

1-1.3-2

AI24BTECH11017-Maanya Sri

Question:

The coordinates of the three consecutive vertices of a parallelogram $ABCD$ are $A(1,3)$, $B(-1,2)$, and $C(2,5)$. Find the coordinates of the fourth vertex D . (10,2021)

Sol:

Given $A = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $B = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, and $C = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

Let D be $\begin{pmatrix} x \\ y \end{pmatrix}$

In a parallelogram, midpoints of diagonals are same.

Therefore midpoint of AC and BD is the same.

$$\text{Midpoint of } AC = \begin{pmatrix} \frac{1+2}{2} \\ \frac{3+5}{2} \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ 4 \end{pmatrix}$$

$$\text{Midpoint of } BD = \begin{pmatrix} \frac{-1+x}{2} \\ \frac{2+y}{2} \end{pmatrix}$$

Now equating both the midpoints we get

$$\begin{pmatrix} \frac{3}{2} \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{-1+x}{2} \\ \frac{2+y}{2} \end{pmatrix}$$

Solving for x and y ,

we get $x=4$ and $y=6$

$$\text{Hence } D = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$