ST-2021-14 to 26

AI24BTECH11017 - MAANYA SRI

1) Let A and B be two events such that $P(B) = \frac{3}{4}$ and $P(A \cup B^c) = \frac{1}{2}$. If A and B are independent, then $P(A)$ equals ______ (round off to 2 decimal places).
2) A fair die is rolled twice independently. Let X and Y denote the outcomes of the first and second roll, respectively. Then $E\left(X + Y \middle| (X - Y)^2 = 1\right)$ equals ______.
3) Let X be a random variable having distribution function

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{a}{2}, & 1 \leq x < 2, \\ \frac{c}{6}, & 2 \leq x < 3, \\ 1, & x \geq 3, \end{cases}$$

where a and c are appropriate constants. Let $A_n = \left[1 + \frac{1}{n}, 3 - \frac{1}{n}\right], n \geq 1$, and $A = \bigcup_{i=1}^{\infty} A_i$. If $P(X \leq 1) = \frac{1}{2}$ and $E(X) = \frac{5}{3}$, then $P(X \in A)$ equals ______ (round off to 2 decimal places).
4) If the marginal probability density function of the k^{th} order statistic of a random sample of size 8 from a uniform distribution on $[0, 2]$ is

$$f(x) = \begin{cases} \frac{7}{32} x^6 (2 - x), & 0 < x < 2, \\ 0, & \text{otherwise}, \end{cases}$$

then k equals ______.
5) For $\alpha > 0$, let $\left\{X_n^{(\alpha)}\right\}_{n \geq 1}$ be a sequence of independent random variables such that

$$P\left(X_n^{(\alpha)} = 1\right) = \frac{1}{n^{2\alpha}} = \quad 1 - P\left(X_n^{(\alpha)} = 0\right).$$

Let $S = \left\{\alpha > 0 : X_n^{(\alpha)} \text{ converges to 0 almost surely as } n \to \infty\right\}$. Then the infimum of S equals ______ (round off to 2 decimal places).
6) Let $\{X_n\}_{n \geq 1}$ be a sequence of independent and identically distributed random variables each having uniform distribution on $[0, 2]$. For $n \geq 1$, let

$$Z_n = -\log_e \left(\prod_{i=1}^{n} (2 - X_i) \right)^{\frac{1}{n}}.$$

Then, as $n \to \infty$, the sequence $\{Z_n\}_{n \geq 1}$ converges almost surely to ______ (round off to 2 decimal places).
7) Let $\{X_n\}_{n \geq 0}$ be a time-homogeneous discrete time Markov chain with state space

$\{0, 1\}$ and transition probability matrix

$$\begin{pmatrix} 0.25 & 0.75 \\ \\ 0.75 & 0.25 \end{pmatrix}$$

If $P(X_0 = 0) = P(X_0 = 1) = 0.5$, then

$$\sum_{k=1}^{100} E\left[(X_{2k})^{2k}\right]$$

equals ______

8) Let $\{0, 2\}$ be a realization of a random sample of size 2 from a binomial distribution with parameters 2 and p, where $p \in (0, 1)$. To test $H_0 : p = \frac{1}{2}$ against $H_1 : p \neq \frac{1}{2}$, the observed value of the likelihood ratio test statistic equals ______ (round off to 2 decimal places).

9) Let X be a random variable having the probability density function

$$f(x) = \begin{cases} \frac{3}{13}(1-x)(9-x), & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then $\frac{4}{3}E\left[X\left(X^2 - 15X + 27\right)\right]$ equals ______ (round off to 2 decimal places).

10) Let (Y, X_1, X_2) be a random vector with mean vector $\begin{pmatrix} 5 \\ 2 \\ 0 \end{pmatrix}$ and variance-covariance matrix

$$\begin{pmatrix} 10 & 0.5 & -0.5 \\ \\ 0.5 & 7 & 1.5 \\ \\ -0.5 & 1.5 & 2 \end{pmatrix}.$$

Then the value of the multiple correlation coefficient between Y and its best linear predictor on X_1 and X_2 equals ______ (round off to 2 decimal places).

11) Let X_1, X_2 and X_3 be a random sample from a bivariate normal distribution with unknown mean vector μ and unknown variance-covariance matrix Σ, which is a positive definite matrix. The p-value corresponding to the likelihood ratio test for testing $H_0 : \mu = 0$ against $H_1 : \mu \neq 0$ based on the realization $\left\{\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -4 \\ -2 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \end{pmatrix}\right\}$ of the random sample equals ______ (round off to 2 decimal places).

12) Let $Y_i = \alpha + \beta x_i + \epsilon_i$, $i = 1, 2, 3$, where x_i's are fixed covariates, α and β are unknown parameters and ϵ_i's are independent and identically distributed random variables with mean zero and finite variance. Let $\hat{\alpha}$ and $\hat{\beta}$ be the ordinary least squares estimators of α and β, respectively. Given the following observations the value of $\hat{\alpha} + \hat{\beta}$ equals

x_i	8.26	26.86	54.02
y_i	3.29	21.53	48.69

______ (round off to 2 decimal places).

Q. 13- Q. 43 Multiple Choice Question (MCQ), carry TWO mark each(for each wrong answer : -2/3).

13) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^3 \sin x, & x = 0 \text{ or } x \text{ is irrational}, \\ \frac{1}{q^3}, & x = \frac{p}{q}, p \in \mathbb{Z} \setminus \{0\}, q \in \mathbb{N} \text{ and } \gcd(p, q) = 1, \end{cases}$$

where $\mathbb{R}$ denotes the set of all real numbers, $\mathbb{Z}$ denotes the set of all integers, $\mathbb{N}$ denotes the set of all positive integers and $\gcd(p, q)$ denotes the greatest common divisor of p and q. Then which one of the following statements is true?

a) f is not continuous at 0
b) f is not differentiable at 0
c) f is differentiable at 0 and the derivative of f at 0 equals 0
d) f is differentiable at 0 and the derivative of f at zero equals 1