Visualising the Projective Geometric Algebra of Lines

Midterm Presentation

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June 28, 2012

Project description

Extend a graphical calculator of geometric algebra to model the projective geometry of lines.

- Much like linear algebra, but uses more geometrically intuitive operators
- Outer product $\wedge$ to create subspaces
- Implementations are fast [Fontijne and Dorst(2003)]

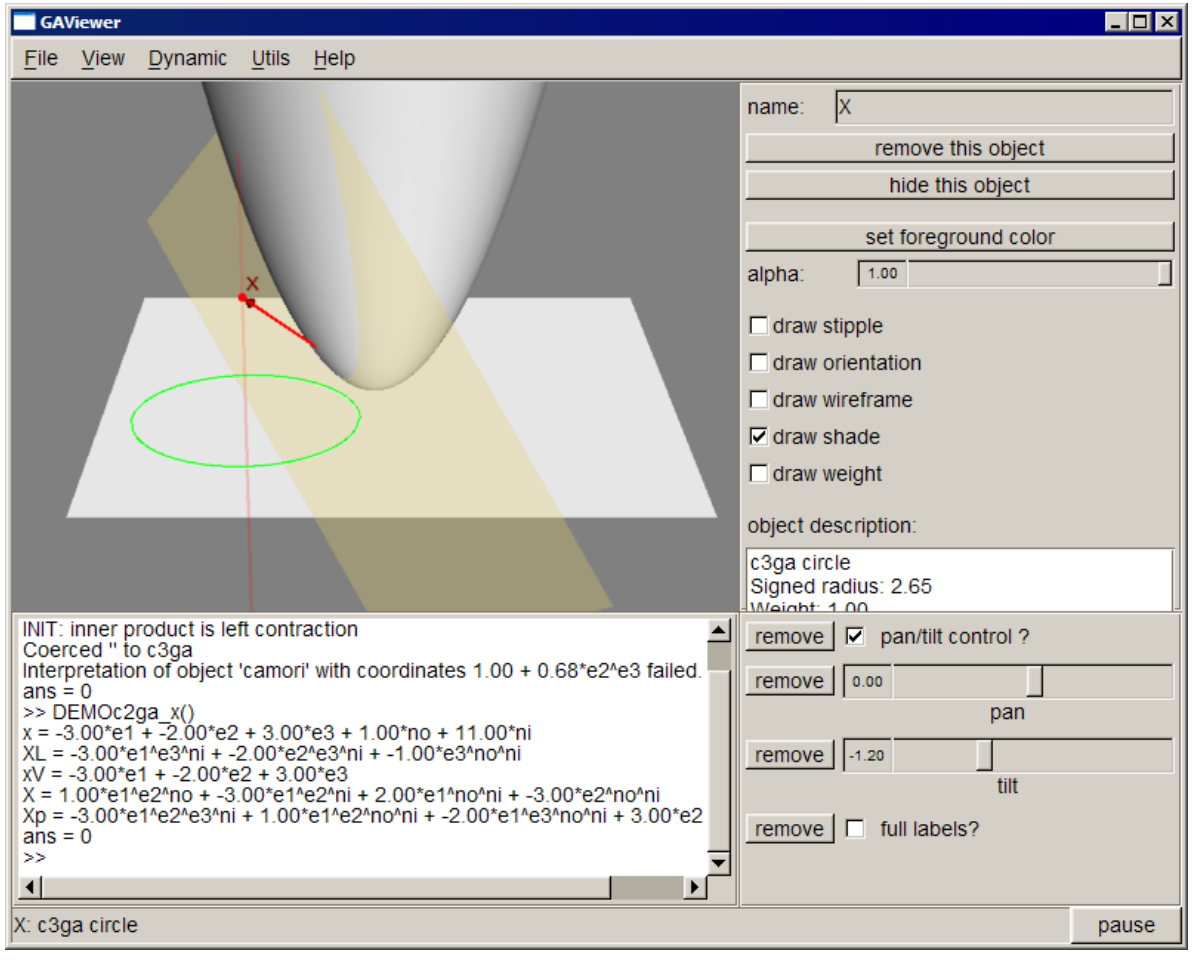
Project description

Extend a graphical calculator of geometric algebra to model the projective geometry of lines.

- Useful for computer vision, graphics, robotics...
- Plücker coordinates: 3D lines are 6D null vectors
  - Representation is homogeneous
- Recently found to work in geometric algebra [Li and Zhang(2011)]

Project description

Extend a graphical calculator of geometric algebra to model the projective geometry of lines.



`http://geometricalgebra.net` → Downloads → GAViewer

Approach

1. Understand Plücker model for GA
2. Compute geometric interpretation
   - First by hand, then find generic formula
3. Extend GAViewer
   - Recognise geometric type of input
   - Computer characteristics (orientation, center, normal. . .)
   - Implement drawing routines

Projective model

Plücker model is really new in GA community

- Original article is badly written
- Must translate from text on other algebras
  - Relate different inner products, $\times$ with $\wedge$
- Big difference in vocabulary
- Some concepts not well defined

Extending GAViewer

Communicating through sockets

- \+ Can use recent libraries
- \+ Socket interface is clear
- − Must write own parser
- − Express every element in terms of (sets of) other algebra's elements
- − End product looks ugly
- − Synchronise `dynamic` variables is difficult

Editing original code

- − Older GA implementation is slower
- − Little documentation
- \+ Parser is given
- \+ Can define completely new shapes
- \+ Final version is easier to use
- \+ User interaction is given

Current status

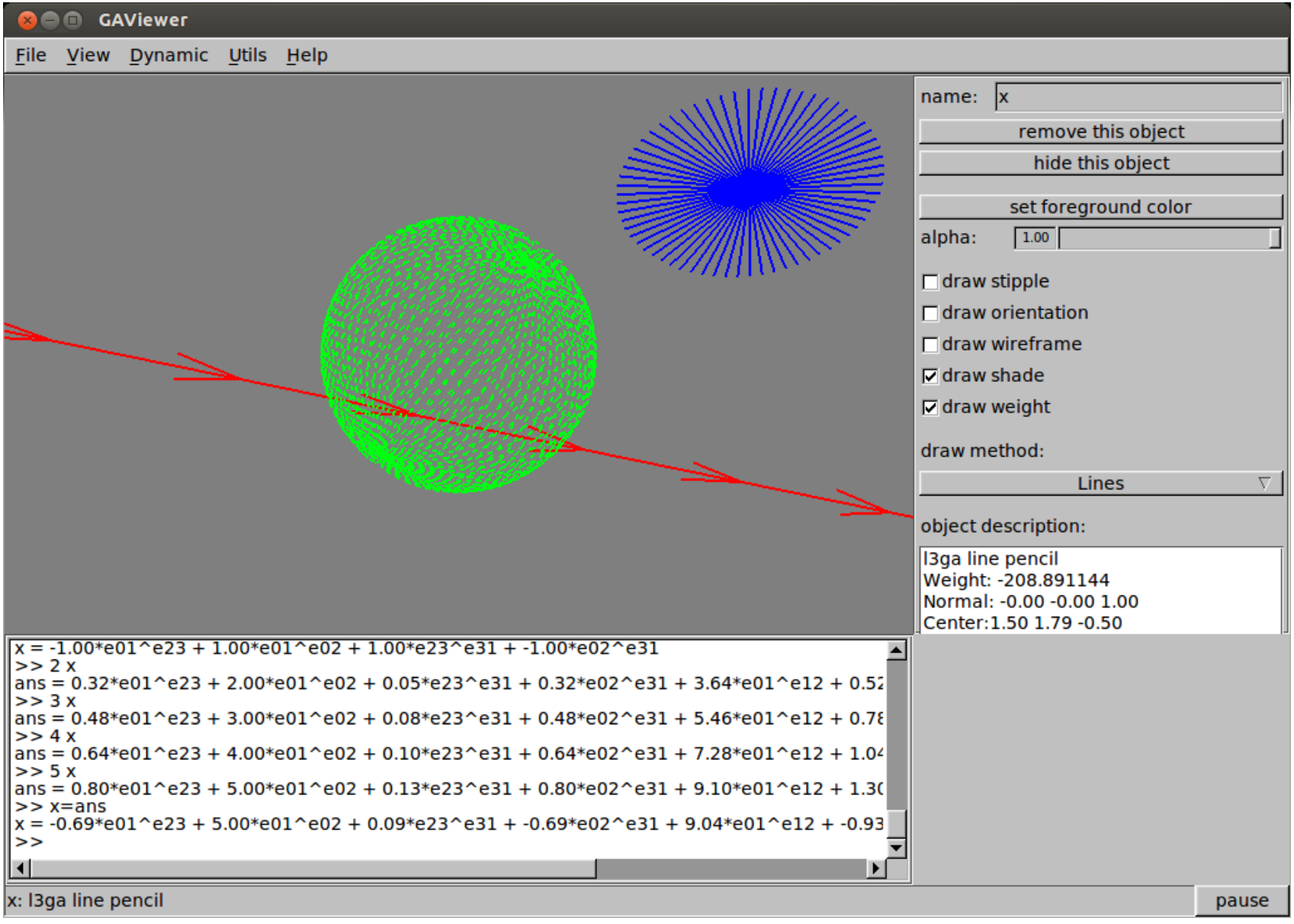
Done:

- Found all spots to add own code
- Added support for 1D, 2D subspaces, and duals
- All objects can be dragged (translation or rotation)
- Casting from and to other algebras

To do:

- Remove last abnormalities from 2D subspaces
- Add support for 3D subspaces
- Add support for direct representation of 4D, 5D subspaces

Questions


GAViewer
File View Dynamic Utils Help
name: x
remove this object
hide this object
set foreground color
alpha: 1.00
draw stipple
draw orientation
draw wireframe
draw shade
draw weight
draw method:
Lines
object description:
l3ga line pencil
Weight: -208.891144
Normal: -0.00 -0.00 1.00
Center:1.50 1.79 -0.50
x = -1.00*e01^e23 + 1.00*e01^e02 + 1.00*e23^e31 + -1.00*e02^e31
>> 2 x
ans = 0.32*e01^e23 + 2.00*e01^e02 + 0.05*e23^e31 + 0.32*e02^e31 + 3.64*e01^e12 + 0.52
>> 3 x
ans = 0.48*e01^e23 + 3.00*e01^e02 + 0.08*e23^e31 + 0.48*e02^e31 + 5.46*e01^e12 + 0.78
>> 4 x
ans = 0.64*e01^e23 + 4.00*e01^e02 + 0.10*e23^e31 + 0.64*e02^e31 + 7.28*e01^e12 + 1.04
>> 5 x
ans = 0.80*e01^e23 + 5.00*e01^e02 + 0.13*e23^e31 + 0.80*e02^e31 + 9.10*e01^e12 + 1.30
>> x=ans
x = -0.69*e01^e23 + 5.00*e01^e02 + 0.09*e23^e31 + -0.69*e02^e31 + 9.04*e01^e12 + -0.93
>>
x: l3ga line pencil
pause

Bibliography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Modeling 3d euclidean geometry.
Computer Graphics and Applications, IEEE, 23(2):68–78, 2003.


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ISBN 978-0-85729-811-9.
URL
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