Calculus 2

Summary

Contents

1. Complex numbers:

1.1. Definition:

$\mathbb{C}$, a complex number is a number that extends the $\mathbb{R}$eal numbers. It is written in the form:

$$a + bi$$

It is formed like this:

$$i^2 = -1$$
$$i = \sqrt{-1}$$

1.2. Operations with complex numbers

let $\mathbb{Z} = a + bi$ and $\mathbb{W} = c + di$

1.2.1. Addition:

$$\mathbb{Z} + \mathbb{W} = (a + c) + (b + d)i$$

1.2.2. Subtract:

$$\mathbb{Z} - \mathbb{W} = (a - c) + (b - d)i$$

Complex numbers behave like vectors. $\mathbb{Z} + \mathbb{W}$ cna be written as $\mathbb{Z} + \mathbb{W} = (a + c, b + d)$ where $\mathbb{Z} = (a, b)$ and $\mathbb{W} = (c, d)$

1.2.3. Multiply:

$$\mathbb{Z} \cdot \mathbb{W} = (ac - bd) + (ad + bc)i$$

Multiply by constant

$$5 \cdot \mathbb{Z} = 5a + 5b \cdot i$$

1.2.4. Complex conjugate

Flip the sign of your complex number:

$$\bar{\mathbb{Z}} = a - bi$$

You can create a $\mathbb{R}$eal number by multiplying your $\mathbb{Z}$ with $\bar{\mathbb{Z}}$

$$\mathbb{Z} \cdot \bar{\mathbb{Z}} = a^2 + b^2$$

1.2.5. Divide:

$$\frac{\mathbb{Z}}{\mathbb{W}} = \frac{\mathbb{Z} \cdot \bar{\mathbb{W}}}{\mathbb{W} \cdot \bar{\mathbb{W}}}$$

1.2.6. Modulus of a complex number:

$$|\mathbb{Z}| := \sqrt{a^2 + b^2} = \sqrt{\mathbb{Z} \cdot \bar{\mathbb{Z}}}$$

1.3. Argument:

$$\arg(\mathbb{Z}) = \tan^{-1}\left(\frac{b}{a}\right)$$

The argument is a infinite set of angles, for example: $\left\{\frac{\pi}{2}, \frac{\pi}{2} + 2\pi, \frac{\pi}{2} + 4\pi, ..., \frac{\pi}{2} + sn\pi\right\}$

2. Formulas:

$$(a + bi)(a - bi) = a^2 + b^2$$