Assignment 3 – literature overview

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With this bachelor thesis, I will try to give insight in the basic elements of the geometric algebra over $\mathbb{R}^{3,3}$. This will be accomplished by first investigating what the geometric interpretation of the basic elements in the space $\bigwedge \mathbb{R}^{3,3}$. After that, I will implement their visualisation in GAViewer. If time runs out, I will do this process for all blades up to a certain grade.

The vector space $\mathbb{R}^{3,3}$ is a six dimensional space, which has three real basis vectors ($\mathbf{v} \cdot \mathbf{v} = 1$), and three basis vectors which square to -1. A k-blade $B \in \bigwedge^k V$ is the outer product on k linear independent vectors from vector space V. The outer product is a linear operation, with the same geometrical intuition as linear algebra's span{} (which, in fact, is not linear).

This representational space for projective geometrical algebra has been found very recently [3]. All null vectors $\mathbf{v}^2 = 0$ of $\mathbb{R}^{3,3}$ represent lines. What the other elements represent has not yet been documented fully.

GAViewer is a tool for calculating and visualising elements of the euclidic and conformal models of geometric algebra, developed at the UvA by Daniel Fontijne. As those models do not contain the same geometrical objects as this projective model...

1 Retry!

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There are already several models of geometry in use in the geometric algebra community, of which the conformal model is the most popular and widely known [1]. Although usable in many cases, this model lacks the projective transformations. The projective transformations are an important class of operations within computer vision and computer graphics.

Recently, researchers have found a way to express these transformations by using a 6 dimensional representation space with 3 basis vectors with squared norm -1 [3]. Using Plücker coordinates, lines are represented in the

representational space by vectors with squared norm 0. Different geometrical objects are generated by the outer product, which can be seen as linear algebra's span{} operator with linear properties. For example, the outer product of two intersecting lines with a third line that intersects only one other line, represents a ruled surface.

GAViewer [2] is a visualisation and computing tool for the Euclidean and conformal models of geometric algebra, developed by Daniel Fontijne at the UvA. These models contain other geometrical entities than the projective model.

In the original paper [3], the authors haven't fully investigated what the geometrical meaning of the different elements of the algebra is. I will show what geometrical entities are represented. Afterwards, I will implement visualisations of these entities.

References

[1] Leo Dorst, Daniel Fontijne, and Stephen Mann. *Geometric Algebra for Computer Science: An Object-Oriented Approach to Geometry (The Morgan Kaufmann Series in Computer Graphics)*. Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, 2007.

[2] Daniel Fontijne. *GAViewer Documentation: Version 0.84*. University of Amsterdam, 3 2010.

[3] Hongbo Li and Lixian Zhang. Line geometry in terms of the null geometric algebra over $\mathbb{R}^{3,3}$, and application to the inverse singularity analysis of generalized stewart platforms. In Leo Dorst and Joan Lasenby, editors, *Guide to Geometric Algebra in Practice*, pages 253–272. Springer London, 2011.