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Graph

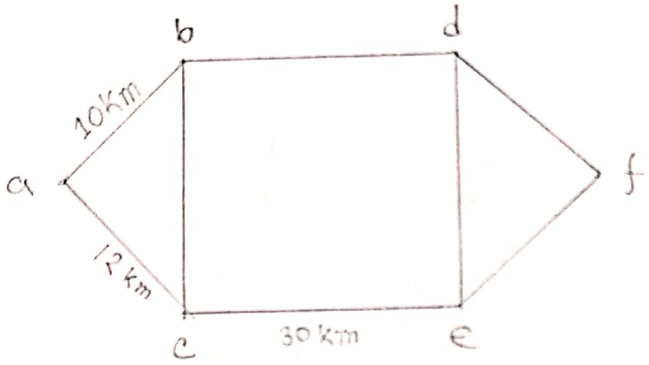
$G_1 = (V, E)$

objects — Relation

V = set of vertices

E = set of edges

1. Friends
2. Roads



$V = \{a, b, c, d, e, f\}$

$F = ((a,b), (b,d), (a,c), (e,f), (c,e), (d,e), (d,f), (b,c))$

1. Order of graph:

No. of vertices of graph $|V| = 6$

2. Size of graph:

edges: $|E| = 8$

1. Order of graph
2. Size of graph

Sub : Lecture - 2 : Graph Representation

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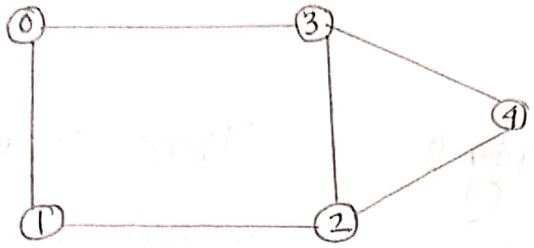


Matrix:

	0	1	2	3	4
0	0	1	0	1	0
1	1	0	1	0	0
2	0	1	0	1	1
3	1	0	1	0	1
4	0	1	0	1	0

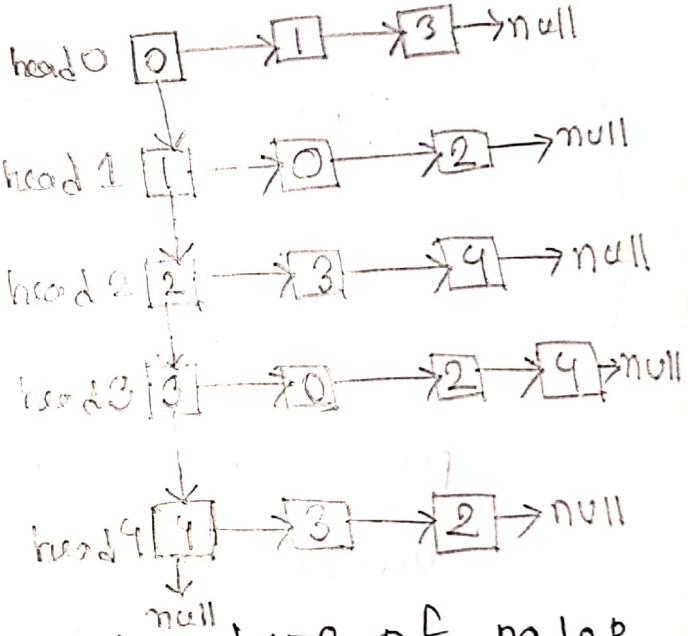
Sparce Matrix

Size = n x n

This Representation waste space.

So, Adjacency List comes.

Adjacency List



Structure of node:

{ int data,
vertex
edge
}

Sub: Lecture - 2 : Graph Representation

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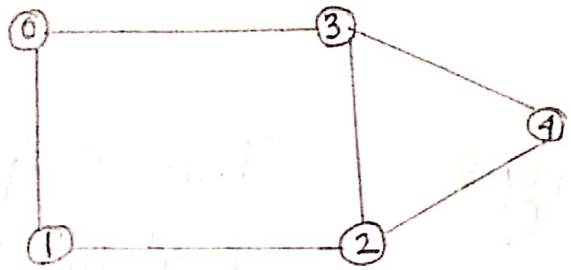


Matrix:

	0	1	2	3	4
0	0	1	0	1	0
1	1	0	1	0	0
2	0	1	0	1	1
3	1	0	1	0	1
4	0	1	0	1	0

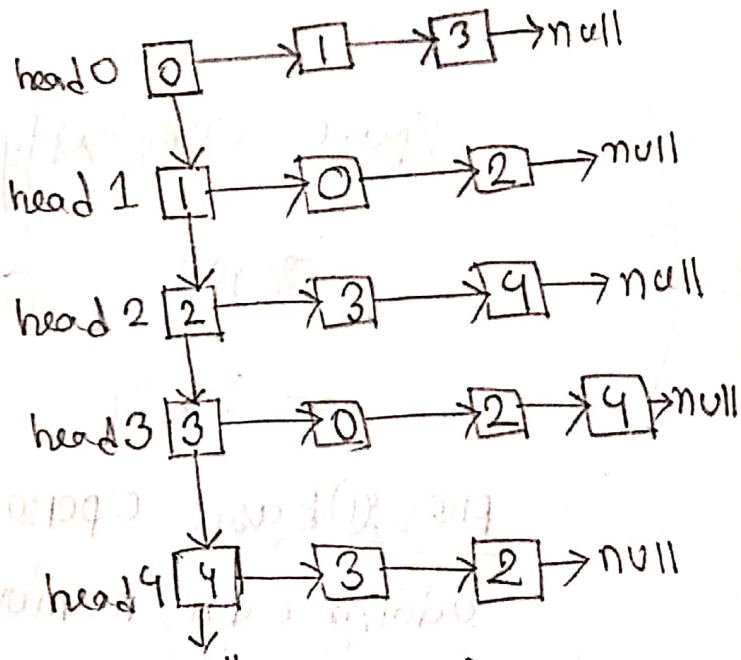
Space Matrix

Size = n x n

This Representation waste space.

So, Adjacency List comes.

Adjacency List



Structure of node:

```
{ int data,
  vertex
  edge
}
```

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Matrix

Time complexity:

$O(1)$

Space complexity:

$O(n^2)$

$= n \times n$

pros: (i) Basic operation: adding edge, removing and checking whether there is an edge from vertex i to vertex j are extremely efficient. constant time

(ii) If the graph is dense and num of edges is close the |V|²

Adjacency List

Time Complexity:

worst case: $O(|V|+|E|)$

Space complexity:

$O(|V|+|E|)$

Sub :

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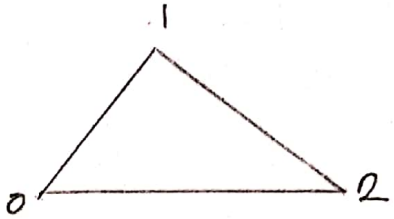
#3. Code for Matrix Representation of graph

1: Adjacency Matrix :



	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

```
public class Graph {
    private boolean adjMatrix[][];
    private int numVertices;
    public Graph(int numVertices) {
        this.numVertices = numVertices;
        adjMatrix = new Boolean[
                numVertices][numVertices];
    }
    public void addEdge(
        adjMatrix[i][j] = true
        adjMatrix[j][i] = true;
    }
```

adj

pub

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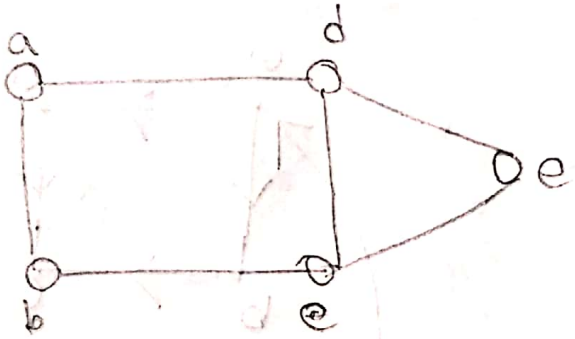
#5 Types of graphs

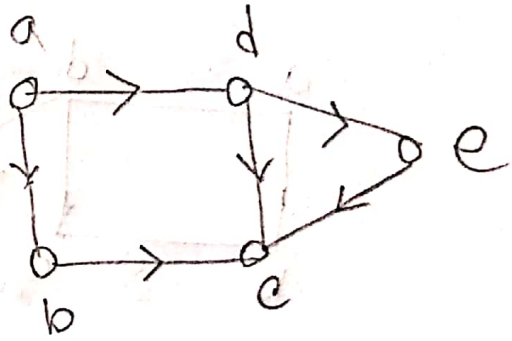
1. Undirected graph



if (x,y) then (y,x)

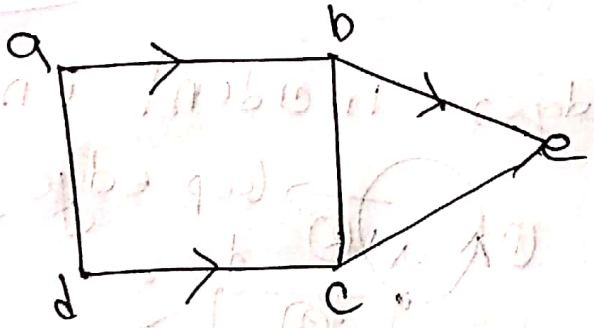
(Fb)

2. Directed graphs



if (x,y), then doesn't mean (y,x)

(Follower)



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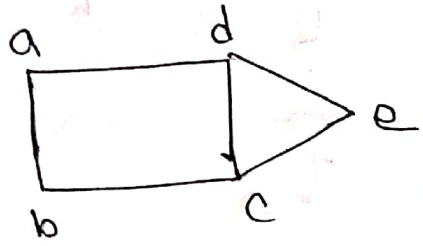
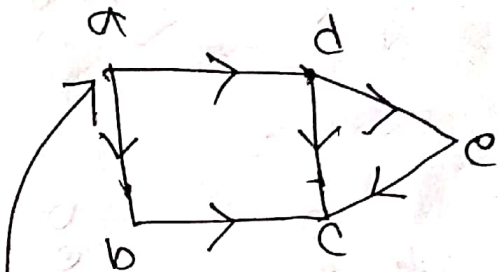
Degree of a vertex :

Undirected



Directed



Indegree

Outdegree

Undirected

Number of edges incident on a vertex

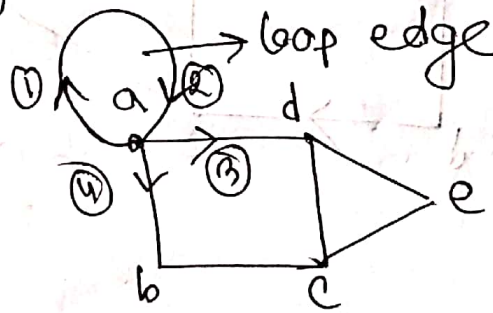
a → 2

b → 2

d → 3

c → 3

e → 2



degree is 1

total degree is = 4

Directed :

a indegree = 0

b outdegree = 1, indegree = 1

c = indegree = 3, outdeg = 0

d(I) = 1, d(O) = 2

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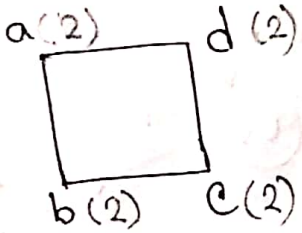
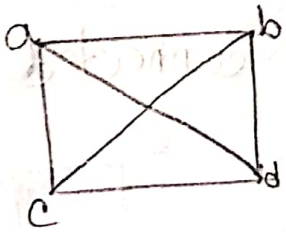
#classes of graph:

① Regular graph:

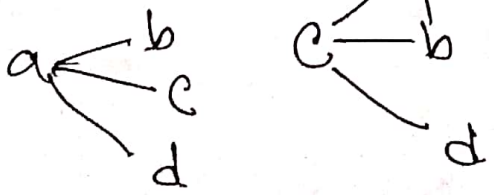


k-regular graph
(every vertex is same)
(2-regular graph)

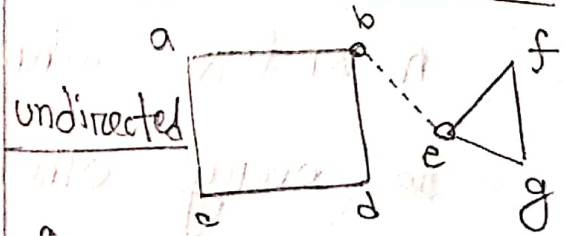
② Complete graph:

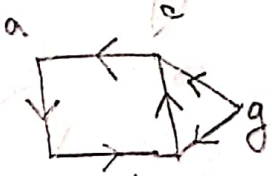


each vertex connected with each other or/ all other



③ Connected Graph:





Directed:

(#connected with a line to go each vertex)

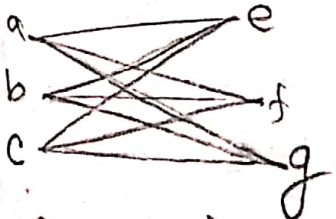
we can go all the each vertex, any vertex.

strongly connected:

4. planner graph:

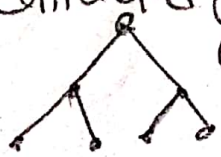
shouldnot be intersect at any edges.

⑤ Bipartite graph:



Devide two sets:

⑥ TREE: connected graph with NO cycle



(NO cycle)

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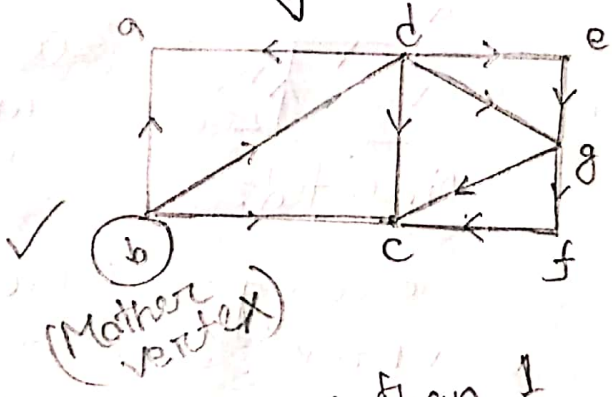
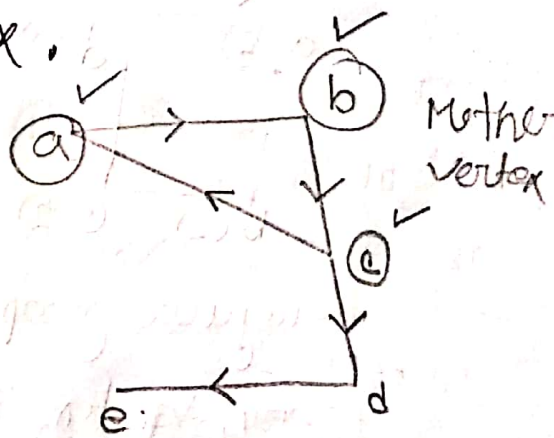
Mother vertex :

A vertex which has a path to reach to every other vertex.

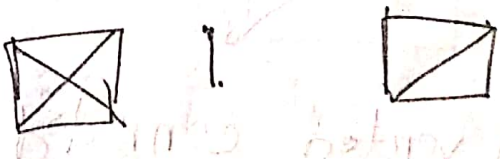


(Mother vertex)

(can more than 1 mother vertex)



Case :
1. Directed graph
2. undirected connected graph
3. undirected disconnected ,,



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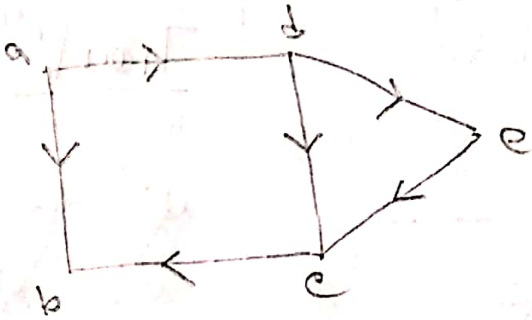
Transative clousure of a graph:



	a	b	c	d	e
a	1	1	1	1	1
b	0	1	0	0	0
c	0	1	1	0	0
d	0	1	1	1	1
e	0	1	1	0	1

Reachability Matrix

DFS algo

Floyd-warshall algo

{ (a,a), (a,b), (a,c), (a,d), (a,e)
(b,b) (b,d), (c,c),
(d,b) (d,c), (d,d), (d,e), (e,b), (e,c), (e,e) }

path from (i,j) কতগুলো path আছে।
প্রত্যেক গ্রাফ কতখানি connected আছে।

Sub: #9. Lecture

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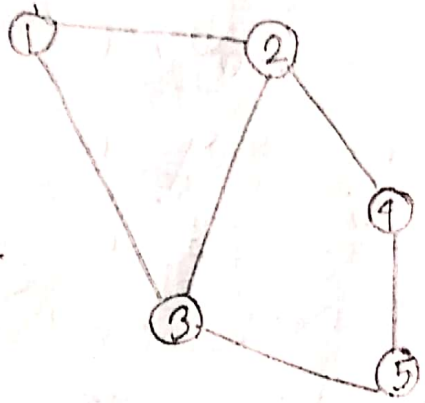
walk, Trail, Circuit, path, circle

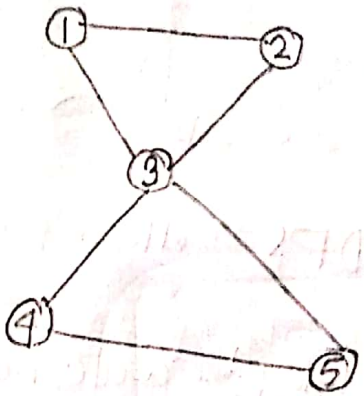
walk:



Rep- vertex
eated edge

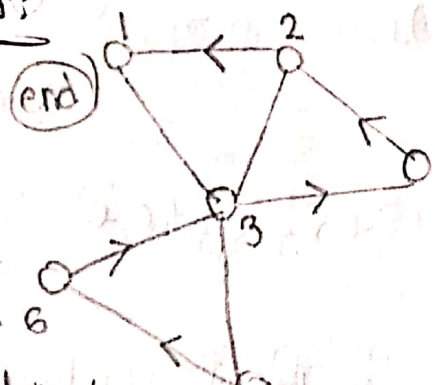
1 → 2 → 4 → 5 → 3 → 2 → 1

Trail:



Repeated vertex ✓
edge ↓
NOT Repeated

path:



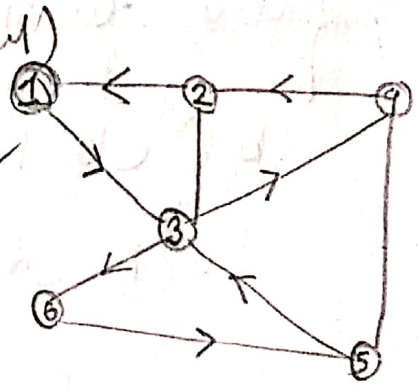
vertex: X
edges: X

No Repeated:

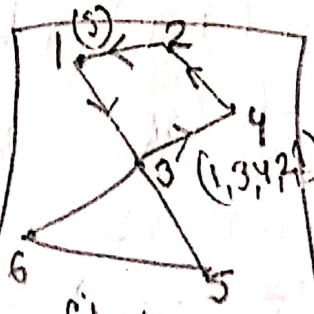
5 (Start) — 6 — 3 — 4 — 2 — 1

✓ (Many path) Containing diff edge and vertices

Circuit: Trail



Vertex ✓
edge



Cycle:

Same as path
But closed (S = end)
✓ vertex Repeated

Starting and ending vertex should be same

1 → 3 → 6 → 5 → 3 → 4 → 2 → 1

edge → Not

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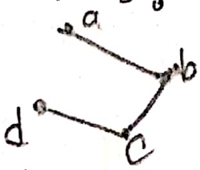
Tree and Its property:

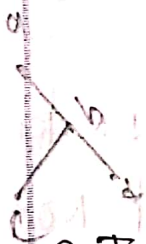
A is an undirected graph. that satisfies:

1. Tree is connected and Acyclic



2. T is acyelic and a simple cycle is formed if any edge is added to T.

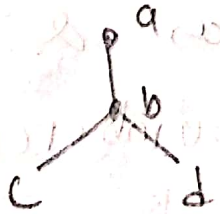
যদি আমরা কোনএকটি edge connected করি তাহলে cycle হয়ে যাবে



(abc) (abd)

3. Two vertices in T can be connected by simple unique path..

(4) T connected and has n-1 edge. (Most important property)

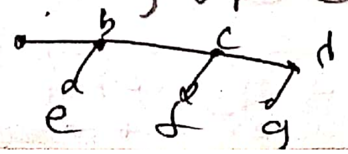


(a,b), (b,c), (b,d)

(4 টা vertices 3 টা edges)

যদি (n-1) এর বেশি একটি edge হয় তাহলে cycle হবে diameter

(5) Internal vertex & external ,, . ↓ (leaf) →



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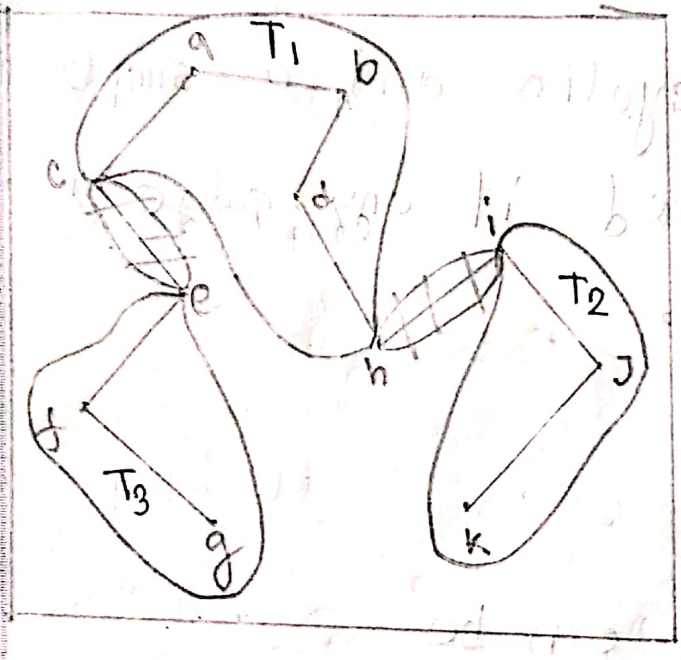
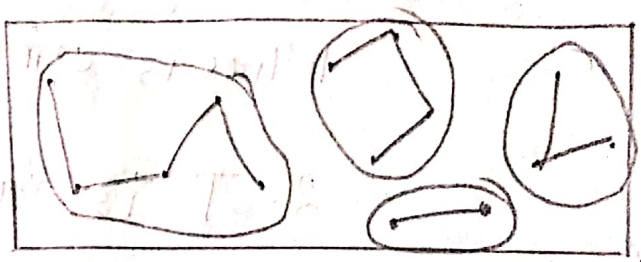
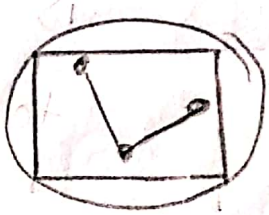
Forest in graph theory :

A forest is the disjoint union of trees.







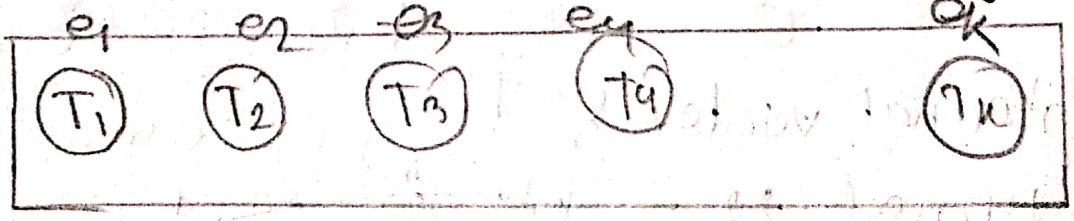
Every tree is forest

Every forest Not Tree

$\{a,b,d,h,c\}$ $\{i,j,k\}$, $\{e,f,g\}$

T_1 T_2 T_3

If there are 'n' vertices & 'k' component in a forest, then number of edges —.



$n_1 + n_2 + n_3 + n_4 + n_k = n$

$e_1 + e_2 + e_3 + e_4 + e_k = e$ or E.

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$(n_1-1)+(n_2-1)+(n_3-1)+(n_4-1)+\ldots(n_k-1) = E_F$

$(n_1+n_2+n_3+n_4+\ldots n_k-(1+1+1+1\ldots+k\text{ time}))$

$= E_F$

$(n-k) = E_F$

Total Number of edges in forest:

$(n-k) = E_F$

Total No. of Trees:

$k = n - E_F$

$= T_V - T_E$

$T_E = 6, \; T_V = 9,$

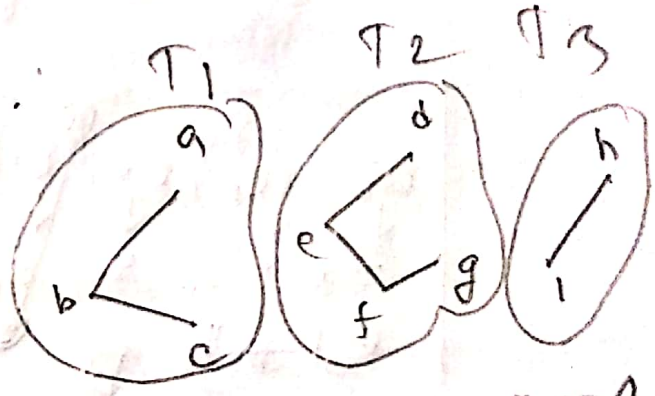
How Many tree

$k = 9 - 6$

$= 3$

n = no of vertices

k = edge vertex



$n_1 = 3 \quad n_2 = 4 \quad n_3 = 2$

$k = 3$

$n = n_1 + n_2 + n_3$

$= 3+4+2$

$= 9$

$E_F = (n-k)$

$= 9-3 = 6$

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Sub :

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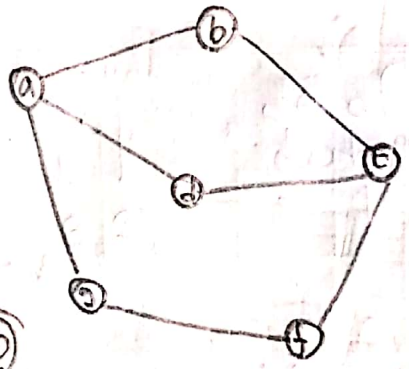
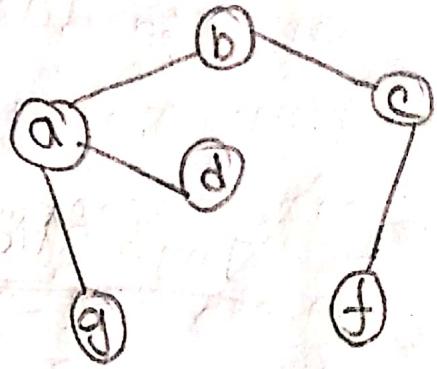
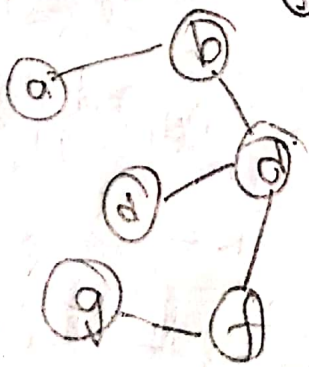
Spanning tree:

A graph which contains all vertices with minimum number of edges

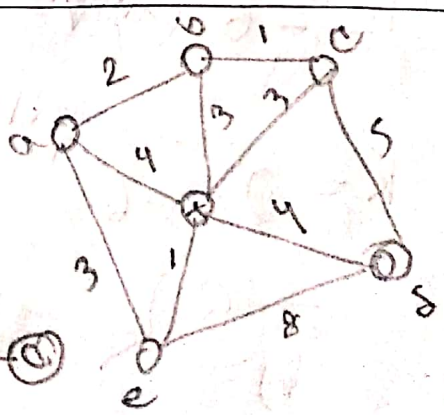
1. A spanning tree subgraph
2. A Tree

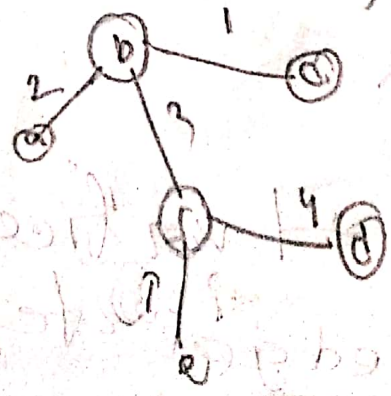






Minimum Spanning tree:





1. Prims Algo
2. Kruskal's algo

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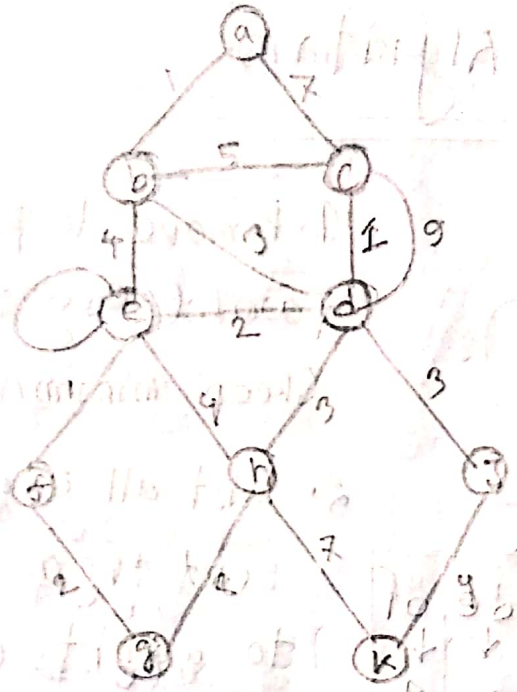
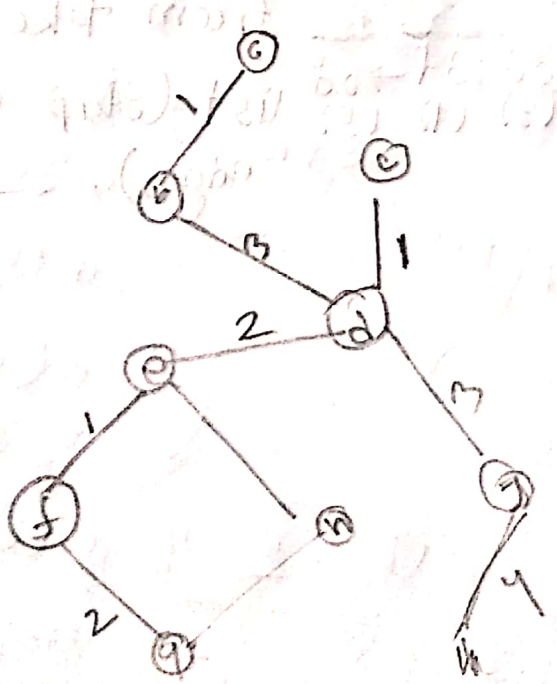
prims Algorithms





steps:

1. **Remove loops:** and parallel edge (keep min weight)
2. while adding new edge, select edge with minimum weight out of the edges from visited vertex (no cycle allowed)
3. stop at (n-1) edges

visited vertices:

a, b, d, c, e, f, g, J

visited edges: ab, ac, bc, db, de, bf, dh, hj, fk, gh, fg

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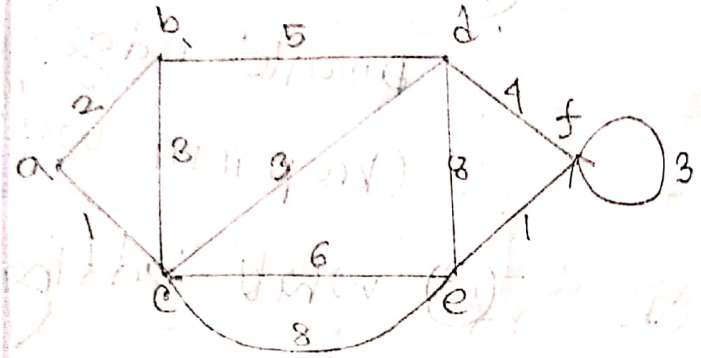
(a)

Kruskal's Algorithm

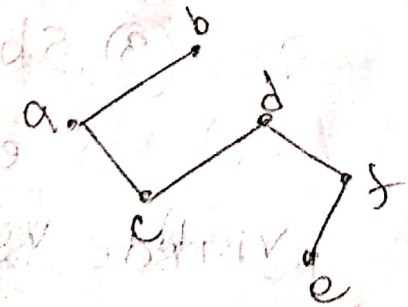


1. Remove loops and parallel edges. (keep minimum weight)
2. List all edges and sort them according to weights (ascending)
3. Take 'n-1' edges from take sorted list (skip cycle making edges).

ab,	ac,	bd,	bc	cd	ce,	de	df
↓	↓	↓	↓	↓	↓	↓	↓
2	1	5	3	3	6	8	4
ef → 1							

ac → ef → ab → bc → cd → df → bd
(1) (1) (2) (3) (3) (4) (5)

ce → de
(6) (8)



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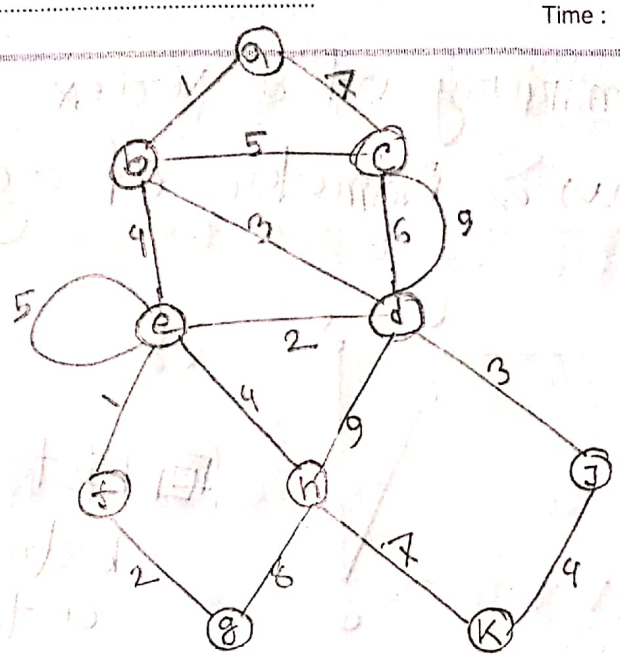
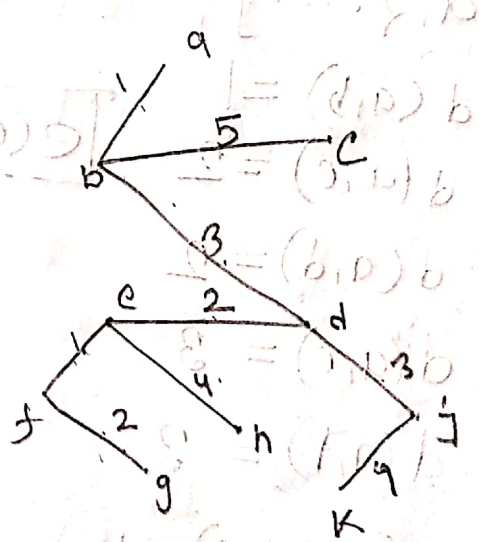


ab – 1	ab – 1
ac – 7	ef – 1
bc – 5	de – 2
bd – 3	fg – 2
be – 4	bd – 3
cd – 6	dj – 3
de – 2	~~be – 4~~
dh – 9	eh – 4
dj – 3	kj – 9
ef – 1	bc – 5
eh – 4	cd – 6
~~fg – 2~~	~~ac – 7~~
gh – 8	~~hk – 7~~
hk – 7	~~cd~~
kj – 4	gh – 8
	dh – 9

n-vertices = 10

No of ~~edges~~ = 9 $|n-1|=9$



$(1+5+3+3+4+2+1+4+2)$

$= 25$

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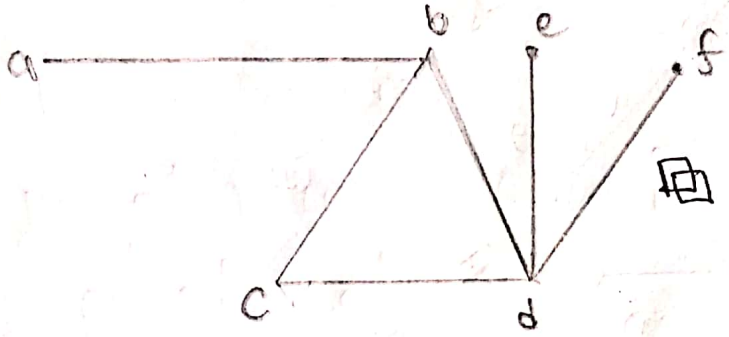
min value of Radius
e(a)(f) = 2

Eccentricity of a vertex
Radius & Diameter of a graph



- Distance between vertex a to f.

Eccentricity — e(a) = maximum distance between a, {. . . . }

- abcdf - 3
- abdf - 4
- d(a,b) shortest path

d(a,b) = 1
d(a,c) = 2
d(a,d) = 2
d(a,e) = 3
d(a,f) = 3

e(a) = 3

e(a) = 3
e(b) = 2
e(c) = 2
e(d) = 2
e(e) = 3
e(f) = 3

R = 2

e(b) { d(b,a) = 1, d(b,c) = 1, d(b,d) = 1, d(b,f) = 2, d(b,e) = 2 } e(b) = 2

e(c) { d(c,a) = 2, d(c,b) = 1, d(c,d) = 1, d(c,f) = 2 } 2

e(d) { d(d,e) = 2, d(d,b) = 1, d(d,c) = 1, d(d,a) = 2, d(d,f) = 1 } = 1

Sub: #16. Eular graph

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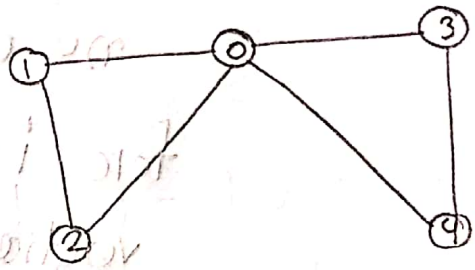
Eular circuit:

A trail which starts and ends at same vertex

Trail – A walk in which no edge is repeated

walk – Any random traversal in graph.



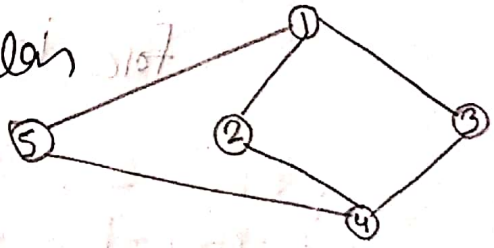
✓

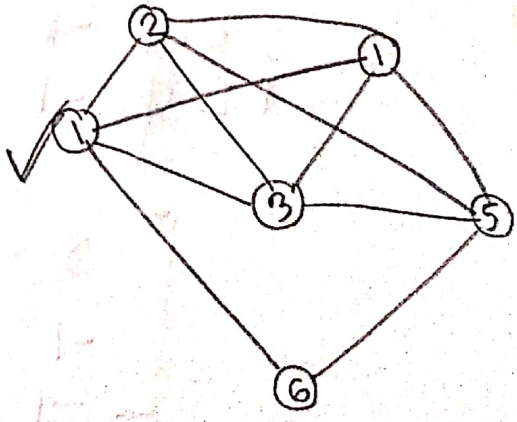
Every vertex in the graph have even degree

even degree – Eular

odd degree – Semi eular

×





Kirchhoff's Matrix-Tree theorem

Find Number of spanning trees.

1. Make a matrix.

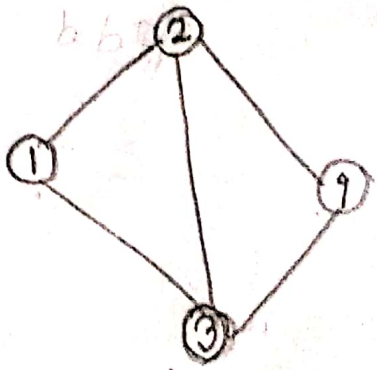
No of rows = no of column = no of vertices

for $i \neq j$, if i & j are adjacent vertices, then $M[i][j] = 1$

if, not $M[i][j] = 0$

for $i = j$, $M[i][j]$ = degree of vertex

i \ j	1	2	3	4
1	2	-1	-1	0
2	-1	3	-1	-1
3	-1	-1	3	-1
4	0	-1	-1	3



2. Delete any one column & find determinant

$$\begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 2 \end{bmatrix} = 3\left[(3\times 2) - (-1)(-1)\right] - (-1)\left[(-1)2 - (-1)(-1)\right] + (-1)\left[(-1)(-1) - (-1)(3)\right]$$

$= 8$