P146-147

#8

Calculate the dot product $u \cdot v$

$u = 2i - 3j + k$
$v = i - 2j + 3k$

$u \cdot v = 2 \cdot 1 + (-3)(-2) + 1 \cdot 3 = 11$

#24

find u_1 & u_2 s.t. $u_1 = \text{proj}_q u$.

Given: $u_1 \perp u_2$

$u = u_1 + u_2$

$u = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

$q = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\frac{u \cdot q}{|q|} \left(\frac{q}{|q|} \right)$ Formula for $\text{proj}_q u$

$u \cdot q = 6 \cdot 1 + 2(-1) = 4$

$|q| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$u_1 = \left(\frac{4}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} \right) \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= \frac{4}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 6 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} + \begin{bmatrix} u_{2x} \\ u_{2y} \end{bmatrix}$

$u = u_1 + u_2$

$\begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} u_{2x} \\ u_{2y} \end{bmatrix}$

Check: $u_1 \perp u_2$, or $u_1 \cdot u_2 = 0$

$\begin{bmatrix} 4 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ $4(2) + 4(-2) = 0$ ✓

$$\boxed{u_1 = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad u_2 = \begin{bmatrix} 4 \\ 4 \end{bmatrix}}$$

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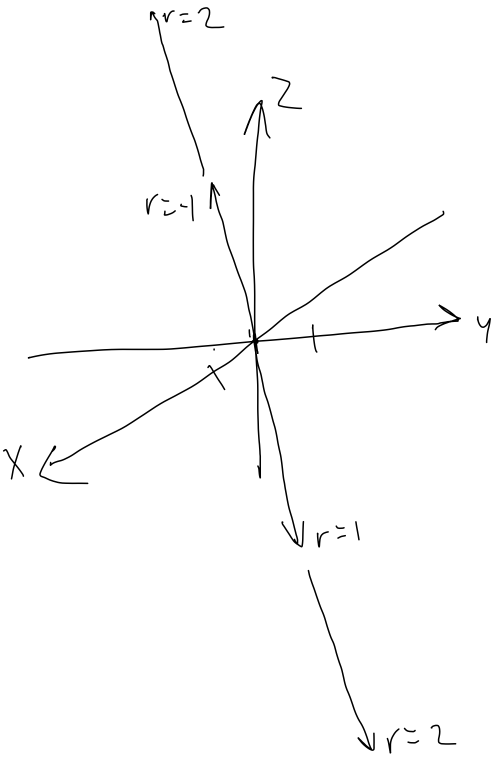
#20

interpret the subset W of R^3 geometrically, by sketching a graph for W

$$W = \left\{ W ; w = r\begin{bmatrix}2\\0\\1\end{bmatrix}, r \text{ any real} \right\}$$



W is a line

P175

#18

Let a be a fixed vector in R^3 and define W to be the subset of R^3 given by

$W = \{x : a^T x = 0\}$

Prove that W is a subspace of R^3.

<u>Pf</u> W is a subset of R^3,

Check 1) and 2)

1) If x_1, x_2 in W, then $x_1 + x_2$ in W.

assume x_1 and x_2 are in W.

$a^T x_1 = 0 = a^T x_2$

$\rightarrow a^T(x_1 + x_2) = 0$

$\rightarrow x_1 + x_2$ is in W. ✓

2) If x_1 is in W then αx_1 is in W.

x_1 is in W.

$a^T x_1 = 0$

$\alpha a^T x_1 = 0 = a^T(\alpha x_1) = 0$

$\rightarrow \alpha x_1$ is in W. ✓

1) and 2) are satisfied, so W is a subspace of R^3.

#32

let U and V be subspaces of R^3 defined by

$U = \{x : a^T x = 0\}$ and $V = \{x : b^T x = 0\}$

$a = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

Show $U \cup V$ is not a subspace of R_3

Pf

1) If x_1 and x_2 are in $U \cup V$ then $x_1 + x_2$ is in $U \cup V$.

$a^T x_1 = 0 = a^T x_2 = b^T x_1 = b^T x_2$

$a^T x_1 - a^T x_2 - b^T x_1 - b^T x_2 = 0$

$a^T (x_1 - x_2) - b^T (x_1 + x_2) = 0$

$a^T (x_1 - x_2) - b^T (x_1 + x_2) = a^T x_1$

$- a^T x_2 - b^T (x_1 + x_2) = 0$

$- a^T x_2 - \underbrace{b^T (2x_1 + x_2)}$

This does not equal $-a^T x_2$ ...

1) cannot be satisfied $\rightarrow$ Not a subspace of R^3