MA-2012-14 to 26

AI24BTECH11017 - MAANYA SRI

1) Let $R = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$ and $I = \mathbb{Z} \times \mathbb{Z} \times \{0\}$. Then which of the following statements is correct?
   a) I is a maximal ideal but not a prime ideal of R.
   b) I is a prime ideal but not a maximal ideal of R.
   c) I is both maximal ideal as well as a prime ideal of R.
   d) I is neither a maximal ideal nor a prime ideal of R.
2) The function $u(r, \theta)$ satisfying the Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0, \quad e < r < e^2$$

   subject to the conditions $u(e, \theta) = 1$, $u\left(e^2, \theta\right) = 0$ is
   a) $\ln\left(\frac{e}{r}\right)$
   b) $\ln\left(\frac{e}{r^2}\right)$
   c) $\ln\left(\frac{e^2}{r}\right)$
   d) $\sum_{n=1}^{\infty}\left(\frac{r-e^2}{e-e^2}\right)\sin n\theta$
3) The functional

$$\int_0^1 \left(y'^2 + (y + 2y')\, y'' + kxyy' + y^2\right) dx$$
$$y(0) = 0,\ y(1) = 1,\ y'(0) = 2,\ y'(1) = 3$$

   is path independent if k equals
   a) 1
   b) 2
   c) 3
   d) 4
4) If a transformation $y = uv$ transforms the given differential equation

$$f(x)\, y'' - 4f'(x)\, y' + g(x)\, y = 0$$

   into the equation of the form $v'' + h(x)\, v = 0$, then u must be
   a) $\frac{1}{f^2}$
   b) xf
   c) $\frac{1}{2f}$
   d) f^2

5) The expression

$$\frac{1}{D_x^2 - D_y^2} \sin(x-y)$$

is equal to

a) $-\frac{x}{2} \cos(x-y)$
b) $-\frac{x}{2} \sin(x-y) + \cos(x-y)$
c) $\frac{x}{2} \cos(x-y) + \sin(x-y)$
d) $\frac{3x}{2} \sin(x-y)$

6) The function $\phi(x)$ satisfying the integral equation

$$\int_0^x e^{x-\xi} \phi(\xi)\, d\xi = \frac{x^2}{2}$$

is:

a) $\frac{x^2}{2}$
b) $x + \frac{x^2}{2}$
c) $x - \frac{x^2}{2}$
d) $1 + \frac{x^2}{2}$

7) Given the data:

x	1	2	3	4	5
y	-1	2	-3	4	-5

If the derivative of $y(x)$ is approximated as:

$$y'(x_k) = \frac{1}{h}\left(\Delta y_k + \frac{1}{2}\Delta^2 y_k - \frac{1}{4}\Delta^3 y_k\right),$$

then the value of $y'(2)$ is

a) 4
b) 8
c) 12
d) 16

8) If

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

then A^{50} is

a) $\begin{pmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{pmatrix}$
b) $\begin{pmatrix} 1 & 0 & 0 \\ 48 & 1 & 0 \\ 48 & 0 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{pmatrix}$

9) If $y = \sum_{m=0}^{\infty} c_m x^{r+m}$ is assumed to be a solution of the differential equation

$$x^2 y'' - xy' - 3\left(1 + x^2\right) y = 0,$$

then the values of r are

a) 1 and 3
b) -1 and 3
c) 1 and -3
d) -1 and -3

10) Let the linear transformation $T : F^2 \to F^3$ be defined by

$$T(x_1, x_2) = (x_1, x_1 + x_2, x_2).$$

Then the nullity of T is

a) 0
b) 1
c) 2
d) 3

11) The approximate eigenvalue of the matrix

$$A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$

obtained after two iterations of Power method, with the initial vector $[1\ 1\ 1]^T$ is

a) 7.768
b) 9.468
c) 10.548
d) 19.468

12) The root of the equation $xe^x = 1$ between 0 and 1, obtained by using two iterations of bisection method, is

a) 0.25
b) 0.50
c) 0.75
d) 0.65

Q. 13 to Q. 55 carry two marks each.

13) Let

$$\int_C \left[\frac{1}{(z-2)^2} + \frac{(a-2)^2}{z} + 4\right] dz = 4\pi,$$

where the close curve C is the triangle having vertices at

$$i, \left(\frac{-1-i}{\sqrt{2}}\right), \quad \text{and} \quad \left(\frac{1-i}{\sqrt{2}}\right)$$

the integral being taken in the anti-clockwise direction. Then one value of a is

a) $1+i$
b) $2+i$
c) $3+i$
d) $4+i$