Math 2568/4568 Homework 5

P146-147

Exercise 8

Solution: First we compute $\cos\theta$:

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{2\times 1 + (-3)\times(-2) + 1\times 3}{\sqrt{2^2+(-3)^2+1^2}\cdot\sqrt{1^2+(-2)^2+3^2}}$$

$$= \frac{11}{14},$$

so $\theta = \arccos\frac{11}{14} \approx 0.6669\ldots$ radian.

Exercise 24

Solution: By equation (4), we have

$$\vec{u}_1 = \mathrm{proj}_{\vec{q}}\,\vec{u} = \frac{\vec{u}\cdot\vec{q}}{\|\vec{q}\|}\left(\frac{\vec{q}}{\|\vec{q}\|}\right)$$

$$= \frac{6\times 1 + 2\times(-1)}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$= 2\begin{pmatrix}1\\-1\end{pmatrix}$$

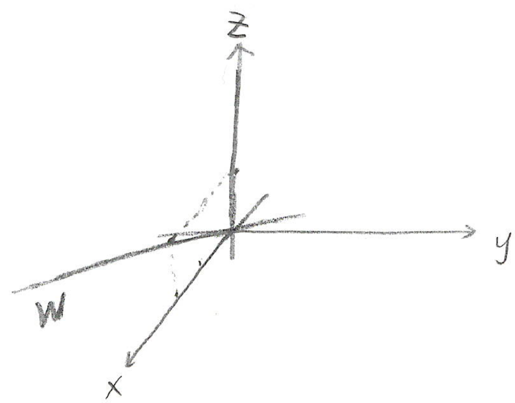
$$= \begin{pmatrix}2\\-2\end{pmatrix}.$$

And then $\vec{u}_2 = \vec{u} - \vec{u}_1 = \begin{pmatrix}6\\2\end{pmatrix} - \begin{pmatrix}2\\-2\end{pmatrix} = \begin{pmatrix}4\\4\end{pmatrix}$.

P_{167}

Exercise 20

Solution: Geometrically, W can be viewed as a line in $\mathbb{R}^3$ with equations $\begin{cases} x = 2r \\ y = 0 \\ z = r \end{cases}$. Below is the sketch of W.



P_{175}

Exercise 18

Solution: To show W is a subspace of $\mathbb{R}^3$, we need to check that (s1), (s2), and (s3) in theorem 2 are satisfied by W.

(s1): For $\vec{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \in \mathbb{R}^3$, $\vec{a}^T \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0$, so $\vec{0} \in \mathbb{R}^3$.

(s2): Let $\vec{x}$ and $\vec{y}$ be two arbitrary vectors in W, then

$$\vec{a}^T \vec{x} = \vec{a}^T \vec{y} = 0.$$

By theorem 9 on page 63,

$$\vec{a}^T (\vec{x} + \vec{y}) = \vec{a}^T \vec{x} + \vec{a}^T \vec{y} = 0 + 0 = 0,$$

so $\vec{x} + \vec{y} \in W$.

(S3): Let $\vec{x}$ be an arbitrary vector in W and c a scalar, then

$$\vec{a}^T \vec{x} = 0.$$

By theorem 8 on page 63, we have

$$\vec{a}^T (c\vec{x}) = c(\vec{a}^T \vec{x}) = c \cdot 0 = 0.$$

so $c\vec{x} \in W$.

Exercise 32

Solution: Geometrically, U and V are two distinct planes in $\mathbb{R}^3$ defined by equations $x+y=0$ and $y-z=0$, respectively, so their union is two planes which is not a subspace of $\mathbb{R}^3$.

To show $U \cup V$ is not a subspace of $\mathbb{R}^3$, we need to show one of (S1)-(S3) in theorem 2 is not satisfied by $U \cup V$.

Consider two vectors $\vec{x}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ and $\vec{x}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, then we have

$$\vec{a}^T \vec{x}_1 = (1 \quad 1 \quad 0) \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 0,$$

and

$$\vec{b}^T \vec{x}_2 = (0 \quad 1 \quad -1) \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 0.$$

so $\vec{x}_1 \in U$ and $\vec{x}_2 \in V$, thus $\vec{x}_1, \vec{x}_2 \in U \cup V$.

Let $\vec{x}_3 = \vec{x}_1 + \vec{x}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, then $\vec{a}^T \vec{x}_3 = (1 \quad 1 \quad 0) \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 1$ and $\vec{b}^T \vec{x}_3 = (0 \quad 1 \quad -1) \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = -1$, so $\vec{x}_3 \notin U$ and $\vec{x}_3 \notin V$, thus $\vec{x}_3 \notin U \cup V$, so (S2) is not satisfied. hence $U \cup V$ is not a subspace of $\mathbb{R}^3$.