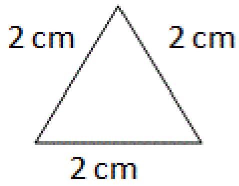
There are three types of triangles based on sides.

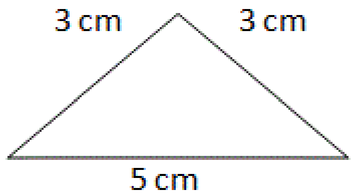
Types of triangles based on sides

Equilateral triangle: A triangle having all the three sides of equal length is an equilateral triangle.



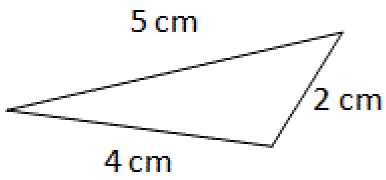
Since all sides are equal, all angles are equal too.

Isosceles triangle: A triangle having two sides of equal length is an Isosceles triangle.

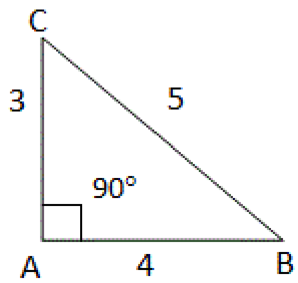


The two angles opposite to the equal sides are equal.

Scalene triangle: A triangle having three sides of different lengths is called a scalene triangle.



Right-angled triangle: A triangle whose one angle is a right-angle is a Right-angled triangle or Right triangle.



In the figure above, the side opposite to the right angle, BC is called the hypotenuse.

For a Right triangle ABC,

$BC^2 = AB^2 + AC^2$

This is called the **Pythagorean Theorem.**

In the triangle above, $5^2 = 4^2 + 3^2$. Only a triangle that satisfies this condition is a right triangle.

Hence, the Pythagorean Theorem helps to find whether a triangle is Right-angled.

Example 2: A triangle has vertices *A*(12,5), *B*(5,3), and *C*(12, 1). Show that the triangle is isosceles.

By the *Distance Formula,*

$AB = \sqrt{(5-12)^2 + (3-5)^2}$	$BC = \sqrt{(12-5)^2 + (1-3)^2}$
$AB = \sqrt{(7^2) + (-2)}$	$BC = \sqrt{7^2 + (-2)^2}$
$AB = \sqrt{49 + 4}$	$BC = \sqrt{49 + 4}$
$AB = \sqrt{53}$	$BC = \sqrt{53}$

Because *AB* =*BC*, triangle *ABC* is isosceles.

Example:

Show that the points (a, a), (- a, - a) and (- a√3, a√3) are the vertices of an equilateral triangle.

The given points are let

A(a, a), B(- a, - a) and C(- a√3, a√3)

$$\begin{aligned} AB &= \sqrt{(-a-a)^2 + (-a-a)^2} \\ &= \sqrt{4a^2 + 4a^2} = 2\sqrt{2}\,a \text{ units.} \\ BC &= \sqrt{(-a\sqrt{3}+a)^2 + (a\sqrt{3}+a)^2} \\ &= \sqrt{3a^2 + a^2 - 2\sqrt{3}a^2 + 3a^2 + a^2 + 2\sqrt{3}a^2} \\ &= \sqrt{8a^2} = 2\sqrt{2}a \text{ units.} \\ \text{and} \quad CA &= \sqrt{(a\sqrt{3}-a)^2 + (-a\sqrt{3}-a)^2} \\ &= \sqrt{3a^2 + a^2 - 2\sqrt{3}a^2 + 3a^2 + a^2 + 2\sqrt{3}a^2} \\ &= \sqrt{8a^2} = 2\sqrt{2}a \text{ units.} \end{aligned}$$

as AB = BC = CA = 2√2a

⇒ ΔABC is an equilateral triangle.

Hence proved.

Example:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Let the given vertices be $A = (1, 6), B = (3, 2)$ and $C = (10, 8)$

We first find the distance between $A = (1, 6)$ and $B = (3, 2)$ as follows:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3-1)^2 + (2-6)^2} = \sqrt{2^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20} = \sqrt{2^2 \times 5} = 2\sqrt{5}$$

Similarly, the distance between $B = (3, 2)$ and $C = (10, 8)$ is:

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(10-3)^2 + (8-2)^2} = \sqrt{7^2 + 6^2} = \sqrt{49+36} = \sqrt{85}$$

Now, the distance between $C = (10, 8)$ and $A = (1, 6)$ is:

$$CA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(10-1)^2 + (8-6)^2} = \sqrt{9^2 + 2^2} = \sqrt{81+4} = \sqrt{85}$$

We also know that If any two sides have equal side lengths, then the triangle is isosceles.

Here, since the lengths of the two sides are equal that is $BC = CA = \sqrt{85}$

Condition of colinearity of three points.

Three points A(x1,y1) B(x2,y2) and C(x3,y3) are co -linear if and only if points are on the same line

Three points are co linear when area of triagle is zero.