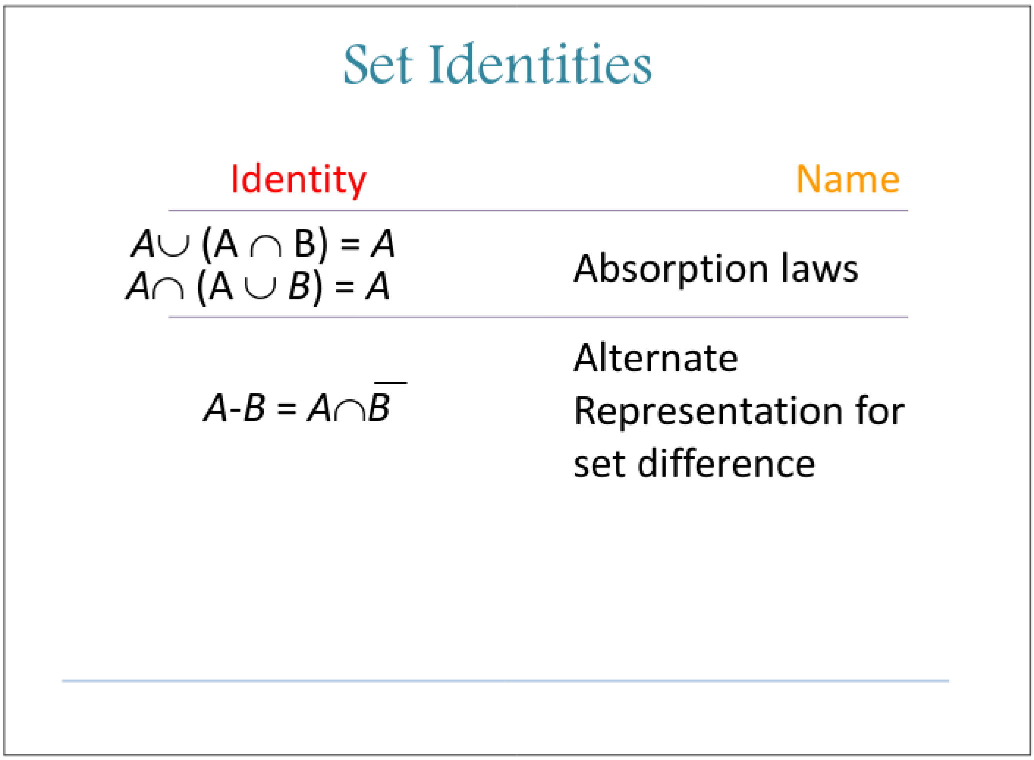
SET THEORY LAWS

Set Identities

Identity	Name
$A \cup \varnothing = A$ $A \cap U = A$	Identity laws
$A \cup U = U$ $A \cap \varnothing = \varnothing$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{\overline{A}} = A$	Double Complement laws

Set Identities

Identity	Name
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive laws
$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$	De Morgan's laws



In above image, bar sign means complemt

- **If A={1, 3, 5}, B = {3, 5, 6} and C = {1, 3, 7}**

(i) Verify that A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

(ii) Verify A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

(distributive law)

(i) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

Solution

L.H.S. = A ∪ (B ∩ C)

B ∩ C = {3}

A ∪ (B ∩ C) = {1, 3, 5} ∪{3} = {1, 3, 5} ……………….. (1)

R.H.S. = (A ∪ B) ∩ (A ∪ C)

$A \cup B = \{1, 3, 5, 6\}$

$A \cup C = \{1, 3, 5, 7\}$

$(A \cup B) \cap (A \cup C) = \{1, 3, 5, 6\} \cap \{1, 3, 5, 7\} = \{1, 3, 5\}$ ……………….. (2)

From (1) and (2), we conclude that;

$A \cup (B \cap C) = A \cup B \cap (A \cup C)$ [*verified*]

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S. $= A \cap (B \cup C)$

$B \cup C = \{1, 3, 5, 6, 7\}$

$A \cap (B \cup C) = \{1, 3, 5\} \cap \{1, 3, 5, 6, 7\} = \{1, 3, 5\}$ ……………….. (1)
R.H.S. $= (A \cap B) \cup (A \cap C)$

$A \cap B = \{3, 5\}$

$A \cap C = \{1, 3\}$

$(A \cap B) \cup (A \cap C) = \{3, 5\} \cup \{1, 3\} = \{1, 3, 5\}$ ……………….. (2)

From (1) and (2), we conclude that;

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [*verified*]

- **Let $A = \{a, b, d, e\}$, $B = \{b, c, e, f\}$ and $C = \{d, e, f, g\}$**

  (i) Verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

  (ii) Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S. $= A \cap (B \cup C)$

$B \cup C = \{b, c, d, e, f, g\}$

$A \cap (B \cup C) = \{b, d, e\}$ ……………….. (1)

R.H.S. = $(A \cap B) \cup (A \cap C)$

$A \cap B = \{b, e\}$

$A \cap C = \{d, e\}$

$(A \cap B) \cup (A \cap C) = \{b, d, e\}$ ……………….. (2)

From (1) and (2), we conclude that;

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [*verified*]

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S. = $A \cup (B \cap C)$

$B \cap C = \{e, f\}$

$A \cup (B \cap C) = \{a, b, d, e, f\}$……………….. (1)

R.H.S. = $(A \cup B) \cap (A \cup C)$

$A \cup B = \{a, b, c, d, e, f\}$

$A \cup C = \{a, b, d, e, f, g\}$

$(A \cup B) \cap (A \cup C) = \{a, b, d, e, f\}$……………….. (2)

From (1) and (2), we conclude that;

$A \cup (B \cap C) = A \cup B \cap (A \cup C)$ [*verified*]

- **If universal set U = {1,2,3,4,5,6}, A={1,3} and B={4,5,6} then prove De morgan's law of intersection.**

  Solution:

  De morgan's law of intersection states that,

  $(A \cap B)' = A' \cup B'$

$(A \cap B)' = (\{1,3\} \cap \{4,5,6\})' = (NULL)' = \{1,2,3,4,5,6\}$(As complement of empty set is the universal set)

$A' \cup B'$
$= (\{1,3\})' \cup (\{4,5,6\})'$

$= \{2,4,5,6\} \cup \{1,2,3\} = \{1,2,3,4,5,6\}$

Hence, it is proved that $(A \cap B)' = \{1,2,3,4,5,6\} = A' \cup B'$