Unit 4 - Function, limits and continuity

Function :
A function ***relates*** an input to an output:

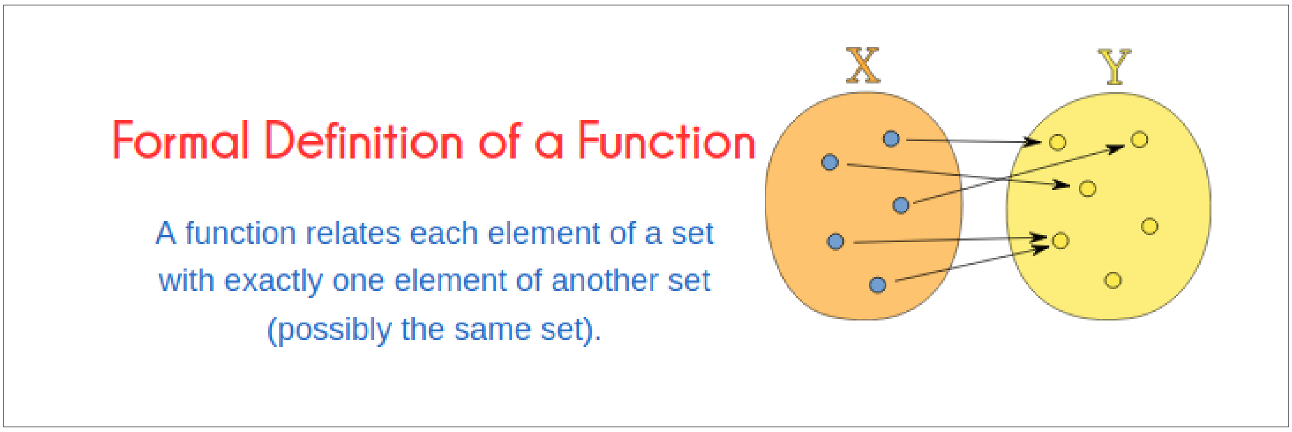
input is 1
Rule is square of input
output is 1*1=1

input is 2
Rule is square of input
output is 2*2 =4

Input set X
X={1,2,3,4,5,-2}

Rule = square of input
y=f(x) = $x^2$ // function form

Output set
y={1,4,9,16,25}

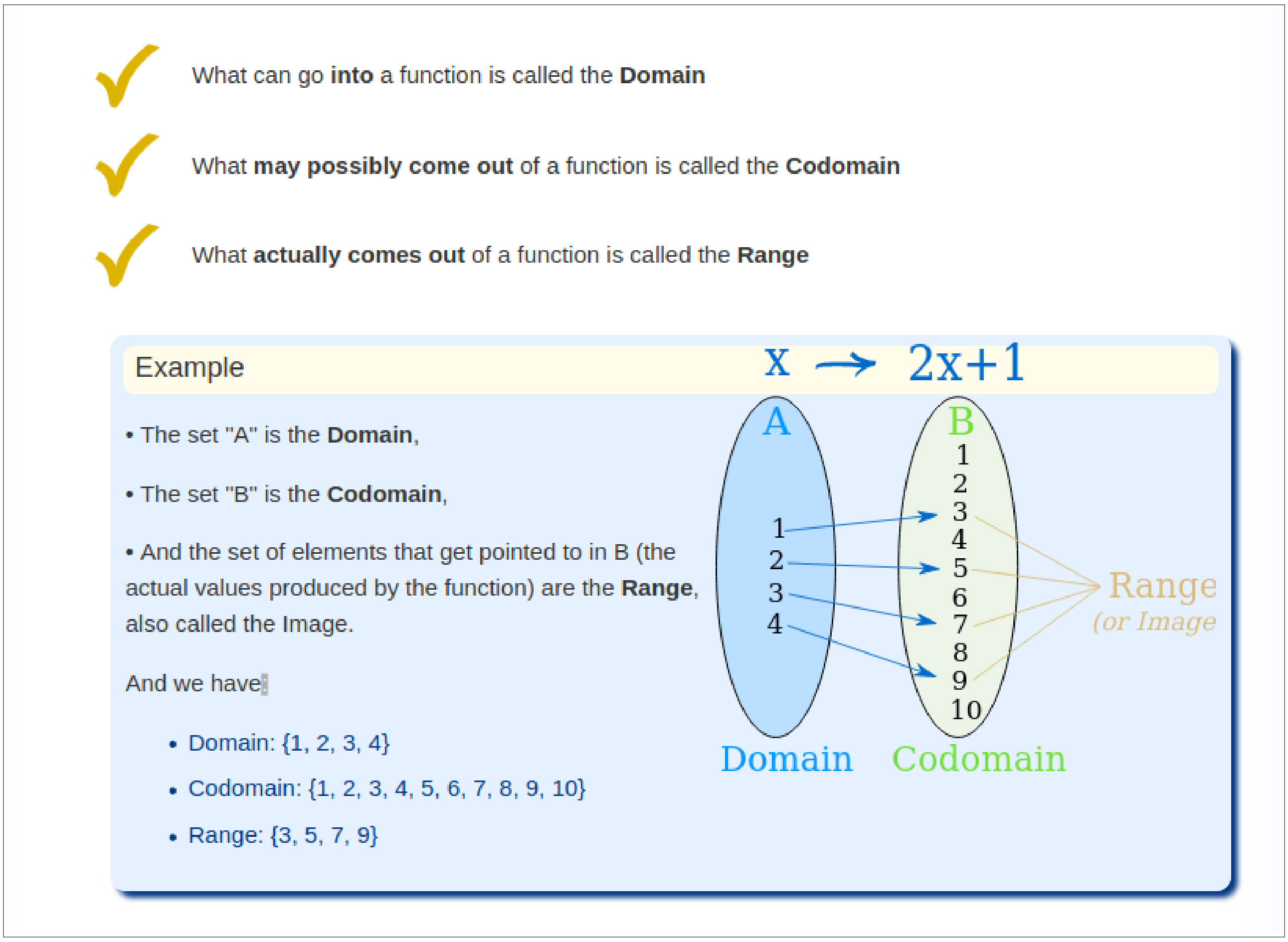



X={2,4,6}
y=f(x)= 5x
f(2)=10
f(4)=20
f(6)=30
Output set
y=f(x)= {10,20,30 }
If f(x)= 2x, then f(-6)= -12

# Domain, Codomain and Range

A= {1,2,3,4}

B= {1,2,3,4,5,6,7,8,9,10}




A= {1,2,3,4} //input set // Domain
B= {1,2,3,4,5,6,7,8,9,10} // possible output set //Co domian

f(x) = 2x+1 // function or rule
f:A->B function defintion // function from A to B
A= input set ( Domain) B= genral Output set (Co – Domain)

f(1) = 2(1) + 1 =3
f(2)= 2(2) + 1= 5
f(3)= 2(3) + 1= 7
f(4)= 2(4) + 1= 9

Range $R_f$ = {3,5,7,9} function images

$f:B\text{->}A$ // function from B to A
B set input (domain)
A set genral output (co domain)
Range

$f(1) = 2(1) + 1 = 3$
$f(2)= 2(2) + 1= 5$
$f(3)= 2(3) + 1= 7$
$f(4)= 2(4) + 1= 9$

Range $R_f = \{3,5,7,9\}$

Domain= set A = {1,2,3,4}
Co Domain= set B = { 1,2,3,4,5,6,7,8,9,10}
Range = { 3,5,7,9}

A function is a rule that assigns each element of a set, called the domain, to exactly one element of a second set, called the codomain.

Notation: $f:X\rightarrow Y$ is our way of saying that the function is called f, the domain is the set X, and the codomain is the set Y

$f(x)=y$ means the element x of the domain (input) is assigned to the element y of the codomain. We say y is an output. Alternatively, we call y the image of x under f.

The range is a subset of the codomain. It is the set of all elements which are assigned to at least one element of the domain by the function. That is, the range is the set of all outputs.

We would write $f:A\rightarrow B$ to describe a function with name f, domain X and codomain Y.
$f:N\rightarrow N$
$f(x)=x^2+3$.
A= {1,2,3,4,5,6,----}
B= {1,2,3,4,5,6,----}
Example 1:
If $f(x) = 2x^2+3x-7$ then find f(1), f(2), f(3), f(-2).

Examle 2:
A= { 2,4,3}
B= { 1,2,3,4,5,6,7,8,9,10----------------------------50}
$f(x)= 2x^2+3x-7$
f: A->B
find domain, co dmain and range
domain= A, codomain= B, range={ 7, 20, 37}
$f(2) = 2.2^2 + 3.2 – 7 = 8+6-7=7$
$f(4) = 2.4^2 + 3.4 – 7 = 32 + 12 – 7= 20$
$f(3)= 2.3^2 + 3.3 – 7 = 18 + 9 +12 = 37$

**Types of function**
Constant
Identity
Moduls
Greatest Integer

A **constant function** is a linear function for which the range does not change no matter which member of the domain is used.
f(x)=C it is constant value.

A={1,2,3,4} f:A->A
f(x)= 3.
find the range.
f(x)=3
f(1)= 3
f(2)= 3
f(3)=3
f(4)=3
Rf= {3,3,3,3}

$f(x)=3$ .

**Identity Function** is defined as the real valued function.

f(x) = x
A={1,2,3}
f(x)= x
find the range.
F(1)=1
f(2)=2
f(3)=3
{1,2,3}

# The Modulus Function

The modulus of any number gives us the magnitude of that number. Using the modulus operation, we can define the modulus function as follows:

f(x)=|x|

A= {1,-1,2,-3}

f(x)= |x|

f(1)= |1| = 1

f(-1)= |-1| = 1

f(2)= |2|=2

f(-3)=|-3|=3

range = {1,2,3}

# Greatest Integer Function

Greatest Integer Function [X] indicates an integral part of the real number $x$ which is nearest and smaller integer to $x$. It is also known as **floor of X** .

*f*(*x*)=[x]

**Input**: X = 2.3= 2-3
**Output**: [2.3] = 2

**Input**: X = -8.0725 = -8 to -9
**Output**: [-8.0725] = -9

**Input**: X = 2
**Output**: [2] = 2

1.5 = 1 to 2 =1

[6.2] = 6 to 7 = 6
[-6.2] = -6 to -7 = -7
in negative, -6 > -7, - 5 > - 6

If f(x)=[x], then f(-4.89)=_____-5_
If f(x)= x, then f(4)=4
If f(x)=|x|, then f(-4)=4
If f(x)= 3, then f(-3)=3
0.75 = 0 to 1
0.25 = -1 to 0

| $x$ | $\lfloor x \rfloor$ |
|---|---|
| $-1.5$ | $-2$ |
| $-1.25$ | $-2$ |
| $-1$ | $-1$ |
| $-0.75$ | $-1$ |
| $-0.5$ | $-1$ |
| $-0.25$ | $-1$ |
| $0$ | $0$ |
| $0.25$ | $0$ |
| $0.5$ | $0$ |
| $0.75$ | $0$ |
| $1$ | $1$ |
| $1.25$ | $1$ |
| $1.5$ | $1$ |