

Matrix Theory 1st Assignment

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- 16) Using mathematical induction, prove that $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \dots + \tan^{-1}\left\{\frac{1}{(n^2+n+1)}\right\} = \tan^{-1}\left\{\frac{n}{(n+2)}\right\}$ (1993-5 Marks)
- 17) Prove that $\sum_{r=1}^k (-3)^{r-1} {}^{3n}C_{2n-1} = 0$, where $k = \frac{(3n)}{2}$ and n is an even positive integer. (1993-5 Marks)
- 18) If x is not an integral multiple of 2π use mathematical induction to prove that : $\cos x + \cos 2x + \dots + \cos nx = \cos \frac{n+1}{2}x \sin \frac{nx}{2} \operatorname{cosec} \frac{x}{2}$ (1994-4 Marks)
- 19) Let n be a positive integer and $(1+x+x^2)^n = a_0 + a_1x + \dots + a_{2n}x^{2n}$. Show that $a_0^2 - a_1^2 + a_2^2 - \dots + a_{2n}^2 = a_n$ (1994-5 Marks)
- 20) Using mathematical induction prove that for every integer $n \geq 1$, $(3^{2n} - 1)$ is divisible by 2^{n+2} but not by 2^{n+3} . (1996-3 Marks)
- 21) Let $0 < A_i < \pi$ for $i = 1, 2, \dots, n$. Use mathematical induction to prove that $\sin A_1 + \sin A_2 + \dots + \sin A_n \leq n \sin\left(\frac{A_1 + A_2 + \dots + A_n}{n}\right)$ where ≥ 1 is a natural number. {You may use the fact that $p \sin x + (1-p) \sin y \leq \sin [px + (1-p)y]$, where $0 \leq p \leq 1$ and $0 \leq x, y \leq \pi$.} (1997-5 Marks)
- 22) Let p be a prime number and m a positive integer. By mathematical induction on m , or otherwise, prove that whenever r is an integer such that p does not divide r , p divides ${}^m p C_r$ [Hint: You may use the fact that $(1+x)^{(m+1)p} = (1+x)^p (1+x)^{mp}$] (1998-8 Marks)
- 23) Let n be any positive integer. Prove that $\sum_{k=0}^m \frac{\binom{2n-k}{k}}{\binom{2n-1}{n}} \cdot \frac{2n-4k+1}{2n-2k+1} 2^{n-2k} = \frac{\binom{n}{m}}{\binom{2n-2m}{n-m}} 2^{n-2m}$ for each non-negative integer $m \leq n$. (Here $\binom{p}{q} = {}^p C_q$). (1999-10 Marks)
- 24) For any positive integer m, n (with $n \geq m$), let $\binom{n}{m} = {}^n C_m$. Prove that $\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \dots + \binom{m}{m} = \binom{n+1}{m+2}$. Hence or otherwise, prove that

$$\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + \dots + (n-m+1)\binom{m}{m} = \binom{n+2}{m+2} \quad (1)$$

(2000-6 Marks)

- 25) For every positive integer n , prove that $\sqrt{(4n+1)} < \sqrt{n} + \sqrt{n+1} < \sqrt{4n+2}$. Hence or otherwise, prove that $\left[\sqrt{n} + \sqrt{(n+1)} \right] = \left[\sqrt{4n+1} \right]$, where $[x]$ denotes the greatest integer not exceeding x . (2000-6 Marks)
- 26) Let a, b, c be positive real numbers such that $b^2 - 4ac > 0$ and let $\alpha_1 = c$. Prove by induction that $\alpha_{n+1} = \frac{aa_n^2}{(b^2 - 2a(\alpha_1 + \alpha_2 + \dots + \alpha_n))}$ is well-defined and $\alpha_{n+1} < \frac{\alpha_n}{2}$ for all $n = 1, 2, \dots$ (Here, 'well-defined' means that the denominator in the expression for α_{n+1} is not zero.) (2001-5 Marks)
- 27) Use mathematical induction to show that $(25)^{n+1} - 24n + 5735$ is divisible by $(24)^2$ for all $n = 1, 2, \dots$ (2002-5 Marks)
- 28) Prove that

$$2^k \binom{n}{0} \binom{n}{k} - 2^{k-1} \binom{n}{1} \binom{n-1}{k-1} + 2^{k-2} \binom{n-2}{k-2} - \dots + (-1)^k \binom{n}{k} \binom{n-k}{0} = \binom{n}{k} \quad (2)$$

(2003-2 Marks)

- 29) A coin has probability p of showing head when tossed. It is tossed n times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that $p_1 = 1, p_2 = 1 - p^2$ and $p_n = (1-p) \cdot p_{n-1} + p(1-p)p_{n-2}$ for all $n \geq 3$. Prove by induction on n , that $p_n = A\alpha^n + B\beta^n$ for all $n \geq 1$, where α and β are the roots of quadratic equation $x^2 - (1-p)x - p(1-p) = 0$ and $A = \frac{p^2 + \beta - 1}{\alpha\beta - \alpha^2}$, $B = \frac{p^2 + \alpha - 1}{\alpha\beta - \beta^2}$. (2000-5 Marks)