

Last time

Church's thesis "right notion"

M_x TM corresponding to x

Universal TM

run M_x on x for t steps

Haltin problem

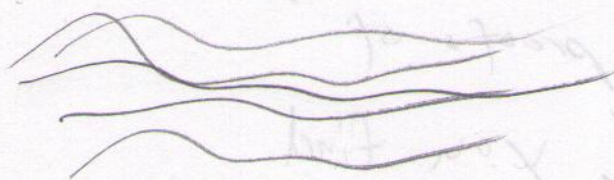
Does M_x halt on x ?

Noncomputable } same
Nonrecursive

$$D_x = \begin{cases} 1 & M_x \text{ halts on } x \text{ with} \\ & \text{output } 0. \\ 0 & \end{cases}$$

M_{x_0} makes mistake on x_0

(and infinitely many more inputs)



Gödel's
Given any (nice) axiomatisation of number theory there are true theorems that are not provable.

Proof system

Axioms

Derivation rules

A true $A \Rightarrow B$
 B true.

Nice: Mechanically decidable if something is an axiom.

A derivation rule is correct.

Proof systems

Complete: A or $\neg A$ provable

Sound: Only true statements prov.

The complete, sound, nice

\Rightarrow The set of true theorems is recursive (computable).

Given A = statement in number theory
generate all proofs of
increasing length until you find
a proof of A or $\neg A$.

If the axiom is strong enough to express
 M_x halts on y we get a contradiction

Efficient computability

Computable in time 2^{1000} or not who cares?


Computable in time $T(n)$, if
computed by TM that runs in
 $T(n)$ steps on inputs of length n .

Given a graph with m nodes and
 n edges is it connected.
 $O(n+m)$ Depth first search.

Given a bipartite graph with
 m nodes and n edges.

Is there a perfect matching?

$O(m^3)$ $O(nm)$



Questions: Does more time allow you to solve more problems?

Can we have arbitrarily complicated problems?

Theorem: Given a function T , there is a problem that requires \geq time T to solve

Proof: $|x| = \text{length of } x$
$$D_T(x) = \begin{cases} 1 & \text{if } M_x \text{ halts on } x \\ & \text{within } T(|x|) \text{ steps with output } 1. \\ 0 & \text{otherwise} \end{cases}$$

Claim: You cannot compute D_T within time $T(n)$ on all inputs of length n

Proof of claim: Suppose M_{x_0} computes D_T in time $T(|x|)$ on all x . Look what happens on input x_0 .

Is P_T computable in time T ,
on input x
1. compute $T(x) = t$
2. Run M_x on input x for t
 T steps (and do the obvious).

As long as T is computable so is P_T .

Assume further that $T(n)$ can
be computed in $T(n)$ steps
certainly true for $n^2, 2^n, \frac{n}{2}, \dots$
... "time constructible".

Suppose T is time constructible then
 P_T is computable in $O(\quad)$ steps.

In the best of all worlds one could
hope to do Σ in time $O(t)$.

How efficient is the universal machine,

It is

If we have two tape T_M , T_N

D_T can be computed in time

$$O(T \log T).$$

Cor:

Given time constructible T

\exists function computable in time

$O(T \log T)$ but not time T .