Introduction to Logic

Propositional Logic

Syntax:

You can make well-formed formulas (**wff**) φ from:

$$
\begin{gathered}
p, \text{a variable} \\
\bot \\
\varphi \wedge \varphi \\
\varphi \vee \varphi \\
\varphi \rightarrow \varphi \\
(\varphi)
\end{gathered}
$$

$\wedge$ and $\vee$ have precedence over $\rightarrow$. This means that the formula

$$r \wedge u \rightarrow w$$

is equal to

$$(r \wedge u) \rightarrow w$$

From this you can derive:

Connective	Definition
$\neg\varphi$	$\varphi \rightarrow \bot$
$\top$	$\neg\bot$
$\neg\varphi$	$\varphi \rightarrow \bot$
$\varphi \leftrightarrow \psi$	$(\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$

Subformulas:

let φ be a formula. The set of *subformulas* of φ is determined like this:

$$
\begin{aligned}
\varphi &= \neg p \wedge q \rightarrow p \rightarrow s \\
\mathrm{Sub}(\varphi) &= \{\varphi\} \cup \mathrm{Sub}(\neg p) \cup \mathrm{Sub}(p \rightarrow s) \\
&= \{\varphi, \neg p \wedge q, \neg p, p, \bot, q, p \rightarrow s, s\}
\end{aligned}
$$

Truth table:

To check if a formula is true or false, you can use a truth table: $p \vee (q \rightarrow p) \rightarrow q \rightarrow p$

p	q	$q \rightarrow p$	$p \vee (q \rightarrow p)$	$p \vee (q \rightarrow p) \rightarrow q \rightarrow p$
0	0	1	1	1
0	1	0	0	1
1	0	1	1	1
1	1	1	1	1

Tautologies:

Symbol	Meaning
PForm	a set of all formulas
Γ	a set of some formulas
$\vDash$	entails
$\Gamma \vDash \varphi$	A set of formulas entails one formula for example: $\{(\text{human} \to \text{mortal}), \text{human}\} \vDash \text{mortal}$, $\Gamma = \{(\text{human} \to \text{mortal}), \text{human}\}$ $\varphi = \text{mortal}$ From Γ you can draw the conclusion that φ is true

For some random formulas φ and ψ, we can say that:

- **satisfiable**, φ is *satisfiable* if there is a way to assign true or false values to each of the variables in φ in such a way that the entire formula becomes true.
- **tautology**, φ is a *tautology* if φ is always true no matter what variables you use.
- **semantically equivalent**, φ and ψ are *semantically equivalent* if every valuation of φ and ψ are the same. This can be written as $\varphi \equiv \psi$, meaning φ is identical to ψ.

Natural deduction:

Symbol	Meaning
$\Gamma \vdash \varphi$	φ can be derived or proved from the set of premises represented by Γ

Rules:

$$\frac{\varphi : \Gamma}{\Gamma \vdash \varphi}\ (\text{Assum}) \qquad \frac{\Gamma \vdash \bot}{\Gamma \vdash \varphi}\ (\bot\text{E})$$

$$\frac{\Gamma \vdash \varphi \land \psi}{\Gamma \vdash \varphi}\ (\land\text{E}_1) \qquad \frac{\Gamma \vdash \varphi \land \psi}{\Gamma \vdash \psi}\ (\land\text{E}_2) \qquad \frac{\Gamma \vdash \varphi \qquad \Gamma \vdash \psi}{\Gamma \vdash \varphi \land \psi}\ (\land\text{I})$$

$$\frac{\Gamma \vdash \varphi}{\Gamma \vdash \varphi \lor \psi}\ (\lor\text{I}_1) \qquad \frac{\Gamma \vdash \psi}{\Gamma \vdash \varphi \lor \psi}\ (\lor\text{I}_2)$$

$$\frac{\Gamma \vdash \varphi \lor \psi \qquad \Gamma, \varphi \vdash \delta \qquad \Gamma, \psi \vdash \delta}{\Gamma \vdash \delta}\ (\lor\text{E})$$

$$\frac{\Gamma, \varphi \vdash \psi}{\Gamma \vdash \varphi \to \psi}\ (\to\text{I}) \qquad \frac{\Gamma \vdash \varphi \to \psi \qquad \Gamma \vdash \varphi}{\Gamma \vdash \psi}\ (\to\text{E})$$

TODO, rules uit video hier uitleggen/neerzetten