MA-2012-14 to 26

AI24BTECH11017 - Maanya

14) Let $R = \mathbb{Z} \times \mathbb{Z}$ and $I = \mathbb{Z} \times \mathbb{Z} \setminus \{0\}$. Then which of the following statements is correct?
   a) I is a maximal ideal but not a prime ideal of R.
   b) I is a prime ideal but not a maximal ideal of R.
   c) I is both maximal ideal as well as a prime ideal of R.
   d) I is neither a maximal ideal nor a prime ideal of R.

15) The function $u(r, \theta)$ satisfying the Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0, \quad e < r < e^2$$

   subject to the conditions $u(e, \theta) = 1$, $u(e^2, \theta) = 0$ is
   a) $\ln(e/r)$
   b) $\ln(e/r^2)$
   c) $\ln(e^2/r)$
   d) $\sum_{n=1}^{\infty} \left(\frac{r-e^2}{e-e^2}\right) \sin n\theta$

16) The functional

$$\int_0^1 \left(y^2 + (y' + 2y')^2 + kyy' + y^2\right) dx, \quad y(0) = 0,\ y(1) = 1,\ y'(0) = 2,\ y'(1) = 3$$

   is path independent if k equals
   a) 1
   b) 2
   c) 3
   d) 4

17) If a transformation $y = uv$ transforms the given differential equation

$$f(x)y'' - 4f'(x)y' + g(x)y = 0$$

   into the equation of the form $v'' + h(x)v = 0$, then u must be
   a) $1/f^2$
   b) xf
   c) $1/2f$
   d) f^2

18) The expression

$$\frac{1}{D_x^2 - D_y^2} \sin(x - y)$$

   is equal to
   a) $-\frac{x}{2}\cos(x - y)$
   b) $-\frac{x}{2}\sin(x - y) + \cos(x - y)$
   c) $\frac{x}{2}\cos(x - y) + \sin(x - y)$
   d) $\frac{3x}{2}\sin(x - y)$

19) The function $\phi(x)$ satisfying the integral equation

$$\int_0^x e^{-t^2}\phi(\xi)\,d\xi = \frac{x^2}{2}$$

is

a) $\frac{x^2}{2}$
b) $x + \frac{x^2}{2}$
c) $x - \frac{x^2}{2}$
d) $1 + \frac{x^2}{2}$

20) Given the data:

x	1	2	3	4	5
y	-1	2	-3	4	-5

If the derivative of $y(x)$ is approximated as:

$$y'(x_i) = \frac{1}{h}\left(\Delta y_i + \frac{1}{2}\Delta^2 y_i - \frac{1}{4}\Delta^3 y_i\right),$$

then the value of $y'(2)$ is

a) 4
b) 8
c) 12
d) 16

21) If

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{pmatrix},$$

then A^{50} is

a) $\begin{pmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{pmatrix}$
b) $\begin{pmatrix} 1 & 0 & 0 \\ 48 & 0 & 0 \\ 48 & 0 & 1 \end{pmatrix}$
c) $\begin{pmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{pmatrix}$
d) $\begin{pmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{pmatrix}$

22) If

$$y = \sum_{n=0}^{\infty} c_n x^n$$

is assumed to be a solution of the differential equation

$$x^2y'' - xy' - 3(1 + x^2)y = 0,$$

then the values of r are

a) 1 and 3

b) -1 and 3
c) 1 and -3
d) -1 and -3

23) Let the linear transformation $T : \mathbb{F}^2 \to \mathbb{F}^3$ be defined by

$$T(x_1, x_2) = (x_1, x_1 + x_2, x_2).$$

Then the nullity of T is:

a) 0
b) 1
c) 2
d) 3

24) The approximate eigenvalue of the matrix

$$A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$

obtained after two iterations of the Power method, with the initial vector $[1\ 1\ 1]^T$, is:

a) 7.768
b) 9.468
c) 10.548
d) 19.468

25) The root of the equation $xe^x = 1$ between 0 and 1, obtained by using two iterations of the bisection method, is:

a) 0.25
b) 0.50
c) 0.75
d) 0.65

Q.26 to Q.55 carry two marks each.

26) Let

$$\oint_C \left(\frac{1}{(z-2)^2} + \frac{(a-2)^2}{z} + 4 \right) dz = 4\pi i,$$

where the closed curve C is the triangle having vertices at:

$$i, \quad \frac{1}{\sqrt{2}}, \quad \text{and} \quad \frac{-1}{\sqrt{2}}\, i,$$

the integral being taken in the anti-clockwise direction. Then one value of a is:

a) $1 + i$
b) $2 + i$
c) $3 + i$
d) $4 + i$