2011-09-sep-02
komplexitetsteori

Last time

Sorry for being late

Weekly homeworks. (Last lecture given out on the web) => Last lecture

Updated course documents (on web)

Basic topics + Maybe advanced topic

- Turing Machines & C-programs
- The basic functions.

Observation 1:

"Everything" is an integer.

C-programs ~ Strings of characters; number written in base 256

M_x machine (or C-program) corresponding to integer x.

Given x & y it is possible to run M_x on input y.

Style machine M_z that given inputs x & y can do this. Also given t it can run it for t steps.

μ_z is the universal Turing machine.

Church's thesis:

Every reasonable definition of mechanically computable gives same class of function.

Functions $\{0,1\}^* \rightarrow \{0,1\}^*$ (finite power)

mostly finite → finite strings

$\mathbb{N} \Rightarrow \{0,1\}$ decision problems

Theorem: There are functions that are not computable.

How many functions do we have?

$|2^{\mathbb{N}}| = (\aleph_1)$ (theoretically)

as many as the real numbers

How many computable functions?

$|\mathbb{N}| = (\aleph_0)$

as many as the natural numbers

The basic non-computable function.
Halting problem

Does M_x halt (ever) on y

Theorem:

The halting problem is not computable
(not recursive)

Define related function

$$D(x) \begin{cases} 1 & M_x \text{ halts on input } x \\ & \text{with output } 0. \\ 0 & \text{otherwise} \end{cases}$$

Theorem: D is not computable.

Proof: Suppose it is, then it is computed by M_{x_0} some x_0

What does M_{x_0} output on x_0?

output $0 \Rightarrow D(x_0) = 1$

output $\neq 0 \quad D(x_0) = 0$ Error

If M_{x_0} computes f correctly on all but one input, then there exists $M_{x'}$ that computes f correctly everywhere

All except y_0 Is input $= y_0$ output $f(y_0)$
else run M_{x_0}
((hardwire))

((there must be an infinite number of exception cases))

(1) Is M_x legal: if no halt output 0

(2) Does M_x halt on x if no (halt) output 0

(3) Simulate M_x on x until halts,
output 1 if answer 0

(1) and (3) is simple, so (2) must be impossible