1-1.3-2

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Question:
The coordinates of the three consecutive vertices of a parallelogram $ABCD$ are A (1,3),B(-1,2), and C(2,5).Find the coordinates of the fourth vertex D. (10,2021)

Sol:

Label	Coordinate
A	(1,3)
B	(-1,2)
C	(2,5)
D	(x,y)

TABLE 0: Variables Used

Given $A=\begin{pmatrix}1\\3\end{pmatrix}$, $B=\begin{pmatrix}-1\\2\end{pmatrix}$, and $C=\begin{pmatrix}2\\5\end{pmatrix}$

Let D be $\begin{pmatrix}x\\y\end{pmatrix}$

In a parallelogram , midpoints of diagonals are same .
Therefore midpoint of AC and BD is the same.
Midpoint of

$$AC = \begin{pmatrix}\frac{1+2}{2}\\\frac{3+5}{2}\end{pmatrix} = \begin{pmatrix}\frac{3}{2}\\4\end{pmatrix} \tag{0.1}$$

Midpoint of

$$BD = \begin{pmatrix}\frac{-1+x}{2}\\\frac{2+y}{2}\end{pmatrix} \tag{0.2}$$

Now equating both the midpoints we get

$$\begin{pmatrix}\frac{3}{2}\\4\end{pmatrix} = \begin{pmatrix}\frac{-1+x}{2}\\\frac{2+y}{2}\end{pmatrix} \tag{0.3}$$

Solving for x and y,
we get x=4 and y=6
Hence $D = \begin{pmatrix}4\\6\end{pmatrix}$