Matrix Theory 1st Assignment

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16) Using mathematical induction, prove that $\tan^{-1}(1/3) + \tan^{-1}(1/7) + \ldots.. \tan^{-1}\{1/\left(n^2+n+1\right)\} = \tan^{-1}\{(n/n+2)\}$ $(1993 - 5Marks)$
17) Prove that $\sum_{r=1}^{k}(-3)^{r-1}\ {}^{3n}C_{2n-1} = 0$, where $k = (3n)/2$ and n is an even positive integer. $(1993 - 5Marks)$
18) If x is not an integral multiple of 2π use mathematical induction to prove that : $\cos x + \cos 2x + \ldots\ldots.. + \cos nx = \cos\frac{n+1}{2}\ x \sin\frac{nx}{2}$ $\operatorname{cosec}\frac{x}{2}$ $(1994 - 4Marks)$
19) Let n be a positive integer and $\left(1+x+x^2\right)^n = a_0 + a_1x + \ldots\ldots\ldots.. + a_{2n}x^{2n}$. Show that $a_0^2 - a_1^2 + a_2^2\ldots\ldots\ldots.. + a_2n^2 = a_n$ $(1994 - 5Marks)$
20) Using mathematical induction prove that for every integer $n \geq 1$, $\left(3^{2n}-1\right)$ is divisible by 2^{n+2} but not by 2^{n+3}. $(1996 - 3Marks)$
21) Let $0 < A_i < \pi$ for $i = 1, 2\ldots., n$. Use mathematical induction to prove that $\sin A_1 + \sin A_2\ldots + \sin A_n \leq n \sin\left(\frac{A_1+A_2+\ldots..+A_n}{n}\right)$ where ≥ 1 is a natural number .{You may use the fact that $p \sin x + (1-p)\sin y \leq \sin[px + (1-p)y]$, where $0 \leq p \leq 1$ and $0 \leq x, y \leq \pi$.} $(1997 - 5Marks)$
22) Let p be a prime number and m a positive integer. By mathematical induction on m, or otherwise, prove that whenever r is an integer such that p does not divide r, p divides ${}^{mp}C_r$ [**Hint:** You may use the fact that $(1+x)^{(m+1)p} = (1+x)^p(1+x)^{mp}$] $(1998 - 8Marks)$
23) Let n be any positive integer. Prove that $\sum_{k=0}^{m}\frac{\binom{2n-k}{k}}{\binom{2n-1}{n}}.\frac{2n-4k+1}{2n-2k+1}2^{n-2k} = \frac{\binom{n}{m}}{\binom{2n-2m}{n-m}}2^{n-2m}$for each non-be gatuve integer $m \leq n.\left(Here\binom{p}{q} = {}^{p}C_q\right)$. $(1999 - 10Marks)$
24) For any positive integer $m, n\,(withn \geq m)$, let $\binom{n}{m} = {}^{n}C_m$. Prove that $\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \ldots.. + \binom{m}{m} = \binom{n+1}{m+2}$. Hence or otherwise, prove that $\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + \ldots. + (n-m+1)\binom{m}{m} = \binom{n+2}{m+2}$. $(2000 - 6Marks)$
25) For every positive integer n, prove that $\sqrt{(4n+1)} < \sqrt{n} + \sqrt{n+1} < \sqrt{4n+2}$. Hence or otherwise, prove that $\left[\sqrt{n} + \sqrt{(n+1)}\right] = \left[\sqrt{4n+1}\right]$, where $[x]$ denotes the greatest integer not exceeding x. $(2000 - 6Marks)$
26) Let a, b, c be positive real numbers such that $b^2 - 4ac > 0$ and let $\alpha_1 = c$. Prove by induction that $\alpha_{n+1} = \frac{a\alpha_n^2}{\left(b^2-2a(\alpha_1+\alpha_2+\ldots\alpha_n)\right)}$ is well-defined and $\alpha_{n+1} < \frac{\alpha_n}{2}$ for all $n = 1,2,\ldots$ (Here,'well-defined' means that the denominator in the expression for α_{n+1} is not zero.) $(2001 - 5Marks)$
27) Use mathematical induction to show that $(25)^{n+1}$ - 24 n + 5735 is divisible by $(24)^2$ for all n= 1,2,..... $(2002 - 5Marks)$
28) Prove that $2^k\binom{n}{0}\binom{n}{k} - 2^{k-1}\binom{n}{2}\binom{n}{1}\binom{n-1}{k-1} + 2^{k-2}\binom{n-2}{k-2} - \ldots..(-1)^k\binom{n}{k}\binom{n-k}{0} = \binom{n}{k}$. $(2003 - 2Marks)$
29) A coin has probability p of showing head when tossed. It is tossed n times.Let p_n denote the probability that no two (or more) consequtive heads occur. Prove that $p_1 = 1, p_2 = 1 - p^2$ and $p_n = (1-p).p_{n-1} + p(1-p)p_{n-2}$ for all n ≥ 3. Prove by induction on n, that $p_n = A\alpha^n + B\beta^n$ for all $n \geq 1$, where α and β are the roots of quadratic equation x^2 - $(1-p)x - p(1-p) = 0$ and A = $\frac{p^2+\beta-1}{\alpha\beta-\alpha^2}$, B = $\frac{p^2+\alpha-1}{\alpha\beta-\beta^2}$. $(2000 - 5Marks)$