Name: ______________________________ Date: __________

§1.7 Linear Independence

Determine if the vectors are linearly independent. Justify each answer.

1. $\begin{bmatrix}5\\1\\0\end{bmatrix}, \begin{bmatrix}7\\2\\-6\end{bmatrix}, \begin{bmatrix}-2\\-1\\6\end{bmatrix}$ 2. $\begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\5\\-8\end{bmatrix}, \begin{bmatrix}-3\\4\\1\end{bmatrix}$ 3. $\begin{bmatrix}1\\-3\end{bmatrix}, \begin{bmatrix}-3\\6\end{bmatrix}$ 4. $\begin{bmatrix}-1\\4\end{bmatrix}, \begin{bmatrix}-2\\8\end{bmatrix}$

Determine if the columns of the matrix form a linearly independent set. Justify each answer

5. $\begin{bmatrix}0 & -8 & 5\\3 & -7 & 4\\-1 & 5 & -4\\1 & -3 & 2\end{bmatrix}$ 6. $\begin{bmatrix}-4 & -3 & 0\\0 & -1 & 4\\1 & 0 & 3\\5 & 4 & 6\end{bmatrix}$ 7. $\begin{bmatrix}1 & 4 & -3 & 0\\-2 & -7 & 5 & 1\\-4 & -5 & 7 & 5\end{bmatrix}$ 8. $\begin{bmatrix}1 & -3 & 3 & -2\\-3 & 7 & -1 & 2\\0 & 1 & -4 & 3\end{bmatrix}$

(a) For what values of h is $\mathbf{v}_3$ in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$ (b) For what values of h is $\{\mathbf{v}_2, \mathbf{v}_2, \mathbf{v}_3\}$ linearly *dependent*? Justify each answer.

9. $\mathbf{v}_1 = \begin{bmatrix}1\\-3\\2\end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix}-3\\10\\-6\end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix}2\\-7\\h\end{bmatrix}$

10. $\mathbf{v}_1 = \begin{bmatrix}1\\-5\\-3\end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix}-2\\10\\6\end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix}2\\-10\\h\end{bmatrix}$

Find the value(s) of h for which the vectors are linearly *dependent*. Justify each answer.

11. $\mathbf{v}_1 = \begin{bmatrix}1\\-1\\4\end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix}3\\-5\\7\end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix}-1\\5\\h\end{bmatrix}$

12. $\mathbf{v}_1 = \begin{bmatrix}2\\-4\\1\end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix}-6\\7\\-3\end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix}8\\h\\4\end{bmatrix}$

13. $\mathbf{v}_1 = \begin{bmatrix}1\\5\\-3\end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix}-2\\-9\\6\end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix}3\\h\\-9\end{bmatrix}$

14. $\mathbf{v}_1 = \begin{bmatrix}1\\-1\\3\end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix}-5\\7\\8\end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix}1\\1\\h\end{bmatrix}$

Determine by inspection whether the following sets of vectors are linearly *independent*. Justify each answer.

15. $\begin{bmatrix}5\\1\end{bmatrix}, \begin{bmatrix}2\\8\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}-1\\7\end{bmatrix}$ 16. $\begin{bmatrix}4\\-2\\6\end{bmatrix}, \begin{bmatrix}6\\-3\\9\end{bmatrix}$

17. $\begin{bmatrix}3\\5\\1\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}-6\\5\\4\end{bmatrix}$

18. $\begin{bmatrix}4\\4\end{bmatrix}, \begin{bmatrix}-1\\3\end{bmatrix}, \begin{bmatrix}2\\5\end{bmatrix}, \begin{bmatrix}8\\1\end{bmatrix}$

19. $\begin{bmatrix}-8\\12\\-4\end{bmatrix}, \begin{bmatrix}2\\-3\\-1\end{bmatrix}$

20. $\begin{bmatrix}1\\4\\-7\end{bmatrix}, \begin{bmatrix}-2\\5\\3\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}$

Mark each statement True or False (**T/F**). Justify each answer on the basis of a careful reading of the text.

21. (**T/F**) The columns of a matrix A are linearly independent if the equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution.

22. (**T/F**) Two vectors are linearly independent if and only if they lie on a line through the origin.

23. (**T/F**) If S is a linearly indpendent set, then each vector is a linear combination of the other vectors in S.

24. (**T/F**) If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent.

25. (**T/F**) The columns of any 4×5 matrix are linearly independent.

26. (**T/F**) If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and if $\mathbf{z}$ is in Span$\{\mathbf{x}, \mathbf{y}\}$, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ linearly dependent.

27. (**T/F**) If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent and if $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent, then $\mathbf{z}$ is in Span$\{\mathbf{x}, \mathbf{y}\}$.

28. (**T/F**) If a set in $\mathbb{R}^n$ is linearly independent, then the set contains more vectors than there are entires in each vector.

Describe the possible echelon forms of the matrix. Use the notation of Example 1 in Section 1.2.

29. A is a 3×3 matrix with linearly independent columns.

30. A is a 2×2 matrix with linearly dependent columns.

31. A is a 4×2 matrix, $A = \begin{bmatrix}\mathbf{a}_1 & \mathbf{a}_2\end{bmatrix}$, and $\mathbf{a}_2$ is a not a multiple of $\mathbf{a}_1$.

32. A is a 4×3 matrix, $A = \begin{bmatrix}\mathbf{a}_1 & \mathbf{a}_2 & \mathbf{v}_3\end{bmatrix}$ such that $\{\mathbf{a}_1, \mathbf{a}_2\}$ is linearly independent and $\mathbf{a}_3$ is not in Span$\{\mathbf{a}_1, \mathbf{a}_2\}$

33. How many pivot columns must a 7×5 matrix have if its columns are linearly independent? Why?

34. How many pivot columns must a 5×7 matrix have if its columns span $\mathbb{R}^{\ntrianglelefteq}$? Why?

35. Construct 3×2 matrices A and B such that $A\mathbf{x} = \mathbf{0}$ has only the trivial solution and $B\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

36. (a) Fill in the blank in the following statement: "If A is an $m \times n$ matrix, then the columns of A are linearly independent if and only if A has _____ pivot columns."

    (b) Explain why the statement in (a) is true.

The following exercises should be solved *without performing row operations.*

37. Given $A = \begin{bmatrix} 2 & 3 & 5 \\ -5 & 1 & -4 \\ -3 & -1 & -4 \\ 1 & 0 & 1 \end{bmatrix}$, observe that the third column is the sum of the first two columns. Find a nontrivial solution to $A\mathbf{x} = \mathbf{0}$.

38. Given $A = \begin{bmatrix} 4 & 1 & 6 \\ -7 & 5 & 3 \\ 9 & -3 & 3 \end{bmatrix}$, observe that the first column plus twice the second column equals the third column. Find a nontrivial solution of $A\mathbf{x} = \mathbf{0}$.

39. this is a question

Solution: Questions 1 This is the solution to question 1

Solution: Question 2 This is the solution to question 2

40. f_x

Describe the possible echelon forms of the matrix. Use the notation of Example 1 in

$$\begin{bmatrix}5\\1\\0\end{bmatrix} \begin{bmatrix}7\\2\\-6\end{bmatrix}, \begin{bmatrix}-2\\-1\\6\end{bmatrix}$$

$$\begin{bmatrix}0\\0\\2\end{bmatrix} \begin{bmatrix}0\\5\\-8\end{bmatrix}, \begin{bmatrix}-3\\4\\1\end{bmatrix}$$

$$A\mathbf{x} = \begin{bmatrix}\mathbf{a}_1 & \mathbf{a}_2 & \dots & \mathbf{a}_n\end{bmatrix} \begin{bmatrix}x_1\\ \vdots \\ x_n\end{bmatrix} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \ldots + x_n\mathbf{a}_n$$

Multiply:

$$\begin{bmatrix}1 & 2 & -1\\ 0 & -5 & 3\end{bmatrix} \begin{bmatrix}4\\3\\7\end{bmatrix}$$

$$\begin{bmatrix}2 & -3\\ 8 & 0\\ -5 & 2\end{bmatrix} \begin{bmatrix}4\\7\end{bmatrix}$$

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Determine if the following sets of vectors are linearly independent columns of the following matrices form a linearly independent set. Justify each answer.

$$\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 6 \end{bmatrix}$$