[z.c.12.3.3.]

$$k\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t} \quad , \qquad 0 < x < L, \quad t > 0.$$

Randvillkor: $\begin{cases} \frac{\partial u}{\partial x}(0;t)=0 \\ \frac{\partial u}{\partial x}(L;t)=0 \end{cases}$ $\qquad t > 0$

Begynnelsevillkor: $u(x;0) = f(x)$, $\quad 0 < x < L$

$$a_0=\frac{2}{L}\int_0^L f(x)\,dx$$

$$a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$$

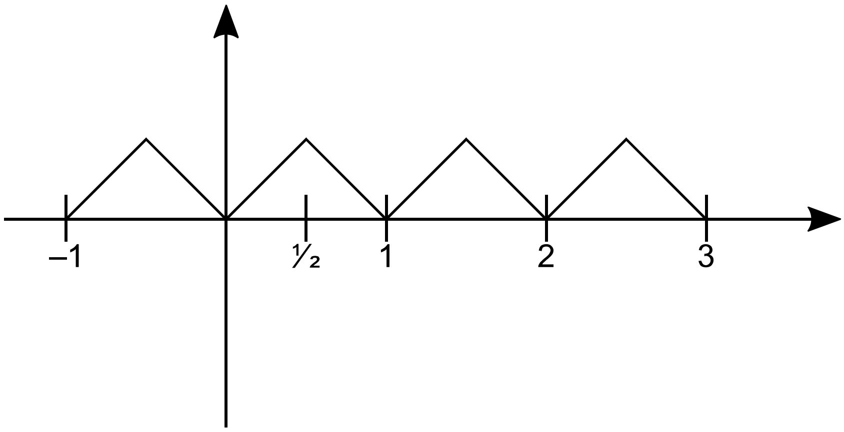
$$u(x;t)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}\, e^{-\left(\frac{n\pi}{L}\right)^2 kt}$$

[z.c.11.3.42.]

$$m\ddot{x}+kx=f(t)=\begin{cases} t & 0<t<\frac{1}{2} \\ 1-1 & \frac{1}{2}\le t<1 \end{cases}$$

$f(t + 1) = f(t)$

$$m=\frac{1}{4}, \quad k=12, \quad \ddot{x}+48x=4f(t)$$



Jämn

Fourierutveckla 4f.

$$4f(x) \sim \mathfrak{F}(4f)(x) = \sum_{n=1}^{\infty} a_n \cos \frac{n\pi t}{1/2} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos 2n\pi t$$

$$a_n = \frac{2}{1/2} \int_0^{1/2} 4t \cos \frac{n\pi t}{1/2} dt = 8 \int_0^{1/2} 2t \cos 2n\pi t \; dt = \{\text{Partiell integration}\} =$$

$$= 8\left(\left[t \frac{\sin 2n\pi t}{n\pi}\right]_0^{1/2} - \int_0^{1/2} 1 \cdot \frac{\sin 2n\pi t}{n\pi} dt\right) =$$

$$= 8\left[\frac{\cos(2n\pi t)}{2(n\pi)^2}\right]_0^{1/2} = 4\frac{\cos(n\pi) - 1}{(n\pi)^2}$$

$$a_0 = \frac{2}{1/2} \int_0^{1/2} 4t \; dt = 8\left[t^2\right]_0^{1/2} = 2$$

$$4f(x) \sim 1 + \sum_{n=1}^{\infty} 4\frac{\cos n\pi - 1}{(n\pi)^2} \cos 2n\pi t$$

Vi ansätter $\quad a_n = \dfrac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos 2n\pi t$

$$\ddot{x}_p = \sum_{n=1}^{\infty} -4n^2\pi^2 A_n \cos 2n\pi t$$

Insättning i den givna ekvationen ger

$$\sum_{n=1}^{\infty} -4n^2\pi^2 A_n \cos 2n\pi t + 48\left(\frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos 2n\pi t\right) =$$

$$= 1 + \sum_{n=1}^{\infty} 4\,\frac{\cos n\pi - 1}{n\pi} \cos 2n\pi t$$

$$48\frac{A_0}{2} - 1 + \sum_{n=1}^{\infty}\left(-4n^2\pi^2 A_n + 48A_n - 4\,\frac{\cos n\pi}{t}\right)\cos 2n\pi t = 0$$

$$\frac{A_0}{2} = \frac{2}{48}, \qquad A_n = \frac{\cos n\pi - 1}{(n\pi)^2(12 - n^2\pi^2)}$$

I ett tabellverk står det att

$$s(x) = \sum_{n=1}^{\infty} \frac{\cos n\pi x}{n^2} \text{ är lka med } \frac{\pi^2}{12}(3x^2 - 6x + 2) \text{ då } 0 < x < 1.$$

Beräkna s(-8/3).

Fourierserie för en jämn funtkion ƒ.

ƒ är periodisk med perioden 2, Det vill säga ƒ(x + 2) = ƒ(x).

$$f(x) = \frac{\pi^2}{12}(3x^2 - 6x + 2) \text{ då } 0 < x < 1.$$

$$f\left(-\frac{8}{3}\right) = f\left(-2 - \frac{2}{3}\right) = f\left(-\frac{2}{3}\right) = \{\text{Jämn}\} = f\left(\frac{2}{3}\right)$$

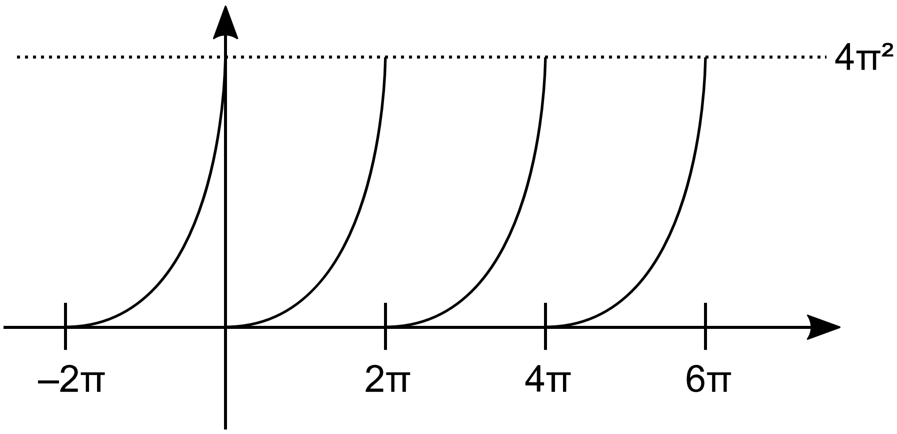
$$s\left(-\frac{8}{3}\right) = f\left(-\frac{2}{3}\right) = f\left(\frac{2}{3}\right)$$

$$f\left(\frac{2}{3}\right)=\frac{\pi^2}{12}\left(3\left(\frac{2}{3}\right)^2 6\left(\frac{2}{3}\right)+2\right)$$

$$s\left(-\frac{8}{3}\right)=\frac{\pi^2}{36}(4-12+6)=-\frac{\pi^2}{18}$$

$f(x) = x^2, \quad 0 < x < \pi$

$f(x + 2\pi) = f(x)$



Varken jämn eller udda.

$$a_n=\frac{1}{\pi}\int_0^{2\pi} x^2 \cos nx \; dx=\{\text{Dubbel partiell integration}\}=\frac{4}{n^2}$$

$$a_0=\frac{1}{\pi}\int_0^{2\pi} x^2 \, dx=\frac{8\pi^2}{3}$$

$$b_n=\frac{1}{\pi}\int_0^{2\pi} x^2 \sin nx \; dx=-\frac{4}{n}$$

$$f\sim\frac{4\pi^2}{3}+\sum_{n=1}^{\infty}\left(\frac{4}{n^2}\cos nx-\frac{4}{n}\sin nx\right)$$

Vi söker $\sum_{n=1}^{\infty} \frac{1}{n^2}$ och $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$.

X = 0:

$$\frac{0+4\pi^2}{3}=\frac{4\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4}{n^2}$$

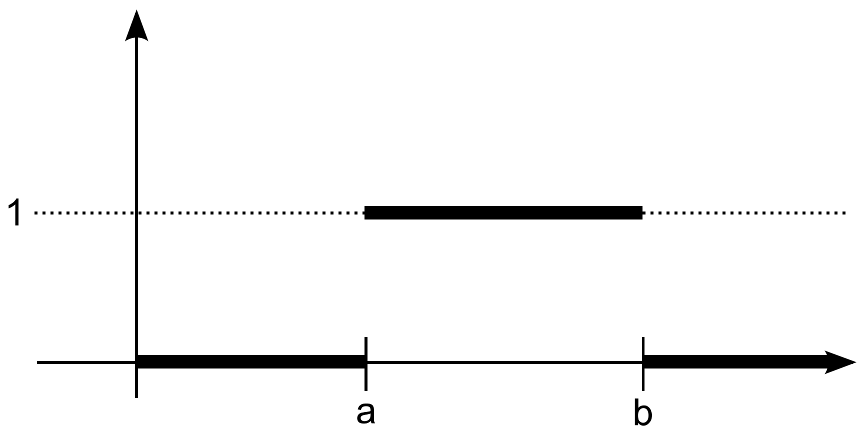
$$\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{2}-\frac{\pi^2}{3}=\frac{\pi^2}{6}$$

x = π:

$$\pi^2=\frac{4\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}=\frac{\pi^2}{4}-\frac{\pi^2}{3}=-\frac{\pi^2}{12}$$

Heavisides funktion (U):



f(t) = U(t – a) – U(t – b)

$$U(t-a)=\begin{cases}1, & t>a\\ 0, & t<a\end{cases}$$