* Integration by Substitution :-

1) Let $u = g(x)$, where $g(x)$ is part of the ~~integral~~ integrand, usually the "inside function" of composite function $f[g(x)]$.

2) Compute $du = g'(x)\,dx$

3) Use the substitution $u = g(x)$ and $du = g'(x)\,dx$ to convert the entire integral into one involving only u.

4) Evaluate the resulting integral.

5) Replace u by $g(x)$ to obtain the final solution as a function of x.

Ex:- Find $\int 3\sqrt{3x+1}\;dx$

Solⁿ:- Let $u = 3x+1$

$$\frac{d}{dx}u = \frac{d}{dx}(3x+1)$$

$$\therefore \frac{du}{dx} = 3$$

$$\therefore \frac{du}{3} = dx$$

Now $\int 3\sqrt{3x+1}\;dx$

$$= \int \cancel{3}\sqrt{u}\cdot\frac{du}{\cancel{3}} = \int u^{1/2}\cdot du$$

$$= \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{u^{3/2}}{3/2} + C = \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{3} \cdot (3x+1)^{3/2} + C$$

Ex:- $\int x^2 (x^3+1)^{3/2} \, dx$

Let $x^3 + 1 = u$

$\therefore \frac{d}{dx}(x^3+1) = \frac{d}{dx} u$

$\therefore 3x^2 = \frac{du}{dx} \Rightarrow dx = \frac{du}{3x^2}$

Now $\int x^2 (x^3+1)^{3/2} dx = \int \cancel{x^2} (u)^{3/2} \cdot \frac{du}{3\cancel{x^2}}$

$$= \frac{1}{3} \int u^{3/2} \, du$$

$$= \frac{1}{3} \cdot \frac{u^{3/2+1}}{3/2+1} = \frac{1}{3} \cdot \frac{u^{5/2}}{5/2} + C$$

$$= \frac{2}{(3)\cdot(5)} \cdot u^{5/2} + C$$

$$= \frac{2(x^3+1)^{5/2}}{15} + C$$

Ex:- $\int e^{-3x} \, dx$

Let $u = -3x \quad \therefore \frac{d}{dx} u = \frac{d}{dx}(-3x)$

$\therefore \frac{du}{dx} = -3 \Rightarrow \frac{du}{-3} = dx$

Now $\int e^{-3x} dx = \int e^{u} \cdot \frac{du}{-3} = \frac{-1}{3} \int e^{u} du$

$$= \frac{-1}{3} \cdot e^{u} + c = -\frac{1}{3} e^{-3x} + c$$

Ex: $\int \frac{x}{3x^2+1} \, dx$

Let $u = 3x^2+1$

$$\frac{d}{dx}(u) = \frac{d}{dx}(3x^2+1)$$

$$\frac{du}{dx} = 3(2x) \Rightarrow \frac{du}{6x} = dx$$

$$= \int \frac{x}{u} \cdot \frac{du}{6\cancel{x}} = \frac{1}{6}\int \frac{1}{u} \, du$$

$$= \frac{1}{6} \log u + c = \frac{1}{6} \log(3x^2+1) + c$$

Ex: Find $\int \frac{(\ln x)^2}{2x} \, dx$

Let $u = \ln x$

$$\frac{d}{dx} u = \frac{d}{dx} (\ln x)$$

$$\frac{du}{dx} = \frac{1}{x} \Rightarrow x \, du = dx$$

$$= \int \frac{(u)^2}{2\cancel{x}} \cdot \cancel{x} \, du = \frac{1}{2} \int u^2 \, du$$

$$= \frac{1}{2} \cdot \frac{u^{2+1}}{2+1} + c = \frac{1}{2} \cdot \frac{u^3}{3} + c$$

$$= \frac{(\ln x)^3}{6} + c$$

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Ex:- 1) $\int 4(4x+3)^4 \, dx$

ans:- $\frac{1}{5}(4x+3)^5 + c$

10) $\int \frac{x}{3x^2-1} \, dx$

ans:- $\frac{1}{6}$

$\log(3x^2-1) + c$

2) $\int (x^3-2x)^2 (3x^2-2) \, dx$

ans:- $\frac{1}{3}(x-2x)^3 + c$

3) $\int \frac{4x}{(2x^2+3)^3} \, dx$

ans:- $\frac{-1}{2(2x^2+3)^2} + c$

4) $\int (x^2-1)^9 x \, dx$

ans:- $\frac{1}{20}(x^2-1)^{10} + c$

5) $\int e^{-2x} \, dx$ ans:- $\frac{-1}{2} e^{-2x} + c$

6) $\int e^{2-x} \, dx$ ans:- $-e^{2-x} + c$

7) $\int \frac{e^x}{1+e^x} \, dx$ ans:- $\log(1+e^x) + c$

8) $\int x e^{-x^2} \, dx$ ans:- $\frac{-1}{2} \cdot e^{-x^2} + c$

9) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$ ans:- $2e^{\sqrt{x}} + c$