

1) Let  $R = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$  and  $I = \mathbb{Z} \times \mathbb{Z} \times \{0\}$ . Then which of the following statements is correct?

- a)  $I$  is a maximal ideal but not a prime ideal of  $R$ .
- b)  $I$  is a prime ideal but not a maximal ideal of  $R$ .
- c)  $I$  is both maximal ideal as well as a prime ideal of  $R$ .
- d)  $I$  is neither a maximal ideal nor a prime ideal of  $R$ .

2) The function  $u(r, \theta)$  satisfying the Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0, \quad e < r < e^2$$

subject to the conditions  $u(e, \theta) = 1$ ,  $u(e^2, \theta) = 0$  is

- a)  $\ln\left(\frac{e}{r}\right)$
- b)  $\ln\left(\frac{e}{r^2}\right)$
- c)  $\ln\left(\frac{e^2}{r}\right)$
- d)  $\sum_{n=1}^{\infty} \left(\frac{r-e^2}{e-e^2}\right) \sin n\theta$

3) The functional

$$\int_0^1 (y'^2 + (y + 2y')y'' + kxyy' + y^2) dx$$

$$y(0) = 0, y(1) = 1, y'(0) = 2, y'(1) = 3$$

is path independent if  $k$  equals

- a) 1
- b) 2
- c) 3
- d) 4

4) If a transformation  $y = uv$  transforms the given differential equation

$$f(x)y'' - 4f'(x)y' + g(x)y = 0$$

into the equation of the form  $v'' + h(x)v = 0$ , then  $u$  must be

- a)  $\frac{1}{f^2}$
- b)  $xf$
- c)  $\frac{1}{2f}$
- d)  $f^2$

5) The expression

$$\frac{1}{D_x^2 - D_y^2} \sin(x - y)$$

is equal to

- a)  $-\frac{x}{2} \cos(x - y)$
- b)  $-\frac{x}{2} \sin(x - y) + \cos(x - y)$
- c)  $\frac{x}{2} \cos(x - y) + \sin(x - y)$
- d)  $\frac{3x}{2} \sin(x - y)$

6) The function  $\phi(x)$  satisfying the integral equation

$$\int_0^x e^{x-\xi} \phi(\xi) d\xi = \frac{x^2}{2}$$

is:

- a)  $\frac{x^2}{2}$
- b)  $x + \frac{x^2}{2}$
- c)  $x - \frac{x^2}{2}$
- d)  $1 + \frac{x^2}{2}$

7) Given the data:

$x$	1	2	3	4	5
$y$	-1	2	-3	4	-5

If the derivative of  $y(x)$  is approximated as:

$$y'(x_k) = \frac{1}{h} \left( \Delta y_k + \frac{1}{2} \Delta^2 y_k - \frac{1}{4} \Delta^3 y_k \right),$$

then the value of  $y'(2)$  is

- a) 4
- b) 8
- c) 12
- d) 16

8) If

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

,then  $A^{50}$  is

- a)  $\begin{pmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{pmatrix}$
- b)  $\begin{pmatrix} 1 & 0 & 0 \\ 48 & 1 & 0 \\ 48 & 0 & 1 \end{pmatrix}$

- c)  $\begin{pmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{pmatrix}$
- d)  $\begin{pmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{pmatrix}$

9) If  $y = \sum_{m=0}^{\infty} c_m x^{r+m}$  is assumed to be a solution of the differential equation

$$x^2 y'' - xy' - 3(1 + x^2)y = 0,$$

then the values of  $r$  are

- a) 1 and 3  
b) -1 and 3  
c) 1 and -3  
d) -1 and -3

10) Let the linear transformation  $T : F^2 \rightarrow F^3$  be defined by

$$T(x_1, x_2) = (x_1, x_1 + x_2, x_2).$$

Then the nullity of  $T$  is

- a) 0  
b) 1  
c) 2  
d) 3

11) The approximate eigenvalue of the matrix

$$A = \begin{pmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{pmatrix}$$

obtained after two iterations of Power method, with the initial vector  $[1 \ 1 \ 1]^T$  is

- a) 7.768  
b) 9.468  
c) 10.548  
d) 19.468

12) The root of the equation  $xe^x = 1$  between 0 and 1, obtained by using two iterations of bisection method, is

- a) 0.25  
b) 0.50  
c) 0.75  
d) 0.65

**Q. 13 to Q. 55 carry two marks each.**

13) Let

$$\int_C \left[ \frac{1}{(z-2)^2} + \frac{(a-2)^2}{z} + 4 \right] dz = 4\pi,$$

where the close curve  $C$  is the triangle having vertices at

$$i, \left( \frac{-1-i}{\sqrt{2}} \right), \quad \text{and} \quad \left( \frac{1-i}{\sqrt{2}} \right)$$

the integral being taken in the anti-clockwise direction. Then one value of  $a$  is

- a)  $1 + i$
- b)  $2 + i$
- c)  $3 + i$
- d)  $4 + i$