Introduction

Infectious diseases impose significant economic and health burdens throughout the world, continuously threatening human quality of life [1]. In order to help alleviate these negative impacts, researchers have utilized mathematical models to aid in the understanding of infectious disease dynamics [2]. These models typically make use of compartments, where individuals are classified according to whether or not they are susceptible to the disease in question, or whether they are infected with the disease and may consequently infect others [3, 4]. These models may be network (graph) based [5], where unlike traditional ordinary differential equation compartmental models which assume homogeneity amongst contact patterns, network based models do not use the assumption that each individual in the population has an equal chance to spread a disease to every other individual. Instead, transmission takes place on a network, where individuals have a specific set of contact(s) through which a disease can spread. In this sense, these models can be more realistic than those that do not utilize networks as contact patterns between individuals are likely heterogeneous [6].

An adaptive network is a network whose topology changes over time with respect to the state of the nodes [7]. These types of networks have been studied extensively due to their interesting properties, and have also been applied to the study of epidemics [8–11]. Here, I study a model that uses the concepts of link rewiring and node positioning to create an adaptive network where nodes who are rewiring a link have a choice of where they wish to rewire to.

In their 2006 paper, Holme and Ghoshal [12] study an adaptive network where nodes compete for a high centrality and low degree. That is, a node with high centrality requires that the path length to any other node in the network is as small as possible. These nodes can be thought of as having a very influential position in the network. A node's score, or fitness, is determined by the strength of their position on the network which they are able to improve by adding and/or removing links each timestep. After certain predetermined intervals, nodes will copy the strategies of neighbours who have accumulated a higher fitness, and mutations of strategy sets randomly occur to drive optimization. This model of competition is of interest when we consider a network whose nodes are susceptible to becoming infected with a disease. On one hand, nodes gain fitness when they have a high centrality on the network. However, a high centrality also ensures that a path length to an infectious node is never large. This introduces a dynamic where accumulating fitness also leaves a node more vulnerable to becoming infected.

The objective of this project will be to study the strategy evolution of the nodes and the resulting effects on disease prevalence, as well as the evolution of the network's topology as the nodes compete for influential positions on the network while trying to maintain a non-infectious state.

Methods

I combine ideas from Gross and Shaw and Schwartz [8–10] and Holme and Ghoshal [12] to create an evolutionary game where nodes compete with each other for a high centrality on a network. To calculate the fitness of a node i, I follow [12] to first calculate its centrality, $c(i)$:

$$c(i) = \sum_{j \in H(i) \setminus \{i\}} \frac{1}{d(i,j)}. \tag{1}$$

Here, $H(i)$ is the subgraph node i belongs to and $d(i,j)$ is the graph distance between node i and node j. Thus, node i's fitness is

$$f(i) = \begin{cases} c(i) & \text{if } k_i > 0 \\ 0 & \text{if } k_i = 0 \end{cases} \tag{2}$$

where k_i is the degree of node i. Beginning with an Erdős-Rényi network model with N nodes and an initial number of infected (I_0) nodes, susceptible nodes who have links to infectious ones may rewire to a different node with probability p. Instead of rewiring to a randomly chosen non-infectious node in the network as in [8–10], I have nodes decide where they would like to redirect their link using a particular strategy. These strategies are to rewire to a non-infectious node with the highest/lowest degree, highest/lowest centrality, or one chosen at random. For this model, I allow nodes to have global information of the network.

The disease on this network will follow a susceptible-infectious-susceptible (SIS) natural history, with an infectious period of 5 time steps and a transmissibility that maintains an intermediate level of prevalence in the population, in the absence of rewiring (Figure 1). Those who become infectious may not rewire any of their edges, and will also see their fitness negatively affected as others may rewire from them.

Every g time steps, each node's cumulative fitness since the beginning of the generation is evaluated. I use 5 tournaments each with size of 20 and the nodes with the 5 worst fitnesses have their rewiring strategies replaced with the 5 strategies from the nodes who obtained the best 5 fitnesses. After this, mutation is applied to the population where a node's strategy may be randomly changed to one of the other 4 strategies. Information regarding baseline parameter values can be found in Table 1.

Simulations are run for 52,000 time steps to obtain the average prevalence in the population, discarding the first 2,000 to avoid including initial fluctuations of the disease invading the population. To study time series changes of strategy use in the population, simulations are run for 100,000 time steps providing data for 1000 strategy change events.

Results

Simulations that tracked the average disease prevalence in the population while using specified strategy types showed that no rewiring gives a higher average prevalence than random rewiring, rewiring to the lowest degree node, and rewiring to the lowest centrality node (Figure 1). Interestingly, allowing nodes to change their strategies to the ones that provide the best fitnesses produces the highest average disease prevalence. This is due to the nodes on the network developing strategies that keeps them as connected as possible. Consequently, the disease can spread much more easily through the population. Here, we can see that with the baseline rewiring rates and infectious period, members of the population choose their links in such a way that enables disease spread.

To investigate this further, we can observe the strategy evolution for the baseline scenario, as well as the degree distribution of the nodes in the network after 100,000 time steps. Strategies to rewire to the highest degree node or the node with the highest centrality dominate in the population (Figure 2). Using these strategies, nodes can ensure that they connect themselves to other nodes that hold important positions in the network, thus increasing their fitness. The resulting degree distributions in these populations at the end of a simulation run show properties similar to that of a scale free network where the node degree distribution follows a power law (Figure 3). In this network, a few hub nodes have high degree, and the majority of the nodes in the network have lower degree.

Given this evolution of the network, it is not necessarily the case that the best performing nodes are the hub nodes with high degree. In fact, very frequently, the winners of tournaments are the nodes with a degree of 10 or less (Figure 4). This is likely due to the fact that high degree nodes have many edges through which the disease can be transmitted to them, thus increasing their infection probability. However, a node may also achieve a high centrality whilst having a low degree. This will ensure that there are only a select few edges through which the disease may be directly transmitted to them.

Two properties of this model that can potentially impact both strategy choice and disease

Parameter	Description	Baseline Value
N	Population Size	200
β	Transmission Rate	0.025
p	Rewiring Rate	0.05
m	Mutation Rate	0.02
g	Generation Length	100
γ	Infectious Period (time steps)	5
I_0	Initial Infected	5

Table 1: Baseline parameter values and their descriptions

prevalence are the rewiring rate and transmissibility of the disease. We may investigate how these parameters alter the dynamics of the model and attempt to gain insight on why and/or when the strategies of rewiring to a node with the highest degree or centrality dominate.

Intuitively, increasing the transmission rate also increases disease prevalence, while increasing the rewiring rate decreases disease prevalence (Figures 5 and 6). However, in these scenarios, the strategies of rewiring to the highest degree node available or the highest centrality node available continue to dominate in the population. With a high rewiring rate, becoming infected will have a large negative impact on a node's fitness. However, susceptible nodes can often effectively rewire their connections away from infected neighbours, avoiding infection themselves. At high rates of disease transmission, becoming infected is more probable. Despite this, nodes continue to use the same strategies for rewiring, indicating that the centrality obtained using these strategies outweighs the punishment for becoming infected.

We can attempt to find scenarios where the costs associated with becoming infected are so extreme, nodes abandon the strategies that we have seen dominate thus far. However, rewiring to nodes with the highest degree and centrality will make up the vast majority of utilized strategies for shorter generation length, longer infectious periods, higher transmissibility, and higher rewiring rate. In all of these cases, the fitness a node accumulates when using these strategies outweighs any loss they will receive from becoming infected. This holds true even when the infectious period (up to 10 time steps) lasts for half of the generation length. Under these circumstances, the disease on the network will much more easily spread due to the population's desire to remain highly connected.

Along the same vein, we can alter the fitness function so that nodes do not accumulate fitness while infected, or gain fitness from an infectious contact. Earlier, fitness gain would be hindered to infectious individuals due to the rewiring of edges alone. If we additionally stop all incoming and outgoing fitness gain during the infectious period, becoming infected will be much more costly to any given node. This alteration can be analogous to individuals not wishing to communicate with infected neighbours, even if transmission pathways that produce sufficient contact remain intact in some form.

With baseline values in this scenario, the model dynamics remain largely unchanged. However, if the infectious period is increased to 10 time steps, strategy choices amongst the population change (Figure 7). Here, a generation length of 20 is used to allow for more generations in a simulation, and the strategy choice amongst the population is shown for the final 1,000 generations. Longer generation lengths do not change these dynamics. Under these new conditions, all 5 strategies are used more equally than what was observed earlier in Figure 2. Now, rewiring to the node with the lowest degree, lowest centrality, or even a random node, can often outperform rewiring to the node with the highest degree or centrality. Thus, if the punishment for getting infected is sufficiently high, we can see strategy choices begin to change, as the benefit from avoiding infection starts to outweigh the immediate benefits of having a high centrality in the network.

Furthermore, the node degree distribution (Figure 8) in the network at the end of a simulation has changed from that of Figure 3. Now, the majority of nodes have degree between 5 to 15, rather than the distribution being heavy on degrees between 0 to 10. Also, the degree of the winning node (Figure 9) is almost exclusively between 5 to 15. This number of connections appears to be the desired amount for allowing nodes to balance a high centrality in the network while simultaneously avoiding infection.

Discussion

We have studied disease dynamics on an adaptive network where nodes rewire their SI edges to SS edges of their choosing at a given rate. The strategy choices of the nodes are governed by an evolutionary game where fitness is obtained by achieving a high centrality in the network. When the rewiring strategies are set to random, we see a decrease in average prevalence from a scenario where no rewiring is present. However, when nodes choose their strategies according to the evolutionary algorithm, average prevalence increases dramatically. This result stems from the dynamic that strategies which achieve high fitnesses also facilitate disease transmission.

The networks in each simulation would arrange themselves to have a scale free topology. That is, the majority of the nodes would have low degree, whereas only a few 'hub' nodes would have high degree. Also, the degree of a tournament winning node would often be one which possessed only a small number of edges.

Typically, the strategies of rewiring to the node with the highest degree or highest centrality dominated in the network. Only when the hindrance to fitness gain from becoming infected was extreme did these strategies begin to give way to the others. In general, however, the fitness achieved using these strategies outweighed the harms of infection.

There are several future research possibilities building off of this work. Firstly, different disease natural histories such as including a recovered class can be tested. Also, the relaxed assumption of nodes having full knowledge of the network can be removed. For example, nodes may only be permitted to rewire to their second neighbourhood, as in [12]. Finally, additional strategy choices may be added to allow for a wider variety of rewiring options.

Adaptive networks provide a rich variety of behaviour stemming from their dynamic topologies. I have drawn from existing research [8–10, 12] to study an adaptive network with disease dynamics and nodes that choose rewiring strategies. We have seen that when achieving a high centrality is a node's priority, the network arranges itself to a scale free topology and also becomes very susceptible to disease spread.

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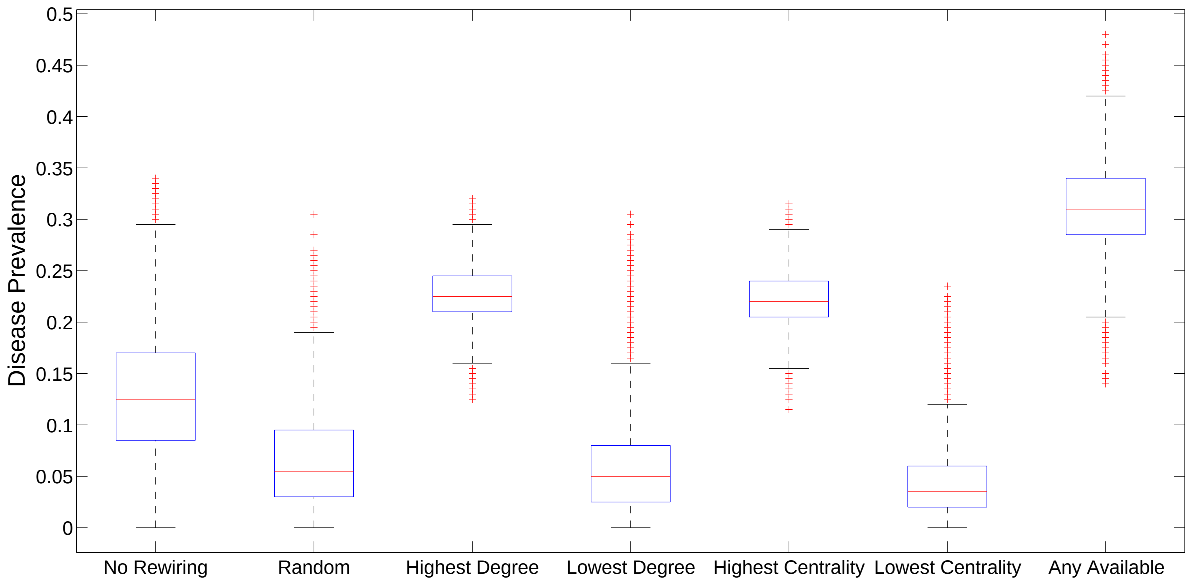

Figure 1: **Average prevalence in a population utilizing various strategies.** Average disease prevalences in the population are shown with box plots, and all members of the population using the specified strategy. 'Any Available' shows the baseline scenario with strategy evolution amongst the population.

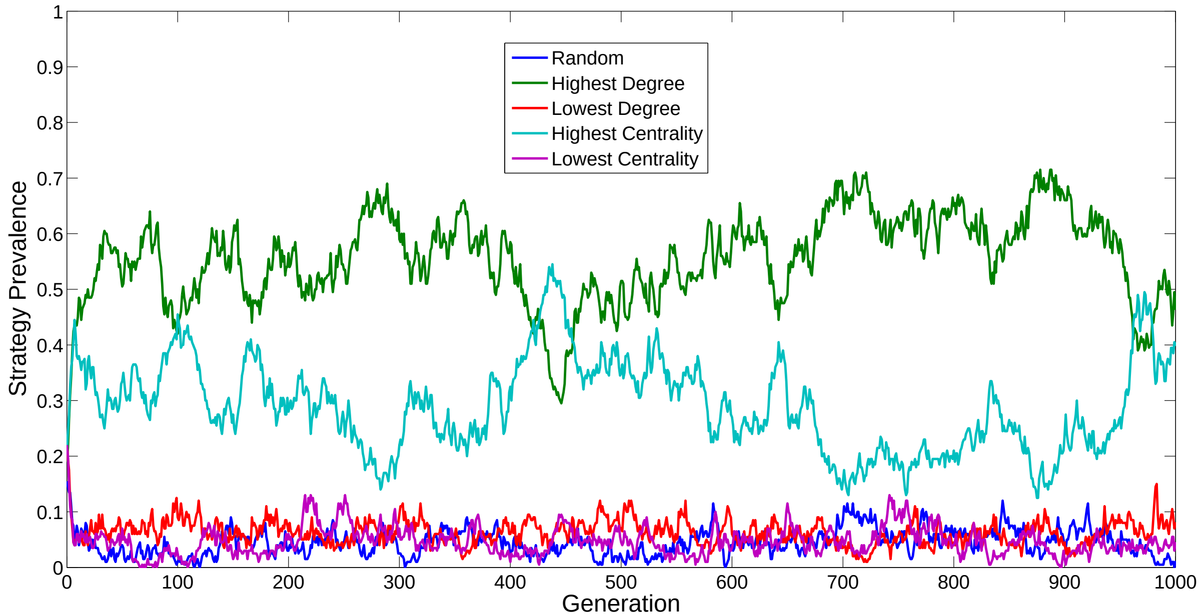

Figure 2: **Strategy evolution in the population.** Prevalence of each strategy in the population in each generation for the first 1,000 generations. These dynamics remain similar for longer run times. (Up to 50,000 generations tested).

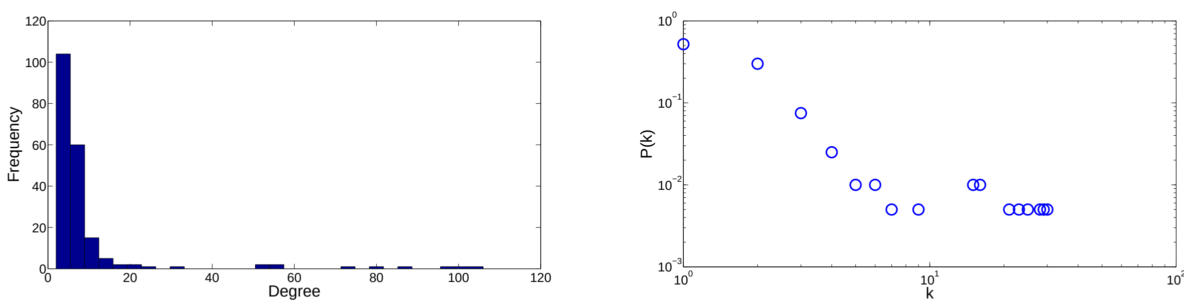

Figure 3: **Frequency of node degrees.** Left: Frequency of node degrees in a network after 1,000,000 time steps. The majority of nodes have a low degree, whereas a select few hub nodes have high degree. Right: Probability distribution of node degree on a log log scale.

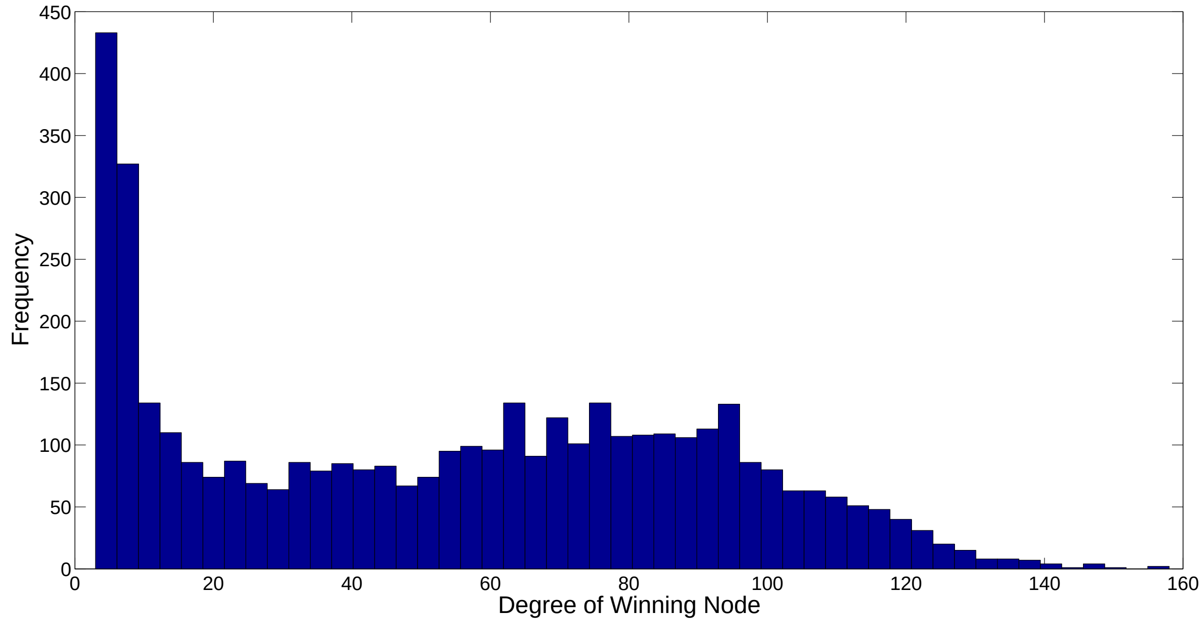

Figure 4: **Degree of tournament winners.** The frequency of tournament winners with the best fitness and their degree in the network.

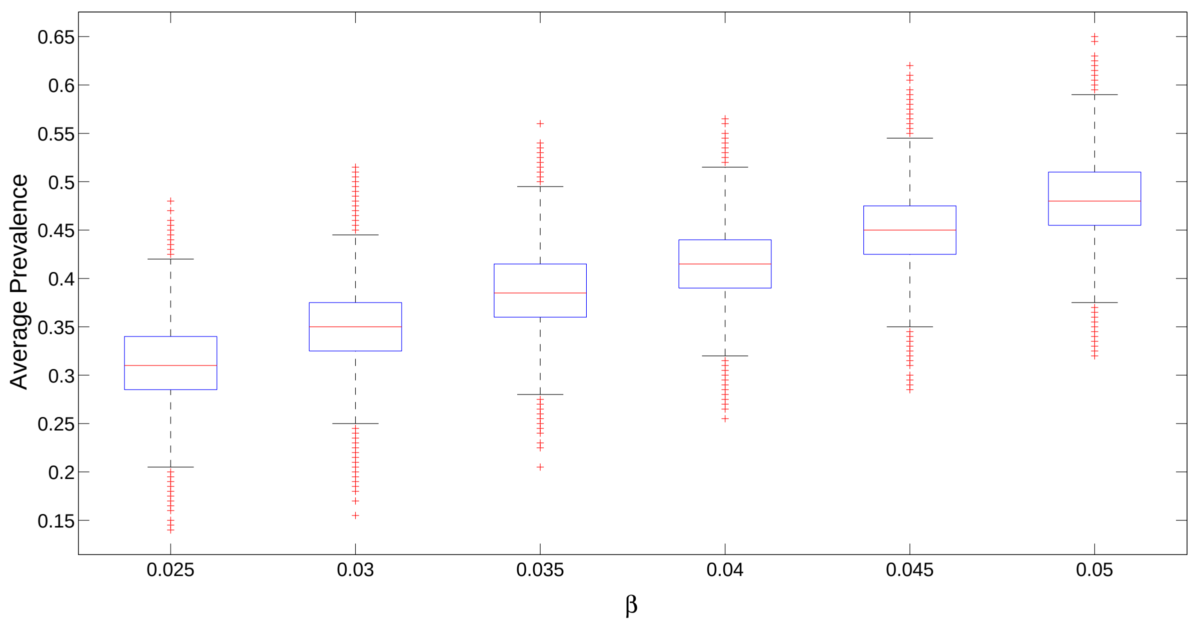

Figure 5: **Change in average prevalence with transmission rate.** Effects of increasing the transmission rate on disease prevalence.

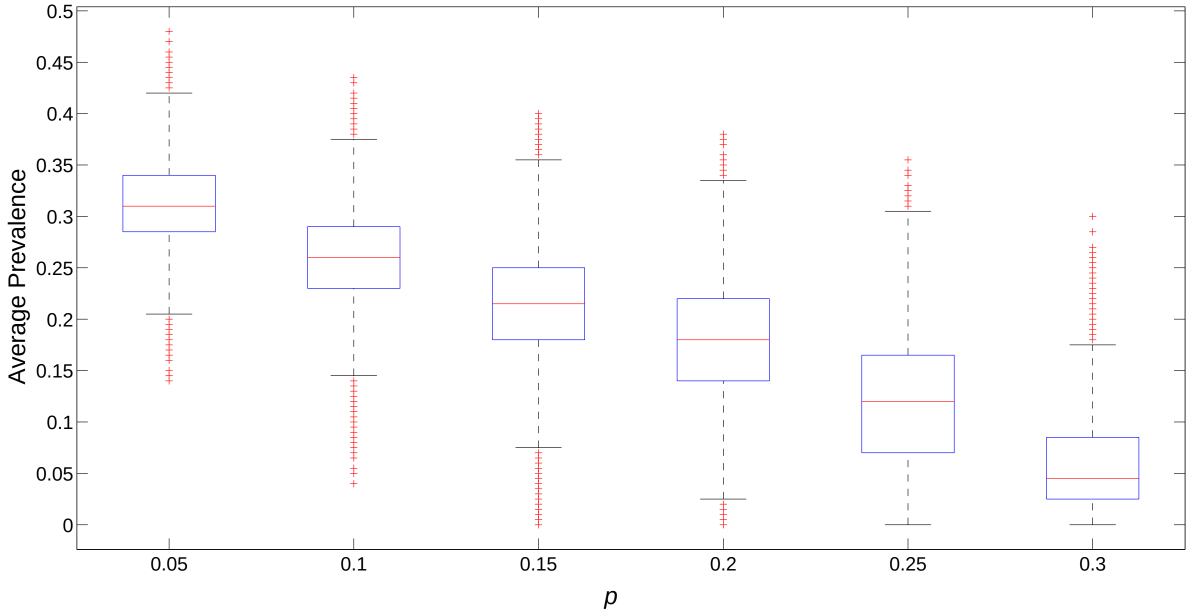

Figure 6: **Change in average prevalence with rewiring rate.** Effects of increasing the rewiring rate on disease prevalence.

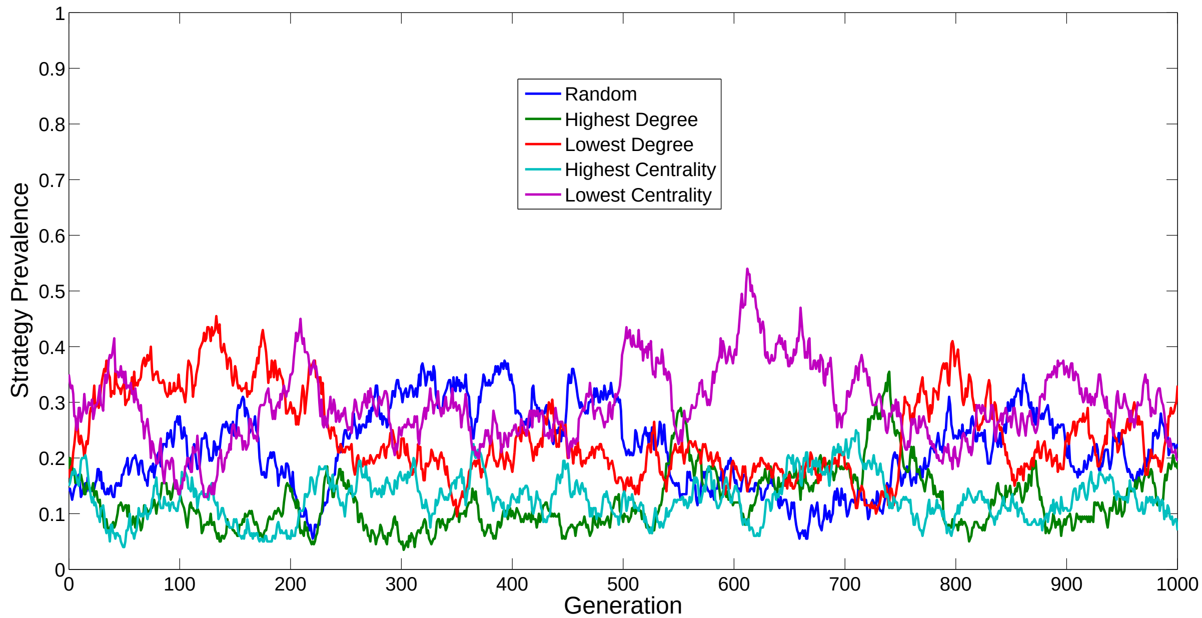

Figure 7: **Strategy evolution in the population with new fitness function.** Prevalence of each strategy in the population in each generation for the last 1,000 generations in a 1,000,000 time step simulation (50,000 generations total).

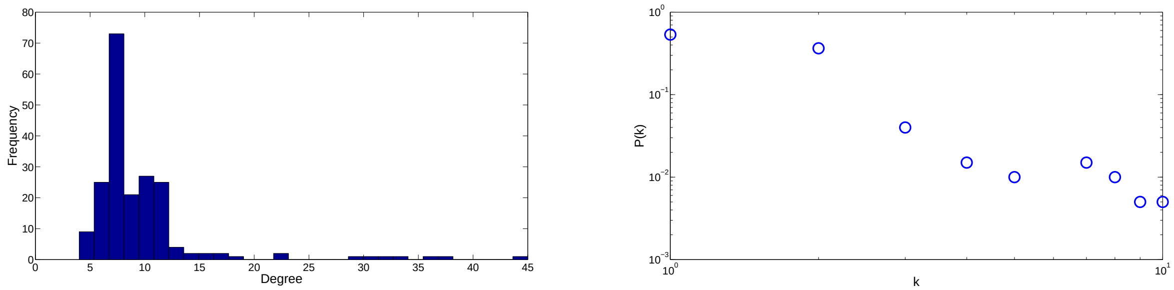

Figure 8: **Frequency of node degrees using the new fitness function.** Left: Frequency of node degrees in a network with the new fitness function after 1,000,000 time steps. The majority of nodes have a low degree, whereas a select few hub nodes have high degree. Right: Probability distribution of node degree on a log log scale.

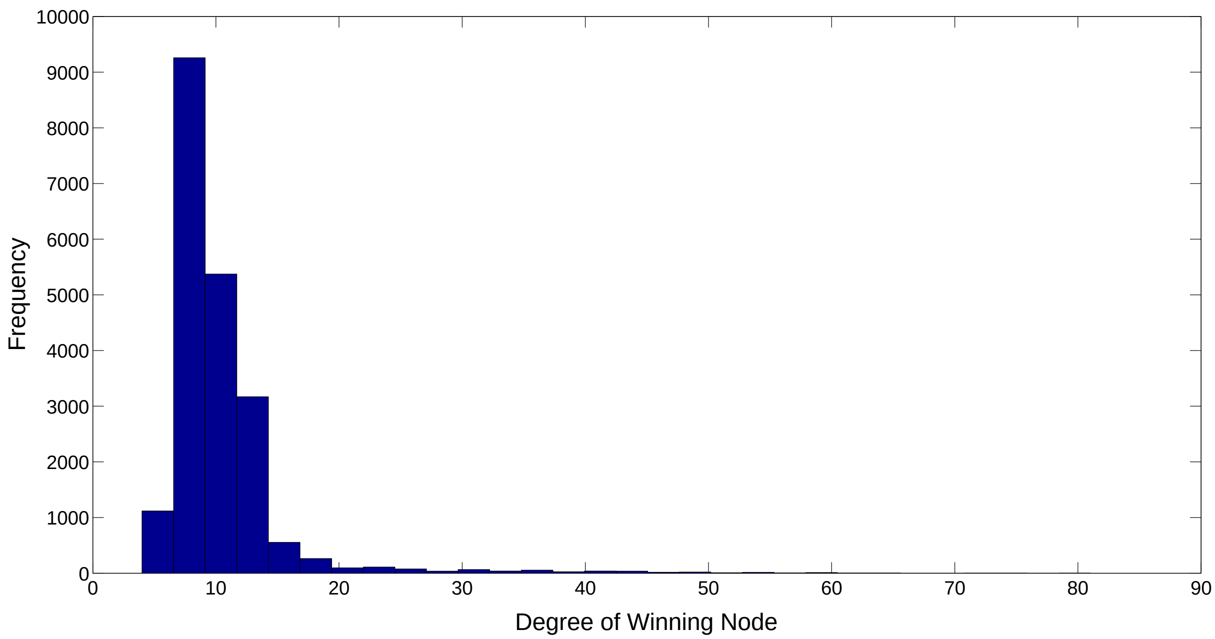

Figure 9: **Degree of tournament winners with new fitness function.** The frequency of tournament winners with the best fitness and their degree in the network.