[z.c.2.2.24.]

$$\frac{dy}{dx}, \quad y(2)=2$$

$$f(x; y)=\frac{y^2-1}{x^2-1}$$

$$\frac{\partial f}{\partial y}=\frac{2y}{x^2-1}$$

x får inte vara ±1.

r kan skapas.

[z.c.3.1.4.]

Antalet bakterier, vid tiden *t*, = *N(t)*.

$$\frac{dN}{dt}=kN(t), \quad k>0$$

$N(t) \neq 0$

$$\frac{1}{N(t)}\cdot\frac{dN}{dt}=k$$

Integrera med avseende på *t*.

ln |N(t)| = kt + ln |C|

$|N(t)| = e^{kt} \cdot C$
$N(t) = \pm Ce^{kt} = Ce^{kt}$

N(3) = 400   ⇔ $Ce^{3k} = 400$
N(10) = 2000   ⇔ $Ce^{10k} = 2000$

$$\frac{2000}{400}=5=\frac{Ce^{10k}}{Ce^{3k}}=e^{7k}$$

$$k = \frac{1}{7} \ln 5$$
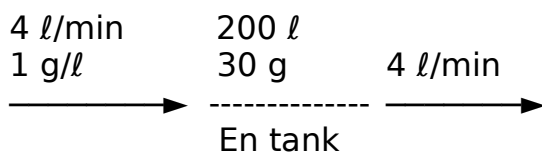
$$400=Ce^{\frac{1}{7}\ln 5}$$

$$C=400\,e^{-\frac{1}{7}\ln 5}=400\cdot 5^{-\frac{3}{7}}$$

$$N(t)=400\cdot 5^{-\frac{3}{7}}\cdot e^{t\frac{1}{7}\ln 5}=400\cdot 5^{\frac{t-3}{7}}$$

$$N(0)=400\cdot 5^{-\frac{3}{7}}\approx 201$$

[z.c.3.1.21.]

4 ℓ/min   200 ℓ
1 g/ℓ   30 g   4 ℓ/min

————→ -------------- ————→

            En tank

Salt i tanken: $A(t)$

$$\frac{dA}{dt}=1\cdot 4-4\cdot \frac{A(t)}{200}$$

$$\frac{dA}{dt}+\frac{1}{50}A(t)=4$$

$A_h = Ce^{-t/50}$
$A_p = 200$

$A(t) = Ce^{-t/50} + 200$

$A(0) = 30$
$30 = C + 200$
$C = -120$

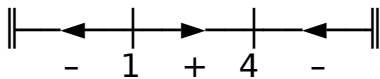$A(t) = 200 - 170e^{-t/50}$

För stora t: $A(5) \approx 200$

Rimligt!

[z.c.3.2.5.]

$$\frac{dP}{dt} = P(a-bP)-h, \quad P(0)=P_0$$

Sätt a = 5, b = 1, h = 4.

$$\frac{dP}{dt} = P(5-P)-4 = 5P-P^2-4 = (P-1)(4-P)$$

Stationära lösningar: P = 1 och P = 4

Lösningar för 1 ≠ P ≠ 4.

Separabel

$$\frac{1}{(P-1)(4-P)} \cdot \frac{dP}{dt} = 1$$

Med handpåläggning

$$\left(\frac{1/3}{P-1} + \frac{1/3}{4-P} \right) \frac{dP}{dt} = 1$$

ln |P – 1| – ln |4 – P| = 3t + ln |C|

$$\ln\left| \frac{P-1}{4-P} \right| = 3t + \ln|C|$$

$$\left| \frac{P-1}{4-P} \right| = |C|e^{3t}$$

$$\frac{P-1}{4-P} = Ce^{3t}$$

Bestäm C

$$P(0)=P_0 \Rightarrow C = \frac{P_0-1}{4-P_0}$$

$P - 1 = 4 \cdot Ce^{3t} - Pe^{3t}$

$$P(t) = \frac{1 + 4 \cdot Ce^{3t}}{1 + Ce^{3t}}$$

Populationen borta (P = 0)

$$\frac{0-1}{4-0} = Ce^{3t_0}$$

$$e^{3t_0} = -\frac{1}{4C}$$

$$t_0 = \frac{1}{3}\ln\frac{-1}{4C} = \frac{1}{3}\ln\frac{4 - P_0}{4(1 - P_0)}$$