3 times 4 = 12

Total number of non-face card out of 52 cards = 52 - 12 = 40

Therefore, probability of getting ‘a non-face card’

$$P(F) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 40/52
= 10/13

(vii) a black face card:

Cards of Spades and Clubs are black cards.

Number of face card in spades (king, queen and jack or knaves) = 3

Number of face card in clubs (king, queen and jack or knaves) = 3

Therefore, total number of black face card out of 52 cards = 3 + 3 = 6

Therefore, probability of getting ‘a black face card’

$$P(G) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 6/52
= 3/26

(viii) a black card:

Cards of spades and clubs are black cards.

Number of spades = 13

Number of clubs = 13

Therefore, total number of black card out of 52 cards = 13 + 13 = 26

Therefore, probability of getting ‘a black card’

$$P(H) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 26/52
= 1/2

(ix) a non-ace:

Number of ace cards in each of four suits namely spades, hearts, diamonds and clubs = 1

Therefore, total number of ace cards out of 52 cards = 4

Thus, total number of non-ace cards out of 52 cards = 52 - 4

= 48

Therefore, probability of getting ‘a non-ace’

$$P(I) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 48/52
= 12/13

(x) non-face card of black colour:

Cards of spades and clubs are black cards.

Number of spades = 13

Number of clubs = 13

Therefore, total number of black card out of 52 cards = 13 + 13 = 26

Number of face cards in each suits namely spades and clubs = 3 + 3 = 6

Therefore, total number of non-face card of black colour out of 52 cards = 26 - 6 = 20

Therefore, probability of getting 'non-face card of black colour'

$$P(J) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}}$$

= 20/52
= 5/13

(xi) neither a spade nor a jack

Number of spades = 13

Total number of non-spades out of 52 cards = 52 - 13 = 39

Number of jack out of 52 cards = 4

Number of jack in each of three suits namely hearts, diamonds and clubs = 3

[Since, 1 jack is already included in the 13 spades so, here we will take number of jacks is 3]

Neither a spade nor a jack = 39 - 3 = 36

Therefore, probability of getting 'neither a spade nor a jack'

$$P(K) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}}$$

= 36/52
= 9/13

(xii) neither a heart nor a red king

Number of hearts = 13

Total number of non-hearts out of 52 cards = 52 - 13 = 39

Therefore, spades, clubs and diamonds are the 39 cards.

Cards of hearts and diamonds are red cards.

Number of red kings in red cards = 2

Therefore, neither a heart nor a red king = 39 - 1 = 38

[Since, 1 red king is already included in the 13 hearts so, here we will take number of red kings is 1]

Therefore, probability of getting ‘neither a heart nor a red king’

$$P(L) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}}$$

= 38/52
= 19/26

Two different coins are tossed randomly. Find the probability of:

(i) getting two heads

(ii) getting two tails

(iii) getting one tail

(iv) getting no head

(v) getting no tail

(vi) getting at least 1 head

(vii) getting at least 1 tail

(viii) getting atmost 1 tail

(ix) getting 1 head and 1 tail

Solution:

When two different coins are tossed randomly, the sample space is given by

S = {HH, HT, TH, TT}

Therefore, n(S) = 4.

(i) getting two heads:

Let E_1 = event of getting 2 heads. Then,

E_1 = {HH} and, therefore, $n(E_1)$ = 1.

Therefore, P(getting 2 heads) = $P(E_1) = n(E_1)/n(S)$ = 1/4.

(ii) getting two tails:

Let E_2 = event of getting 2 tails. Then,

E_2 = {TT} and, therefore, $n(E_2)$ = 1.

Therefore, P(getting 2 tails) = $P(E_2) = n(E_2)/n(S)$ = 1/4.

(iii) getting one tail:

Let E_3 = event of getting 1 tail. Then,

E_3 = {TH, HT} and, therefore, $n(E_3) = 2$.

Therefore, P(getting 1 tail) = $P(E_3) = n(E_3)/n(S) = 2/4 = 1/2$

(iv) getting no head:

Let E_4 = event of getting no head. Then,

E_4 = {TT} and, therefore, $n(E_4) = 1$.

Therefore, P(getting no head) = $P(E_4) = n(E_4)/n(S)$ = ¼.

(v) getting no tail:

Let E_5 = event of getting no tail. Then,

E_5 = {HH} and, therefore, $n(E_5) = 1$.

Therefore, P(getting no tail) = $P(E_5) = n(E_5)/n(S)$ = ¼.

(vi) getting at least 1 head:

Let E_6 = event of getting at least 1 head. Then,

E_6 = {HT, TH, HH} and, therefore, $n(E_6) = 3$.

Therefore, P(getting at least 1 head) = $P(E_6) = n(E_6)/n(S)$ = ¾.

(vii) getting at least 1 tail:

Let E_7 = event of getting at least 1 tail. Then,

E_7 = {TH, HT, TT} and, therefore, $n(E_7) = 3$.

Therefore, P(getting at least 1 tail) = $P(E_2) = n(E_2)/n(S)$ = ¾.

(viii) getting atmost 1 tail:

Let E_8 = event of getting atmost 1 tail. Then,

E_8 = {TH, HT, HH} and, therefore, $n(E_8) = 3$.

Therefore, P(getting atmost 1 tail) = $P(E_8) = n(E_8)/n(S)$ = ¾.

(ix) getting 1 head and 1 tail:

Let E_9 = event of getting 1 head and 1 tail. Then,

E_9 = {HT, TH } and, therefore, $n(E_9) = 2$.

Therefore, P(getting 1 head and 1 tail) = $P(E_9) = n(E_9)/n(S)= 2/4 = 1/2$.