Matrix Theory 1st Assignment

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16) Using mathematical induction, prove that $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\ldots..\tan^{-1}\left\{\frac{1}{(n^2+n+1)}\right\}=\tan^{-1}\left\{\frac{n}{(n+2)}\right\}$ (1993-5 Marks)

17) Prove that $\sum_{r=1}^{k}(-3)^{r-1}\,{}^{3n}C_{2n-1}=0$, where $k=\frac{(3n)}{2}$ and n is an even positive integer. (1993-5 Marks)

18) If x is not an integral multiple of 2π use mathematical induction to prove that : $\cos x+\cos 2x+\ldots\ldots..+\cos nx=\cos\frac{n+1}{2}x\sin\frac{nx}{2}\operatorname{cosec}\frac{x}{2}$ (1994-4 Marks)

19) Let n be a positive integer and $\left(1+x+x^2\right)^n=a_0+a_1x+\ldots\ldots\ldots..+a_{2n}x^{2n}$. Show that $a_0^2-a_1^2+a_2^2\ldots\ldots\ldots..+a_2n^2=a_n$ (1994-5 Marks)

20) Using mathematical induction prove that for every integer $n\geq 1,\left(3^{2n}-1\right)$ is divisible by 2^{n+2} but not by 2^{n+3}. (1996-3 Marks)

21) Let $0<A_i<\pi$ for $i=1,2\ldots.,n$. Use mathematical induction to prove that $\sin A_1+\sin A_2\ldots+\sin A_n\leq n\sin\left(\frac{A_1+A_2+\ldots..+A_n}{n}\right)$ where ≥ 1 is a natural number .{You may use the fact that $p\sin x+(1-p)\sin y\leq\sin\left[px+(1-p)y\right]$, where $0\leq p\leq 1$ and $0\leq x,y\leq\pi$.} (1997-5 Marks)

22) Let p be a prime number and m a positive integer. By mathematical induction on m, or otherwise, prove that whenever r is an integer such that p does not divide r, p divides ${}^{mp}C_r$ [**Hint:** You may use the fact that $(1+x)^{(m+1)p}=(1+x)^p(1+x)^{mp}$] (1998-8 Marks)

23) Let n be any positive integer. Prove that $\sum_{k=0}^{m}\frac{\binom{2n-k}{k}}{\binom{2n-1}{n}}\cdot\frac{2n-4k+1}{2n-2k+1}2^{n-2k}=\frac{\binom{n}{m}}{\binom{2n-2m}{n-m}}2^{n-2m}$for each non-be gatuve integer $m\leq n$. (Here $\binom{p}{q}={}^pC_q$). (1999-10 Marks)

24) For any positive integer m,n(with $n\geq m$), let $\binom{n}{m}={}^nC_m$. Prove that $\binom{n}{m}+\binom{n-1}{m}+\binom{n-2}{m}+\ldots..+\binom{m}{m}=\binom{n+1}{m+2}$. Hence or otherwise, prove that

$$\binom{n}{m}+2\binom{n-1}{m}+3\binom{n-2}{m}+\ldots.+(n-m+1)\binom{m}{m}=\binom{n+2}{m+2} \tag{1}$$

. (2000-6 Marks)

25) For every positive integer n, prove that $\sqrt{(4n+1)}<\sqrt{n}+\sqrt{n+1}<\sqrt{4n+2}$. Hence or otherwise, prove that $\left[\sqrt{n}+\sqrt{(n+1)}\right]=\left[\sqrt{4n+1}\right]$, where $[x]$ denotes the greatest integer not exceeding x. (2000-6 Marks)

26) Let a,b,c be positive real numbers such that $b^2-4ac>0$ and let $\alpha_1=c$. Prove by induction that $\alpha_{n+1}=\frac{a\alpha_n^2}{\left(b^2-2a(\alpha_1+\alpha_2+\ldots\alpha_n)\right)}$ is well-defined and $\alpha_{n+1}<\frac{\alpha_n}{2}$ for all $n=1,2,\ldots$ (Here,'well-defined' means that the denominator in the expression for α_{n+1} is not zero.) (2001-5 Marks)

27) Use mathematical induction to show that $(25)^{n+1}-24n+5735$ is divisible by $(24)^2$ for all n= 1,2,..... (2002-5 Marks)

28) Prove that

$$2^k\binom{n}{0}\binom{n}{k}-2^{k-1\binom{n}{2}}\binom{n}{1}\binom{n-1}{k-1}+2^{k-2}\binom{n-2}{k-2}-\ldots..(-1)^k\binom{n}{k}\binom{n-k}{0}=\binom{n}{k} \tag{2}$$

. (2003-2 Marks)

29) A coin has probability p of showing head when tossed. It is tossed n times.Let p_n denote the probability that no two (or more) consequtive heads occur. Prove that $p_1=1,p_2=1-p^2$ and $p_n=(1-p).p_{n-1}+p(1-p)p_{n-2}$ for all $n\geq 3$. Prove by induction on n, that $p_n=A\alpha^n+B\beta^n$ for all $n\geq 1$, where α and β are the roots of quadratic equation $x^2-(1-p)x-p(1-p)=0$ and A = $\frac{p^2+\beta-1}{\alpha\beta-\alpha^2}$, B = $\frac{p^2+\alpha-1}{\alpha\beta-\beta^2}$. (2000-5 Marks)