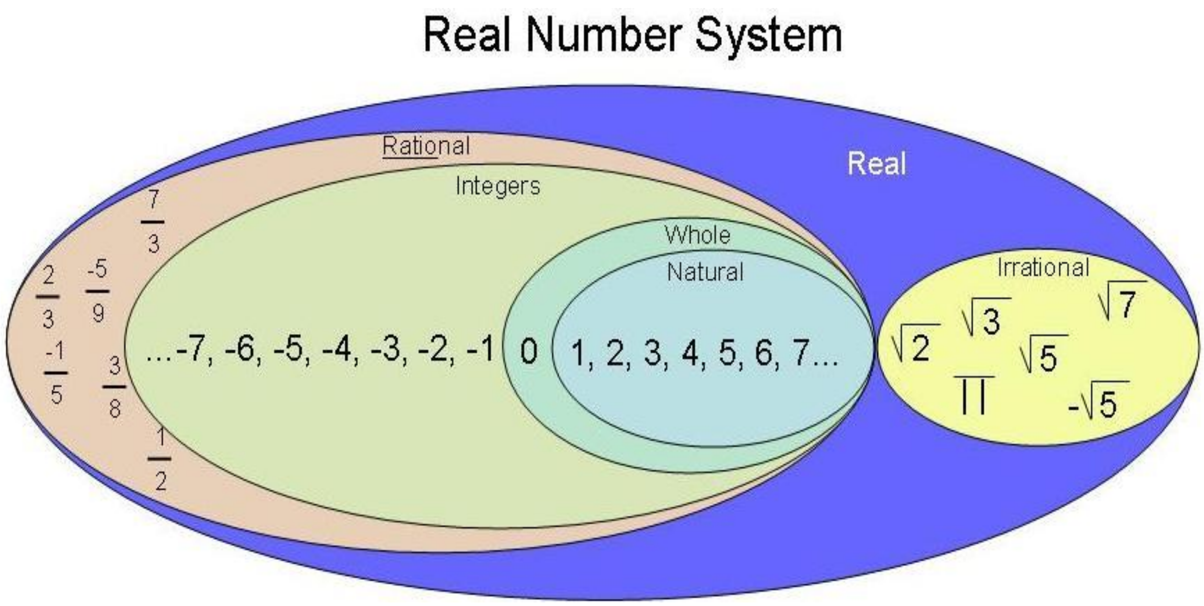
NUMBER SYSTEM

Work with the world of numbers

Numbers and its types



How to find if a number is prime or not?

N is a prime number if it is not divisible by numbers lesser than √N.

Example: 191 is a prime number since it is not divisible by 2, 3, 5, 7, 11 and 13 [numbers less than √191 (≈14)].

Note: Prime numbers will always be in the form **(6k±1)** where k= 1, 2, 3....

But not all (6k±1) will be a prime number.

Divisibility rules:

- A number is divisible by **2** if the last digit is even.
- A number is divisible by **3** if the sum of the digits is divisible by 3.
- A number is divisible by **4** if the last two digits of the number is divisible by 4.
- A number is divisible by **5** if the last digit is 5 or 0.
- A number is divisible by **6** if the number is divisible by **both** 3 and 2.
- A number is divisible by **7** if the number formed by subtracting twice the last digit with the number formed by rest of the digits is divisible by 7**.**
  Example: 343. 34-(3x2) = 28 is divisible by 7.
- A number is divisible by **8** if the last three digits of a number is divisible by 8.
- A number is divisible by **9** if the sum of the digits is divisible by 9.
- A number is divisible by **10** if the last digit of number is 0.
- A number is divisible by **11** if the difference between **sum of digits in even places** and the **sum of the digits in odd places** is 0 or divisible by 11.
  Example: 365167484
  (3+5+6+4+4) – (6+1+7+8) = 0
  ∴ 365167484 is divisible by 11.
- A number is divisible by **12** if the number is divisible by **both** 3 and 4.
- Any other numbers can be written in terms of the numbers whose divisibility is already known.
  Example: 15 = 3 x 5 ;
  18 = 2 x 9 ;
  33 = 3 x 11

Note: The numbers expressed should be co-prime numbers;

That is, **the HCF of the two numbers should be 1**

Example: 40 = 4 x 10 is wrong because HCF(4,10) is 2.

∴ 40 = 5 x 8 because HCF(5,8) is 1.

Unit Digit Concept

Unit digit is the last digit of any number.

- **1,0,5 and 6** forms a set.
- Any number ending with these digits, the unit digits remains the same. **Example:** 111^555 ends in 1(1^555)
  435^440 ends in 5(5^440)
  856^999 ends in 6(6^999)
- **4 and 9** forms a set.
- For 4, if the power is odd unit digit is 4 and if power is even unit digit is 6.
- For 9, if the power is odd unit digit is 9 and if power is even unit digit is 1.
  Example:334^440 ends in 6(4^440-even power)
  679^331 ends in 9(9^331-odd power)
- **2,3,7 and 8** forms a set.
  For 2, 2^1 ends in 2, 2^2 ends in 4, 2^3 ends in 8, 2^4 ends in 6 and again 2^5 ends in 2 and the cyclicity repeats. So**, the length of the cycle is 4.**
  This case is same for 3, 7 and 8 also.

	Power			
Base	1	2	3	4
2	2	4	8	6
3	3	9	7	1
7	7	9	3	1
8	8	4	2	6
4	4	6		
9	9	1		

Number	Cyclicity
1	1
2	4
3	4
4	2
5	1
6	1
7	4
8	4
9	2
10	1

Note: The last digit of an expression will always depend on the unit digit of the values.

Example 1: The unit digit of 123 x 456 x 789 = 3 x 6 x 9

= 18 x 9 = 8 x 9 = 2

Example 2: What is the unit digit of (123)^42?

The unit digit pattern of 3 repeats four times. So, find the remainder when **the power value is divided by 4**.

42/4= R(**2**)

2^{nd} value in 3 cycle is 9.

∴ Unit digit of (123)^42 is 9

Example 3: What is the unit digit of (337)^27?

By dividing the power 27 by 4, 27/4 = R(3)

7^27 = 3^{rd} value in 7 cycle is 3.

Hence, the unit digit is 3.

Remainder Theorems:

Type 1: Numerator in terms of powers

The remainder pattern should be found starting from the power of 1. The same procedure should be followed as done in the unit digit concept.

Example: What is the remainder when 2^202 is divided by 7?

2^1/7 = R(2)

2^2/7= R(4)

2^3/7 = R(1)

The next three remainder values will be the same because **the cycle repeats when the remainder is 1**. i.e., The remainder pattern is 2,4,1, 2,4,1, 2,4,1.....

The size of the pattern is 3.

Now divide the power by number of repeating values (3) i.e., the length of the cycle to choose the remainder.

Choose the nth value in the cycle if the remainder is n except for the last value whose remainder should be 0.

202/3 = R(**1**).

The 1^{st} value in the cycle is 2.

Note: While finding the remainder pattern if the remainder becomes 1, then the process can be stopped as it will always repeat after 1.

∴ 2^202/7 = R(2)

Type 2: Different numerator values

Replace each of the values of the numerator by its remainder when divided by the denominator and then simplify.

Example: What is the remainder when 13 x 14 x 16 is divided by 6.

13/6 = R(1) ∴ replace 13 by 1

Similarly replace 14 and 16 by 2 and 4 respectively.

∴ $(13 \times 14 \times 16)/6 = R(13)/6 + R(14)/6 + R(16)/6 = 1/6 + 2/6 + 4/6$

$= 7/6$

$= R(1)$

Note: Do not cancel any numerator value with the denominator value as the remainder will differ.

$R(6/4) \neq R(3/2)$

$6/4 = R(2)$

But $3/2 = R(1)$

The above question has multiplication values. The same method applies for addition and subtraction also.

FACTORS

- **Factor** of a number are the values that divides the number completely.

Example: Factors of 10 are 1, 2, 5 and 10.

- **Multiple** of a number is the product of that number and any other whole number.

Example: multiples of 10 are 10, 20, 30,…..

Finding Number of factors:

Example: 3600

Step 1: Prime factorize the given number

$3600 = 36 \times 100$

$= 6^2 \times 10^2$

$= 2^2 \times 3^2 \times 2^2 \times 5^2$

$= 2^4 \times 3^2 \times 5^2$

Step 2: Add 1 to the powers and multiply.

$(4+1) \times (2+1) \times (2+1)$

$= 5 \times 3 \times 3$

$= 45$

∴ Number of factors of 3600 is 45.

Finding Sum of factors:

Example: 45

Step 1: Prime factorize the given number

45 = 3^2 x 5^1

Step 2: Split each prime factor as sum of every distinct factors.

(3^0 + 3^1 + 3^2) x (5^0 + 5^1)

The following result will be the sum of the factors

= 78

Factors will occur in pairs for the numbers except perfect squares.

Example 1: A non perfect square number- 10

1 x 10 = 10 ; 2 x 5 = 10

'∴ Factors of 10 are 1, 2, 5 and 10.

Non perfect squares will have **even number of factors**

Example 2: A **perfect square number-** 16

1 x 16 = 16 ; 2 x 8 = 16 ; 4^2 = 16

∴ Factors of 16 are 1, 2, 4, 8 and 16.

Every **perfect square** will have **odd number of factors** because its square root number will pair with itself.

This has odd number of factors because 4 will pair with itself.

Example 3: A prime square number- 49

The factors of 49 are 1, 7 and 49.

Prime square number will have exactly **3 factors** (1, the number itself and the square root of that number).

If **N** is a **prime square number** then the **factors are 1, N and √N.**

<u>Finding the maximum power of a number:</u>

Example 1: Find the number of 2's in 50!

Step 1: Divide 50 by 2.

50/2 = **25** →25/2 = **12** → 12/2 = **6** → 6/2 = **3** → 3/2 = **1**

Note: Leave the remainders when dividing since the question is to find number of 2's in the value.

Step 2: Add: 25+12+6+3+1 = 47

There are 47 2's in 50!. Hence, the maximum power of 2 in 50! is 47.

Example 2: Find the number of 6's in 50!

Step 1: Prime factorize 6 as 2 and 3 since the dividend has to be divided only with a prime number.

Step 2: Divide 50 by 2.

50/2 = **25** →25/2 = **12** → 12/2 = **6** → 6/2 = **3** → 3/2 = **1 Total = 47**

Then, Divide 50 by 3.

50/3 = **16** → 16/3 = **5** → 5/3 = **1 Total = 22**

Step 3: A 2 and a 3 is required to make a 6 since the question is about 6. So, out of 47 2's and 22 3's, only only 22 6's can be formed.

Hence, the number of 6 or maximum power of 6 in 50! Is 22.

Example 4: How many zeros are there in 50!

50 have to be divided by only prime numbers. So, the only possibility to get 0 by multiplying 2 prime numbers is 5 and 2.

Number of 2's in 50! is 47 and number of 5's in 50! is 12.

Hence, the number of zeros in 50! is 12.

HCF & LCM

HCF

The greatest number that will exactly divide a, b and c is **HCF(a, b, c)**.

The greatest number that will divide a, b and c leaving remainder of x, y and z respectively is **HCF(a-x, b-y, c-z)**.

The greatest remainder which when it divides a, b and c will leave the same remainder in each case is **HCF(a-b, b-c, c-a)**.

LCM

The least number which is exactly divisible by a, b and c is **LCM(a, b, c)**.

The least number which when divided by a, b and c leaves the same reminder r in each case is **LCM(a, b, c) + r**.

The least number which when divided by a, b and c leaves the remainder x, y and z respectively is **LCM(a, b, c) – K**.

This is possible only if a-x = b-y = c-z = K.

FINDING THE H.C.F. OF LARGER NUMBERS

Step 1: Find all prime factors of both numbers.

Step 2: Write both numbers as a multiplication of prime numbers.

Step 3: Find which factors are repeating in both numbers and multiply them to get H.C.F.

FINDING THE L.C.M. OF LARGER NUMBERS

Step 1: Find all the prime factors of both numbers.

Step 2: Multiply all the prime factors of the larger number by those prime factors of the smaller number that are not already included.

To determine LCM of 14, 42, 21.

7	14,	42,	21
2	2,	6,	3
3	1,	3,	3
	1,	1,	1

$\therefore$ LCM of 14, 42, 21 = $7 \times 2 \times 3 = 42$

To determine HCF of 33, 55, 22

33) 55 (1
33
22) 33 (1
22
11) 22 (2
22
×
11) 22 (2
22
×

$\therefore$ HCF of 33, 55, 22 = 11

Hence, Required LCM = $\frac{42}{11}$

Important formulae:

$$LCM(a,b) = \frac{a \times b}{HCF(a,b)}$$

- Product of Two numbers = LCM X HCF
- HCF of fractions = $\frac{HCF\ OF\ numerators}{LCM\ OF\ denominators}$
- LCM of fractions = $\frac{LCM\ of\ numerators}{HCF\ of\ denominators}$