

MA-2012-14 to 26

AI24BTECH11017 - Maanya sri

1) Let $R = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$ and $I = \mathbb{Z} \times \mathbb{Z} \times \{0\}$. Then which of the following statements is correct?

- a) I is a maximal ideal but not a prime ideal of R .
- b) I is a prime ideal but not a maximal ideal of R .
- c) I is both maximal ideal as well as a prime ideal of R .
- d) I is neither a maximal ideal nor a prime ideal of R .

2) The function $u(r, \theta)$ satisfying the Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0, \quad e < r < e^2 \quad (1)$$

subject to the conditions $u(e, \theta) = 1, u(e^2, \theta) = 0$ is

- a) $\ln\left(\frac{e}{r}\right)$
- b) $\ln\left(\frac{e}{r^2}\right)$
- c) $\ln\left(\frac{e^2}{r}\right)$
- d) $\sum_{n=1}^{\infty} \left(\frac{r-e^2}{e-e^2}\right) \sin n\theta$

3) The functional

$$\int_0^1 (y'^2 + (y + 2y')y'' + kxyy' + y^2) dx \quad (2)$$

$$y(0) = 0, y(1) = 1, y'(0) = 2, y'(1) = 3 \quad (3)$$

is path independent if k equals

- a) 1
- b) 2
- c) 3
- d) 4

4) If a transformation $y = uv$ transforms the given differential equation

$$f(x)y'' - 4f'(x)y' + g(x)y = 0 \quad (4)$$

into the equation of the form $v'' + h(x)v = 0$, then u must be

- a) $\frac{1}{f^2}$
- b) xf

- c) $\frac{1}{2f}$
- d) f^2

5) The expression

$$\frac{1}{D_x^2 - D_y^2} \sin(x - y) \quad (5)$$

is equal to

- a) $-\frac{x}{2} \cos(x - y)$
- b) $-\frac{x}{2} \sin(x - y) + \cos(x - y)$
- c) $\frac{x}{2} \cos(x - y) + \sin(x - y)$
- d) $\frac{3x}{2} \sin(x - y)$

6) The function $\phi(x)$ satisfying the integral equation

$$\int_0^x e^{x-\xi} \phi(\xi) d\xi = \frac{x^2}{2} \quad (6)$$

is:

- a) $\frac{x^2}{2}$
- b) $x + \frac{x^2}{2}$
- c) $x - \frac{x^2}{2}$
- d) $1 + \frac{x^2}{2}$

7) Given the data:

x	1	2	3	4	5
y	-1	2	-3	4	-5

If the derivative of $y(x)$ is approximated as:

$$y'(x_k) = \frac{1}{h} \left(\Delta y_k + \frac{1}{2} \Delta^2 y_k - \frac{1}{4} \Delta^3 y_k \right), \quad (7)$$

then the value of $y'(2)$ is

- a) 4
- b) 8
- c) 12
- d) 16

8) If

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (8)$$

then A^{50} is

a) $\begin{pmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 0 & 0 \\ 48 & 1 & 0 \\ 48 & 0 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{pmatrix}$

- 9) If $y = \sum_{m=0}^{\infty} c_m x^{r+m}$ is assumed to be a solution of the differential equation

$$x^2 y'' - xy' - 3(1+x^2)y = 0, \quad (9)$$

then the values of r are

- a) 1 and 3
- b) -1 and 3
- c) 1 and -3
- d) -1 and -3

- 10) Let the linear transformation $T : F^2 \rightarrow F^3$ be defined by

$$T(x_1, x_2) = (x_1, x_1 + x_2, x_2). \quad (10)$$

Then the nullity of T is

- a) 0
- b) 1
- c) 2
- d) 3

- 11) The approximate eigenvalue of the matrix

$$A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix} \quad (11)$$

obtained after two iterations of Power method, with the initial vector $[1 \ 1 \ 1]^T$ is

- a) 7.768
- b) 9.468
- c) 10.548
- d) 19.468

- 12) The root of the equation $xe^x = 1$ between 0 and 1, obtained by using two iterations of bisection method, is

- a) 0.25
- b) 0.50
- c) 0.75
- d) 0.65

Q. 26 to Q. 55 carry two marks each.

- 13) Let

$$\int_C \left[\frac{1}{(z-2)^2} + \frac{(a-2)^2}{z} + 4 \right] dz = 4\pi, \quad (12)$$

where the close curve C is the triangle having vertices at

$$i, \left(\frac{-1-i}{\sqrt{2}} \right), \text{ and } \left(\frac{1-i}{\sqrt{2}} \right) \quad (13)$$

the integral being taken in the anti-clockwise direction. Then one value of a is

- a) $1+i$
- b) $2+i$
- c) $3+i$
- d) $4+i$