Storyline

Chapter 10:
Rotation of a Rigid Object About a Fixed Axis

Physics for Scientists and Engineers, 10e

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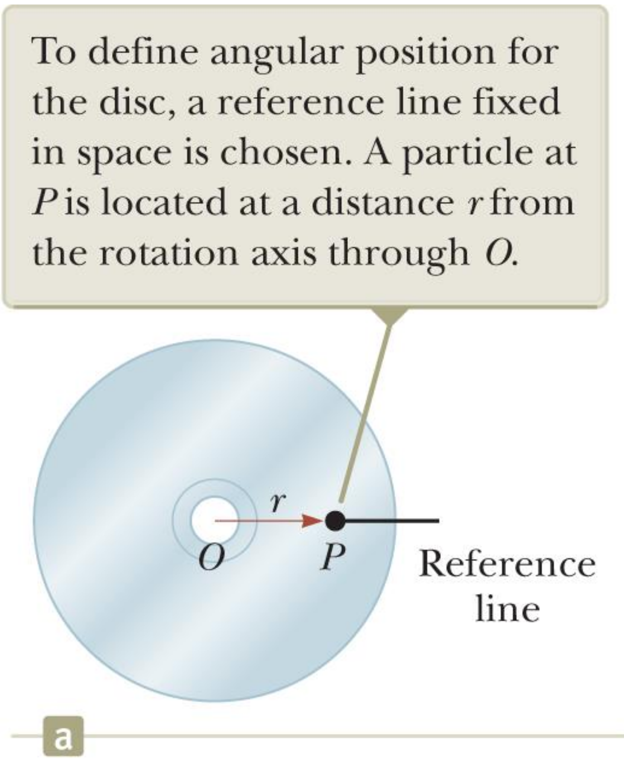
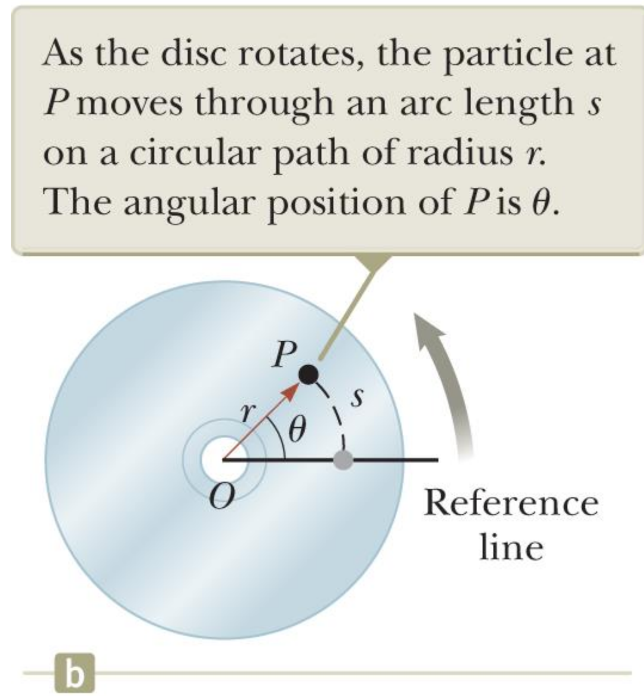
Angular Position



$$\theta = \frac{s}{r}$$

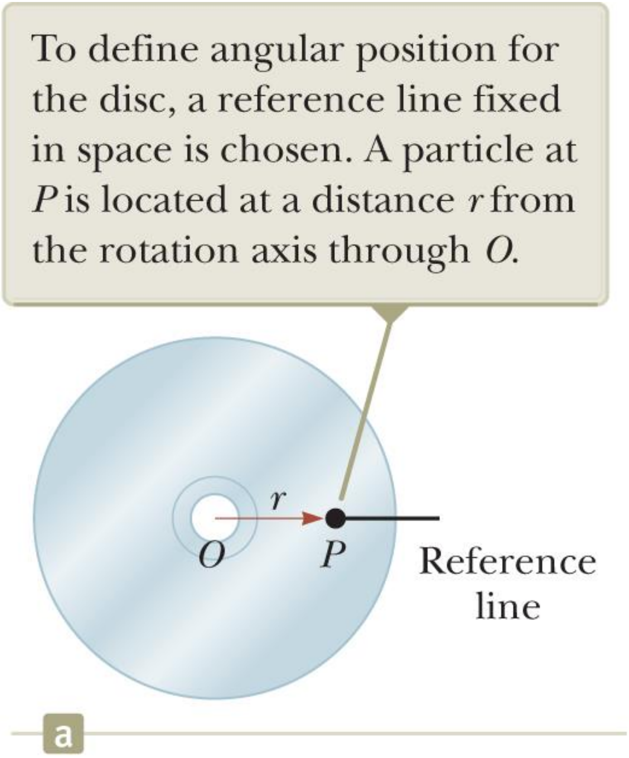
$$s = r\theta$$



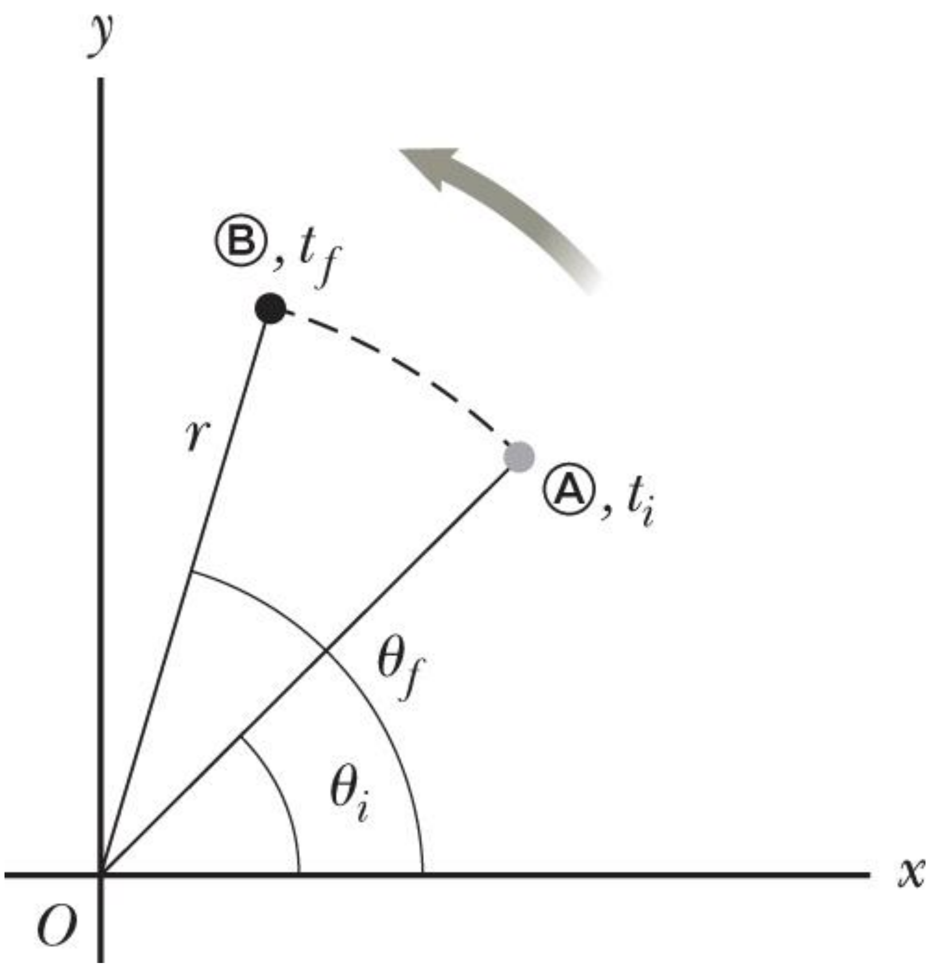
$$C = 2\pi r \Rightarrow 360° \text{ corresponds to } \frac{2\pi r}{r} = 2\pi \text{ rad}$$

$$1 \text{ rad} = \frac{360°}{2\pi} \approx 57.3° \qquad \theta(\text{rad}) = \frac{\pi}{180°}\theta(\text{deg})$$

Angular Position



Angular Speed



$$\Delta\theta = \theta_f - \theta_i$$

$$\Delta\theta \equiv \theta_f - \theta_i$$

$$\omega_{\text{avg}} \equiv \frac{\Delta\theta}{\Delta t}$$

$$\boxed{\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}}$$

Quick Quiz 10.1 Part I

A rigid object rotates in a counterclockwise sense around a fixed axis. Each of the following pairs of quantities represents an initial angular position and a final angular position of the rigid object.

Which of the sets can *only* occur if the rigid object rotates through more than 180°?

(a) 3 rad, 6 rad

(b) –1 rad, 1 rad

(c) 1 rad, 5 rad

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Quick Quiz 10.1 Part II

A rigid object rotates in a counterclockwise sense around a fixed axis. Each of the following pairs of quantities represents an initial angular position and a final angular position of the rigid object.

Suppose the change in angular position for each of these pairs of values occurs in 1 s. Which choice represents the lowest average angular speed?

(a) 3 rad, 6 rad

(b) –1 rad, 1 rad

(c) 1 rad, 5 rad

Quick Quiz 10.1 Part II

A rigid object rotates in a counterclockwise sense around a fixed axis. Each of the following pairs of quantities represents an initial angular position and a final angular position of the rigid object.

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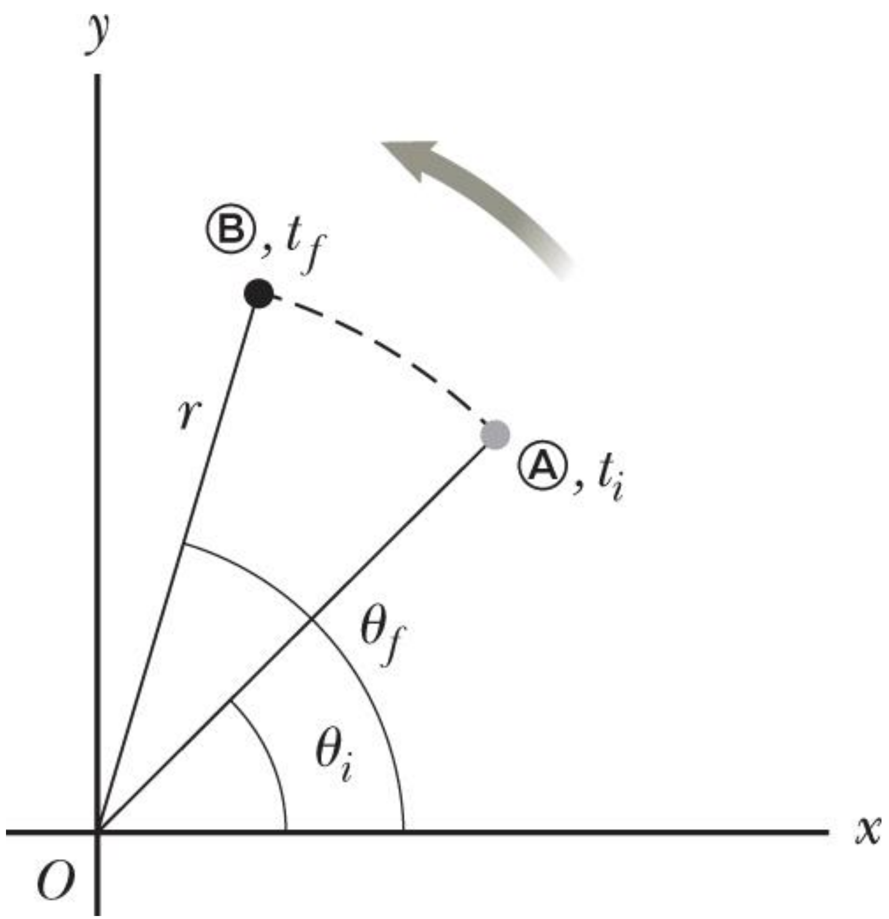
Average and Instantaneous Angular Acceleration

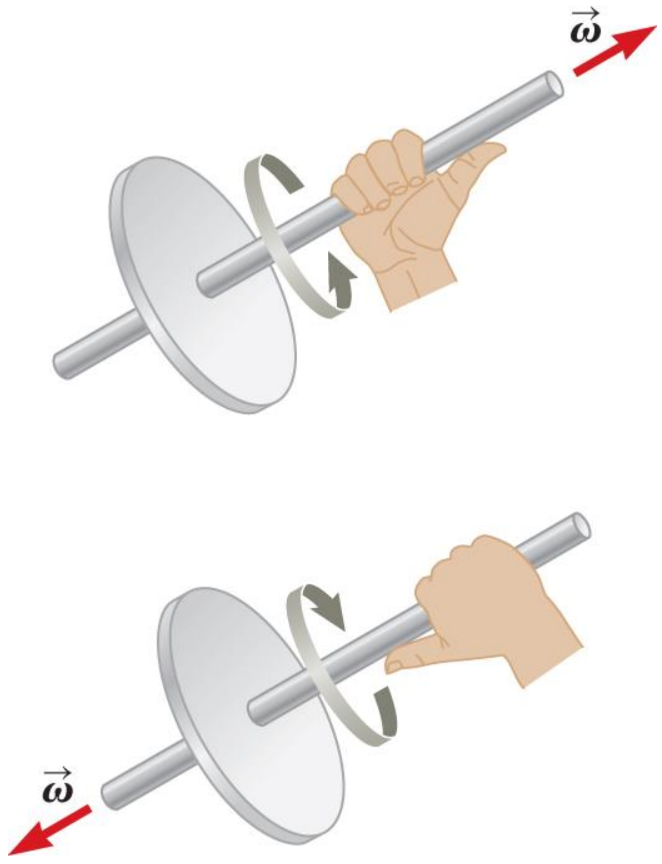
$$\alpha_{\text{avg}} \equiv \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i}$$

$$\boxed{\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}}$$

$$\theta \to x \quad \omega \to v \quad \alpha \to a$$

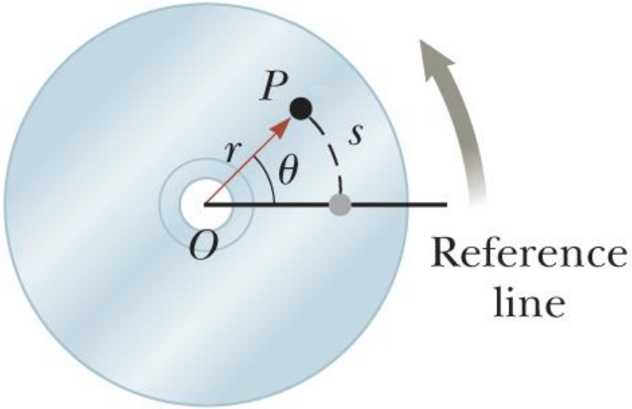
Directions of Angular Vectors





Analysis Model: Rigid Object Under Constant Angular Acceleration

$$\alpha = \frac{d\omega}{dt} \Rightarrow d\omega = \alpha\, dt \Rightarrow \Delta\omega = \int_0^t \alpha\, dt$$



$$\boxed{\omega_f = \omega_i + \alpha t \quad (\text{for constant } \alpha)}$$

$$\omega = \frac{d\theta}{dt} \Rightarrow d\theta = \omega\, dt \Rightarrow \Delta\theta = \int_0^t (\omega_i + \alpha t)\, dt$$

$$\boxed{\theta_f = \theta_i + \omega t + \frac{1}{2}\alpha t^2 \quad (\text{for constant } \alpha)}$$

Analysis Model: Rigid Object Under Constant Angular Acceleration

$$\omega_f = \omega_i + \alpha t \quad \text{and} \quad \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$t = \frac{\omega_f - \omega_i}{\alpha} \rightarrow \theta_f = \theta_i + \omega_i\left(\frac{\omega_f - \omega_i}{\alpha}\right) + \frac{1}{2}\alpha\left(\frac{\omega_f - \omega_i}{\alpha}\right)^2$$

$$2\alpha\left(\theta_f - \theta_i\right) = 2\omega_i\left(\omega_f - \omega_i\right) + \left(\omega_f - \omega_i\right)^2 \Rightarrow$$

$$2\alpha\left(\theta_f - \theta_i\right) = \cancel{2\omega_i\omega_f} - 2\omega_i^2 + \omega_f^2 + \omega_i^2 - \cancel{2\omega_i\omega_f}$$

$$\boxed{\omega_f^2 = \omega_i^2 + 2\alpha\left(\theta_f - \theta_i\right) \quad \left(\text{for constant } \alpha\right)}$$

Analysis Model: Rigid Object Under Constant Angular Acceleration

$$\omega_f = \omega_i + \alpha t \quad \text{and} \quad \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\alpha = \frac{\omega_f - \omega_i}{t} \Rightarrow \theta_f = \theta_i + \omega_i t + \frac{1}{2}\left(\frac{\omega_f - \omega_i}{t}\right)t^2$$

$$\theta_f = \theta_i + \omega_i t + \frac{\omega_f t}{2} - \frac{\omega_i t}{2} \Rightarrow$$

$$\boxed{\theta_f = \theta_i + \frac{1}{2}\left(\omega_i + \omega_f\right)t \quad \left(\text{for constant } \alpha\right)}$$

Kinematic Expressions

TABLE 10.1 Kinematic Equations for Rotational and Translational Motion

Rigid Object Under Constant Angular Acceleration		Particle Under Constant Acceleration	
$\omega_f = \omega_i + \alpha t$	(10.6)	$v_f = v_i + at$	(2.13)
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	(10.7)	$x_f = x_i + v_i t + \frac{1}{2}at^2$	(2.16)
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	(10.8)	$v_f^2 = v_i^2 + 2a(x_f - x_i)$	(2.17)
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	(10.9)	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$	(2.15)

Quick Quiz 10.2

A rigid object rotates in a counterclockwise sense around a fixed axis. If the object starts from rest at the initial angular position, moves counterclockwise with constant angular acceleration, and arrives at the final angular position with the same angular speed in all three cases, for which choice is the angular acceleration the highest?

(a) 3 rad, 6 rad

(b) –1 rad, 1 rad

(c) 1 rad, 5 rad

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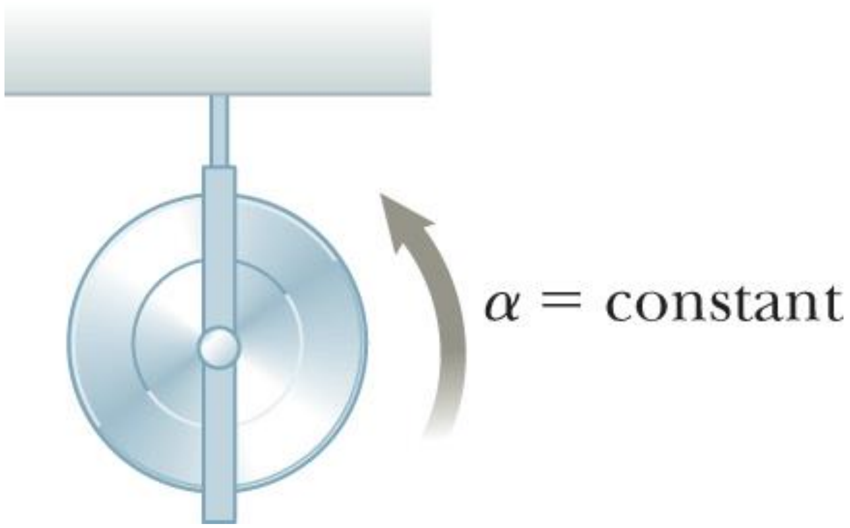
Analysis Model: Rigid Object Under Constant Angular Acceleration

$$\omega_f = \omega_i + \alpha t$$

$$\theta_f = \theta_i + \omega t + \frac{1}{2}\alpha t^2$$

$$\omega_f^{\,2} = \omega_i^{\,2} + 2\alpha\left(\theta_f - \theta_i\right)$$

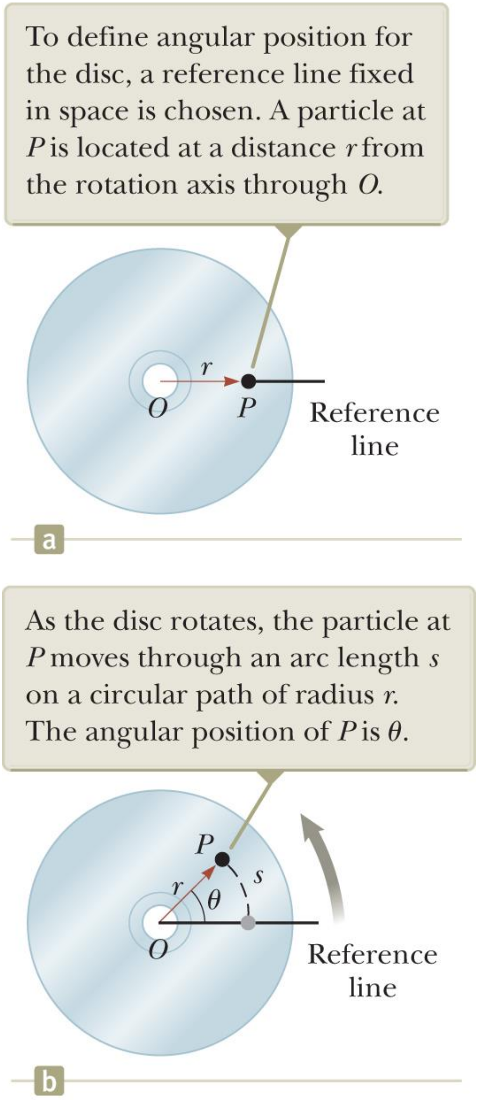
$$\theta_f = \theta_i + \frac{1}{2}\left(\omega_i + \omega_f\right)t$$



Example 10.1: Rotating Wheel

A wheel rotates with a constant angular acceleration of 3.50 rad/s^2.

(A) If the angular speed of the wheel is 2.00 rad/s at $t_i = 0$, through what angular displacement does the wheel rotate in 2.00 s?



Example 10.1: Rotating Wheel

$$\Delta\theta = \theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = (2.00\text{ rad/s})(2.00\text{ s}) + \frac{1}{2}(3.50\text{ rad/s}^2)(2.00\text{ s})^2$$

$$= \boxed{11.0\text{ rad}} = (11.0\text{ rad})(180°/\pi\text{ rad}) = \boxed{630°}$$

Example 10.1: Rotating Wheel

(B) Through how many revolutions has the wheel turned during this time interval?

$$\Delta\theta = 630^\circ\left(\frac{1\ \text{rev}}{360^\circ}\right) = \boxed{1.75\ \text{rev}}$$

Example 10.1: Rotating Wheel

(C) What is the angular speed of the wheel at t = 2.00 s?

$$\omega_f = \omega_i + \alpha t$$

$$= 2.00 \text{ rad/s} + \left(3.50 \text{ rad/s}^2\right)\left(2.00 \text{ s}\right) = 9.00 \text{ rad/s}$$

Example 10.1: Rotating Wheel

Suppose a particle moves along a straight line with a constant acceleration of 3.50 m/s^2. If the velocity of the particle is 2.00 m/s at $t_i = 0$, through what displacement does the particle move in 2.00 s? What is the velocity of the particle at $t = 2.00$ s?

$$\Delta x = x_f - x_i = v_i t + \frac{1}{2} a t^2$$

$$= (2.00\text{ m/s})(2.00\text{ s}) + \frac{1}{2}(3.50\text{ m/s}^2)(2.00\text{ s})^2 = 11.0\text{ m}$$

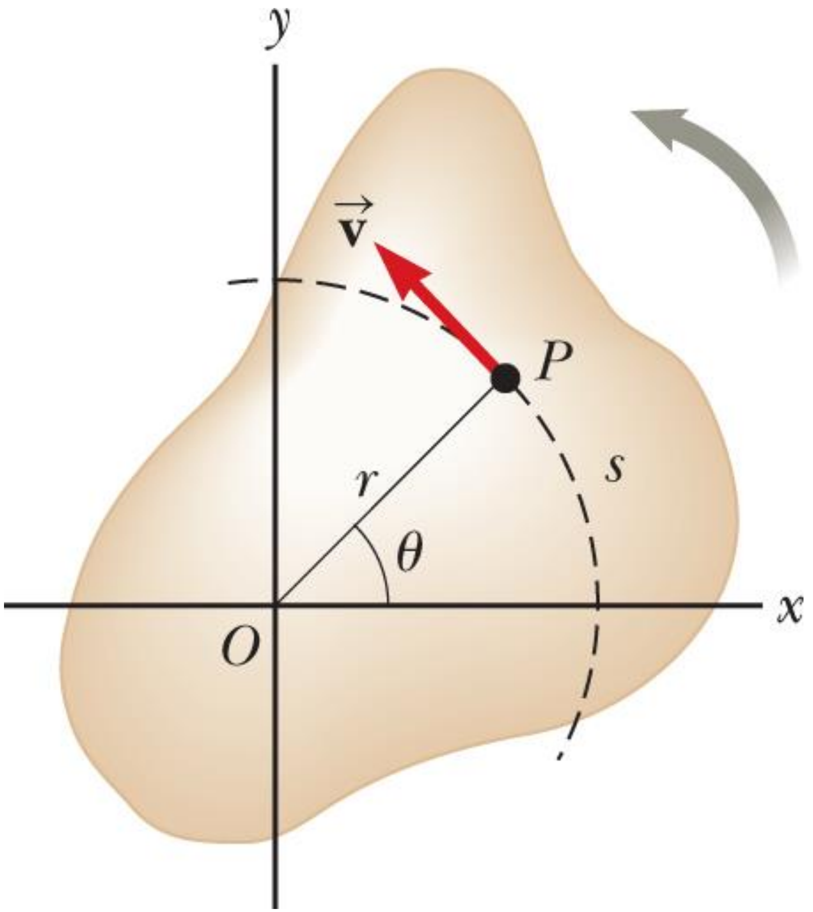
$$v_f = v_i + at = 2.00\text{ m/s} + (3.50\text{ m/s}^2)(2.00\text{ s})^2 = 9.00\text{ m/s}$$

Angular and Translational Quantities

$$s = r\theta$$

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$\boxed{v = r\omega}$$



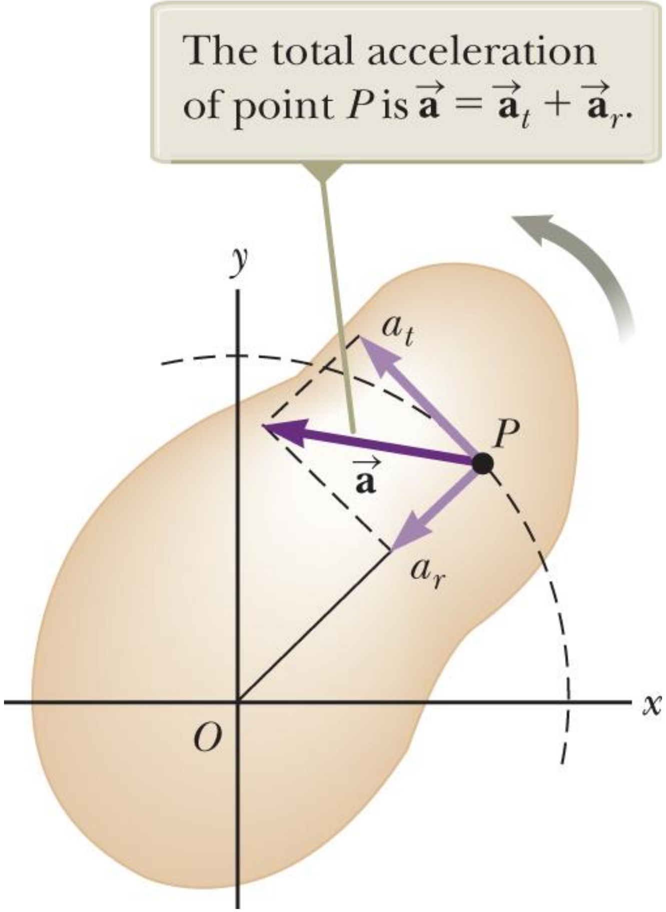
Angular and Translational Quantities

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt}$$

$$\boxed{a_t = r\alpha}$$

$$\boxed{a_c = \frac{v^2}{r} = r\omega^2}$$

$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_t + \vec{\mathbf{a}}_r$$



$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{r^2\alpha^2 + r^4\omega^2} = r\sqrt{\alpha^2 + \omega^4}$$

Quick Quiz 10.3 Part I

Ethan and Rebecca are riding on a merry-go-round. Ethan rides on a horse at the outer rim of the circular platform, twice as far from the center of the circular platform as Rebecca, who rides on an inner horse.

When the merry-go-round is rotating at a constant angular speed, what is Ethan's angular speed?

(a) twice Rebecca's

(b) the same as Rebecca's

(c) half of Rebecca's

(d) impossible to determine

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(c) half of Rebecca's

(d) impossible to determine

Quick Quiz 10.3 Part II

Ethan and Rebecca are riding on a merry-go-round. Ethan rides on a horse at the outer rim of the circular platform, twice as far from the center of the circular platform as Rebecca, who rides on an inner horse.

When the merry-go-round is rotating at a constant angular speed, describe Ethan's tangential speed.

(a) twice Rebecca's

(b) the same as Rebecca's

(c) half of Rebecca's

(d) impossible to determine

Quick Quiz 10.3 Part II

Ethan and Rebecca are riding on a merry-go-round. Ethan rides on a horse at the outer rim of the circular platform, twice as far from the center of the circular platform as Rebecca, who rides on an inner horse.

When the merry-go-round is rotating at a constant angular speed, describe Ethan's tangential speed.

(a) twice Rebecca's

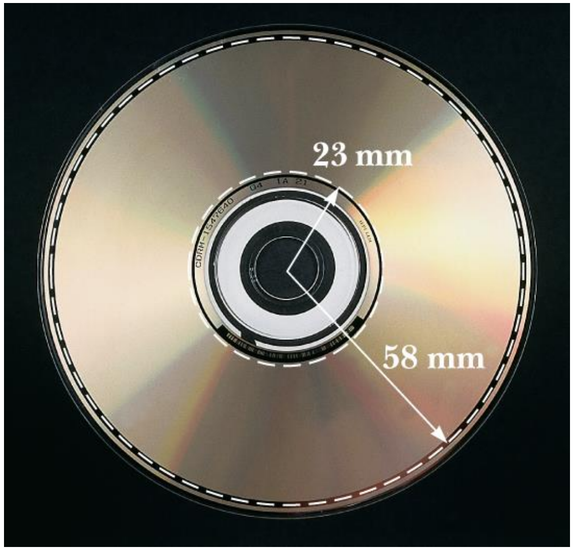
(b) the same as Rebecca's

(c) half of Rebecca's

(d) impossible to determine

Example 10.2
CD Player

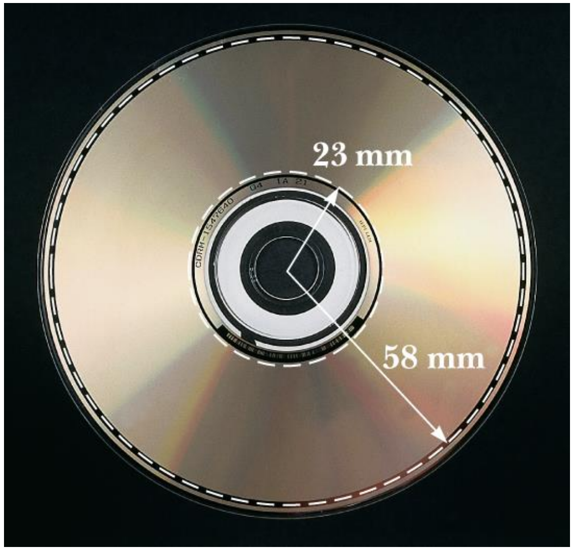
Despite the availability of music in digital form, the compact disc, or CD, remains a popular format for music and data. On a CD, audio information is stored digitally in a series of pits and flat areas on the surface of the disc. The alternations between pits and flat areas on the surface represent binary ones and zeros to be read by the CD player and converted back to sound waves. The pits and flat areas are detected by a system consisting of a laser and lenses. The length of a string of ones and zeros representing one piece of information is the same everywhere on the disc, whether the information is near the center of the disc or near its outer edge.



Example 10.2
CD Player

So that this length of ones and zeros always passes by the laser–lens system in the same time interval, the tangential speed of the disc surface at the location of the lens must be constant. According to Equation 10.10, the angular speed must therefore vary as the laser–lens system moves radially along the disc. In a typical CD player, the constant speed of the surface at the point of the laser–lens system is 1.3 m/s.

(A) Find the angular speed of the disc in revolutions per minute when information is being read from the innermost first track (r = 23 mm) and the outermost final track (r = 58 mm).



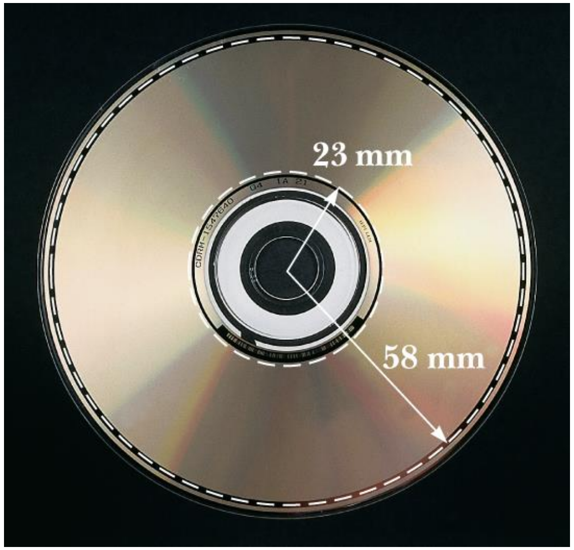
Example 10.2:
CD Player

$$\omega_i = \frac{v}{r_i} = \frac{1.3 \text{ m/s}}{2.3\times10^{-2} \text{ m}} = 57 \text{ rad/s}$$

$$= (57 \text{ rad/s})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{5.4\times10^{2} \text{ rev/min}}$$

$$\omega_f = \frac{v}{r_f} = \frac{1.3 \text{ m/s}}{5.8\times10^{-2} \text{ m}}$$

$$= 22 \text{ rad/s} = \boxed{2.1\times10^{2} \text{ rev/min}}$$



Example 10.2: CD Player

(B) The maximum playing time of a standard music disc is 74 min and 33 s. How many revolutions does the disc make during that time?

$$t = (74\text{ min})(60\text{ s/min}) + 33\text{ s} = 4\,473\text{ s}$$

$$\Delta\theta = \theta_f - \theta_i = \frac{1}{2}(\omega_i + \omega_f)t$$

$$= \frac{1}{2}(57\text{ rad/s} + 22\text{ rad/s})(4\,473\text{ s}) = 1.8\times10^5\text{ rad}$$

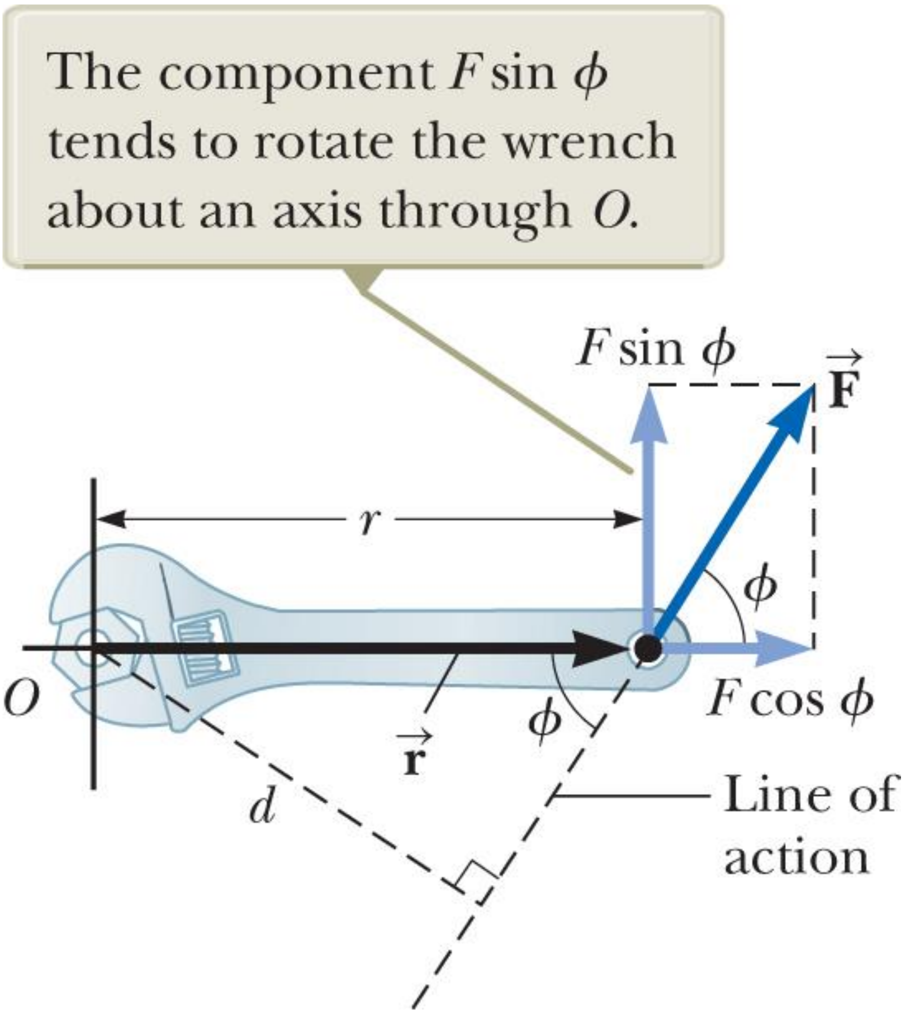
$$\Delta\theta = (1.8\times10^5\text{ rad})\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{2.8\times10^4\text{ rev}}$$

Example 10.2: CD Player

(C) What is the angular acceleration of the compact disc over the 4 473-s time interval?

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{22 \text{ rad/s} - 57 \text{ rad/s}}{4\,473 \text{ s}} = -7.6 \times 10^{-3} \text{ rad/s}^2$$

Torque

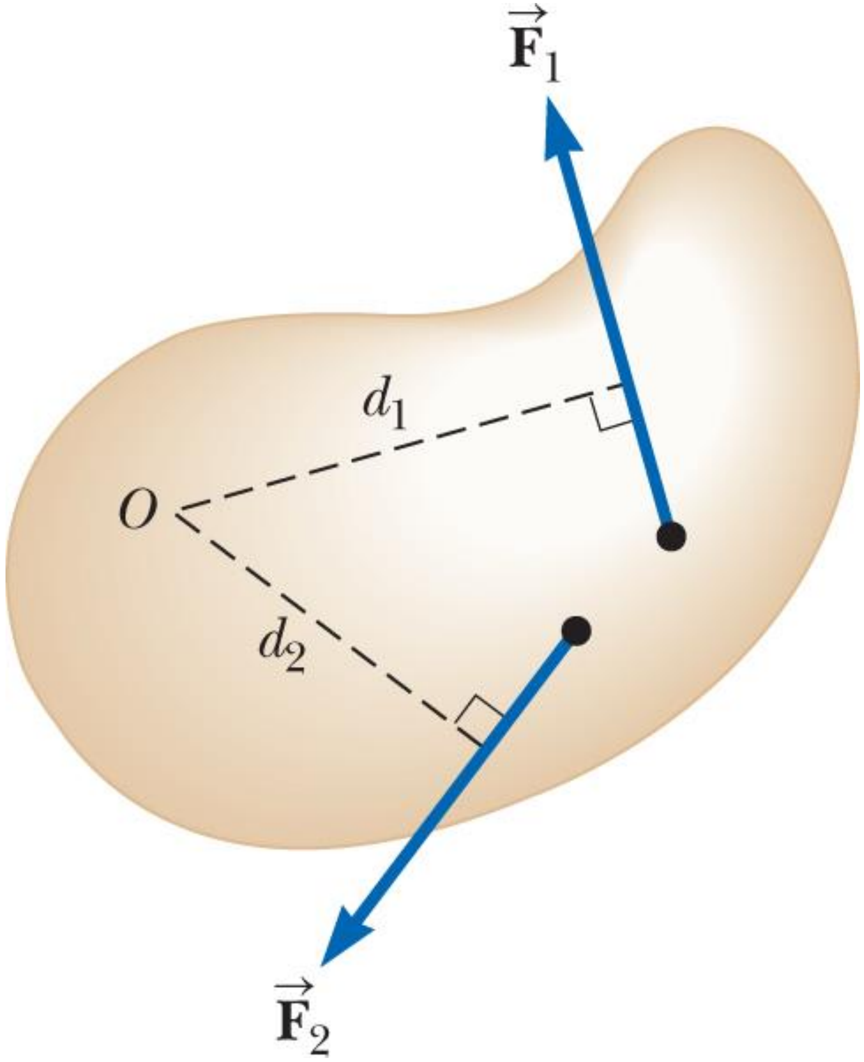


Torque (τ): vector quantity measuring changes in rotational motion of object about some axis

$$\tau \equiv rF\sin\theta = Fd$$

$$d = r\sin\phi$$

Torque



$$\sum \tau = \tau_1 + \tau_2$$
$$= F_1 d_1 - F_2 d_2$$

Quick Quiz 10.4

You are trying to loosen a stubborn screw from a piece of wood with a screwdriver and fail,. You should find a screwdriver for which the handle is

(a) longer.

(b) fatter.

Quick Quiz 10.4

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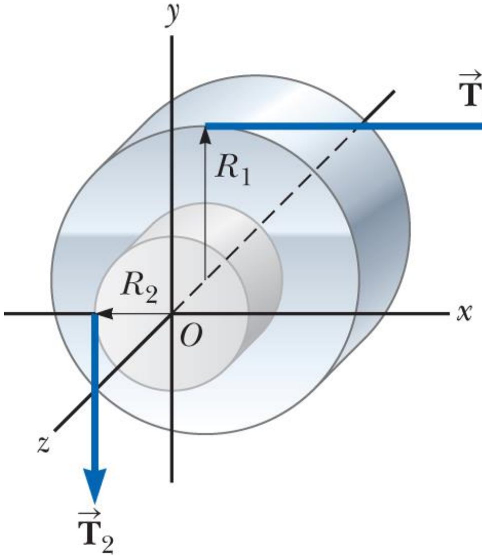
Example 10.3: The Net Torque on a Cylinder

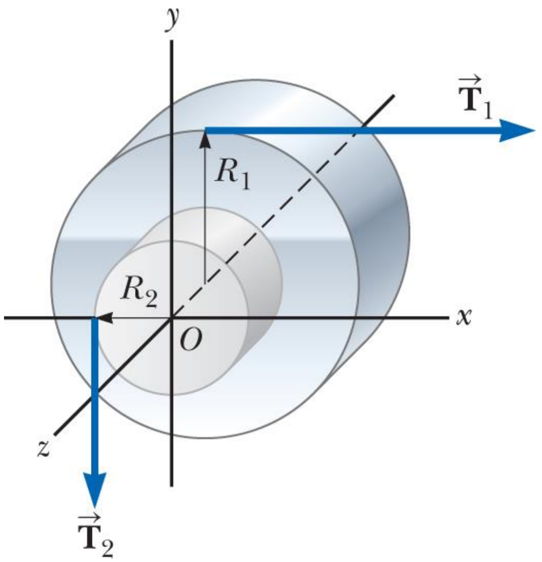
A one-piece cylinder is shaped as shown in the figure, with a core section protruding from the larger drum. The cylinder is free to rotate about the central z axis shown in the drawing. A rope wrapped around the drum, which has radius R_1, exerts a force $\vec{\mathbf{T}}_1$ to the right on the cylinder. A rope wrapped around the core, which has radius R_2, exerts a force $\vec{\mathbf{T}}_2$ downward on the cylinder.

(A) What is the net torque acting on the cylinder about the rotation axis (which is the z axis in the figure)?



Example 10.3:
The Net Torque on a Cylinder

$$\sum \tau = \tau_1 + \tau_2 = R_2 T_2 - R_1 T_1$$



Example 10.3: The Net Torque on a Cylinder

(B) Suppose T_1 = 5.0 N, R_1 = 1.0 m, T_2 = 15 N, and R_2 = 0.50 m. What is the net torque about the rotation axis, and which way does the cylinder rotate starting from rest?

$$\sum \tau = (0.50\text{ m})(15\text{ N}) - (1.0\text{ m})(5.0\text{ N}) = \boxed{2.5\text{ N}\cdot\text{m}}$$

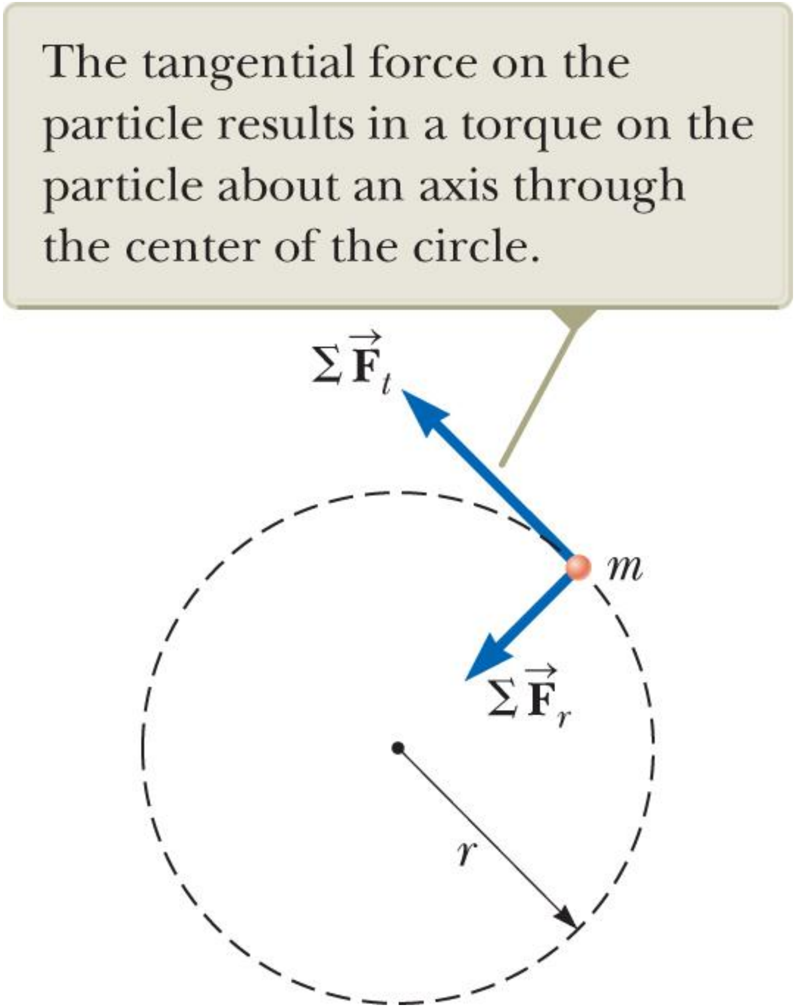
Analysis Model: Rigid Object Under a Net Torque

$$\sum F_t = ma_t$$

$$\sum \tau = \sum F_t r = (ma_t)r$$

$$\sum \tau = (mr\alpha)r = (mr^2)\alpha$$

$$\sum \tau = I\alpha$$

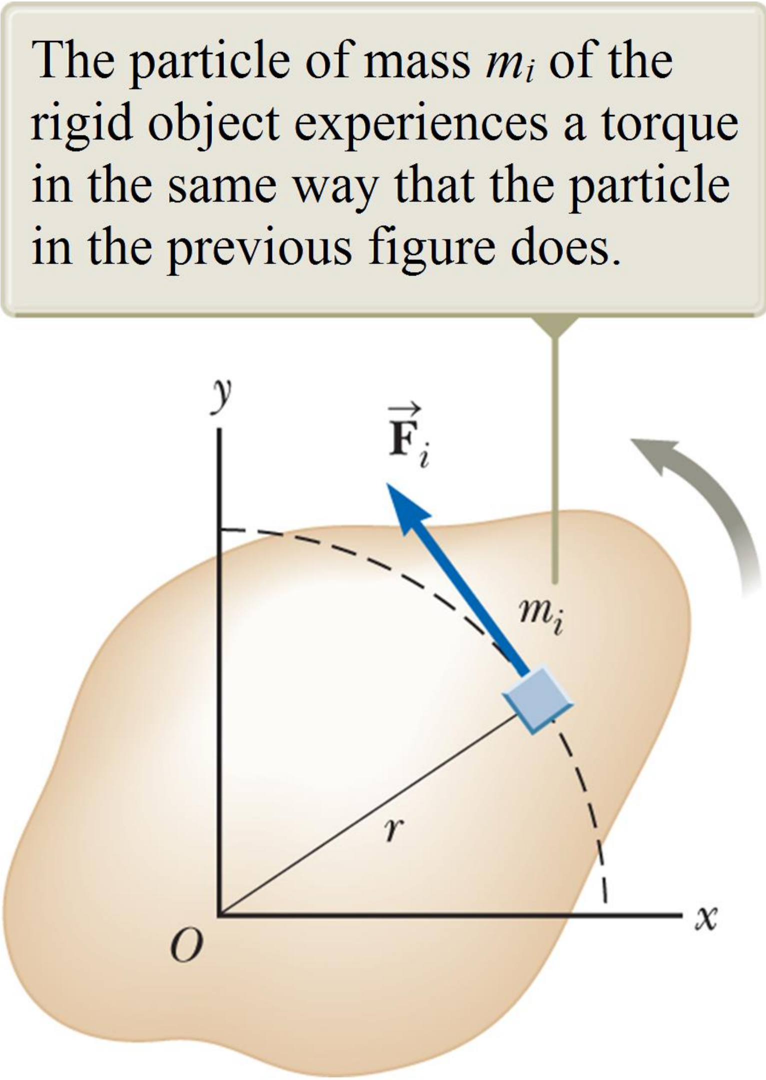


Analysis Model: Rigid Object Under a Net Torque

$$F_i = m_i a_i$$

$$\tau_i = r_i F_i = r_i m_i a_i$$

$$\tau_i = m_i r_i^2 \alpha$$



Analysis Model: Rigid Object Under a Net Torque

$$\sum \tau_{\text{ext}} = \sum_i \tau_i = \sum_i m_i r_i^2 \alpha = \left(\sum_i m_i r_i^2 \right) \alpha$$

$$\boxed{\sum \tau_{\text{ext}} = I\alpha}$$

$$\text{where } I = \sum_i m_i r_i^2$$

Moment of Inertia

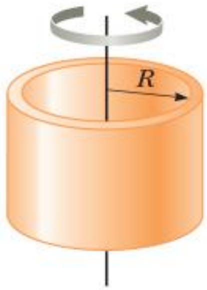
$$I = \sum_i m_i r_i^{\,2}$$

$$\sum \tau_{\text{ext}} = I\alpha \qquad \sum \vec{\mathbf{F}}_{\text{ext}} = M\vec{\mathbf{a}}_{\text{CM}}$$

Moment of Inertia

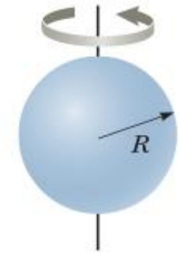
TABLE 10.2 Moments of Inertia of Homogeneous Rigid Objects with Different Geometries

Hoop or thin cylindrical shell
$I_{CM} = MR^2$

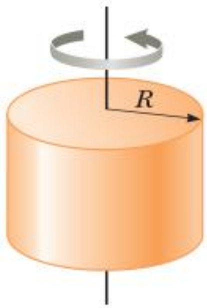


Long, thin rod with rotation axis through center
$I_{CM} = \frac{1}{12}ML^2$

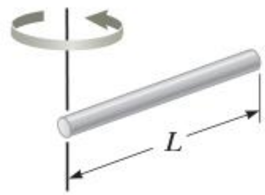
Solid sphere
$I_{CM} = \frac{2}{5}MR^2$



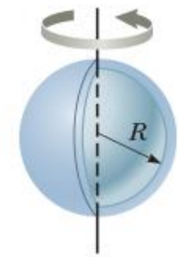
Solid cylinder or disk
$I_{CM} = \frac{1}{2}MR^2$



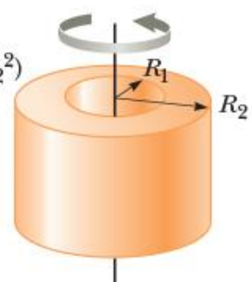
Long, thin rod with rotation axis through end
$I = \frac{1}{3}ML^2$



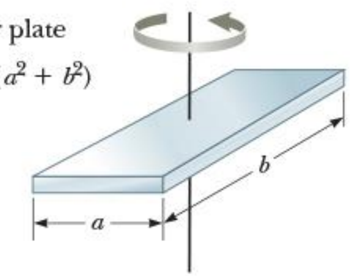
Thin spherical shell
$I_{CM} = \frac{2}{3}MR^2$



Hollow cylinder
$I_{CM} = \frac{1}{2}M(R_1^2 + R_2^2)$



Rectangular plate
$I_{CM} = \frac{1}{12}M(a^2 + b^2)$



Quick Quiz 10.5

You turn off your electric drill and find that the time interval for the rotating bit to come to rest due to frictional torque in the drill is Δt. You replace the bit with a larger one that results in a doubling of the moment of inertia of the drill's entire rotating mechanism. When this larger bit is rotated at the same angular speed as the first and the drill is turned off, the frictional torque remains the same as that for the previous situation. What is the time interval for this second bit to come to rest?

(a) $4\Delta t$ (b) $2\Delta t$ (c) Δt (d) $0.5\Delta t$ (e) $0.25\Delta t$

(f) impossible to determine

Quick Quiz 10.5

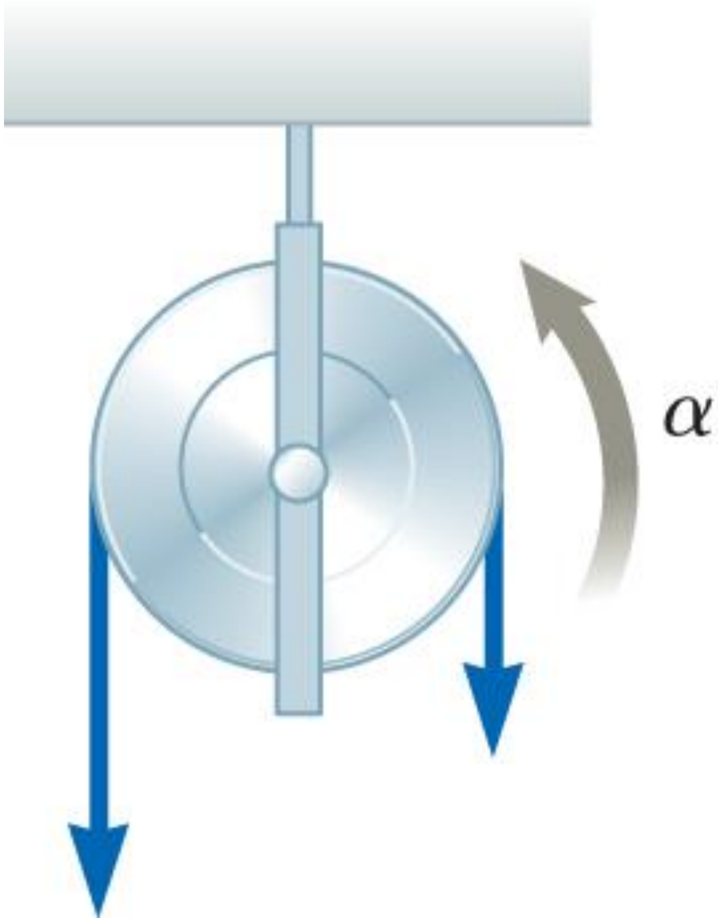
You turn off your electric drill and find that the time interval for the rotating bit to come to rest due to frictional torque in the drill is Δt. You replace the bit with a larger one that results in a doubling of the moment of inertia of the drill's entire rotating mechanism. When this larger bit is rotated at the same angular speed as the first and the drill is turned off, the frictional torque remains the same as that for the previous situation. What is the time interval for this second bit to come to rest?

(a) $4\Delta t$ **(b) $2\Delta t$** (c) Δt (d) $0.5\Delta t$ (e) $0.25\Delta t$

(f) impossible to determine

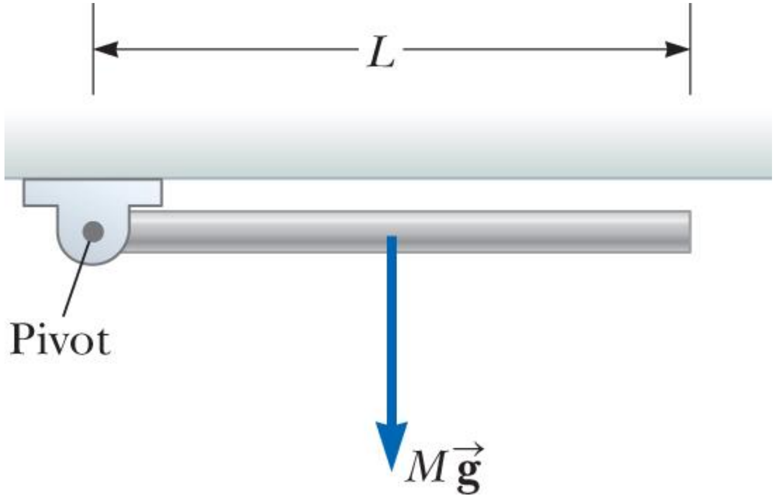
Analysis Model: Rigid Object Under a Net Torque

$$\sum \tau_{\text{ext}} = I\alpha$$



Example 10.4: Rotating Rod

A uniform rod of length L and mass M is attached at one end to a frictionless pivot and is free to rotate about the pivot in the vertical plane as in the figure. The rod is released from rest in the horizontal position. What are the initial angular acceleration of the rod and the initial translational acceleration of its right end?

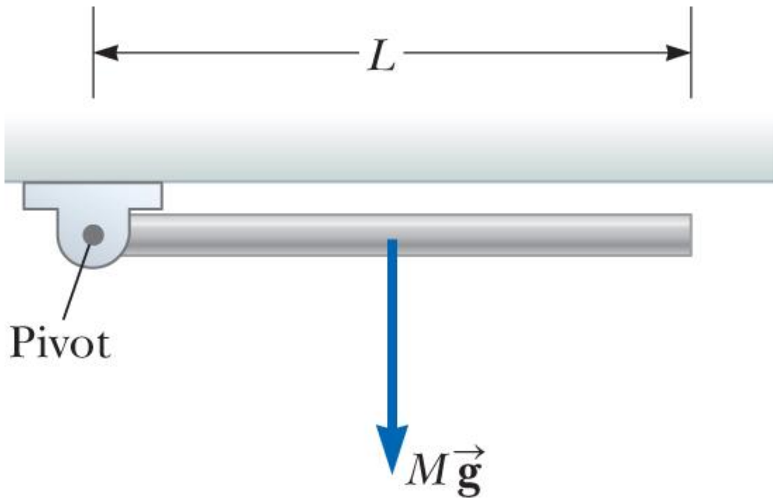


Example 10.4: Rotating Rod

$$\sum \tau_{\text{ext}} = Mg\left(\frac{L}{2}\right)$$

$$a = \frac{\sum \tau_{\text{ext}}}{I} = \frac{Mg\left(L/2\right)}{\frac{1}{3}ML^2} = \boxed{\frac{3g}{2L}}$$

$$a_t = L\alpha = \boxed{\frac{3}{2}g}$$



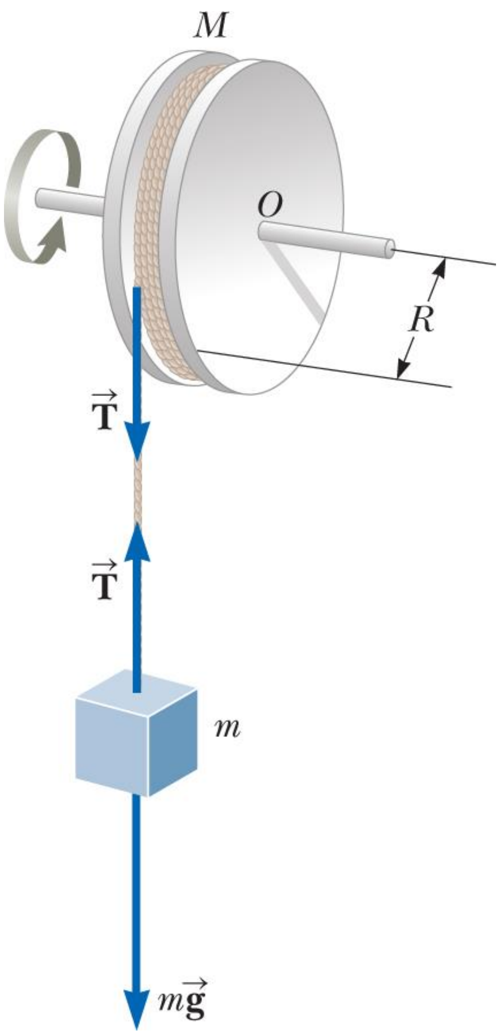
Conceptual Example 10.5: Falling Smokestacks and Tumbling Blocks

When a tall smokestack falls over, it often breaks somewhere along its length before it hits the ground as shown in the figure. Why?

$$a_t = r\alpha$$

Example 10.6: Angular Acceleration of a Wheel

A wheel of radius *R*, mass *M*, and moment of inertia *I* is mounted on a frictionless, horizontal axle as in the figure. A light cord wrapped around the wheel supports an object of mass *m*. When the wheel is released, the object accelerates downward, the cord unwraps off the wheel, and the wheel rotates with an angular acceleration. Find expressions for the angular acceleration of the wheel, the translational acceleration of the object, and the tension in the cord.



Example 10.6:
Angular Acceleration of a Wheel

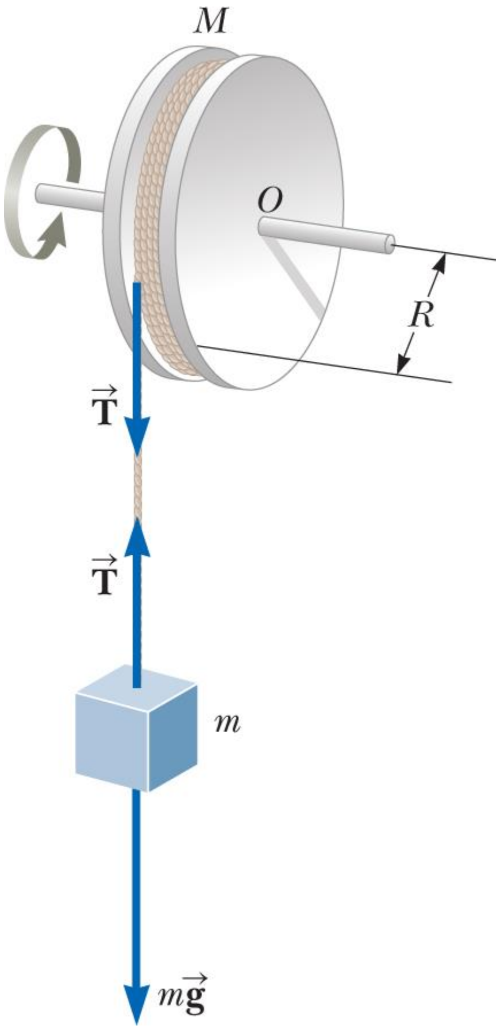
$$\sum \tau_{\text{ext}} = I\alpha \qquad \rightarrow \alpha = \frac{\sum \tau_{\text{ext}}}{I} = \frac{TR}{I}$$

$$\sum F_y = mg - T = ma \qquad \rightarrow a = \frac{mg - T}{m}$$

$$a = R\alpha = \frac{TR^2}{I} = \frac{mg - T}{m}$$

$$T = \boxed{\frac{mg}{1 + \left(mR^2/I\right)}} \qquad a = \boxed{\frac{g}{1 + \left(I/mR^2\right)}}$$

$$\alpha = \frac{a}{R} = \boxed{\frac{g}{R + \left(I/mR\right)}}$$



Example 10.6:
Angular Acceleration of a Wheel

What if the wheel were to become very massive so that I becomes very large? What happens to the acceleration a of the object and the tension T ?

$$a = \lim_{I \to \infty} \frac{g}{1 + \left(I / mR^2 \right)} = 0$$

$$T = \lim_{I \to \infty} \frac{mg}{1 + \left(mR^2 / I \right)} = mg$$

Calculation of Moments of Inertia

$$I = \sum_i r_i^{\,2} \Delta m_i \quad \left(\text{system of discrete particles}\right)$$

Continuous rigid object: $\boxed{I = \lim_{\Delta m_i \to 0} \sum_i r_i^{\,2} \Delta m_i = \int r^2 \, dm}$

$$\rho = \frac{m}{V} \qquad \Rightarrow dm = \rho \, dV$$

$$I = \int \rho r^2 \, dV$$

Calculation of Moments of Inertia

Calculation of Moments of Inertia

$$\sigma = \frac{m}{A} = \rho t \ \left(\text{mass per unit area}\right)$$

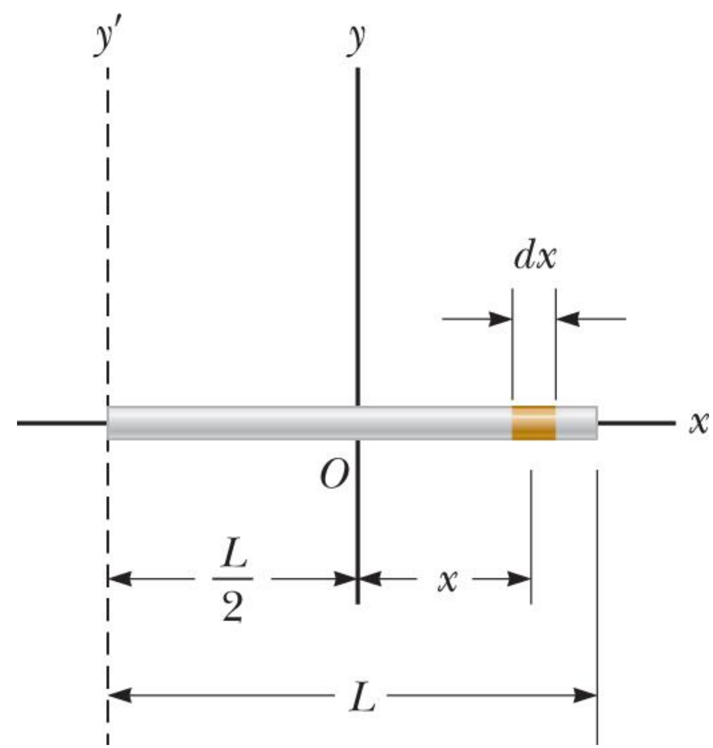
$$\lambda = \frac{m}{L} = \rho A \ \left(\text{mass per unit length}\right)$$

Example 10.7:
Uniform Rigid Rod

Calculate the moment of inertia of a uniform thin rod of length L and mass M about an axis perpendicular to the rod (the y axis) and passing through its center of mass.

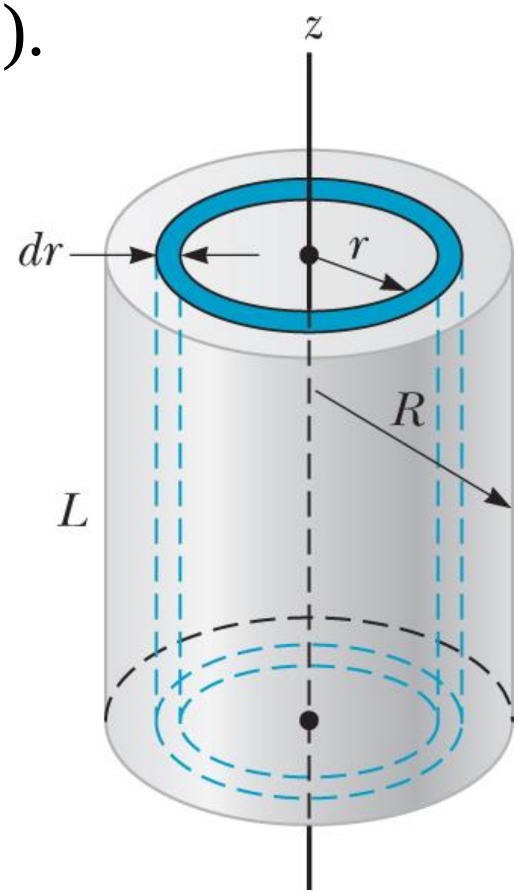
$$dm = \lambda\, dx = \frac{M}{L}\, dx$$

$$\begin{aligned} I_y &= \int r^2\, dm = \int_{-L/2}^{L/2} x^2 \frac{M}{L}\, dx \\ &= \frac{M}{L}\int_{-L/2}^{L/2} x^2 dx \\ &= \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{L/2} = \boxed{\frac{1}{12} ML^2} \end{aligned}$$



Example 10.8:
Uniform Solid Cylinder

A uniform solid cylinder has a radius *R*, mass *M*, and length *L*. Calculate its moment of inertia about its central axis (the *z* axis in in the figure).



$$dV = LdA = L(2\pi r)dr$$

$$dm = \pi dV = \pi L(2\pi r)dr$$

$$I_z = \int r^2 dm = \int r^2 \left[\rho L(2\pi r)dr\right]$$

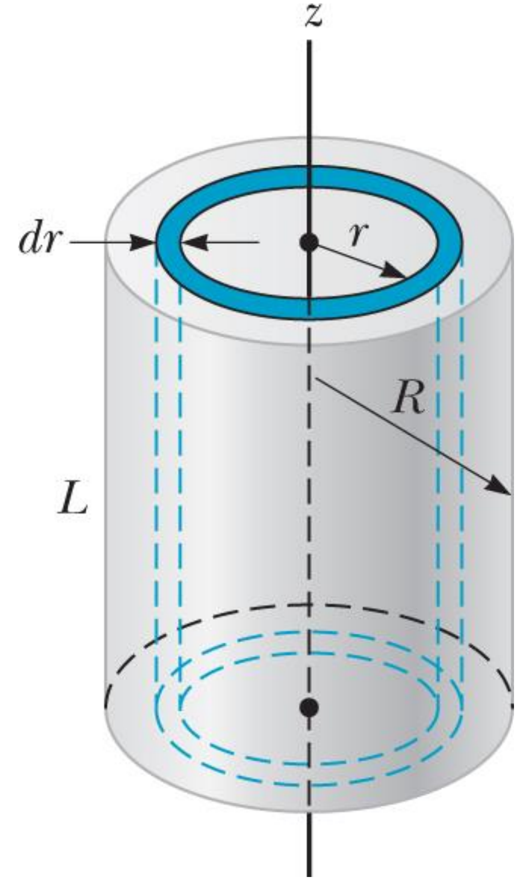
$$= 2\pi\rho L \int_0^R r^3 dr = \frac{1}{2}\pi\rho L R^4$$

$$\rho = \frac{M}{V} = \frac{M}{\pi R^2 L}$$

$$I_z = \frac{1}{2}\pi\left(\frac{M}{\pi R^2 L}\right)LR^4 = \boxed{\frac{1}{2}MR^2}$$

Example 10.8:
Uniform Solid Cylinder

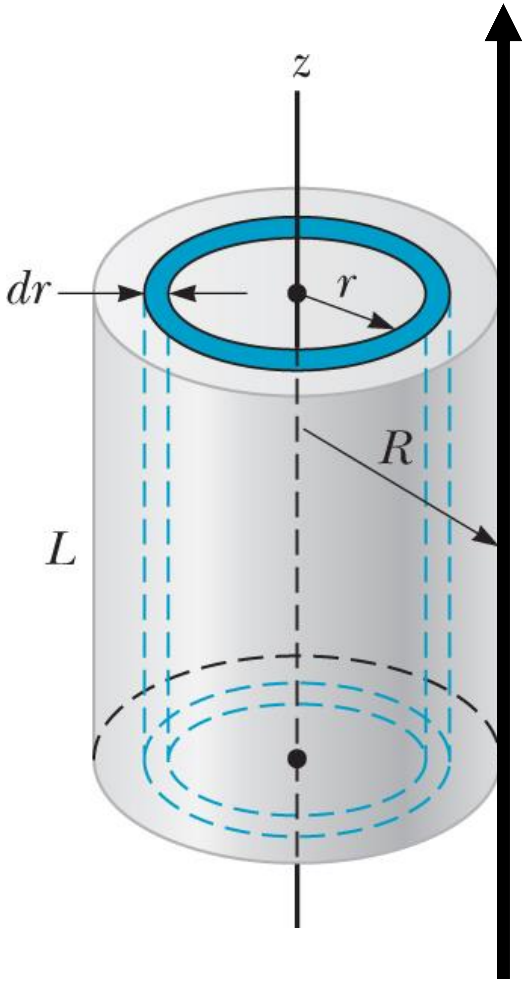
What if the length of the cylinder in the figure is increased to 2L, while the mass M and radius R are held fixed (The density becomes half as large)? How does that change the moment of inertia of the cylinder?



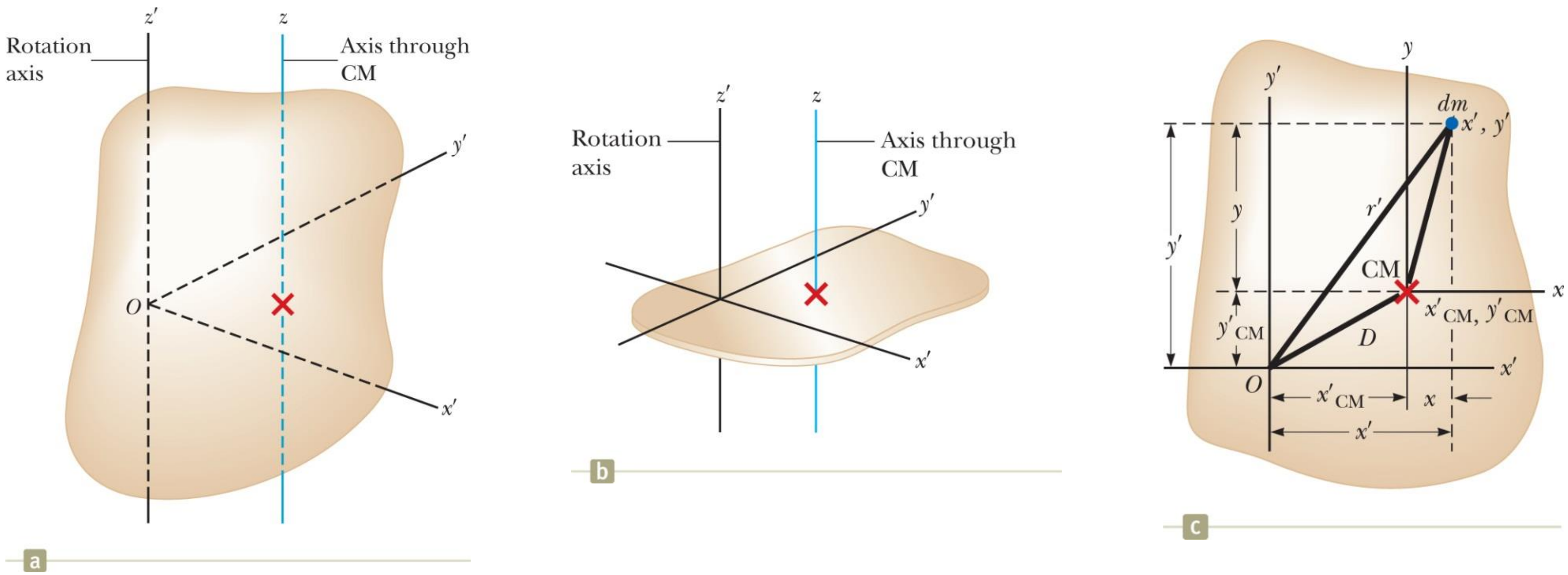
$$I_z = \frac{1}{2} MR^2$$

$\boldsymbol{I_{\text{cylinder}}}$ **is not affected**

Parallel-Axis Theorem



Parallel-Axis Theorem



$$r = \sqrt{(x')^2 + (y')^2}$$

$$I = \int (r')^2\, dm = \int \left[(x')^2 + (y')^2 \right] dm$$

Parallel-Axis Theorem

$$x' = x + x'_{\mathrm{CM}} \quad y' = y + y'_{\mathrm{CM}} \quad z' = z = 0$$

$$I = \int \left[\left(x + x'_{\mathrm{CM}} \right)^2 + \left(y + y'_{\mathrm{CM}} \right)^2 \right] dm$$

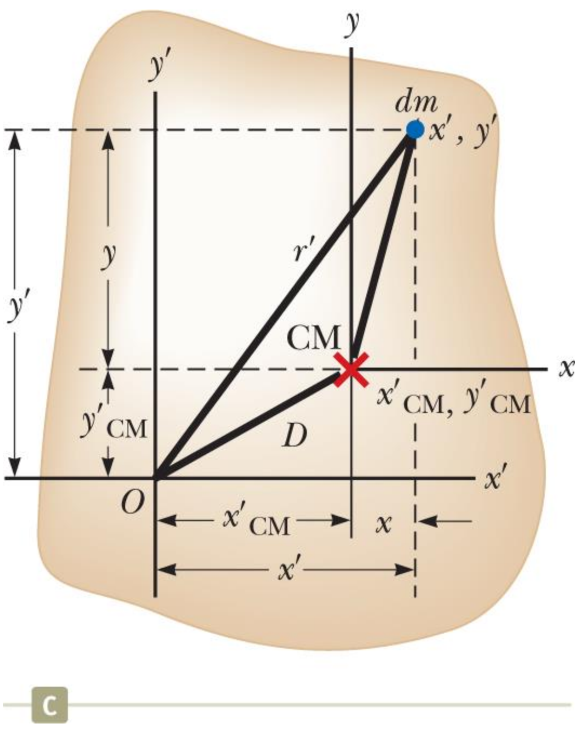
$$= \int \left(x^2 + y^2 \right) dm + \cancel{2x'_{\mathrm{CM}} \int x\, dm}$$

$$+ \cancel{2y'_{\mathrm{CM}} \int y\, dm} + \left({x'_{\mathrm{CM}}}^2 + {y'_{\mathrm{CM}}}^2 \right) \int dm$$

$$\int x\, dm = \int y\, dm = 0$$

$$\int dm = M \ \text{ and } D^2 = {x'_{\mathrm{CM}}}^2 + {y'_{\mathrm{CM}}}^2$$

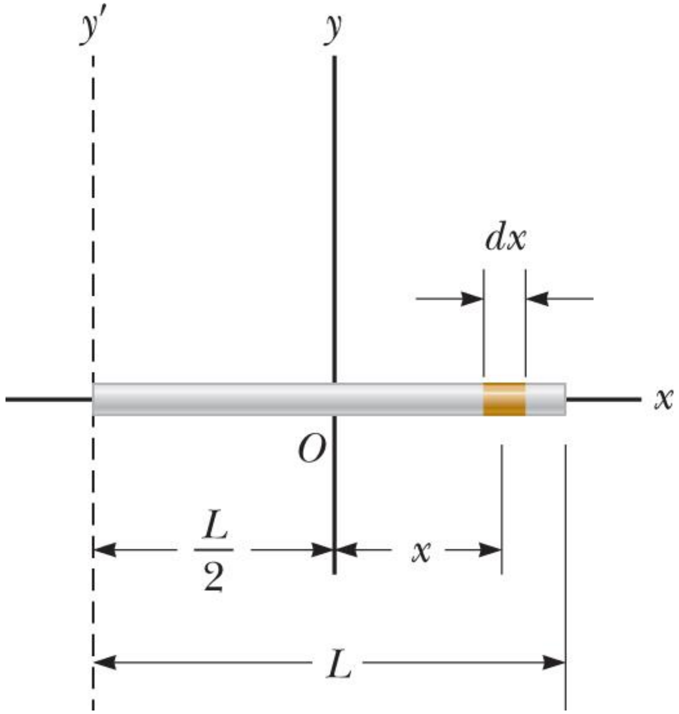
$$\boxed{I = I_{\mathrm{CM}} + MD^2}$$



Example 10.9: Applying the Parallel-Axis Theorem

Consider once again the uniform rigid rod of mass *M* and length *L* shown in the figure. Find the moment of inertia of the rod about an axis perpendicular to the rod through one end (the *y* axis in the figure).

$$I = I_{\mathrm{CM}} + MD^2$$
$$= \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2$$
$$= \boxed{\frac{1}{3}ML^2}$$



Rotational Kinetic Energy

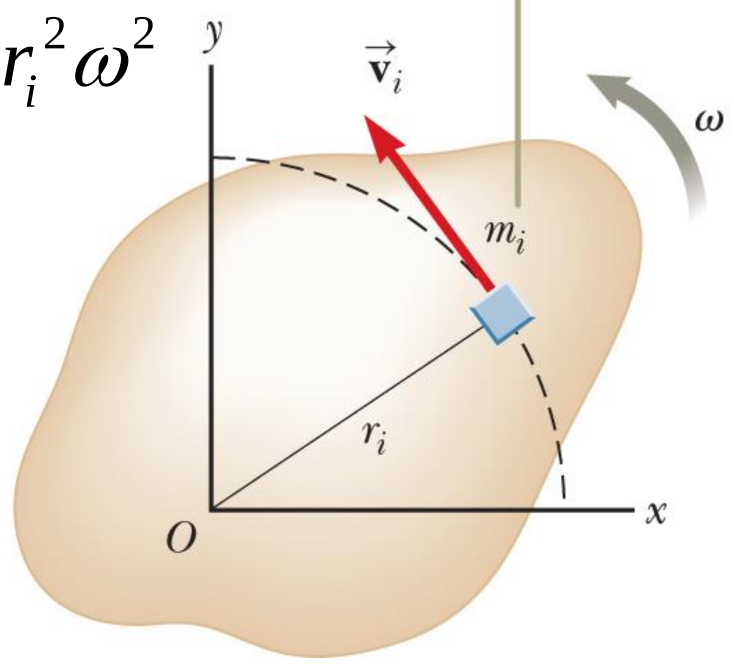
$$K_i = \frac{1}{2} m_i v_i^2$$

$$K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i v_i^2 = \frac{1}{2} \sum_i m_i r_i^2 \omega^2$$

$$K_R = \frac{1}{2} \left(\sum_i m_i r_i^2 \right) \omega^2$$

$$\boxed{K_R = \frac{1}{2} I \omega^2}$$

The particle of mass m_i of the rigid object has the same kinetic energy as if it were moving through space with the same speed.



Quick Quiz 10.6

A section of hollow pipe and a solid cylinder have the same radius, mass, and length. They both rotate about their long central axes with the same angular speed. Which object has the higher rotational kinetic energy?

(a) The hollow pipe does.

(b) The solid cylinder does.

(c) They have the same rotational kinetic energy.

(d) It is impossible to determine.

Quick Quiz 10.6

A section of hollow pipe and a solid cylinder have the same radius, mass, and length. They both rotate about their long central axes with the same angular speed. Which object has the higher rotational kinetic energy?

(a) The hollow pipe does.

(b) The solid cylinder does.

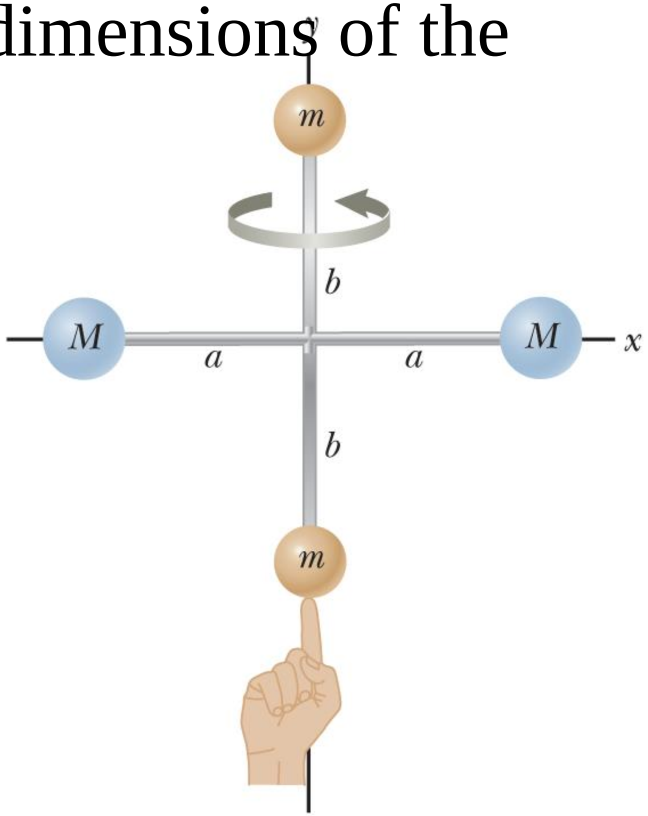
(c) They have the same rotational kinetic energy.

(d) It is impossible to determine.

Example 10.10:
An Unusual Baton

Four tiny spheres are fastened to the ends of two rods of negligible mass lying in the *xy* plane to form an unusual baton (see figure). We shall assume the radii of the spheres are small compared with the dimensions of the rods.

(A) If the system rotates about the *y* axis with an angular speed ω, find the moment of inertia and the rotational kinetic energy of the system about this axis.

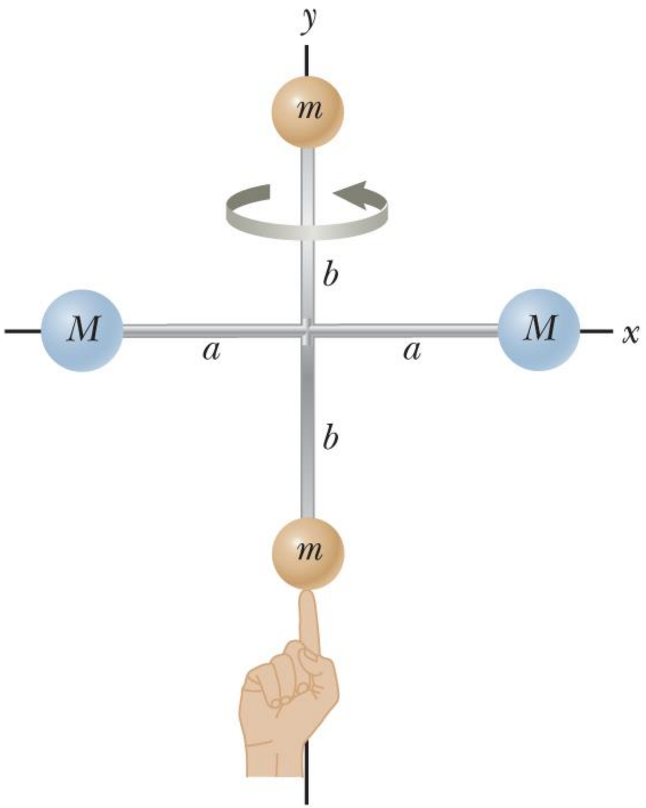


Example 10.10:
An Unusual Baton

$$I_y = \sum_i m_i r_i^2 = Ma^2 + Ma^2 = \boxed{2Ma^2}$$

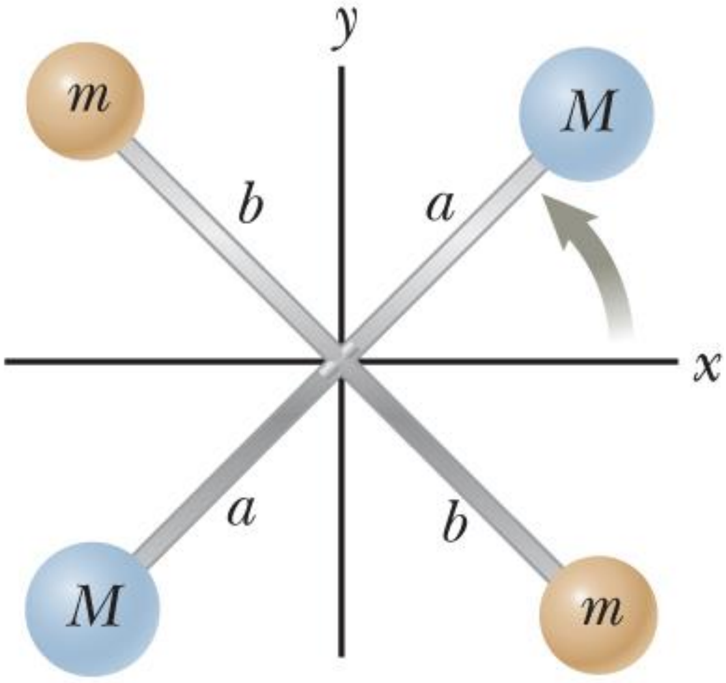
$$K_R = \frac{1}{2} I_y \omega^2$$

$$= \frac{1}{2}\left(2Ma^2\right)\omega^2 = \boxed{Ma^2\omega^2}$$



Example 10.10:
An Unusual Baton

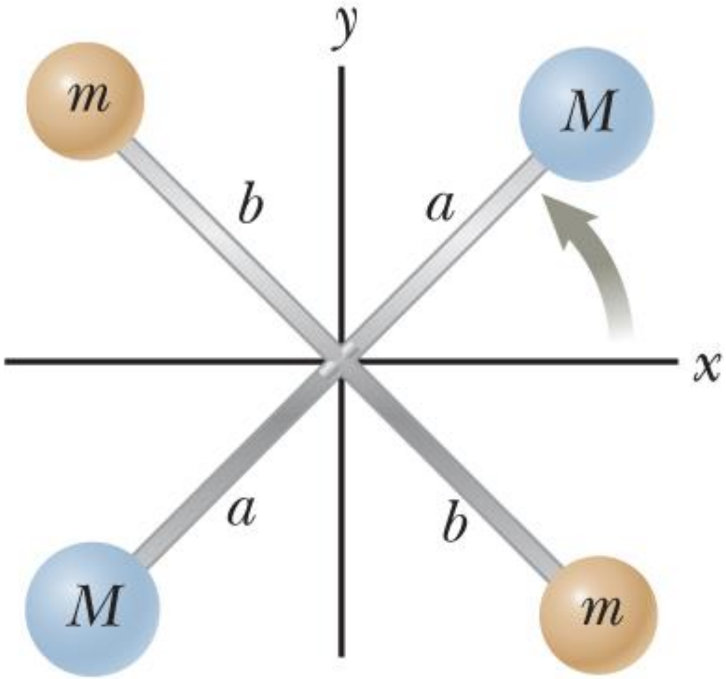
(B) Suppose the system rotates in the *xy* plane about an axis (the *z* axis) through the center of the baton. Calculate the moment of inertia and rotational kinetic energy about this axis.



Example 10.10: An Unusual Baton

$$I_z = \sum_i m_i r_i^2 = Ma^2 + Ma^2 + mb^2 + mb^2$$
$$= \boxed{2Ma^2 + 2mb^2}$$

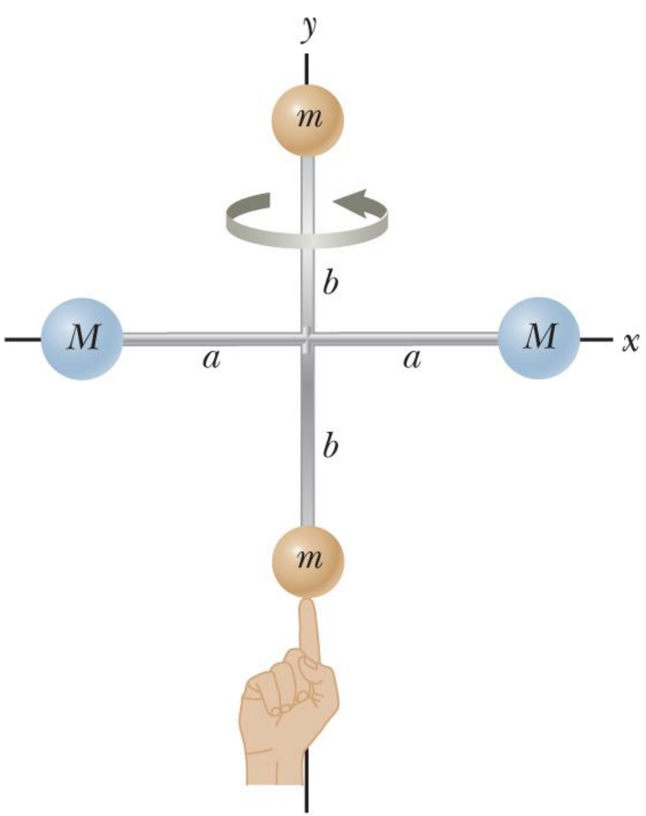
$$K_R = \frac{1}{2} I_z \omega^2$$
$$= \frac{1}{2}\left(2Ma^2 + 2mb^2\right)\omega^2$$
$$= \boxed{\left(Ma^2 + mb^2\right)\omega^2}$$



Example 10.10: An Unusual Baton

What if the mass *M* is much larger than *m*? How do the answers to parts (A) and (B) compare?

$$I_z = 2Ma^2 \text{ and } K_R = Ma^2\omega^2$$



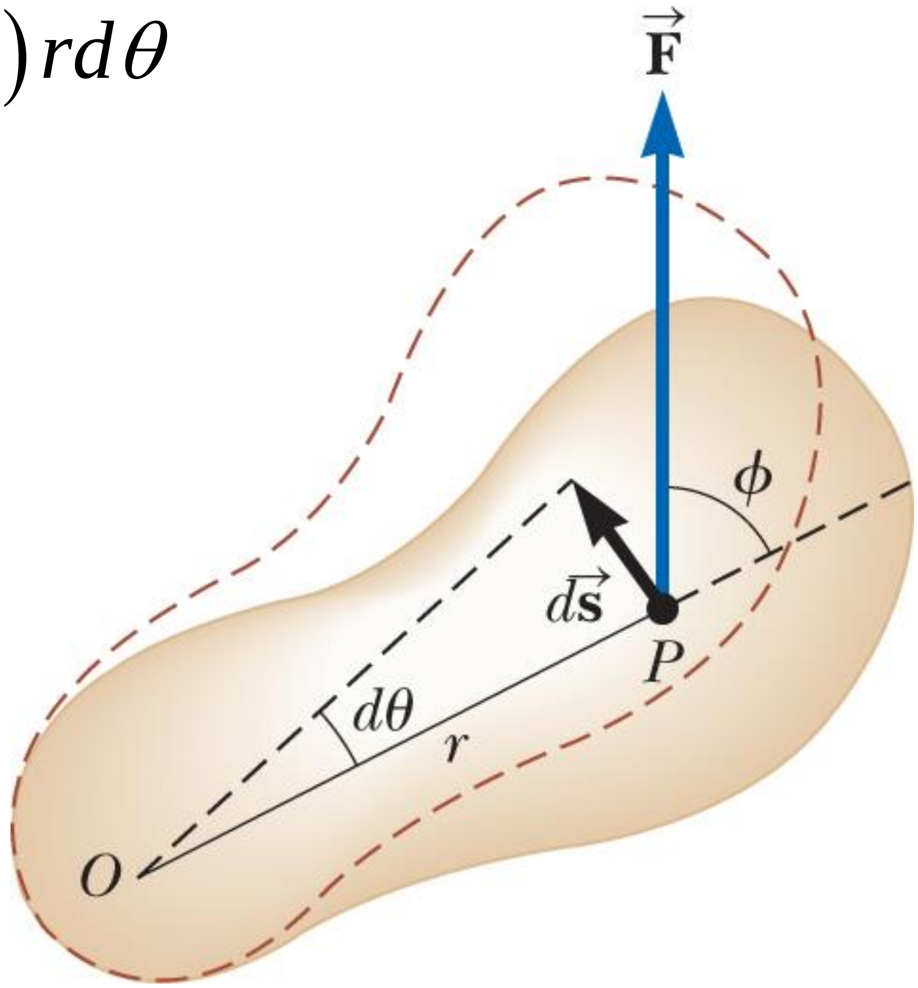
Energy Considerations in Rotational Motion

$$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = (F \sin\phi) r d\theta$$

$$dW = \tau d\theta$$

$$\frac{dW}{dt} = \tau \frac{d\theta}{dt}$$

$$P = \frac{dW}{dt} = \tau\omega$$



Energy Considerations in Rotational Motion

$$\sum \tau_{\text{ext}} = I\alpha$$

$$\sum \tau_{\text{ext}} = I\alpha = I\frac{d\omega}{dt} = I\frac{d\omega}{d\theta}\frac{d\theta}{dt} = I\frac{d\omega}{d\theta}\omega$$

$$\sum \tau_{\text{ext}} d\theta = dW = I\omega\, d\omega$$

$$\int_{\omega_i}^{\omega_f} I\omega\, d\omega = \frac{1}{2}I\omega_f{}^2 - \frac{1}{2}\omega_i{}^2$$

Rotational and Translational Motion Expressions

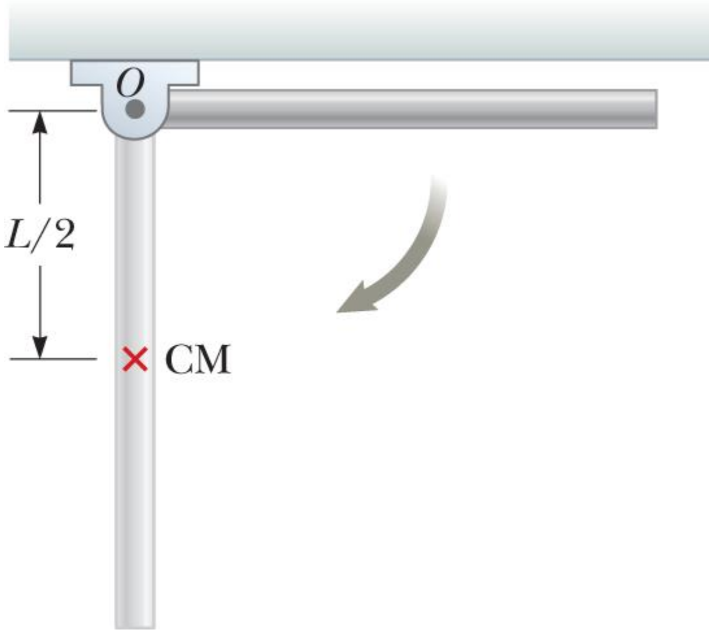
TABLE 10.3 Useful Equations in Rotational and Translational Motion

Rotational Motion About a Fixed Axis	Translational Motion
Angular speed $\omega = d\theta/dt$	Translational speed $v = dx/dt$
Angular acceleration $\alpha = d\omega/dt$	Translational acceleration $a = dv/dt$
Net torque $\Sigma\tau_{\text{ext}} = I\alpha$	Net force $\Sigma F = ma$
If $\alpha =$ constant $\begin{cases} \omega_f = \omega_i + \alpha t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$	If $a =$ constant $\begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$	Work $W = \int_{x_i}^{x_f} F_x \, dx$
Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$	Kinetic energy $K = \frac{1}{2}mv^2$
Power $P = \tau\omega$	Power $P = Fv$
Angular momentum $L = I\omega$	Linear momentum $p = mv$
Net torque $\Sigma\tau = dL/dt$	Net force $\Sigma F = dp/dt$

Example 10.11: Rotating Rod Revisited

A uniform rod of length L and mass M is free to rotate on a frictionless pin passing through one end. The rod is released from rest in the horizontal position.

(A) What is its angular speed when the rod reaches its lowest position?



$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2} I\omega^2 - 0\right) + \left(0 - \frac{1}{2} mgL\right) = 0$$

$$\omega = \sqrt{\frac{MgL}{I}} = \sqrt{\frac{MgL}{\frac{1}{3}ML^2}} = \boxed{\sqrt{\frac{3g}{L}}}$$

Example 10.11: Rotating Rod Revisited

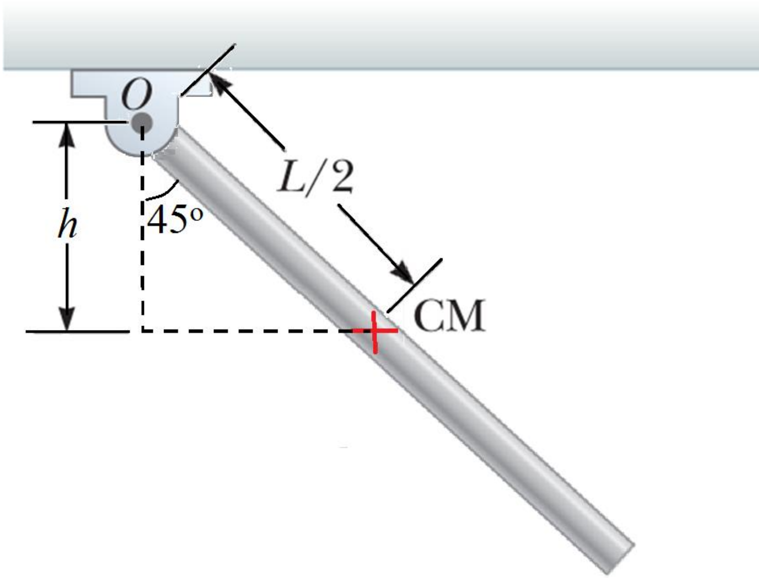
(B) Determine the tangential speed of the center of mass and the tangential speed of the lowest point on the rod when it is in the vertical position.

$$v_{\mathrm{CM}} = r\omega = \frac{L}{2}\omega = \boxed{\frac{1}{2}\sqrt{3gL}}$$

$$v = 2v_{\mathrm{CM}} = \boxed{\sqrt{3gL}}$$

Example 10.11: Rotating Rod Revisited

What if we want to find the angular speed of the rod when the angle it makes with the horizontal is 45.0°? Because this angle is half of 90.0°, for which we solved the problem in the previous slide, is the angular speed at this configuration half the previous answer, that is, $\frac{1}{2}\sqrt{3g/L}$?

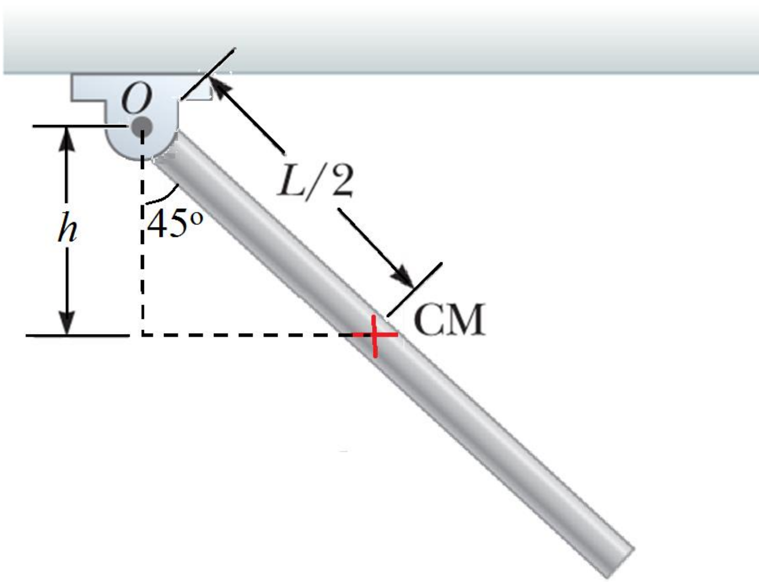


Example 10.11: Rotating Rod Revisited

$$\frac{L}{2}\cos 45° = h = \frac{\sqrt{2}L}{4}$$

$$0 = \frac{1}{2}I\omega^2 - Mgh$$

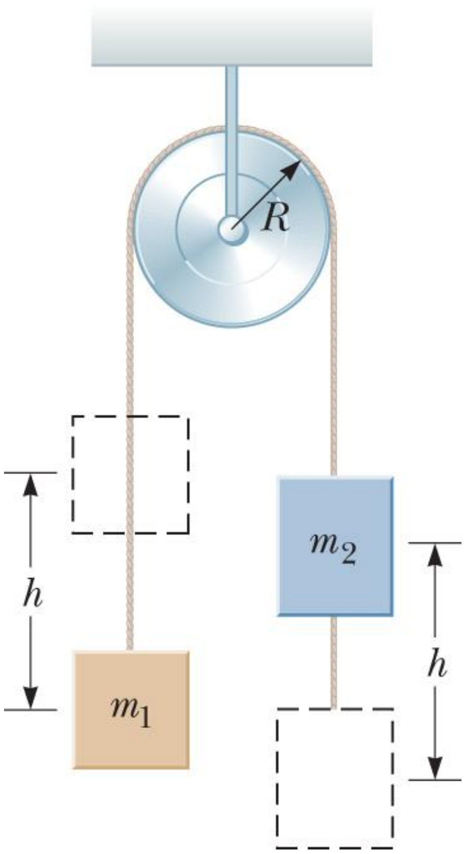
$$= \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 - Mg\left(\frac{\sqrt{2}L}{4}\right)$$



$$\omega = \frac{1}{2^{1/4}}\sqrt{\frac{3g}{L}} \approx 0.841\sqrt{\frac{3g}{L}}$$

Example 10.12:
Energy and the Atwood Machine

Two blocks having different masses m_1 and m_2 are connected by a string passing over a pulley. The pulley has a radius R and moment of inertia I about its axis of rotation. The string does not slip on the pulley, and the system is released from rest. Find the translational speeds of the blocks after block 2 descends through a distance h and find the angular speed of the pulley at this time.



Example 10.12:
Energy and the Atwood Machine

$$\Delta K + \Delta U = 0$$

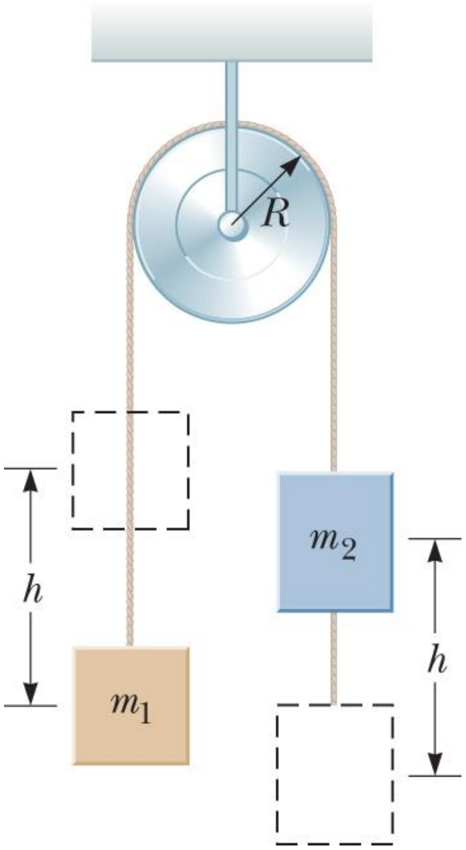
$$\left[\left(\frac{1}{2}m_1 v_f^{\ 2} + \frac{1}{2}m_2 v_f^{\ 2} + \frac{1}{2}I\omega_f^{\ 2}\right) - 0\right] + \left[\left(m_1 gh - m_2 gh\right) - 0\right] = 0$$

$$\frac{1}{2}m_1 v_f^{\ 2} + \frac{1}{2}m_2 v_f^{\ 2} + \frac{1}{2}I\frac{v_f^{\ 2}}{R^2} = m_2 gh - m_1 gh$$

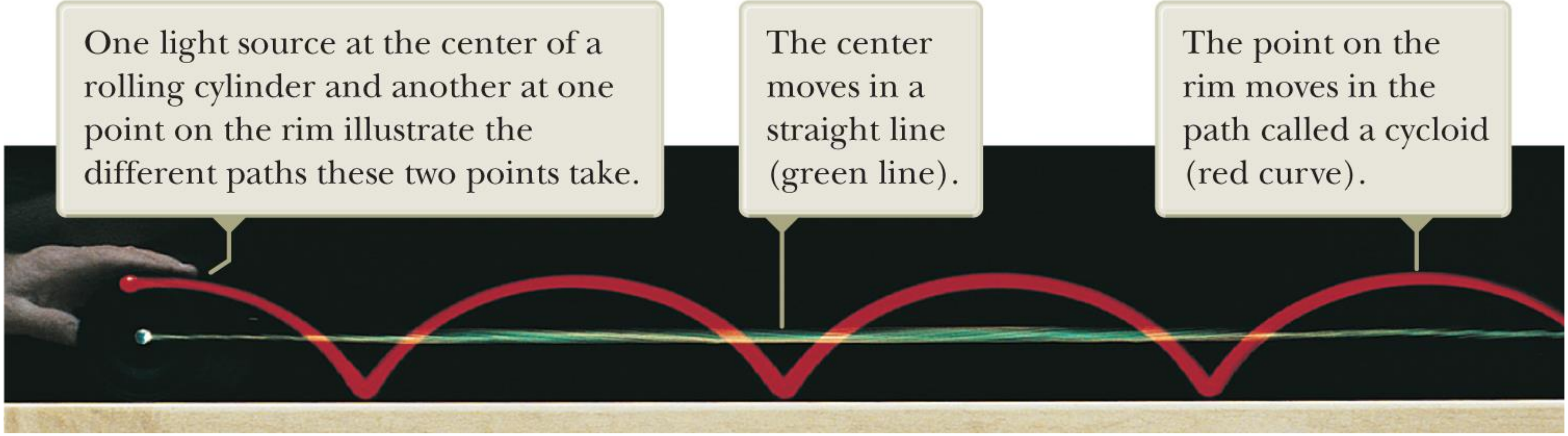
$$\frac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)v_f^{\ 2} = \left(m_2 - m_1\right)gh$$

$$v_f = \boxed{\left[\frac{2\left(m_2 - m_1\right)gh}{m_1 + m_2 + I/R^2}\right]^{1/2}}$$

$$\omega_f = \frac{v_f}{R} = \boxed{\frac{1}{R}\left[\frac{2\left(m_2 - m_1\right)gh}{m_1 + m_2 + I/R^2}\right]^{1/2}}$$



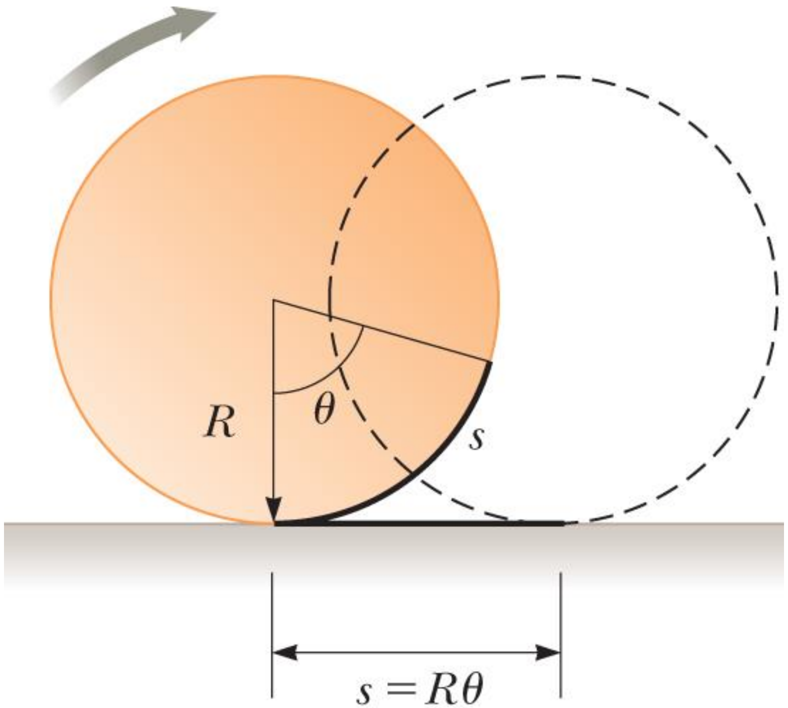
Rolling Motion of a Rigid Object



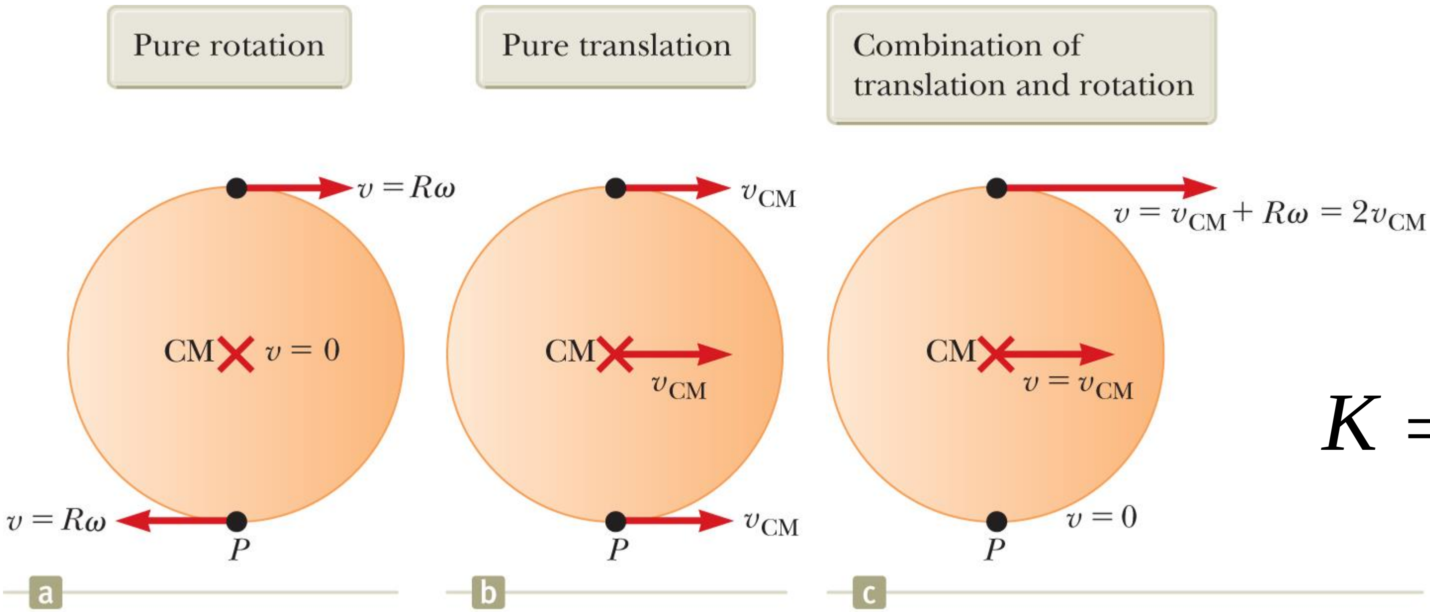
Rolling Motion of a Rigid Object

$$v_{\mathrm{CM}} = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\alpha$$

$$a_{\mathrm{CM}} = \frac{dv_{\mathrm{CM}}}{dt} = R\frac{d\omega}{dt} = R\alpha$$



Kinetic Energy of a Rolling Object



$$K = \frac{1}{2} I_P \omega^2$$

$$K = \frac{1}{2} I_{\text{CM}} \omega^2 + \frac{1}{2} M R^2 \omega^2$$

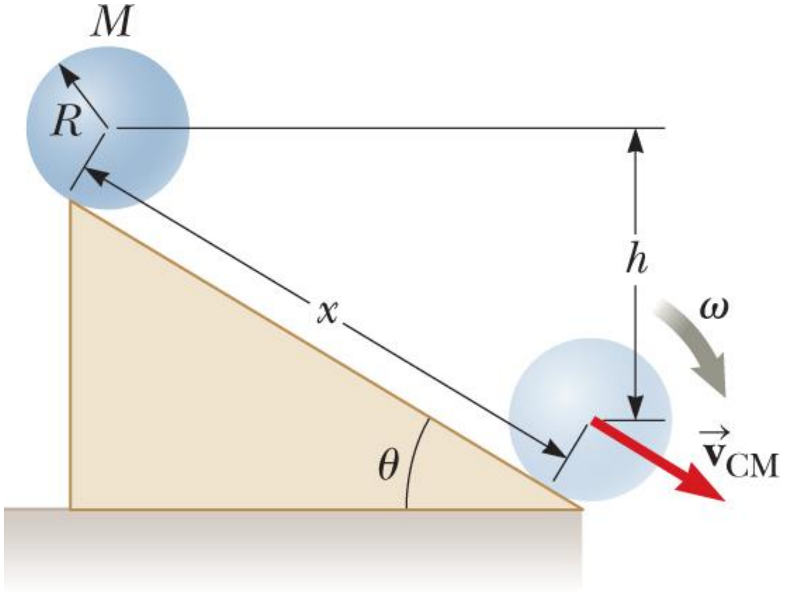
$$\boxed{K = \frac{1}{2} I_{\text{CM}} \omega^2 + \frac{1}{2} M v_{\text{CM}}^2}$$

Analysis of Rolling Object

$$K = \frac{1}{2} I_{\mathrm{CM}} \left(\frac{v_{\mathrm{CM}}}{R} \right)^2 + \frac{1}{2} M v_{\mathrm{CM}}{}^2 \Rightarrow K = \frac{1}{2} \left(\frac{I_{\mathrm{CM}}}{R^2} + M \right) v_{\mathrm{CM}}{}^2$$

$$\Delta K + \Delta U = 0 \Rightarrow \left[\frac{1}{2} \left(\frac{I_{\mathrm{CM}}}{R^2} + M \right) v_{\mathrm{CM}}{}^2 - 0 \right] + (0 - Mgh) = 0$$

$$v_{\mathrm{CM}} = \left[\frac{2gh}{1 + \left(I_{\mathrm{CM}} / MR^2 \right)} \right]^{1/2}$$



Quick Quiz 10.7

A ball rolls without slipping down incline A, starting from rest. At the same time, a box starts from rest and slides down incline B, which is identical to incline A except that it is frictionless. Which arrives at the bottom first?

(a) The ball arrives first.

(b) The box arrives first.

(c) Both arrive at the same time.

(d) It is impossible to determine.

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(a) The ball arrives first.

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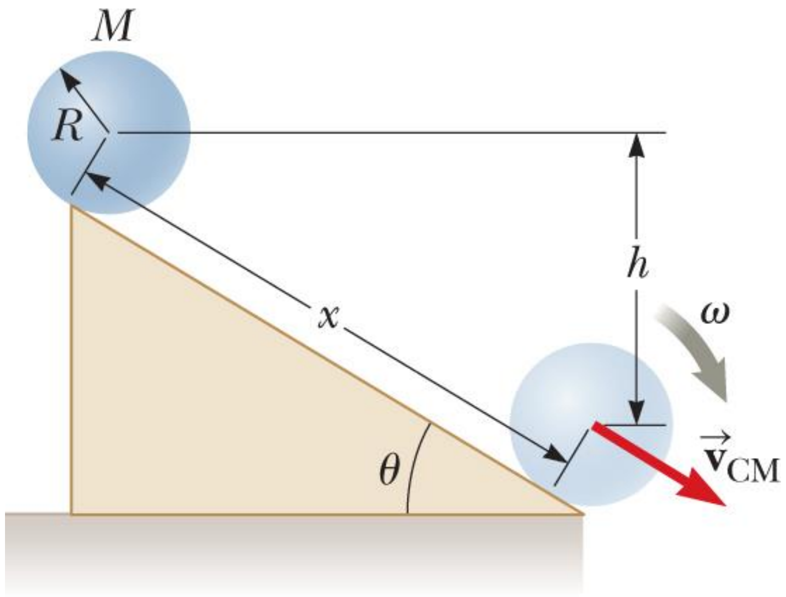
(d) It is impossible to determine.

Example 10.13: Sphere Rolling Down and Incline

Suppose the sphere shown in the figure is solid and uniform. Calculate the translational speed of the center of mass at the bottom of the incline and the magnitude of the translational acceleration of the center of mass.



Example 10.13: Sphere Rolling Down and Incline

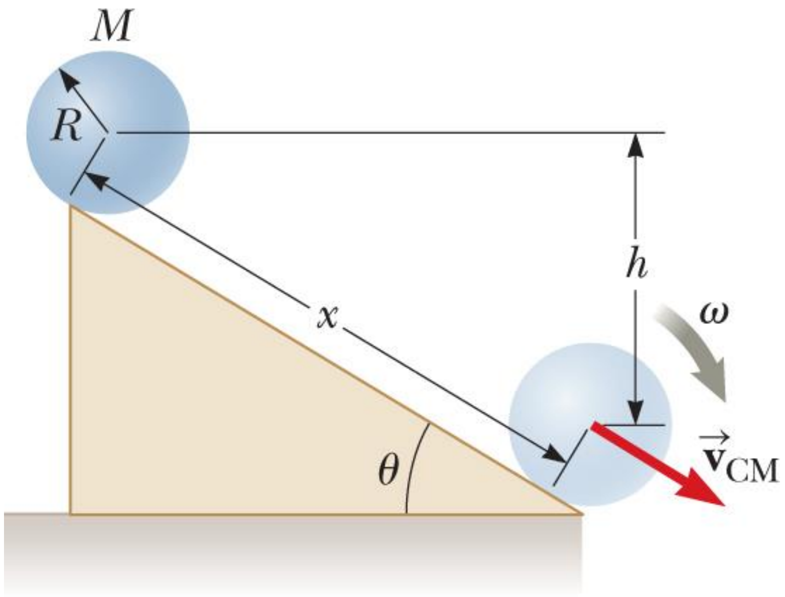
$$v_{\mathrm{CM}} = \left[\frac{2gh}{1 + \left(\frac{2}{5} MR^2 / MR^2 \right)} \right]^{1/2} = \boxed{\left(\frac{10}{7} gh \right)^{1/2}}$$

$$h = x \sin\theta$$

$$v_{\mathrm{CM}}{}^2 = \frac{10}{7} g x \sin\theta$$

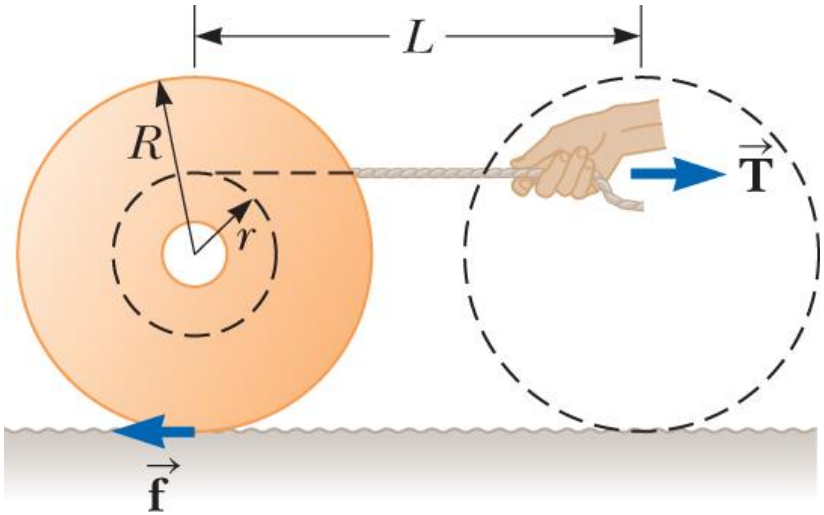
$$v_{\mathrm{CM}}{}^2 = 2 a_{\mathrm{CM}} x$$

$$a_{\mathrm{CM}} = \frac{5}{7} g \sin\theta$$



Example 10.14:
Pulling on a Spool

A cylindrically symmetric spool of mass m and radius R sits at rest on a horizontal table with friction. With your hand on a light string wrapped around the axle of radius r, you pull on the spool with a constant horizontal force of magnitude T to the right. As a result, the spool rolls without slipping a distance L along the table with no rolling friction.



(A) Find the final translational speed of the center of mass of the spool.

Example 10.14: Pulling on a Spool

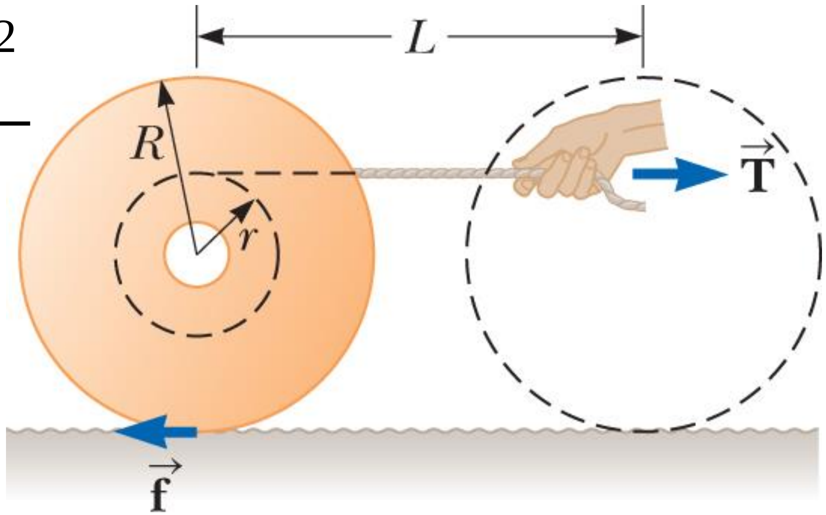
$$W = \Delta K = \Delta K_{\text{trans}} + \Delta K_{\text{rot}} \qquad \ell = r\theta = \frac{r}{R}L$$

$$\ell + L = L\left(1 + r/R\right)$$

$$W = TL\left(1 + \frac{r}{R}\right) = \frac{1}{2}m{v_{\text{CM}}}^2 + \frac{1}{2}I\omega^2$$

$$TL\left(1 + \frac{r}{R}\right) = \frac{1}{2}m{v_{\text{CM}}}^2 + \frac{1}{2}I\frac{{v_{\text{CM}}}^2}{R^2}$$

$$v_{\text{CM}} = \boxed{\sqrt{\frac{2TL\left(1 + r/R\right)}{m\left(1 + I/mR^2\right)}}}$$



Example 10.14: Pulling on a Spool

(B) Find the value of the friction force *f*.

$$m\left(v_{\mathrm{CM}}-0\right)=\left(T-f\right)\Delta t \rightarrow mv_{\mathrm{CM}}=\left(T-f\right)\Delta t$$

$$\Delta t=\frac{L}{v_{\mathrm{CM,avg}}}=\frac{2L}{v_{\mathrm{CM}}}$$

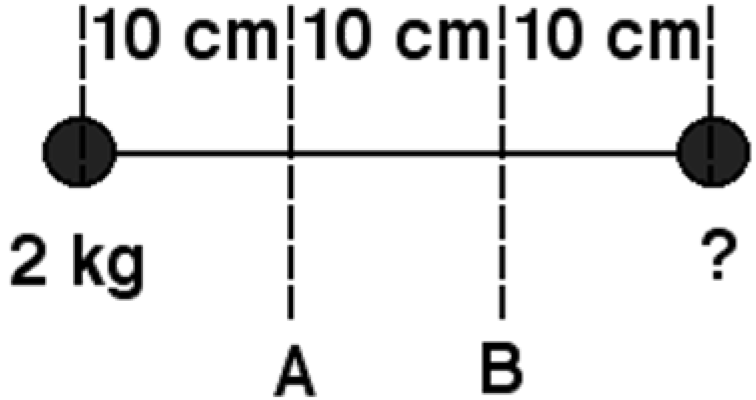
$$mv_{\mathrm{CM}}=\left(T-f\right)\frac{2L}{v_{\mathrm{CM}}} \rightarrow f=T-\frac{m{v_{\mathrm{CM}}}^{2}}{2L}$$

$$f=T-\frac{m}{2L}\left[\frac{2TL\left(1-r/\mathrm{R}\right)}{m\left(1+I/mR^{2}\right)}\right]=T-T\,\frac{\left(1+r/R\right)}{\left(1+I/mR^{2}\right)}=T\left[\frac{1-mrR}{1+mR^{2}}\right]$$

Assessing to Learn

The rotational inertia of the dumbbell (see figure) about axis A is twice the rotational inertia about axis B. The unknown mass is:

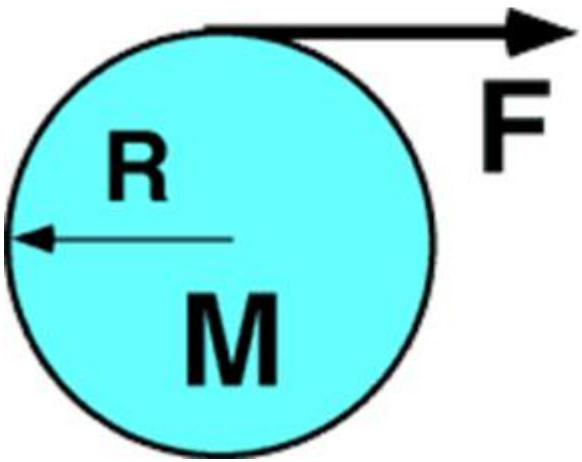
1. 4/7 kg
2. 2 kg
3. 4 kg
4. 5 kg
5. 7 kg
6. 8 kg
7. 10 kg
8. None of the above
9. Cannot be determined
10. The rotational inertia cannot b different axes.



Assessing to Learn

A disk, with radius 0.25 m and mass 4 kg, lies flat on a smooth horizontal tabletop. A string wound about the disk is pulled with a force of 8 N. What is the acceleration of the disk?

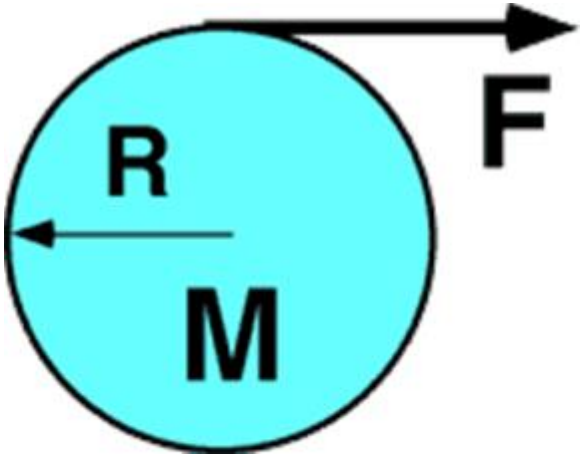
1. 0
2. 0.5 m/s^2
3. 1 m/s^2
4. 2 m/s^2
5. 4 m/s^2
6. None of the above.
7. Cannot be determined.



Assessing to Learn

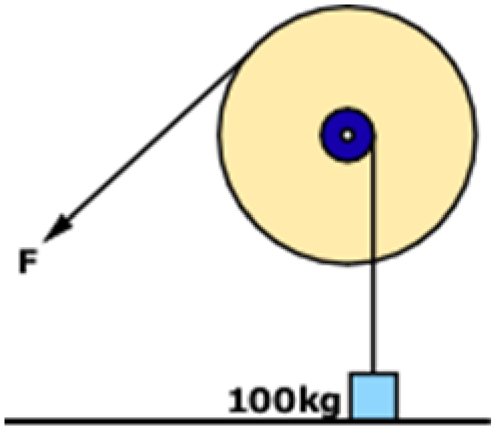
A disk, with radius 0.25 m and mass 4 kg, lies flat on a smooth horizontal tabletop. A string wound about the disk is pulled with a force of 8 N. What is the *angular* acceleration of the disk?

1. 0
2. 0.5 m/s^2
3. 1 m/s^2
4. 2 m/s^2
5. 4 m/s^2
6. None of the above.
7. Cannot be determined.



Assessing to Learn

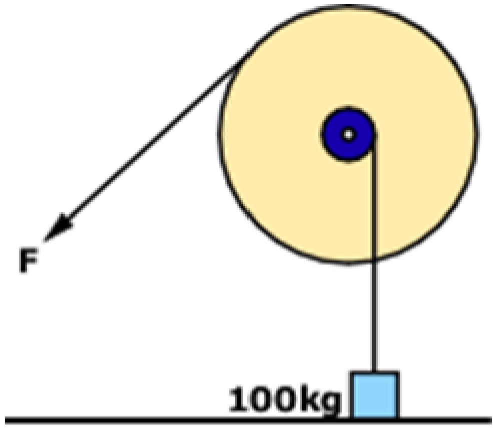
A 100 kg crate is attached to a rope wrapped around the inner disk as shown. A person pulls on another rope wrapped around the outer disk with force *F* to lift the crate. What force *F* is needed to lift the crate 2 m?



1. about 20 N
2. about 50 N
3. about 100 N
4. about 200 N
5. about 500 N
6. about 1,000 N
7. about 2,000 N
8. about 5,000 N
9. Impossible to determine without knowing the radii
10. Impossible to determine for some other reason(s)

Assessing to Learn

A 100-kg crate is attached to a rope wrapped around the inner disk as shown. A person pulls on another rope wrapped around the outer disk with force *F* to lift the crate. What force *F* is needed to lift the crate 2 m?

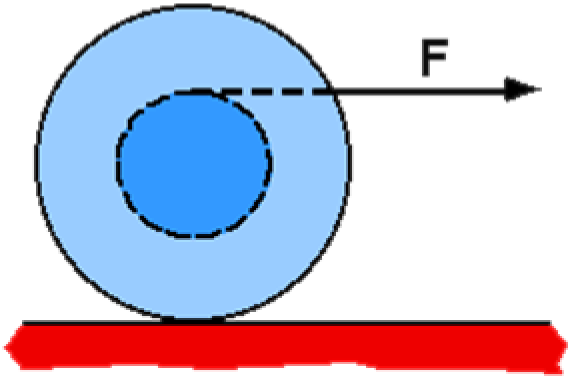


1. about 400 J
2. slightly less than 2,000 J
3. Exactly 2,000 J
4. slightly more than 2,000 J
5. much more than 2,000 J
6. Impossible to determine without knowing *F*
7. Impossible to determine without knowing the radii
8. Impossible to determine without knowing the mass of the pulley
9. Impossible to determine for two or more of the reasons given in 6, 7, and 8 above
10. Impossible to determine for some other reason(s)

Assessing to Learn

A spool has string wrapped around its center axle and is sitting on a horizontal surface. If the string is pulled in the horizontal direction when tangent to the top of the axle, the spool will...

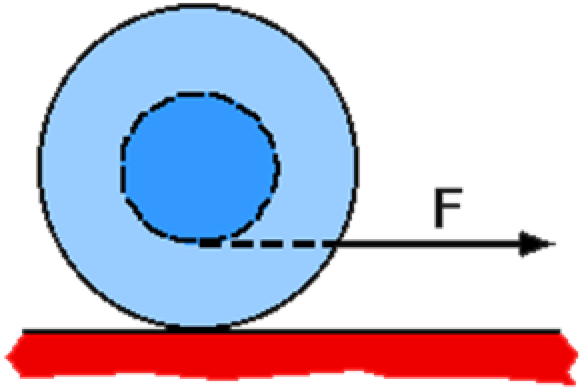
1. ... roll to the right.
2. ... not roll, only slide to the right.
3. ... spin and slip, without moving left or right.
4. ... roll to the left.
5. None of the above
6. The motion cannot be determined.



Assessing to Learn

A spool has string wrapped around its center axle and is sitting on a horizontal surface. If the string is pulled in the horizontal direction when tangent to the bottom of the axle, the spool will...

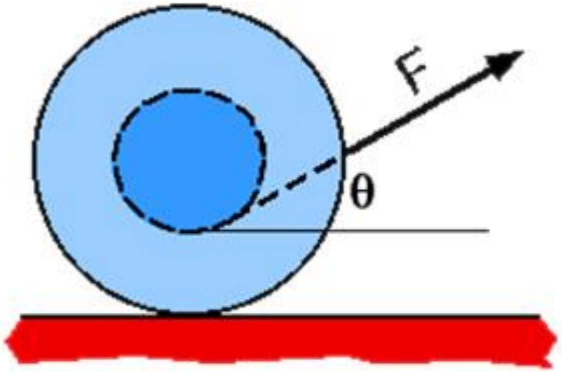
1. ... roll to the right.
2. ... not roll, only slide to the right.
3. ... spin and slip, without moving left or right.
4. ... roll to the left.
5. None of the above
6. The motion cannot be determined.



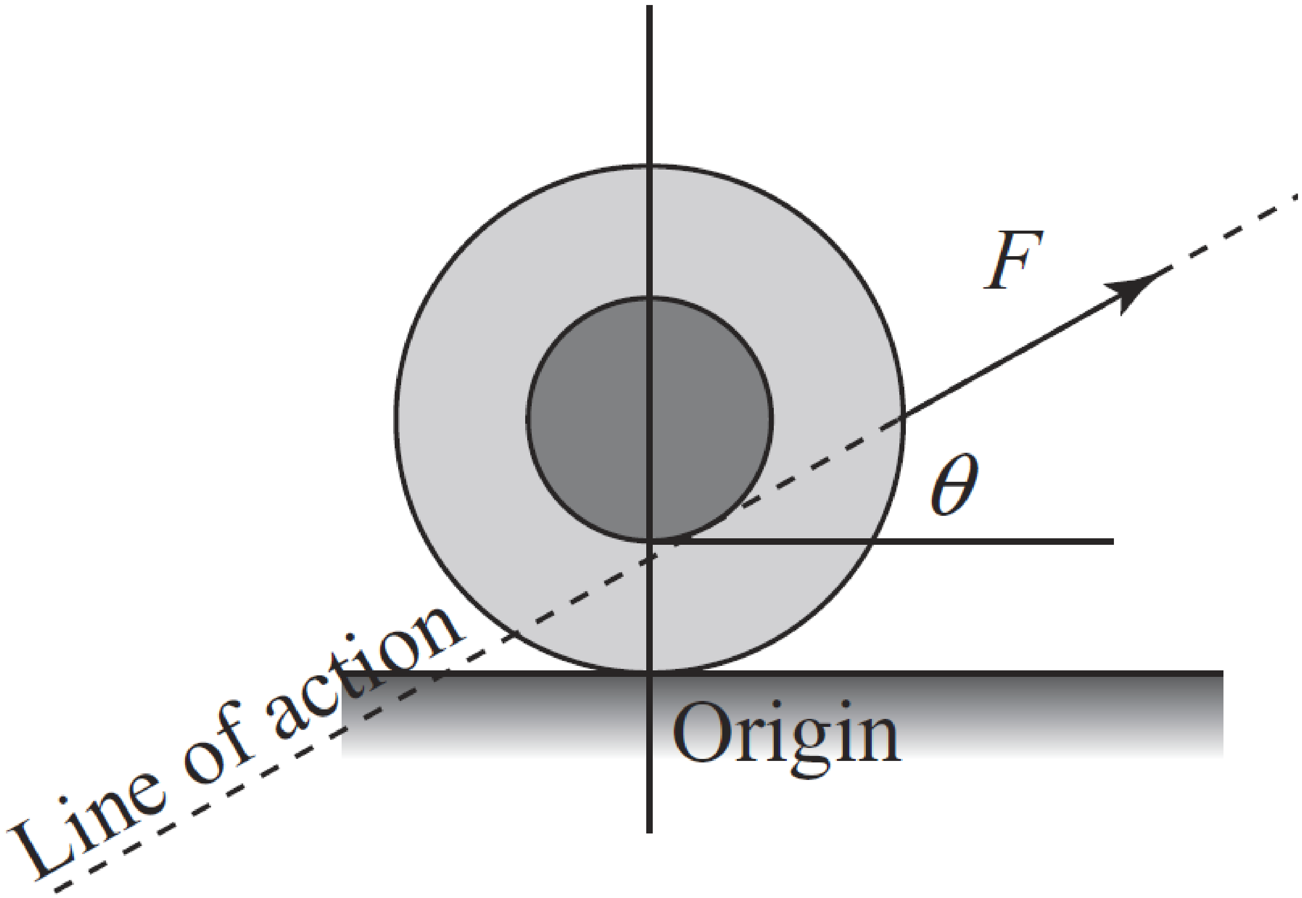
Assessing to Learn

A spool has string wrapped around its center axle and is sitting on a horizontal surface. If the string is pulled at an angle to the horizontal when drawn from the bottom of the axle, the spool will...

1. ... roll to the right.
2. ... not roll, only slide to the right.
3. ... spin and slip, without moving left or right.
4. ... roll to the left.
5. None of the above
6. The motion cannot be determined.



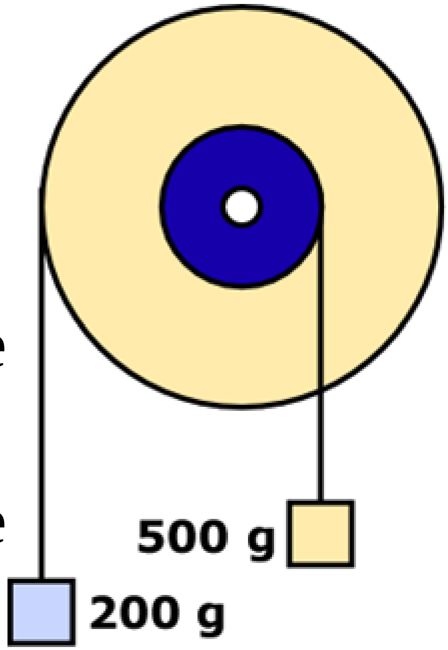
Assessing to Learn



Assessing to Learn

Two blocks hang from strings wound around different parts of a double pulley as shown. Assuming the system is **not** in equilibrium, what happens to the system's *potential energy* when it is released from rest?

1. It remains the same.
2. It decreases.
3. It increases.
4. Impossible to determine without knowing the radii of the two pulleys.
5. Impossible to determine without knowing the ratio of the radii of the two pulleys.
6. Impossible to determine for some other reason.



Assessing to Learn

Two blocks hang from strings would around different parts of a 2 kg double pulley as shown. The pivot exerts a normal force F_N supporting the double pulley. Assuming the system is **not** in equilibrium, which statement about F_N is true after the system is released from rest? (Use g = 10 N/kg.)

1. $F_N = 20$ N
2. 20 N $< F_N <$ 27 N
3. $F_N = 27$ N
4. $F_N >$ 27N
5. It is impossible to predict what the normal force on the double pulley will be.