Symmetric Matrix

A **[square Matrix](#) A** is said to be **symmetric** if $\mathbf{aij=aji}$for all i and j, whereaij is an element present at $(i,j)th$ position (ith row and jth column in **matrix A**) and aji is an element present at $(j,i)th$ position (jth row and ith column in **matrix A**).

In other words, we can say that **matrix A** is said to be **symmetric** if **transpose** of **matrix A** is equal to **matrix A** itself ($A^T=A$).

A=

1	2	5
2	5	-7
5	-7	3

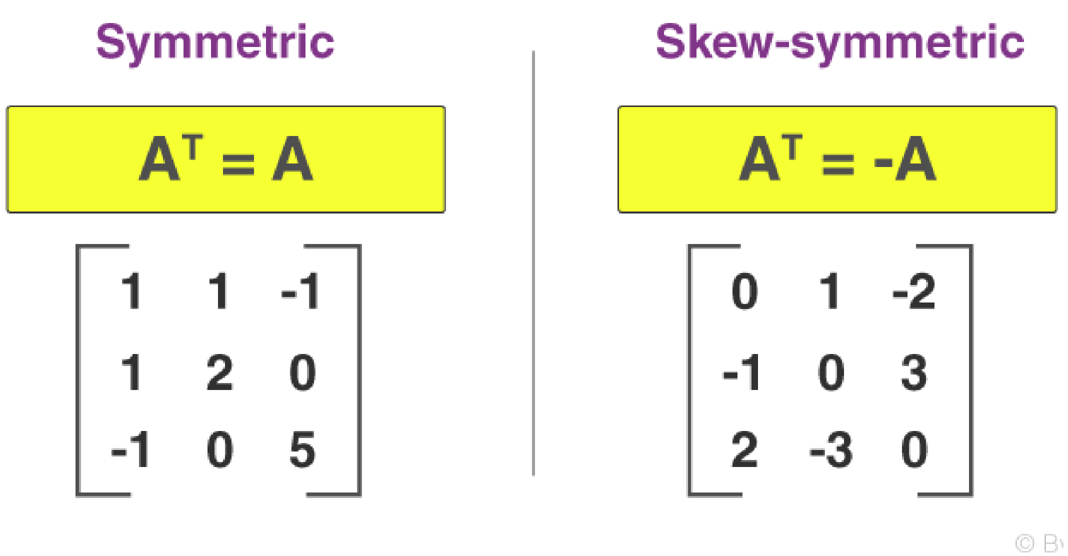
It is **symmetric matrix** because$aij=aji$for all i and j
Example, $a12=a21=2,a13=a31=5$ and$a23=a32=-7$

In other words, **transpose** of **Matrix A** is equal to **matrix A**itself ($A^T=A$) which means **matrix A** is **symmetric**.

Skew-Symmetric Matrix

Square matrix A is said to be skew-symmetric if $a_{ij}=-a_{ji}$ for all i and j. In other words, we can say that matrix A is said to be skew-symmetric if transpose of matrix A is equal to negative of matrix A i.e ($A^T=-A$). Note that all the main diagonal elements in the skew-symmetric matrix are zero.

0	-5	4
5	0	-1
-4	1	0



Above is skew-symmetric matrix because $a_{ij} = -a_{ji}$ for all i and j and leading diagonal elements are zero.

$A = -A^T$ and leading diagonal elements are zero.

Singular and Non singular Matrix

Non Singular Matrix

A square matrix A is said to be non-singular if $|A| = \det(A) \neq 0$

Singular Matrix

A matrix is singular if its determinant is zero.

A 2 x 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is singular if its determinant $ad - bc = 0$

A 3 x 3 matrix $\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$ is singular if its determinant

$$a_1 \bullet \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \bullet \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \bullet \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = 0$$

$$a_1 (b_2 c_3 - b_3 c_2) - a_2 (b_1 c_3 - b_3 c_1) + a_3 (b_1 c_2 - b_2 c_1) = 0$$

Properties of determinant

- If all the elements of a row (or column) are zeros, then the value of the determinant is zero.
- Determinant of a Identity matrix (I_n) is 1
- If two rows (or columns) of a matrix are identical the value of the determinant is zero.
- If rows and columns are interchanged then value of determinant remains same (value does not change). Therefore, det(A) = det(A^T), here A^T is transpose of matrix A.

  $|A| = |A^T|$
- Let A and B be two matrix, then **det(AB) = det(A)*det(B)**.

  |AB|=|A|*|B|

Determinant of a Matrix

The determinant of a matrix is a **special number** that can be calculated from a square matrix.

Determinant can be calculated only and only of square matrix.

A Matrix is an array of numbers:

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix}$$

leading diagonal elements are: 3 and 6

A Matrix
(This one has 2 Rows and 2 Columns)

The determinant of that matrix is (calculations are explained later):

$3\times6 - 8\times4 = 18 - 32 = -14$

Symbol

The **symbol** for determinant is two vertical lines either side.

Example:

|A| means the determinant of the matrix A

For a 2×2 Matrix

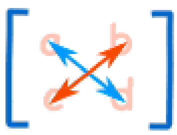
For a 2×2 matrix (2 rows and 2 columns):

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The determinant is:

first multiply leading diagonal values – multiply other diagonal values

determinant of A= det(A)=|A| = ad − bc



- Blue is positive (+ad),
- Red is negative (−bc)

Example:

$$B = \begin{bmatrix} 4 & 6 \\ 3 & 8 \end{bmatrix}$$

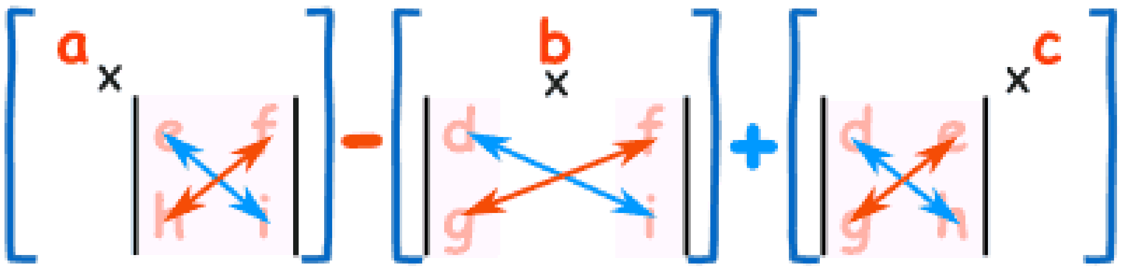
$$\begin{aligned} |B| &= 4\times8 - 6\times3 \\ &= 32 - 18 \\ &= 14 \end{aligned}$$

For a 3×3 Matrix

For a 3×3 matrix (3 rows and 3 columns):

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

The determinant is:



$$|A| = a \cdot \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \cdot \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \cdot \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

|A| = a.(ei − fh) − b.(di − fg) + c.(dh − eg)

$$|A| = a \cdot \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \cdot \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \cdot \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$C = \begin{bmatrix} 6 & 1 & 1 \\ 4 & -2 & 5 \\ 2 & 8 & 7 \end{bmatrix}$$

$$\begin{aligned} |C| &= 6\times(-2\times7 - 5\times8) - 1\times(4\times7 - 5\times2) + 1\times(4\times8 - (-2\times2)) \\ &= 6\times(-54) - 1\times(18) + 1\times(36) \\ &= \mathbf{-306} \end{aligned}$$

Example:

$$\begin{aligned} \det\begin{bmatrix} -5 & 0 & -1 \\ 1 & 2 & -1 \\ -3 & 4 & 1 \end{bmatrix} &= -5\cdot\det\begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix} - (0)\cdot\det\begin{bmatrix} 1 & -1 \\ -3 & 1 \end{bmatrix} + (-1)\cdot\det\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} \\ &= -5[2-(-4)] - 0[1-(3)] - 1[4-(-6)] \\ &= -5(2+4) - 0 - 1(4+6) \\ &= -5(6) - 1(10) \\ &= -30 - 10 \\ &= -40 \checkmark \end{aligned}$$

Minors and co factors of matrix

Write Minors and Cofactors of the elements of following determinants: (i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$$\text{Minor of } a_{11} = M_{11} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1(1) - 0 = 1$$

$$\text{Minor of } a_{12} = M_{12} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$$

$$\text{Minor of } a_{13} = M_{13} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$$

$$\text{Minor of } a_{21} = M_{21} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$$

$$\text{Minor of } a_{22} = M_{22} = \begin{vmatrix} 1 & \cancel{0} & 0 \\ \cancel{0} & \cancel{1} & \cancel{0} \\ 0 & \cancel{0} & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$\text{Minor of } a_{23} = M_{23} = \begin{vmatrix} 1 & 0 & \cancel{0} \\ \cancel{0} & \cancel{1} & \cancel{0} \\ 0 & 0 & \cancel{1} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$$

$$\text{Minor of } a_{31} = M_{31} = \begin{vmatrix} \cancel{1} & 0 & 0 \\ \cancel{0} & 1 & 0 \\ \cancel{0} & \cancel{0} & \cancel{1} \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0$$

$$\text{Minor of } a_{32} = M_{32} = \begin{vmatrix} 1 & \cancel{0} & 0 \\ 0 & \cancel{1} & 0 \\ \cancel{0} & \cancel{0} & \cancel{1} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$$

$$\text{Minor of } a_{33} = M_{33} = \begin{vmatrix} 1 & 0 & \cancel{0} \\ 0 & 1 & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{1} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

Minor matrix:

1	0	0
0	1	0
0	0	1

This matrix is fixed

+1	-1	+1
-1	+1	-1
+1	-1	+1

Co factor matrix:

1	-0=0	0
-0=0	1	-0=0
0	-0=0	1

1	0	0
0	1	0
0	0	1

Adjoint matrix= transpose of co factor matrix

transpose means = interchange of rows and columns

1st row of co factor matrix = 1st column of adjoint matrix
2nd row of co factor matrix = 2nd column of adjoint matrix
3rd row of co factor matrix = 3rd column of adjoint matrix

Adjoint matrix=

1	0	0
0	1	0
0	0	1

Minor, co factor and adjoint of 2X 2 matrix
A=

2	3
1	5

Minor Matrix=

swap the elements of both diagonals

5	1
3	2

You have to change the sign in Elements of other diagonal (not leading digonal)

+1	-1
-1	+1

Co factors =

5	-1
-3	2

Adjoint matrix= transpose of co factor matrix

5	-3
-1	2

A formula for finding the inverse

$$\boxed{A^{-1} = \frac{\operatorname{adj} A}{|A|}}$$

A square matrix A has an inverse, if |A| is not equal to 0.

To find inverse of matrix:

- Matrix shoud be square
- Det(MAtrix) is not equal to 0 0r |A| Not equal to 0