Due: Friday, 22 September 2025
Directions:

1. Provide an $\varepsilon - \delta$ proof of the following:
$$\lim_{x \to 5} 3x - 7 = 8$$

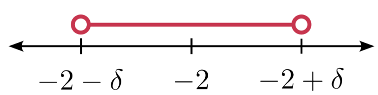
2. Provide an $\varepsilon - \delta$ proof of the following:
$$\lim_{x \to 3} \frac{3x^2 + 10x - 8}{x + 4} = 7$$

3. Provide an $\varepsilon - \delta$ proof of the following:
$$\lim_{x \to -2} 2x^2 - 5x - 12 = 6$$

Let ε and suppose $0 < |x - (-2)| = |x + 2| < \delta$ where $\delta = \min\left(1, \frac{\varepsilon}{15}\right)$. Then,

$$\begin{aligned} |f(x) - L| &= |(2x^2 - 5x - 12) - 6| \\ &= |2x^2 - 5x - 18| \\ &= |2x^2 + 4x - 9x - 18| \\ &= |2x(x + 2) - 9(x + 2)| \\ &= |(2x - 9)(x + 2)| \\ &= |x + 2| \cdot |2x - 9| \\ &< \delta \cdot |2x - 9| \end{aligned}$$

We attempt to place an upper bound on $|2x - 9|$:



Assume $\delta \leq 1$. Then,

$$\begin{aligned} |x - (-2)| &\leq 1 \\ |x + 2| &\leq 1 \\ -1 \leq x + 2 &\leq 1 \\ -3 \leq x &\leq -1 \\ -6 \leq 2x &\leq -2 \\ -15 \leq 2x - 9 &\leq -11 \\ |2x - 9| &\leq 15 \end{aligned}$$

Alternatively, again assuming $\delta \leq 1$

$$\begin{aligned} |2x - 9| &= |2(x + 2) - 13| \\ &\leq 2|x + 2| + 13 \text{ (Triangle Inequality)} \\ &\leq 2(1) + 13 \\ &= 15 \end{aligned}$$

Continuing from earlier,

$$\begin{aligned}
\ldots &< \delta \cdot |2x - 9| \\
&\leq 15\delta \\
&\leq 15\left(\frac{\varepsilon}{15}\right) \\
&\leq \varepsilon
\end{aligned}$$