Matrix Theory 1st Assignment

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16) Using mathematical induction, prove that $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + ... \tan^{-1}\left\{\frac{1}{(n^2+n+1)}\right\} = \tan^{-1}\left\{\frac{n}{(n+2)}\right\}$ (1993-5 Marks)

17) Prove that $\sum\limits_{r=1}^{k} (-3)^{r-1}\, {}^{3n}C_{2n-1} = 0$, where $k = \frac{(3n)}{2}$ and n is an even positive integer. (1993-5 Marks)

18) If x is not an integral multiple of 2π use mathematical induction to prove that : $\cos x + \cos 2x + ... + \cos nx = \cos \frac{n+1}{2} x \sin \frac{nx}{2} \operatorname{cosec} \frac{x}{2}$ (1994-4 Marks)

19) Let n be a positive integer and $\left(1 + x + x^2\right)^n = a_0 + a_1 x + ... + a_{2n} x^{2n}$. Show that $a_0^2 - a_1^2 + a_2^2 .... + a_2 n^2 = a_n$ (1994-5 Marks)

20) Using mathematical induction prove that for every integer $n \geq 1, \left(3^{2n} - 1\right)$ is divisible by 2^{n+2} but not by 2^{n+3}. (1996-3 Marks)

21) Let $0 < A_i < \pi$ for $i = 1, 2...., n$. Use mathematical induction to prove that $\sin A_1 + \sin A_2 ... + \sin A_n \leq n \sin\left(\frac{A_1 + A_2 + ... + A_n}{n}\right)$ where ≥ 1 is a natural number .{You may use the fact that $p \sin x + (1-p) \sin y \leq \sin\left[px + (1-p) y\right]$, where $0 \leq p \leq 1$ and $0 \leq x, y \leq \pi$.} (1997-5 Marks)

22) Let p be a prime number and m a positive integer. By mathematical induction on m, or otherwise, prove that whenever r is an integer such that p does not divide r, p divides ${}^{mp}C_r$ [**Hint:** You may use the fact that $(1 + x)^{(m+1)p} = (1 + x)^p (1 + x)^{mp}$] (1998-8 Marks)

23) Let n be any positive integer. Prove that $\sum\limits_{k=0}^{m} \frac{\binom{2n-k}{k}}{\binom{2n-1}{n}} \cdot \frac{2n-4k+1}{2n-2k+1} 2^{n-2k} = \frac{\binom{n}{m}}{\binom{2n-2m}{n-m}} 2^{n-2m}$for each non-be gatuve integer $m \leq n$. (Here $\binom{p}{q} = {}^{p}C_q$). (1999-10 Marks)

24) For any positive integer m, n(with $n \geq m$), let $\binom{n}{m} = {}^{n}C_m$. Prove that $\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + ... + \binom{m}{m} = \binom{n+1}{m+2}$. Hence or otherwise, prove that

$$\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + ... + (n-m+1)\binom{m}{m} = \binom{n+2}{m+2} \tag{1}$$

. (2000-6 Marks)

25) For every positive integer n, prove that $\sqrt{(4n+1)} < \sqrt{n} + \sqrt{n+1} < \sqrt{4n+2}$. Hence or otherwise, prove that $\left[\sqrt{n} + \sqrt{(n+1)}\right] = \left[\sqrt{4n+1}\right]$, where $[x]$ denotes the greatest integer not exceeding x. (2000-6 Marks)

26) Let a, b, c be positive real numbers such that $b^2 - 4ac > 0$ and let $\alpha_1 = c$. Prove by induction that $\alpha_{n+1} = \frac{a\alpha_n^2}{\left(b^2 - 2a(\alpha_1 + \alpha_2 + ... \alpha_n)\right)}$ is well-defined and $\alpha_{n+1} < \frac{\alpha_n}{2}$ for all n = 1,2,... (Here,'well-defined' means that the denominator in the expression for α_{n+1} is not zero.) (2001-5 Marks)

27) Use mathematical induction to show that $(25)^{n+1} - 24n + 5735$ is divisible by $(24)^2$ for all n= 1,2,..... (2002-5 Marks)

28) Prove that

$$2^k \binom{n}{0}\binom{n}{k} - 2^{k-1\binom{n}{2}}\binom{n}{1}\binom{n-1}{k-1} + 2^{k-2}\binom{n-2}{k-2} - ..... (-1)^k \binom{n}{k}\binom{n-k}{0} = \binom{n}{k} \tag{2}$$

. (2003-2 Marks)

29) A coin has probability p of showing head when tossed. It is tossed n times.Let p_n denote the probability that no two (or more) consequtive heads occur. Prove that $p_1 = 1, p_2 = 1 - p^2$ and $p_n = (1-p).p_{n-1} + p(1-p)p_{n-2}$ for all $n \geq 3$. Prove by induction on n, that $p_n = A\alpha^n + B\beta^n$ for all $n \geq 1$, where α and β are the roots of quadratic equation $x^2 - (1-p)x - p(1-p) = 0$ and A = $\frac{p^2+\beta-1}{\alpha\beta-\alpha^2}$, B = $\frac{p^2+\alpha-1}{\alpha\beta-\beta^2}$. (2000-5 Marks)