### BOOK OF ABSTRACTS

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### Preface

A very warm welcome to all the participants of the conference MAA 2020, as well as established friends and colleagues. It was always fun and challenging to organize a Conference physically. Nevertheless, it is harder to embrace ourselves for a virtual conference. However with the support of our Department of Mathematics, National Institute of Technology Jamshedpur, we are trying to fill the void of physical conference. The aim of the Conference MAA 2020 is to bring together mathematicians and researchers who work in the fields of Mathematical Analysis and its applications in various fields of science and engineering and to encourage collaboration and exchange of interdisciplinary ideas among the participants. Even though there are many virtual conferences being held in all parts of the world, our Conference MAA 2020, received a huge support from many researchers across the world.

Apart from eminent speakers from reputed institutes from India, we have speakers from across the globe to share their expertise to young researchers. There are nearly 80 researchers including researchers from abroad contributed for paper presentation from Institutes of Excellence in India and abroad. The substantial number of abstracts for paper presentation indicates implicitly the success of the conference.

While talking about the success of the conference, it would be incomplete if we forget to thank our Institution to support and encourage us immensely to organize this conference. We would like to thank our honorable Director Prof. K. K. Shukla. Also, we would like to thank Dr. Sunil Kumar, Head, Department of Mathematics and our colleagues for continuous support from the commencing of conference preparation. We would like to propose our sincere thanks to Prof. Ram N. Mohapatra from University of Central Florida for his support. We would Further, we extend our sincere thanks to Dr. A. Swaminathan from Indian Institute of Technology Roorkee for his continuous help and moral support. We extend grateful thanks to our keynote and invited speakers; in spite of their busy schedule they accepted our invitation to share their valuable knowledges with us. Our special thanks to our participants who have attend this conference. Without their helping hand it would be impossible to complete this conference. We owe our heartful thanks to our organizing committee members, advisory committee members, reviewers, supporting staffs, students and faculty members for their support and entire effort to complete this conference. Once again, we thank one and all who supported directly or indirectly to make this conference a reality.

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### **Keynote Speakers**

- Prof. Ram N. Mohapatra, University of Central Florida, USA
- Prof. P. D. Srivastava, Indian Institute of Technology Bhilai, India
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- Dr. Armin Straub, University of South Alabama, USA
- Dr. A. Swaminathan, Indian Institute of Technology Roorkee, India
- Dr. Bappaditya Bhowmik, Indian Institute of Technology Kharagpur, India

### **Invited Speakers**

- Dr. Khaled Mehrez, Université Tunis El Manar, Tunisia
- Dr. Ratikanta Behera, University of Central Florida, USA
- Dr. Kanailal Mahato, Institute of Science, Banaras Hindu University, India

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### Keynote Talks

#### Equilibrium Problems and Variational Inequalities

#### R. N. Mohapatra

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The purpose of this talk is to inform the audience about developments in Variational Analysis in general and Variational Inequalities in particular. Variational Analysis has been used by Applied Mathematicians and Engineers for a very long time. Variational Inequalities got more attention after Kinedeleherer and Stampachia published their monograph on variational inequalities and showed how abstract mathematics can be used to solve practical problems. Equilibrium problems have been studied by Economists and Engineers for a very long time. Mathematicians consider variational inequalities as a part of Nonlinear Functional Analysis and use tools from Functional Analysis. There are new approaches to solve complimentarity problems and variational inequalities. In this talk, we will present some aspects of our research on two problems of our interest in Equilibrium Problems and Variational Inequalities. If time permits, I shall mention other results of Gianessi, Chadli, Anasari, and my work with Chadli, Pani and Sahu.

This presentation is a joint work with my colleague, Dr. Gayatri Pany, University of Central Florida. She has kindly helped me in preparing the talk and present our joint research despite her other commitments and duties.

## Spectrum and fine spectrum of generalized lower triangular triple band matrices over the sequence space $l_p$

#### P. D. Srivastava

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The spectrum of triangular band matrices defined on the sequence spaces where the entries of each band are a constant or convergent sequence is well studied. In this article, the spectrum and fine spectrum of a new generalized difference operator defined by a lower triangular triple band matrix on the sequence space  $l_p(1 \le p < \infty)$  are obtained where the bands are considered as periodic sequences. The approximate point spectrum, defect spectrum, compression spectrum and the Goldberg

classification of the spectrum are also discussed. Suitable examples are given in order to supplement the results. Several special cases of our findings are discussed which confirm that our study is more general and extensive.

(The talk is based on joint work with Arnab Patra (Indian Institute of Technology Kharagpur).)

**Keywords:** Spectrum, infinite matrices, sequence spaces, difference equation.

**2010** AMS Subject Classification: 47A10, 47B37.

### On the removable singularities theorem for harmonic functions

#### G. K. Srinivasan

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The theorem is a generalization of the well known theorem of Riemann on removability of isolated singularities of holomorphic functions under suitable local integrability assumptions. In this talk we shall prove the analogous theorem for harmonic functions. Although the result and proofs are folk-lore, it is not as widely available in the text book literature as it ought to be. We provide here an argument which is different from the available ones in the literature known to us. The proof has some didactic values and is of some interest.

## The generalized Stieltjes transform and special functions Henrik L. Pedersen

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This talk reports on some recent results about the connection between generalized Stieltjes functions and classical special functions, among those Nielsen's beta function, Euler's Gamma function and Barnes' Gamma functions.

Nielsen's beta-function is closely related to Euler's gamma function. By its very definition it is completely monotonic, and it is actually logarithmically completely monotonic.

A method utilizing Laplace transforms of a class of periodic functions gives a number of concrete examples of logarithmically completely monotonic functions. Among these are ratios of gamma functions and the results for these functions are extended in the framework of entire functions of finite genus having their zeros on the negative half line.

Finally, if time permits, the family of functions  $f_a(x) = (1+1/x)^{ax}$  is investigated and this leads to the introduction of the class of Horn-Bernstein functions.

The talk is based on joint work with Christian Berg (University of Copenhagen), Stamatis Koumandos (University of Cyprus) and Dimitrios Askitis (University of Copenhagen).

**Keywords:** Nielsen's beta function, Generalized Stieltjes function, Laplace transform.

**2010 AMS Subject Classification:** 26A48, 33B15, 44A10.

## Gaussian binomial coefficients with negative arguments Armin Straub

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In the early 90s, Loeb showed that a natural extension of the usual binomial coefficient to negative (integer) entries continues to satisfy many of the fundamental properties. In particular, he gave a uniform binomial theorem as well as a combinatorial interpretation in terms of choosing subsets of sets with a negative number of elements. We tell this remarkably little known story and show that all of it can be extended to the case of Gaussian binomial coefficients. This talk is, in parts, based on joint work with Sam Formichella.

**Keywords:** binomial coefficients, q-binomial coefficients, hybrid sets, Lucas congruences

**2010** AMS Subject Classification: 05A10, 05A30, 11B65, 11A07

## Geometric Properties of Analytic Functions Associated with Nephroid domain

#### A. Swaminathan

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In this talk, the Carathéodory function  $\varphi_{Ne}(z) = 1 + z - z^3/3$  which maps the unit circle  $\{z : |z| = 1\}$  onto a 2-cusped curve called nephroid given by  $\left((u-1)^2 + v^2 - \frac{4}{9}\right)^3 - \frac{4v^2}{3} = 0$ , and the function class  $\mathcal{S}_{Ne}^*$  defined as

$$\mathcal{S}_{Ne}^* := \left\{ f \in \mathcal{A} : \frac{zf'(z)}{f(z)} \prec \varphi_{Ne}(z) \right\},$$

where  $\prec$  denotes subordination are considered. Apart from discussing the characteristic properties of the region bounded by this nephroid, the structural formulas, extremal functions, growth and distortion results, inclusion results, coefficient bounds

and Fekete-Szegö problems are discussed for the classes  $\mathcal{S}_{Ne}^*$  and  $\mathcal{C}_{Ne}$ . Further, sharp estimates on  $\beta \in \mathbb{R}$  so that the first-order differential subordination

$$1 + \beta \frac{zp'(z)}{p^{j}(z)} \prec \mathcal{P}(z), \quad j = 0, 1, 2$$

implies  $p \prec \varphi_{Ne}$ , where  $\mathcal{P}(z)$  is certain Carathéodory function with nice geometrical properties and p(z) is analytic satisfying p(0) = 1 are explained. As applications, sufficient conditions for  $f \in \mathcal{A}$  to be in the class  $\mathcal{S}_{Ne}^*$  are established. In continuation, sharp  $\mathcal{S}_{Ne}^*$ -radii for several geometrically defined function classes introduced in the recent past are outlined. Moreover, radii problems related to the families defined in terms of ratio of functions are also exhibited with graphical illustrations for all the possible situations. Results from other related literature are also provided. Problems for further research are highlighted at the end.

(This talk is based on the joint work with Lateef Ahmad Wani)

**Keywords:** Starlike and convex functions, Subordination, Fekete-Szegö problem, Cardioid, Nephroid, Lemniscate of Bernoulli.

**2010** AMS Subject Classification: 30C45, 30C50, 30C80.

### On harmonic univalent mappings with nonzero pole Bappaditya Bhowmik

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We consider the class  $A_H(p)$  of all sense preserving harmonic functions f in the open unit disc having a simple pole at  $z=p\in(0,1)$  with the normalizations  $f(0)=f_z(0)-1=0$ . We first talk about a sufficient condition for univalence of such functions. Next we consider the class  $S_H^0(p)$  which consists of all functions belonging to  $A_H(p)$  that are univalent with the additional normalization  $f_{\bar{z}}(0)=0$ . We discuss about the class  $S_H^0(p)$  in the view point of geometric function theory. This talk is based on the following article: B. Bhowmik and S. Majee: On harmonic univalent mappings with nonzero pole, Journal of Mathematical Analysis and Applications, 482(1), 13pp, 2020.

**Keywords:** Harmonic univalent functions, Harmonic shear, Taylor coefficients.

**2010** AMS Subject Classification: 31A05, 30C50, 30C55.

### Invited Talks

### Certain generating function involving the incomplete Fox-Wright function and its consequences

#### Khaled Mehrez

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The main focus of this note is to present certain generating functions for the incomplete Fox-Wright function when their terms contain the Appell function of the first kind are established. Finally, by means of the generating functions obtained here, new summation formula for the incomplete Fox-Wright function in terms of the H-function of two variables is made.

**Keywords:** Incomplete Fox-Wright function, Appel function, Generating function

**2010** AMS Subject Classification: 33C20, 33E20, 26D07, 26A42, 44A10.

## Wavelets for Evolution Problems with Localized Structures Ratikanta Behera

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Mathematical modeling of problems in science and engineering typically involves solving partial differential equations. However, this becomes particularly challenging for problems having localized structures or sharp transitions. The numerical solution of such problems on uniform grids is impractical since high-resolution computations are required only in regions where sharp transitions occur. An adaptive wavelet collocation method provides a robust method for controlling spatial grid adaptation - fine grid spacing in regions where a solution varies greatly (i.e., near steep gradients, or near-singularities) and a much coarser grid where the solution goes slowly. In this talk, we will discuss an adaptive wavelet collocation method with the solution of a linear advection equation and a non-divergent barotropic vorticity equation on the sphere.

**Keywords:** Wavelets; Partial differential equations; Adaptive grid; Numerical

2010 AMS Subject Classification: 65T60, 65DXX

## Composition of wavelet transform on some function spaces Kanailal Mahato

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The main goal of this present talk is with the construction and study of wavelet transform associated with fractional Hankel transform on some function spaces. Basic properties of wavelet transform are discussed. Parseval's relation is obtained. Plancherel's formula is proposed. Furthermore we defined the composition of wavelet transform and derived the Parseval's and Plancherel's identity. Moreover we obtained some fruitful estimates for the composition of wavelet transform on some suitably designed function spaces.

**Keywords:** Wavelet transform, Hankel transform, Fractional Hankel transform.

**2010** AMS Subject Classification: 42C40, 65T60, 43A32.

### Contributed Talks

### Rolle's Mean Value Theorem for Holomorphic Functions of a Generalized Complex Variable

#### Nirmal Kotal

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Rolle's mean value theorem, which is one of the most fundamental results in real analysis, may not hold true for a complex-valued holomorphic function, even if the function is entire. However, in 1992, Evard and Jafari showed that the real part and the imaginary part of a complex-valued holomorphic function satisfy the Rolle's mean value theorem individually. Generalized complex number systems generalize all unital real algebras of dimension two, including complex numbers, dual numbers, and double numbers. In this article, we have shown that the real part and the imaginary part of a holomorphic function of a generalized complex variable satisfy the Rolle's mean value theorem individually. Thus we have extended the result of Evard and Jafari to a larger space.

**Keywords:** Generalized complex number, Rolle's mean value theorem.

2010 AMS Subject Classification: 30G35, 26A24.

#### Sufficient Conditions Concerning the Unified Class of Starlike and Convex Functions

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Let  $\mathcal{A}_n$  be the class of analytic functions  $f(z) = z + \sum_{k=n+1}^{\infty} a_k z^k$ ,  $n \in \mathbb{N}$ , defined in the open unit disk  $\mathbb{D}$ . We use differential subordinations to obtain sufficient conditions in terms of third-order differential inequalities for a function  $f \in \mathcal{A}_n$  to be in the unified class of starlike and convex functions

$$\mathcal{S}^*\mathcal{C}_n(\alpha,\beta) := \left\{ f \in \mathcal{A}_n : \operatorname{Re}\left(\frac{zf'(z) + \beta z^2 f''(z)}{\beta z f'(z) + (1-\beta)f(z)}\right) > \alpha \right\},\,$$

where  $0 \le \alpha < 1$  and  $0 \le \beta \le 1$ . As applications, we construct certain members of  $\mathcal{S}^*\mathcal{C}_n(\alpha,\beta)$  involving triple-integrals and also derive conditions for the Pascu class of functions. Apart from obtaining new results, some of the already known results concerning starlikeness of  $f \in \mathcal{A}_n$  are obtained as special cases.

**Keywords:** Differential subordination, Starlikenss, Convexity, Integral operators, Pascu class.

**2010** AMS Subject Classification: 30C45, 30C80.

## Uniqueness result of meromorphic functions and some sharing problems

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In this paper, we have dealt with some uniqueness problems of meromorphic functions sharingtwo sets and improved a result of *Chen-Chen* [Bull. Malays. Math. Sci.Soc., 35(3)(2012), 765-774.] reducing the lower bound of a set. The result is: Let  $n \geq 6$  and  $p \geq 1$  be integers. Let  $S = \left\{ w \mid aw^n - n(n-1)w^2 + 2n(n-2)bw - (n-1)(n-2)b^2 = 0 \right\}$ , where  $a, b \in C - \{0\}$  such that  $ab^{n-2} \neq 2$ . Suppose that f and hence  $F_{a,b} = af + b$  is a non-constant meromorphic function of finite order satisfying  $E_{F_{a,b}}(S) = E_{\Delta_c^p f}(S)$  and  $E_{F_{a,b}}(\{\infty\}) = E_{\Delta_c^p f}(\{\infty\})$ ,  $a(\neq 0), b \in C$ . If  $N\left(r, \frac{1}{\Delta_c^p f}\right) = T(r, f) + S(r, f)$ , then  $F_{a,b} \equiv \Delta_c^p f$ . Some constructive examples exhibited to validate the main results of the paper. At last some questions have been raised for future investigation in this direction.

**Keywords:** Meromorphic functions, Difference operator, Gross question, finite order, shared sets.

**2010** AMS Subject Classification: 30D35.

## Starlikeness of the $B_{\kappa}^{c}$ -operator associated with the exponential function

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Several differential subordination implications are derived for analytic functions involving Bessel function and the operator introduced by Baricz *et al.* [Differential subordinations involving generalized Bessel functions, Bull. Malays. Math. Sci. Soc. **38** (2015), no. 3, 1255–1280]. These results are obtained by constructing suitable class of admissible functions. Examples involving trigonometric and hyperbolic functions are provided to illustrate the obtained results.

**Keywords:** Differential Subordination, Exponential Function, Generalized Bessel Function, Linear Operator.

**2010** AMS Subject Classification: 33C10, 33B10, 30C45, 30C80.

#### A one dimensional parametrized test functions space of entire functions

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Inspired by the construction of Kondratiev test functions in infinite dimensional analysis, this paper constructs a nuclear space of entire test functions. More pre-

cisely, for 
$$p = 0, 1, 2, \ldots$$
 and  $0 \le \beta \le 1$ , the inner product  $(f, g)_p = \sum_{n=0}^{\infty} a_n \bar{b_n} e^{pn} (n!)^{1+\beta}$ 

defines a countable system of nondecreasing Hilbertian norms and constructs a family of Hilbert spaces  $\{\mathcal{H}_p\}_{p=0}^{+\infty}$  of entire functions. The functions in  $\mathcal{H}_p$  are entire of at most  $\frac{2}{1+\beta}$  order growth and type not more than  $\frac{(1+\beta)}{2}e^{\frac{-p}{1+\beta}}$ . Moreover, the linear space  $\mathcal{E} = \bigcap_p \mathcal{H}_p$  is countably Hilbert and Fréchet space. Also,  $\mathcal{E}$  is a nuclear space

of entire functions of order growth at most  $\frac{2}{1+\beta}$  and of minimal type, endowed with the projective limit topology.

**Keywords:** White noise theory, topological spaces of test functions, special classes of entire functions and growth estimates.

**2010 AMS Subject Classification:** 60H40, 46F05, 30D15.

#### On the Bertrand Pairs of Open Non Uniform B-Spline Curves

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In this paper the second and third derivatives of open non-uniform B-spline curves at the points  $t = t_d$  and  $t = t_{m-d}$  and the Frenet vector fields and curvatures of these curves are given. In addition, the control points of the second open non-uniform B-spline curve are given in terms of the control points of the first open non-uniform B-spline curve when given two curves occurred a Bertrand curve pairs.

**Keywords:** Frenet vectors, Bertrand curve pairs, open non-uniform B-spline curves

**2010 AMS Subject Classification**: 65D17, 68U01, 53A04.

## Application of Shehu-Adomian Decomposition Method for solving dispersive KdV-like equations

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In this paper, we propose a new method called Shehu-Adomian decomposition method to solve homogeneous and non-homogeneous partial differential equations, in particular, Korteweg-de Vries-like equations. The Shehu-Adomian decomposition method is a hybrid of Shehu's transform and Adomian Decomposition perturbation method. To confirm the efficiency and validity of the method, the computation of some test problems of dispersive linearized KdV-like equations are presented. The proposed solutions are obtained in series form, converges very fast. The scheme seems very reliable, effective and efficient powerful technique for solving various type of physical models arising in sciences and engineering.

**Keywords:** Shehu transform, Adomian decomposition method, dispersive linearized KdV equations.

2010 AMS Subject Classification: 35A22, 35A25, 34A45.

## On the Weinstein-Wigner transform and localization operators

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In this talk, we define and study the Weinstein-Wigner transform and we prove its inversion formula. Then, we define and study the notion of localization operators associated with the Weinstein-Wigner transform. Next, we prove that they are in the trace class  $S_1$  and give a trace formula for them. At last, we study their boundedness and compactness on the space  $L^p_{\alpha}(\mathbb{R}^{d+1}_+)$ ,  $1 \leq p \leq \infty$ .

**Keywords:** Weinstein operator; Inversion formula; Weinstein-Wigner transform; localization operators.

2010 AMS Subject Classification: 43A32, 44A15.

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## Some results on the $\mathcal{R}$ -function associated with generalized fractional calculus operators

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In this paper, we establish some fractional properties of  $\mathcal{R}$ -function using generalized fractional calculus operators. Further we introduce an integral operator  $\mathcal{R}_{\alpha,\beta,\mathbf{u_p},\mathbf{v_q},\omega;a+}$  which contains  $\mathcal{R}$ -function as a kernel in the space L(a,b). We also establish the composition of the operator  $\mathcal{R}_{\alpha,\beta,\mathbf{u_p},\mathbf{v_q},\omega;a+}$  with the Riemann-Liouville fractional operators and Hilfer fractional derivative operator. Some composition properties of integral operator  $\mathcal{R}_{\alpha,\beta,\mathbf{u_p},\mathbf{v_q},\omega;a+}$  and their inversion have also been exhibited.

**Keywords:** Riemann-Liouville fractional calculus operators, Hilfer derivative operator,  $\mathcal{R}$ -function.

**2010** AMS Subject Classification: 26A33, 33E12, 33C15, 47B38, 47G10, 33C60.

## Boundedness, compactness and essential norm estimates of weighted composition operators on $\mathcal{D}^p_{\alpha}$ spaces

#### Sanjay Kumar

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Let  $\mathbb{D}$  denote the open unit disk in the complex plane  $\mathbb{C}$ . Let  $\varphi, \psi \in H(\mathbb{D})$ , the space of complex valued holomorphic functions on the open unit disk  $\mathbb{D}$  be such that  $\varphi(\mathbb{D}) \subseteq \mathbb{D}$ . Then the weighted composition operator  $W_{\varphi,\psi}$  acting on  $H(\mathbb{D})$  is defined as

$$W_{\varphi,\psi}(f)(z) = \psi(z)f(\varphi(z)).$$

When  $\psi \equiv 1$ , we just have the composition operator  $C_{\varphi}$ , defined by  $C_{\varphi}(f) = f \circ \varphi$ . Also if  $\varphi \equiv I$ , the identity function, then we get the multiplication operator  $M_{\psi}$  defined by  $M_{\psi}(f)(z) = \psi(z)f(z)$ . Let  $dA_{\alpha}(z) = (1+\alpha)(1-|z|^2)^{\alpha} dA(z)$  denote the weighted Lebesgue area measure on  $\mathbb{D}$ . For  $0 and <math>-1 < \alpha < \infty$ , the spaces of Dirichlet type  $\mathbb{D}_{\alpha}^p$  consist of those functions f holomorphic on  $\mathbb{D}$  such that

$$||f||_{\mathbb{D}^p_{\alpha}} = \left(|f(0)|^p + \int_{\mathbb{D}} |f'(z)|^p dA_{\alpha}(z)\right)^{1/p} < \infty.$$

 $Q_{\alpha}$  is the Banach space of function  $f \in \mathcal{D}_{\alpha}^2$  with the norm

$$||f||_{Q_{\alpha}} = |f(0)| + \sup_{\omega \in \mathbb{D}} ||f \circ \varphi_{\omega} - f||_{\mathbb{D}_{\alpha}^{2}} < \infty,$$

where  $\varphi_{\omega} = \frac{\omega - z}{1 - \bar{\omega}z}$ .

In this work we characterize the boundedness and the compactness of weighted composition operators acting between Dirichlet type spaces by using Carleson measures. We also find the essential norm estimates for these operators.

**Keywords:** Boundedness, compactness essential norm, weighted composition operators.

**2010** AMS Subject Classification: 47G10, 31C25, 47B38.

#### Tikhonov Type Regularization for unbounded operators

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In this paper, we introduce a Tikhonov type regularization method for an illposed operator equation Tx = y where T is a closed densely defined unbounded operator on a Hilbert space H.

**Keywords:** Densely defined operator, closed operator, Tikhonov type regularization.

**2010** AMS Subject Classification: 47A10, 47A52.

### On Differences of Positive Linear Operators

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In the recent years, several researchers have studied problems regarding the difference of two linear positive operators having same basis functions as well as different basis functions. In the present paper, we deal with the general quantitative theorems and error estimates for the difference of positive linear operators in both cases i.e. same basis functions and different basis functions.

**Keywords:** Difference of operators, modulus of continuity, error estimate, quantitative theorem.

2010 AMS Subject Classification: 41A25; 41A30.

## Some classes of analytic functions bound with Kober operators in q-calculus

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The conflict of the existing consideration is to express the q-analogue of kober differential and integral operators for the functions of complex variable  $\xi$  which are analytic in open unit disk and further with the aid of these operators, we also outline certain classes of analytic functions. The findings evaluated for these classes of functions admit the coefficient inequality with certain distortion theorems.

**Keywords:** Analytic functions, fractional q-calculus operators, coefficient bounds and distortion theorems.

2010 AMS Subject Classification: Primary 30C45; Secondary 26A33, 33D15.

## Power of a meromorphic function whose certain differential polynomial share a set with its derivative

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In this article, we deal with the unicity problem of the power of a meromorphic function whose differential polynomial share a set with its derivative. This paper improves some of the results on some constraints.

**Keywords:** Meromorphic function, Differential polynomial, Weighted set sharing, Uniqueness

2010 AMS Subject Classification: Primary 30D35.

### Weighted value sharing and uniqueness of entire and meromorphic functions of a linear differential polynomial

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In this research article, we deal with the uniqueness of entire and meromorphic functions when two linear differential polynomial share a non-zero value and obtain some results.

**Keywords:** Entire function, Linear Differential Polynomial, Meromorphic Function, uniqueness, Weighted Sharing.

**2010 AMS Subject Classification:** Primary 30D35.

## On a subclass of harmonic mappings with restricted analytic part

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In this article, a new subclass of harmonic mappings with restricted analytic part is introduced. Coefficient bounds and Fekete-Szegö inequality for both the analytic and co-analytic parts of functions in this subclass are obtained. An integral representation and bounds for the growth of functions in this subclass are found along with the covering theorem. Upper and lower bounds of area of the image of a subdisk inside the unit disk under the functions in this subclass are established together with the distortion theorem for the co-analytic parts. Finally we prove that this subclass of harmonic mappings is closed under convex combination.

**Keywords:** Analytic part, co-analytic part, coefficient estimates, growth theorem, convex combination.

**2010** AMS Subject Classification: 30C45, 30C50, 58E20.

#### Meromorphic function of a homogeneous differential polynomials sharing a set of the roots of unity

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In this research article, we deal with the uniqueness of homogeneous differential polynomial of a non-constant meromorphic function f of degree d. This article refines the prior outcomes obtained by Indrajit Lahiri and Kalyan Sinha.

**Keywords:** Nevanlinna Theory, Homogeneous Differential Polynomial, Meromorphic Function.

2010 AMS Subject Classification: Primary 30D35.

## Some results concerning to homogeneous differential polynomial with weighted sharing

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In this article we prove some of the results concerning to the homogeneous differential polynomial of degree  $\bar{d}(p)$  with weight  $\Gamma_p$  that shares (a, k). This article extends and generalises several earlier results of Dilip Chandra Pramanik and Jayanta Roy.

**Keywords:** Nevanlinna Theory, Homogeneous Differential Polynomial, Meromorphic Function.

2010 AMS Subject Classification: Primary 30D35.

### An Introduction to New Special Function Using Fractional Calculus

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Fractional calculus and special functions have contributed a lot to the science and engineering. In view of great importance and usefulness of fractional-calculus operators in different directions, I propose a new Special Function using Fractional Calculus named as Ganesh M-function. My approach is based on the use of modified Bessel function of the first kind and modified Struve function. In the present paper, my aim is to develop Ganesh M-function which may be regarded as a special function.

**Keywords:** Fractional calculus, Special functions, Ganesh M-function, Modified Bessel function, Modified Struve function.

2010 AMS Subject Classification: 41A30.

### Numerical solution of fractional Klein-Gordon equation with Caputo-Fabrizio fractional operator

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In this present work the fractional homotopy perturbation transform method (FHPTM) on linear and nonlinear fractional Klein-Gordon equation is applied via Caputo-Fabrizio derivative. This method was used directly to solve the fractional models without being linearization and discretization. The obtained results confirm that the proposed method is valid and reliable for approximate analytic treatment of a wide classes of nonlinear phenomena.

**Keywords:** Fractional Klein-Gordon equation, Caputo-Fabrizio derivative, Laplace transform, Approximate solution.

2010 AMS Subject Classification: 44A99, 35Q99.

#### A New Approach for Denoising Images using Diffusion Equation and Wavelets

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This study is based on two filtering process, diffusion equation and wavelets. In this paper, we solved diffusion equation by using finite difference method by taking noisy image as initial solution of the diffusion equation and then we applied wavelet denoising scheme based on soft thresholding. To validate the developed scheme we compared peak signal to noise ratio (PSNR) with wavelet method, diffusion equation method and wavelet coefficients via diffusion equation method.

**Keywords:** Diffusion equation; Finite Difference; Image denoising; Wavelet Transform.

**2010 AMS Subject Classification:** 65T60, 65N06, 68U10.

## Some Double integral Formulae Associated with Q Function and Galue Type Struve Function

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In this study, with the aid of Edward's double integral formula, we establish some double integral formula; our results are associated with Q Function and Galue type Struve function. We often examine their special cases in the form of recognized functions such as the Generalized Mittag-Leffler function and the Generalized Struve function. The findings of our present paper would both be useful and helpful in the study of applied science and engineering problems.

**Keywords:** Mittag Leffler function, Galue type Struve function, Generalized Struve function, Q function.

2010 AMS Subject Classification: 33E12, 33C20, 33E99.

#### Positivity of Hadamard powers of tridiagonal matrices

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We give a necessary and sufficient condition for a nonnegative tridiagonal positive semidefinite matrix to be infinitely divisible. We characterize all infinitely divisible matrices of order 3. It is well known that if A is a nonnegative positive semidefinite matrix of order n, then  $A^{\circ r} = [a_{ij}^r]$  is positive semidefinite when  $r \geq n-2$  or r is a positive integer. We improve this lower bound from n-2 to 1, if we restrict to nonnegative tridiagonal matrices. We give a similar result for a particular nonnegative pentadiagonal matrix.

**Keywords:** Infinitely divisible matrices, tridiagonal matrices, Hadamard powers, pentadiagonal matrices, chain sequences, totally nonnegative matrices.

**2010 AMS Subject Classification:** 15B48, 47B36, 15B36.

## A system of coupled two-term fractional differential equations

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In this paper, we study a new boundary value problem of coupled nonlinear two-term fractional differential equations. We make use of Banach's contraction principle and Schaefer's fixed point theorem to derive the existence and uniqueness results, which are well illustrated with examples.

**Keywords:** Multi-term fractional differential equations, Coupled system, Fixed point theorem

**2010 AMS Subject Classification:** Primary 26A33, Secondary 34B15.

## Complex symmetry of weighted composition operators on the space $\mathcal{H}^2_{\alpha,\beta}(\mathbb{D})$

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Let  $\mathbb{D}$  denote the open unit disc in the complex plane  $\mathbb{C}$ . For each pair  $\alpha, \beta \in \mathbb{C}$  with  $|\alpha|^2 + |\beta|^2 = 1$ , denote by  $\mathcal{H}^2_{\alpha,\beta}(\mathbb{D})$  the codimension one subspace of the Hardy space having the set  $\{\alpha + \beta z, z^2, z^3, \cdots\}$  as its orthonormal basis. An anti-linear map  $\mathcal{C}$  defined on a separable complex Hilbert space  $\mathcal{H}$  into itself is said to be a conjugation if it is both involutive and isometric. A bounded linear operator T on  $\mathcal{H}$  is said to be complex symmetric if there exists a conjugation  $\mathcal{C}$  such that  $T = \mathcal{C}T^*\mathcal{C}$ . We have studied the complex symmetric structure of the weighted composition operators  $W_{\psi,\phi}$  on  $\mathcal{H}^2_{\alpha,\beta}(\mathbb{D})$ , where  $\psi: \mathbb{D} \to \mathbb{C}$  and  $\phi: \mathbb{D} \to \mathbb{D}$  are analytic maps, with respect to a certain conjugation. We have also explored a few isometric properties of the complex symmetric operator  $W_{\psi,\phi}$  on  $\mathcal{H}^2_{\alpha,\beta}(\mathbb{D})$ .

**Keywords:** Complex symmetry, Composition operator, Conjugation.

**2010** AMS Subject Classification: 47B33, 47B38.

## On a new subclass of Sakaguchi type functions using $(\mathfrak{p},\mathfrak{q})$ -derivative operator

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The authors have introduced a new subclass of bi-univalent functions consisting of Sakaguchi type functions involving (p,q)-derivative operator. Further, the estimation of bounds for initial coefficients  $a_2$  and  $a_3$  have been obtained. The authors have stated few examples in this article.

**Keywords:** Analytic function, Bi-Univalent function,  $(\mathfrak{p}, \mathfrak{q})$ - derivative operator, Sakaguchi type function, Subordination.

**2010** AMS Subject Classification: 30C45, 30C15.

#### Hermite Wavelet Based Numerical Scheme for Solving Benjamina-Bona-Mohany Partial Differential Equations

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In this paper, a new Hermite wavelet based numerical scheme is developed to find the numerical solutions of the Benjamin-Bona-Mahony partial differential equations. The scheme is based on the time discretization of Hermite wavelets series approximations with collocation technique. Hermite wavelet based numerical scheme convergence analysis is also presented. In order to test an efficiency of the proposed scheme, it is applied on well known Benjamina-Bona-Mohany partial differential equations. Numerical solutions are obtained by preparing MATLAB codes of present scheme. The space-time graph of the Hermite wavelets series numerical solution is comparision with exact solution is presented in figures and tables at different time levels and absolute error is also calculated at the different time levels and presented in figures and tables to demonstrate the accuracy and applicability of the present scheme.

**Keywords:** Hermite wavelet; Benjamin-Bona-Mahony partial differential equation; Collocation technique, time discretization of Hermite wavelet series, Convergence analysis.

**2010** AMS Subject Classification: 65M50, 35M70, 65N22.

#### An Information Theoretic Entropy Related to Ihara $\zeta$ Function and Billiard Dynamics

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We shall initiate our discussion by illustrating a connection between the billiard dynamical system and the admissible graphs in the Ihara zeta function literature. Then we shall formally define the Ihara zeta function with its different representations. The power series representation of  $\zeta$  function is utilized to define a universal group entropy, a generalized information-theoretic entropy satisfying the Shannon-Khinchin axiom. This presentation will establish a connection between the billiard dynamical system and information theory.

**Keywords:** Billiard dynamics, Ihara zeta function, Shannon-Khinchin axioms, Entropy, Formal group theory

**2010** AMS Subject Classification: 11M36, 37B40, 13F25, 05C31, 94A17.

#### Some Curvature Conditions on Riemannian Manifolds

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In this paper we obtain necessary and sufficient condition for a generalized projective 2-recurrent Riemannian manifold to be a generalized 2-recurrent Riemannian manifold. We also prove that an Einstein generalized projective 2-recurrent Riemannian manifold is of constant curvature.

**Keywords:** Generalized projective 2-recurrent Riemannian manifold, Einstein manifold.

**2010 AMS Subject Classification**: 53B15, 53C20, 53C25

#### Generalization of the Hypergeometric Fractional Integral Operators in Relationship with R-Function

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The object of this paper is to investigate the Marichev-Saigo Maeda fractional integral operators, R-function and its generalization introduced by Robotnov and Hartley's. Representations of such relations are obtained in terms of Wright type function and Hypergeometric functions. Some interesting special cases of the established results associated with R-Function are obtained. Moreover, certain special cases of the main results are pointed out in the last section of this paper.

**Keywords:** Generalized fractional integral operators, H-Function, I-Function, and R-Function.

**2010** AMS Subject Classification: 26A33, 33C05, 33C10, 33C20.

## (p,q)-Extended incomplete and complete elliptic-type integrals

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We introduce a very general family of (p,q)-extended elliptic-type integrals for which the usual properties and representations of various known generalizations of the (classical) Legendre elliptic integrals are extended in simple manner. This family involves a number of special cases and, in particular, some of these extensions are related with (p,q)-extended Gauss' hypergeometric function and (p,q)-extended Appell's double hypergeometric function  $F_1$ . In addition, we deduce several special values, establish numerous differentiation and integral formulas, Mellin transforms and obtain some infinite series representations containing Laguerre polynomials.

**Keywords:** (p, q)-extended Gauss' hypergeometric functino, (p, q)-extended elliptic-type integrals, Mellin transforms.

**2010** AMS Subject Classification: 33C05, 33C60, 33C75.

## Continuous fractional Hankel wavelet transform on some function spaces

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In this article we discuss the continuity properties of fractional Hankel transform and fractional Hankel wavelet transform on the ultradifferentiable function spaces. Further, we introduced some Beaurling type spaces and studied the continuity of wavelet transform associated with fractional Hankel transform on this spaces.

**Keywords:** Wavelet transform, Fractional Hankel wavelet transform.

**2010** AMS Subject Classification: 42C40, 65T60, 46F12.

## Approximation of fixed point and its application to fractional differential equation

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In this paper, we prove convergence results for generalized  $\alpha$ -Reich-Suzuki non-expansive mappings via a fast iterative scheme. We validate our result by constructing a non-trivial numerical example. Also, we compare our results with that of the other well known iterative schemes utilizing MATLAB R2018a software and show the fastness of newly proposed iterative scheme by changing the set of parameters and initial points. Finally, by using our main result, we approximate the solution of nonlinear fractional differential equation.

**Keywords:** Generalized  $\alpha$ -Reich-Suzuki nonexpansive mappings; Nonlinear fractional differential equation; Fixed point.

2010 AMS Subject Classification: 47H10, 54H25.

#### Existence of Solution for Neutral Caputo-Hadamard Fractional Differential Equation via the Method of Upper and Lower Solution

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This paper is devoted to consider the existence of solutions for a class of nonlinear fractional neutral differential equations using Caputo-Hadamard fractional derivative with integral terms is given as follows

$${}^{c}D_{1}^{\alpha}[\vartheta(t) - g(t,\vartheta(a(t)))] = \chi(t,\vartheta(t),I_{1}^{\alpha}\vartheta(t)), \quad t \in [1,e], \quad 0 \le a(t) \le t$$
 (1)

$$\vartheta(1) = \vartheta_0, \tag{2}$$

where  ${}^cD_1^{\alpha}$  and  $I_1^{\alpha}$  denotes it by Caputo-Hadamard derivative operator and Hadamard integral operator, respectively,  $\chi:[1,e]\times R\times R\to R$ .

To establish the condition of solvability for the purposed problem, we shall apply the semi-group property of Hadamard fractional integral operator. To prove our result, we shall use the method of upper and lower solutions for fractional integral equation with Arzelá-Ascoli theorem.

**Keywords:** Caputo-Hadamard fractional derivative, neutral differential equations, upper and lower solutions, monotone sequences, Arzelá-Ascoli theorem.

**2010** AMS Subject Classification: 26A33, 34A12.

# Trajectory Controllability of Fractional Integro-differential Systems of Order $\alpha \in (1,2]$ with Nonlocal Condition in a Banach Space

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In this paper, we consider a fractional Integro-differential control system of order  $\alpha \in (1,2]$  in a Banach space X with nonlocal initial conditions. To study the trajectory controllability for the proposed control problem, the theory of fractional calculus, Gronwal's inequality and  $\alpha$ -order cosine family is used. At the end of the paper, we provide an example to illustrate our main results.

**Keywords:** Integro-differential equation, Trajectory controllability, Gronwal's inequality,  $\alpha$ -order cosine family.

**2010 AMS Subject Classification**: 34A08, 34K30, 93B05.

#### New Fixed Point Theorems in Operator Valued Extended Hexagonal b-Metric Spaces

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In the current work, we broaden the class of  $C^*$ -algebra-valued hexagonal b-metric spaces and  $C^*$ -algebra-valued extended b-metric spaces by defining the class of  $C^*$ -algebra-valued extended hexagonal b-metric spaces and demonstrate a fixed point theorem with distinct contractive conditions. In addition, an application is presented in the later part to demonstrate the existence and uniqueness of a particular type of operator equation in order to elucidate our results.

**Keywords:**  $C^*$ -algebra;  $C^*$ -algebra-valued extended hexagonal b-metric spaces; Contractive mapping; Fixed point

2010 AMS Subject Classification: 47H10, 54H25

### Certain properties of a p-deformed Jacobi polynomial Nidhi R Joshi

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The work incorporates the study of certain properties of a p-deformation of the classical Jacobi polynomial. This includes the three-term Recurrence relation, the differential equation, Rodrigues formula and the orthogonality. The companion matrix of the monic form of the polynomial is illustrated and MATLAB programming is used to compute the zeros of the polynomial of a particular degree.

**Keywords:** Jacobi polynomial; p-deformation; orthogonality; zeros; recurrence relation; Rodrigues formula

**2010** AMS Subject Classification: 11C08; 33C05; 33C45; 33E99.

### $(\Delta^m_v,f)\text{-lacunary statistical convergence of order }\alpha$

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In this paper, we define the space  $S^{\alpha}_{\theta}(\Delta^m_v,f)$  of all  $\Delta^m_v$ -lacunary statistical convergent sequences of order  $\alpha$  with the help of unbounded modulus function f, lacunary sequence  $(\theta)$ , generalized difference operator  $\Delta^m_v$  and real number  $\alpha$  with  $0<\alpha\leq 1$ . Properties related to this space are studied. We also introduce the space  $\omega^{\alpha}_{\theta}(\Delta^m_v,f)$  of all strong  $(\Delta^m_v,f)$ -lacunary summable sequences of order  $\alpha$ . Inclusion relations between spaces  $S^{\alpha}_{\theta}(\Delta^m_v,f)$  and  $\omega^{\alpha}_{\theta}(\Delta^m_v,f)$  are established under certain conditions.

**Keywords:** Statistical convergence, Difference sequence space, Modulus function.

**2010 AMS Subject Classification:** 40A35, 40C05, 46A45.

TO ANIS Subject Classification: 40A55, 40C05, 40A45.

### Iterated Function System in the Intuitionistic M- Fuzzy Metric Spaces

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Fractals are an exciting area in the research that offers many research possibilities in applied and pure mathematics. Most of the researchers are doing lots of work in this area. The Hutchinson-Barnsley theory (HB theory) to define and construct a fractal set as its unique fixed point in complete metric space. In this paper, we study the concept of Intuitionistic M-Fuzzy metric spaces and give some of their properties. The objective of this study is to further investigate the fractals generated by the iterated function system of fuzzy contractions in the intuitionistic M-fuzzy metric spaces by generalizing the Hutchinson-Barnsley theory. Our results contain some of the recent results reported in the literature.

**Keywords:** Intuitionistic M- Fuzzy Metric Space, Fuzzy Contraction, Hausdorff Fuzzy metric, Attractor, Iterated Function System.

2010 AMS Subject Classification: 28A80, 54H25,26E50.

#### Brent's Method and Its Improvements

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This study presents Brent's Method and its improvements by reconstruction. Brent's method is a root finding algorithm that combines aspects of the Bisection method, the Secant method and Inverse Quadratic Interpolation. The 1st and 2nd improved method consists of a combination of Bisection method and Steffensen method/Kurchatov method. The Modified Brent's method is an improved method that determines the next iteration interval from three subsections constructed by four given points, whereas the original Brent's method determines the next iteration interval from two subsections. The 3rd and 4th improved method consists of a combination of this Modified Brent's method with Steffensen method/Kurchatov method.

All four methods are tested on a number of numerical examples and results obtained show that all the methods are effective in different parameters.

**Keywords:** Brent's method, Bisection Method, Secant Method, Inverse Quadratic Interpolation, Steffensen Method, Kurchatov method.

**2010 AMS Subject Classification**: 65H05, 90C53, 26C10.

### On the domain of q-Euler matrix in $c_0$ and cTaja Yaying

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In this study, we present Banach spaces  $e_0^q$  and  $e_c^q$  obtained by the domain of q-analog  $E^q$  of Euler matrix of order 1 in the spaces  $c_0$  and c, respectively. We exhibit certain topological properties and inclusion relations of these spaces. We obtain the bases and determine Köthe duals of the spaces  $e_0^q$  and  $e_c^q$ . We characterize certain classes of matrix mappings from the spaces  $e_0^q$  and  $e_c^q$  to the space  $\mu \in \{\ell_\infty, c, c_0, \ell_1, bs, cs, cs_0\}$ .

**Keywords:** Sequence space, q-Euler matrix, Köthe-Toeplitz duals, Matrix mappings.

2010 AMS Subject Classification: 46A45, 46B45.

#### (m, n)-paranormal Composition Operators

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In this paper, we prove some characterizations for the class of (m, n)-paranormal operators acting on the complex Hilbert space  $\mathcal{H}$ . The class of (m, n)-paranormal operators is characterized in terms of the Radon–Nikodym derivative of the measure  $\lambda T^{-1}$  with respect to  $\lambda$ . Moreover, we discuss the conditions under which the classes of composition operators, weighted composition operators, multiplication composition operators are (m, n)-paranormal.

**Keywords:** (m, n)-paranormal operator, composition operator, weighted composition operator, multiplication composition operator.

2010 AMS Subject Classification: Primary 47B38; Secondary 47B20, 47B33.

#### On Generalized Weaving Frames in Hilbert Spaces

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Generalized frames (in short g-frames) are natural generalization of standard frames in separable Hilbert spaces. Motivated by the concept of weaving frames in separable Hilbert spaces by Bemrose et al. in the context of distributed signal processing, we study weaving properties of g-frames. Firstly, we present necessary and sufficient conditions for weaving g-frames in Hilbert spaces. We extend some results of Bemrose et al. and Casazza and Lynch about standard weaving frames to g-weaving frames. Some Paley-Winer type perturbation results for weaving g-frames are obtained. Finally, we give necessary and sufficient conditions for weaving g-Riesz bases.

**Keywords:** Frame, Generalized Frames, Weaving frames, Riesz basis, Perturbation.

**2010** AMS Subject Classification: 42C15, 42C30, 42C40.

# Investigation of the existence criteria for the solution of the Functional Integral Equation in the $L^p(R_+)$ space

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This work manifests the credibility of Darbo's fixed point theory towards the solvability of nonlinear functional convolution integral equation with deviating argument. The solution space is taken to be the space of Lebesgue integrable functions defined on  $R_+$ , with the standard norm,  $||x||_p = (\int_0^\infty |x(t)|^p dt)^{\frac{1}{p}}$ .

The concept of measure of noncompactness in correlation with the compactness criterion i.e., Kolmogorov-Riese compactness theorem in  $L^p(R_+)$  space has been taken. Then under certain suitable hypothesis and by the assistance of Darbo's fixed point theory, sufficient conditions for the existence of the solution have been introduced. Finally, some examples have been taken in order to justify the result.

**Keywords:** Fixed Point Theory, Functional Integral Equation, Measure of Noncompactness.

**2010 AMS Subject Classification:** 45E10, 45G10, 45D05.

### Bounds of the Zeros of a Polynomial with Quaternionic Variable

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Let  $p(q) = \sum_{l=0}^{n} q^{l} a_{l}$ , where q = x + yi + zj + wk;  $x, y, z, w \in \mathbb{R}$  and i, j, k are the fundamental quaternion units and  $a_{l}$ ,  $0 \le l \le n$  are either real or quaternions. In this paper, we obtain some results concerning the location of zeros of polynomial p(q), which generalize and refine some known Eneström-Kakeya type bounds.

**Keywords:** Polynomial, Quaternion, Zeros, Eneström-Kakeya Theorem.

2010 AMS Subject Classification: Primary 30E10; Secondary 30A01, 30C01.

### Extremal mild solutions of Hilfer fractional Impulsive Systems

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The well established monotone iterative technique that is used to study the existence and uniqueness of fractional impulsive system is extended to Hilfer fractional order. The results are derived using the method of upper and lower solution and Gronwall inequality. Also, conditions on non-compactness of measure is used effectively to prove the main result.

**Keywords:** Lower and upper solution, Impulsive system, Hilfer fractional derivative, Non-compact measure.

**2010** AMS Subject Classification: 26A33, 34K30, 34K45, 47D06.

# Characterization of Psuedo scaling function on Vilenkin group

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The study of wavelets, originated from it's applications in diverse fields, was combined together by it's Mathematical properties. Initially, all the wavelets and it's variants were explored in the real space  $\mathbb{R}^n$ . But now, these are being studied in different abstract settings. The present paper also contributes to this extension. Vilenkin groups, introduced by F. Ya Vilenkin, form a class of locally compact abelian groups. In this paper, Parseval frame multiwavelets associated to multiresolution analysis (MRA) are characterized in  $L^2(G)$ , where G is the Vilenkin group. Further, we introduce the pseudo-scaling function along with a class of generalized low pass filters and study their properties in Vilenkin group.

**Keywords:** Walsh function, psuedo scaling function, Parseval frame multiwavelet.

**2010 AMS Subject Classification:** 42C15, 42C40, 42C10.

### Spherical Fuzzy Set Approach of Graphs based on Dombi Operator

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One of the main problems in practice is to deal with membership functions. Many decision makers ask for a graphical representation to visualize the results. In this research article, the novel idea of Spherical fuzzy set way of graphs based on Dombi operator is introduced. The implementation of the new concept is known as the Spherical Dombi Fuzzy Graph (SDFG). Moreover, we introduce spherical fuzzy terminology of several basic graph-theoretical concepts using Dombi operator.

**Keywords:** Fuzzy set, Spherical fuzzy sets, T-spherical fuzzy sets, Dombi fuzzy graph.

**2010** AMS Subject Classification: 03E72, 68R10, 05C72, 94C15, 90B50.

#### Orthogonality and other properties of an M-LaguerrePolynomial

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The present work incorporates an extension of the generalized Laguerre polynomial by introducing a parameter M (> 0) in its differential equation. The polynomial solution of this is referred to as the M-Laguerre polynomial. It is shown that this polynomial is orthogonal with respect to the weight  $x^{\alpha+2M-2} \exp\left(-x^M/M\right)$  over  $(0,\infty)$ . The non zero integral corresponding to the same degree of the polynomials is evaluated by proving the Rodrigues formula. Also, the generating function relation is derived and the Companion matrix of the monic polynomial is illustrated.

**Keywords:** M-Laguerre polynomial, Orthogonality, Rodrigues formula.

**2010 AMS Subject Classification:** 33C45, 33E99, 35A09.

### An investigation on existence of warped product slant lightlike submanifolds of indefinite Kenmotsu manifolds

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In the present paper, we investigate whether a slant submanifold of indefinite Kenmotsu manifold can be written in the form of warped product submanifold or not. In this context, we establish some important results to justify our claim investigation. Further, we prove that "there does not exist any slant lightlike submanifold of indefinite Kenmotsu manifold".

**Keywords:** Indefinite Kenmotsu manifold, Slant lightlike submanifold, Warped product submanifolds.

**2010** AMS Subject Classification: 53C15, 53C40, 53C50.

### Convergence analysis of a sixth-order method under weak continuity condition with first-order Fréchet derivative

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This article discusses the semi-local convergence analysis of the well-established sixth-order method in Banach spaces. The analysis has been done under Hölder continuity condition with the help of the recurrence relation technique. The significance of our study lies in the fact that there are many examples that do not satisfy the Lipschitz continuity but satisfy the Hölder continuity. A convergence theorem has been established for the existence-uniqueness of the solution. A priori error bound expression is also derived. Finally, the convergence analysis is carried out on a variety of examples. These examples include Hammerstein and Fredholm integral equation and a boundary value problem that justified the theoretical developments.

**Keywords:** Banach spaces, recurrence relation, Hölder continuity, semi-local convergence.

2010 AMS Subject Classification: 65H10, 65D30, 65J15.

# Study on some particular class of non linear integral equation with a hybridized approach

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This article is entirely devoted to a particular class of integral equation involving pure delay term. The existence of solution is described using fixed point theory. Moreover a hybridized scheme is proposed to investigate approximate solution. In this context boundary element method is used with piecewise linear interpolation. Also an algorithm is there for error estimation and in support of the considered numerical method stability analysis is done. This testimony completely demonstrates the comprehensive study of considered class of integral equation and understanding the behaviour of approximate solution in presence of delay term.

**Keywords:** Fredholm integral equation, Pure delay term, Fixed point theory, Boundary element method, Piecewise linear interpolation.

**2010** AMS Subject Classification: 45G10, 45B05, 65R20.

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# Integral representation of non-negative q-superharmonic functions in Schrödinger network

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In the generalized infinite network called Schrödinger network, q-harmonic and q-superharmonic functions are defined and its properties are discussed. In the Schrödinger network a unique integral representation for a non-negative q-harmonic function is obtained by using the Choquet integral representation theorem. A similar representation for a non-negative q-superharmonic function is obtained by identifying the extremal points on a base of the cone of non-negative q-superharmonic functions.

**Keywords:** q-harmonic functions, q-superharmonic functions, Schrödinger network, integral representation.

**2010** AMS Subject Classification: 31C20, 31A05, 31A10.

### Semilocal convergence of fifth order Chebyshev-type methods under a mild condition

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In this paper, the semilocal convergence of a family of fifth order Chebyshevtype methods used for solving nonlinear operator equations in Banach spaces has been studied. This has been done under the assumption that the second Fréchet derivative of the operator satisfies a mild  $\omega$ -continuity condition. Using recurrence relations a convergence theorem is given that establishes the R-order of the method to be (3+2p),  $p \in (0,1]$  and existence-uniqueness ball for the solution. A number of numerical examples have been given to show efficacy of our result.

**Keywords:** Fifth order Chebyshev-type methods,  $\omega$ -continuity condition, Recurrence relations, R-order of convergence, Semilocal convergence.

**2010** AMS Subject Classification: 47H10, 41A25, 65Q05.

### On approximating fixed point in CAT(0) spaces

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In this paper, we construct a new iteration process in the setting of CAT(0) spaces involving generalized  $\alpha$ -nonexpansive mapping. We prove strong and delta convergence results for approximating fixed point via newly defined iteration process. Further, we reconfirm our results by non trivial example and tables.

**Keywords:** CAT(0) space, Fixed point,  $\Delta$ -convergence, Generalized  $\alpha$ -nonexpansive mapping.

2010 AMS Subject Classification: 47H10, 54H25.

### Oscilations and asymptotic stability of entire solutions of linear delay differential equations

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Consider, the linear differential equation,

$$x'(t) + \sum_{n=1}^{k} P_n(t)x(t - \tau_n) = 0, t \ge t_0$$
(1)

where  $P_n \in C([t_0, \infty), R)$  and  $\tau_n \geq 0$  for n = 1, 2, ....k. By investigating the oscillatory solutions of the linear delay differential equations, we present new sufficient condition for the asymptotic stability of the solutions of (1). The results improve and complement some earlier ones in the literature.

**Keywords:** Linear delay differential equations; oscillation; stability of entire solution.

2010 AMS Subject Classification: Primary 39A10, Second 30D35, 39A12.

### A fourth order derivative free iteration method for multiple roots of nonlinear equations

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In this paper, a derivative free iterative method of fourth order is proposed for finding multiple roots of nonlinear equations  $\varkappa(t)=0$ , where  $\varkappa:[l,m]\subset\mathbb{R}\to\mathbb{R}$  is a continuously differentiable function. This is done by first describing the proposed method for finding simple roots of  $\varkappa(t)=0$ . Then by using some suitable transformation, the given function  $\varkappa(t)=0$  with multiple roots is transformed to  $\bar{\varkappa}(t)=0$  with simple roots. Then the method starts with a suitably chosen initial approximation  $t_0$  near the simple root  $\xi$  of  $\bar{\varkappa}(t)=0$  and generates two sequences, one sequence of successive iterates  $\{t_n\}$ , and other sequence of intervals  $\{[l_n,m_n]\}$  containing the root  $\xi$ . It is shown that, simultaneously both the sequence of diameters  $\{(m_n-l_n)\}$  and the sequence of iterates  $\{(t_n-\xi)\}$  has fourth order of convergence. The method is tested on a number of numerical examples which show its applicability.

**Keywords:** Nonlinear equation; Bisection Method; Multiple roots; Iterative methods.

2010 AMS Subject Classification: 65Hxx, 65H04, 65H05.

### On existence of integral solutions for a class of mixed Volterra-Fredholm integro fractional differential equations

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Fractional differential equation has emerged as an important area of investigation due to its growing number of applications in various areas of applied sciences and engineering. It has been found that the differential equations involving fractional derivatives are more realistic for describing many physical phenomena than those of integer order derivatives. There are various ways of interpolating the definition of integer order to non-integer order. In this work, we study the existence of integral solution of a class of neutral fractional integro-differential equation of mixed type involving Caputo fractional derivative under the assumption that the associated operator A is not dense. Utilizing semigroup theory, fractional calculus, fixed point theorems and measure of noncompactness, a set of sufficient conditions are established which ensure the existence of integral solutions of our problem, when the associated  $C_0$ -semigroup generated by the part of A in  $\overline{D(A)}$  is compact or noncompact.

**Keywords:** Hille-Yosida condition, integral solution, fixed-point theorem, measure of noncompactness.

**2010** AMS Subject Classification: 26A33, 34G20, 47H10, 47H08.

#### On Weyl-Heisenberg Frames

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In this paper we have generalized and studied the K-Weyl-Heisenberg frames, where K is a bounded linear operator on  $L^2(\mathbb{R}^d)$ . We have obtained necessary and sufficient conditions for a certain system to be a K-Weyl-Heisenberg frame. We have also given the invariance property of these K-Weyl-Heisenberg frames.

**Keywords:** Frames, K-Frames, Weyl-Heisenberg frames.

**2010** AMS Subject Classification: 42C15, 46B15.

### On Retro Frame Associated With Measurable Space

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Frames are redundant system which are useful in the reconstruction of certain classes of spaces. Duffin and Schaeffer introduced frames for Hilbert spaces, while addressing some deep problems in nonharmonic Fourier series. The dual of a frame (Hilbert) always exists and can be obtained in a natural way. In this paper we introduce the notion of  $\Omega_0$ -type duality of retro  $(\Omega, \mu)$ -frames are given. Necessary and sufficient conditions for the existence of the dual of retro  $(\Omega, \mu)$ - frames are obtained. A special class of retro  $(\Omega, \mu)$ -frames which always admit a dual frame is discussed.

**Keywords:** Frames,  $(\Omega, \mu)$ -frames, Retro  $(\Omega, \mu)$ -frames.

**2010** AMS Subject Classification: 42C15, 42C30, 46B15, 47B48.

### Stronger forms of sensitivity in uniformly convergent non-autonomous dynamical systems

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We prove equivalence of different stronger forms of sensitivity under certain conditions for uniformly convergent non-autonomous dynamical systems and provide examples wherever equivalence is not true in general. We also prove that if a non-minimal, topologically transitive non-autonomous system having the set of almost periodic points dense converges uniformly, then it is thickly syndetically sensitive.

**Keywords:** Non-autonomous dynamical systems, Multi-sensitivity, Thick sensitivity.

**2010** AMS Subject Classification: 37B20, 37B55, 54B20, 54H20.

# Sufficiency and duality of set-valued fractional programming problems via second-order contingent epiderivative

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In optimization theory, many authors have been interested to study fractional programming problems. This type of problems arises in a large number of applications in different fields of mathematics, economics and engineering. In this paper, we study a set-valued fractional programming problem for the case, where the objective function and functions attached to constraints are set-valued maps.

We consider the following set-valued fractional programming problem:

where X is a real normed space, A is a nonempty subset of X, and  $F: X \to 2^{\mathbb{R}^m}$ ,  $G: X \to 2^{\mathbb{R}^m}$ , and  $H: X \to 2^{\mathbb{R}^k}$  are set-valued maps.

We establish second-order sufficient KKT optimality conditions of the set-valued fractional programming problem (FP) under second-order generalized cone convexity assumptions. We also prove duality results between the primal problem (FP) and second-order dual problems of parametric, Mond-Weir, Wolfe, and mixed types via the notion of second-order contingent epiderivative.

**Keywords:** Convex cone, Set-valued map, Contingent epiderivative, Duality.

**2010** AMS Subject Classification: 26B25, 49N15.

#### Time-dependent analytical and computational study of an M/M/1 queue with disaster failure and multiple working vacations

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An M/M/1 working vacation queueing model with disaster failure is considered to examine time-dependent behavior. When the system is in busy mode, it can fail such that the customers in the system flushed out and never returns; such type of failure is known as disaster failure. The server is allowed to go for a WV after each busy period of a random duration of time. In the duration of WV, the server reduced the service rate rather than halting the service. After completing the vacation period, the server can take any number vacation until he found some customers waiting in the queue; this vacation policy is known as multiple vacation policy. The transient analytical formulae for the queue size distributions are formulated by solving the Chapman-Kolmogorov equations using continued fractions, modified Bessel function and probability generating function methods. Moreover, various queueing performance measures are given, and real-time performance is evaluated by computing the performance measures numerically.

Keywords: Transient queue, System disaster, Working Vacation, Repair, Continued fraction, Modified Bessel function

2010 AMS Subject Classification: 60K25, 30B70.

### Chaos in a three-cell population cancer model with variable-order fractional derivative with power, exponential and Mittag-Leffler memories

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In this work, a three-dimensional cancer model which includes the interactions between tumor cells, healthy tissue cells, and activated immune system cells was considered via Liouville-Caputo, Caputo-Fabrizio, Atangana-Baleanu, and fractional conformable derivative. We show a numerical method based on two-step Lagrange polynomial interpolation to achieve numerical approximations to these derivatives. Besides, also we analyze the dynamics observed via sensitivity to initial conditions, Lyapunov exponent estimation, square sum error, and phase-space diagrams. Novel attractors were obtained and all of them depicted novel chaotic behaviors by choosing a fractional variable-order.

**Keywords:** Fractional calculus; Cancer model; Variable-order fractional derivatives; Chaotic systems.

2010 AMS Subject Classification: 26A33, 34A08, 34H10.

# Complex chaotic systems and its complexity Ajit K. Singh

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Chaos is a complex behavioural pattern associated with nonlinear systems. Its association with the analytical complexity of the generating system is thought to be minimal. Experiments show that this is not necessarily true and that, with the increase of analytical complexity, newer classes of dynamical behaviour can be seen. Chaotic regions and periodic windows are explored as well as different types of motion shown along the routes to chaos. This constructs a novel coupling term by replacing the coupling parameter with a coupling matrix. Numerical experiments by means of phase portraits, bifurcation diagrams and the largest Lyapunov exponent are involved. Based on these results, a synchronization scheme in fractional-order complex Lorenz systems is presented and the corresponding numerical simulations demonstrate the effectiveness and feasibility of the scheme. The results show that expressing the Lorenz system in terms of complex variables leads to more distinguished behaviours, which could not be achieved in the Lorenz system with real variables.

**Keywords:** Analytical complexity, dynamic property, nonlinear coupling method.

2010 AMS Subject Classification: 32-XX, 37Fxx, 37D45, 39A33.

### Complex Delay-differential Equations of Malmquist type

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In this paper, we investigates some results on complex delay-differential equations of the classical Malmquist theorem. A classic illustrations of their results states us that if a complex delay equation

$$w(t+1) + w(t-1) = R(t, w)$$

with R(t, w) rational in both arguments admits (concede) a transcendental meromorphic solution of first order, then  $deg_wR(t, w) \leq 2$ . Development and upgrade of such results are presented in this paper. In addition, Borel exceptional zeros and poles seem to appear in special solutions.

**Keywords:** Complex Delay-differential Equations; Value distribution; Nevanlinna Characteristic function; Zeros and poles; Borel Exceptional Values

2010 AMS Subject Classification: Primary 39A10, Secondary 30D35, 39A12.

# Solvability of the system of implicit generalized order complementarity problems

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In this paper, we introduce the notion of exceptional family for the system of implicit generalized order complementarity problems in vector lattice. We present some alternative existence results of the solutions for the system of implicit generalized order complementarity problems via topological degree aspects. The new developments in this paper generalize and improve some known results in the literature.

**Keywords:** Order exceptional family, System of implicit generalized order complementarity problems, Existence theorem, Topological degree

**2010** AMS Subject Classification: 90C33, 47N10.

# Usual stochastic ordering results for series and parallel systems with components having Exponentiated Chen distribution

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In this paper we have discussed the usual stochastic ordering relations between two system. Each system consists of n mutually independent components. The components follow Exponentiated (Extended) Chen distribution with three parameters  $\alpha, \beta, \lambda$ . Two popular system are taken into consideration, one is the series system and another is the parallel system. The results in this paper are obtained by varying one parameter and the other parameters are kept constant. The hazard rate ordering or reversed hazard rate ordering relations that are not possible for series or parallel systems have been demonstrated with the help of counterexamples.

**Keywords:** Exponentiated Chen distribution, Majorization, Parallel system, Series system, Usual stochastic order.

2010 AMS Subject Classification: 62N05.

# A matrix analogue of a general inversion pair and associated matrix polynomials

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Since two decades the matrix analogues of several classical polynomials have been introduced and their various properties have been studied. In the present work, we establish a general inverse matrix series relation and thereby invert the matrix analogues of several polynomials belonging to the Askey scheme.

**Keywords:** Inverse matrix series relation, Matrix polynomials

**2010** AMS Subject Classification: 15A16, 15A24, 33C45, 33C99.

### On Nearly Quasi Einstein Manifolds

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In this work we have attained satisfactory conditions for a generalized Ricci recurrent manifold to be a nearly quasi-Einstein manifold. Non trivial examples can proved presence of nearly quasi-Einstein manifold. We also worked on semi-Riemannian  $GR_n$  in it.

**Keywords:** Generalized Ricci recurrent manifold, Ricci recurrent manifolds, recurrent manifold, quasi-Einstein manifold, nearly quasi-Einstein manifold

2010 AMS Subject Classification: 53C25, 53C35.

### On entropy with respect to two topologies

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Complexity is a part of natural systems. There are many ways to measure the complexity of a dynamical system involving only one topology and entropy is one of them. But till now there is not any measure to find the complexity of a system involving two topologies. In this paper we introduce the notion of entropy with respect to two topologies as a measure of complexity and discuss some results related to entropy from the perspective of bitopological dynamical systems [Acharjee S, Goswami K, Sarmah HK. On entropy in bitopological dynamical systems (communicated)].

**Keywords:** Entropy, Bitopology.

2010 AMS Subject Classification: 54E55, 37B20, 37B99.

### Reductions of ideals in a Noetherian local ring

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Let (R, m) be a Noetherain local ring and  $I \subseteq R$  be an ideal of R. We give theory of reduction of an ideal which has recently played primary role in a homogeneous affine co-ordinate ring of fiber over the closed point m in the blow-up of spec(R) along the subscheme R/I. In the present study, we investigate the properties of reductions of ideals using Zariski open sets.

**Keywords:** Reductions of ideals, Fiber-cone, Zariski open set.

**2010 AMS Subject Classification:** 13A30, 13B22, 13A15.

# Modeling the effect of predator defense in a plankton-fish system with gestation delay

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In the present article, we propose and evaluate a delayed plankton-fish model with the functional response of Monod-Haldane type. Numerous prey organisms have adopted predation defense mechanisms. Predators also display ways to cope with variable environmental conditions which are adapted in the event of a scarcity of prey. We assume that prey and middle predators are capable of defending their predators. As an adaptation to deal with variable environmental conditions, time delay is integrated into the middle predator. Analytically, the feasibility and boundedness restrictions of the model system are studied by comparision principle along with local and global stability conditions. We study the effect of defense and time delay on the system's stability behavior. Our numerical results indicate that the increase in time delay shows unstable and stable both kinds of behavior under different parametric limitations. We observe that extinction in high predators due to defensive capacity in prey and middle predators.

**Keywords:** Plankton, Gestation delay, Functional response, Defense, Chaos.

**2010** AMS Subject Classification: 37C10, 37C75, 92B05.

#### Properties and Matrix Sequences of Derived K-Jacobsthal, Derived k-Jacobsthal Lucas

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In this paper We analyze a couple of properties of Derived k-Jacobsthal and Derived k-Jacobsthal Lucas. Then by utilizing these arrangements we depict Derived k-Jacobsthal Matrix Sequences, Derived k-Jacobsthal Lucas Matrix Sequences. We characterise few properties and discuss some connection between these grid successions.

**Keywords:** Derived k-Jacobsthal, Derived k-Jacobsthal Lucas, Derived k-Jacobsthal Matrix Sequences, Derived k-Jacobsthal Lucas Matrix Sequences

**2010 AMS Subject Classification:** 15A24, 05A15, 11B37, 11B39, 11B83, 40C05, 11K31.

### TransResV-Net: Deep Learning Based Brain Tumor Segmentation

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Tumor is a deadly disease, more than thousand people per day die due to this in India. The main cause is sudden growth of mass in organs. Benign and malignant, two types of tumors are there. The main issue is that the size, shape is different for different person. So perfect segmentation is very important to diagnosis this at the early stage. This is only way to prevention. Here a Computer aided diagnosis system is described to segment the tumor cell using deep learning based approach, ransResV-Net, which is the extension of V-net. For pixel based segmentation, V-net is very successful than the conventional network and for limited data set this is one of the best way of segmentation. To implement the segmentation, this model is applied on the available data set containing MRI images from Kaggle. The parameters are taken for results are accuracy, Intersection over union, dice coefficient. The result shown a massive improvement when it is compared with the well known and benchmark methods.

**Keywords:** Tumor, deep learning, V-net, MRI.

2010 AMS Subject Classification: 94A08, 68U10, 65D18.

### Numerical Solution of Stochastic Differential Equations by Using Third Order Improved Runge-Kutta Methods

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In this paper, we consider numerical solutions of stochastic initial value problems by the improved Runge-Kutta method of the third order. A random mean value theorem is established and the mean square convergence of these methods is proved. The expectation and variance of the solution are derived. Numerical example shows that the inexact arrangements.

**Keywords:** Stochastic differential equation (SDE), Stochastic processes, Mean value theorem.

**2010** AMS Subject Classification: 03E72, 68R10, 05C72, 94C15, 90B50.

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