

SET Theory Word Problems

\cup -----> union (or)

\cap -----> intersection (and)

The number of elements in a finite set is called cardinal number of a set or cardinality of set.

Represented by $n()$.

$$A = \{1, 3, 4\}$$

$$n(A) = \text{cardinality of set } A = 3$$

$$B = \{1, 3, 5, 7, 8\}$$

$$n(B) = \text{cardinality of set } B = 5$$

Theorem 1 :

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$A = \{1, 2, 3\}$$

$$B = \{3, 4, 5\}$$

$$A \cup B = \{1, 2, 3, 4, 5\}$$

$$n(A \cup B) = 5$$

$$n(A) = 3$$

$$n(B) = 3$$

$$n(A \cap B) = 1$$

A and B are sets

$n(A \cup B)$ = Total number of elements related to any of the two sets A & B.

$n(A)$ = Total number of elements related to set A.

$n(B)$ = Total number of elements related to set B.

$n(A \cap B)$ = Total number of elements related to both sets A & B

For two events A & B, we have

$$n(A - B) = n(A) - n(A \cap B)$$

Total number of elements related to set A only.

$$n(B - A) = n(B) - n(A \cap B)$$

Total number of elements related to set B only.

$$n(A \cup B)' = n(U) - n(A \cup B)$$

Total no of elements which are neither in set A nor in set B

Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$, find $n(A \cap B)$.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B). \text{ then}$$

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$= 20 + 28 - 36$$

$$= 48 - 36$$

$$= 12$$

In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coldrink and hot drink?

Let A = Set of people who like cold drinks B = Set of people who like hot drinks Given,
 $n(A \cup B) = 60$

$$n(A) = 27$$

$$n(B) = 42$$

$$n(A \cap B) = ?$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$60 = 27 + 42 - n(A \cap B)$$

$$n(A \cap B) = 27 + 42 - 60$$

$$= 27 + 42 - 60$$

$$= 69 - 60 = 9$$

$$= 9$$

Therefore, 9 people like both tea and coffee.

In a group of 100 persons, 72 people can speak English and 43 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French?

Solution: Let A be the set of people who speak English.

B be the set of people who speak French.

A - B be the set of people who speak English and not French.

B - A be the set of people who speak French and not English.

$A \cap B$ be the set of people who speak both French and English.

Given,

$$n(A) = 72$$

$$n(B) = 43$$

$$n(A \cup B) = 100$$

$$\text{Now, } n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$= 72 + 43 - 100$$

$$= 115 - 100$$

$$= 15$$

Therefore, Number of persons who speak both French and English = 15

$$n(A - B) = n(A) - n(A \cap B)$$

$$= 72 - 15$$

$$= 57$$

and

$$n(B - A) = n(B) - n(A \cap B)$$

$$= 43 - 15$$

$$= 28$$

Therefore, Number of people speaking English only = 57

Number of people speaking French only = 28

In a school, all pupils play either Hockey or Football or both. 300 play Football, 250 play Hockey and 110 play both the games. find :

(i) The number of pupils who play Football only; $n(F-H)$

(ii) The number of pupils who play Hockey only; $n(H-F)$

(iii) The total number of pupils in the school. $n(H \cup F)$

H = Hockey and F = Football

$$n(H) = 250 \quad n(F) = 300$$

$$n(H \cap F) = 110$$

The number of pupils who only play Football = $n(F - H)$

$$n(F - H) = n(F) - n(H \cap F)$$

$$= 300 - 110$$

$$= 190$$

The number of pupils who only play Hockey = $n(H - F)$

$$n(H - F) = n(H) - n(H \cap F)$$

$$= 250 - 110$$

$$= 140$$

The total number of pupils in school

$$= n(H) + n(F) - n(H \cap F)$$

$$= 250 + 190 - 110$$

$$= 330$$

In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee? Find number of students who taking tea only and coffee only.

Only tea = 50 (T-C)

only coffee = 125 (C-T)

neither tea nor coffee = 325

T = tea

C = coffee

$$n(T) = 150$$

$$n(C) = 225$$

$$n(T \cap C) = 100$$

$$n(T \cup C) = n(T) + n(C) - n(T \cap C) = 150 + 225 - 100$$

$$n(T \cup C) = 275$$

$$n(T \cup C)' = n(U) - n(T \cup C)$$

$$= 600 - 275 = 325$$

**Of the 100 people of a hostel 80 drink tea, 40 drink coffee and 25 drink tea and coffee both. How many of them drink neither tea nor coffee?
Find no of students take only tea and only coffee?**

$$n(T)=80$$

$$n(C)=40$$

$$n(T \cap C)=25$$

$$n(U)=100$$

$$n(T \cup C)' = ?$$

$$n(T \cup C)=95$$

$$\text{only coffee } 15$$

$$\text{only tea } 55$$

$$n(T \cup C)' = 5$$

n

Total number of students = U ; T = tea and C = coffee

$$n(U) = 600 \quad n(T) = 150 \quad n(C) = 225 \quad \text{and} \quad n(T \cap C) = 100$$

Students who take only tea = $n(T - C)$

$$n(T - C) = n(T) - n(T \cap C)$$

$$= 150 - 100$$

$$n(T - C) = 50$$

Students who take only coffee = $n(C - T)$

$$n(C - T) = n(C) - n(T \cap C)$$

$$= 225 - 100$$

$$n(C - T) = 125$$

Students were taking neither tea nor coffee is a complement of $n(T \cup C) = n(T \cup C)'$

$$n(T \cup C) = n(T) + n(C) - n(T \cap C)$$

$$= 150 + 225 - 100 = 275$$

$$n(T \cup C)' = n(U) - n(T \cup C)$$

$$= 600 - 275$$

$$n(T \cup C)' = 325$$

At a school of 500 students, there are 125 students enrolled in Algebra, 257 students who play sports and 52 students that are enrolled in Algebra and play sports.