Replies to Leo's notes

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1 2-blades

Theorem 1. *Show that* any *2-blade of* $\mathbb{R}^{3,3}$ *contains at least two* Lines

Lemma 1. *A 2-blade of two intersecting* Lines $\ell_1 \wedge \ell_2$ *contains at least two* Lines.

See Leo's notes, 2.2, April 19. Also: per definition.

Lemma 2. *A 2-blade of two skew* Lines $\ell_1 \wedge \ell_2$ *contains at least two* Lines.

See Leo's notes, 2.2, April 19. Also: per definition.

Lemma 3. *A 2-blade of two non-*Lines $k_1 \wedge k_2$ *contains at least two* Lines.

We are looking for Lines. As every null vector is a Line, we have the constraint $x^2 = 0$.

$$\begin{aligned} & x \wedge (k_1 \wedge k_2) = 0 && \text{and} \quad x^2 = 0 \\ \Leftrightarrow \quad & x = \alpha k_1 + \beta k_2 && \text{and} \quad 2\alpha\beta k_1 \cdot k_2 = 0 \end{aligned}$$

For $2\alpha\beta k_1 \cdot k_2 = 0$ to hold, α or β should be 0. $k_1 \cdot k_2 \neq 0$, as that would mean k_1 and k_2 would be intersecting lines, which they are not by definition.

This would mean that $x = \alpha k_1$ or $x = \beta k_2$. As the representation is homgeneous, $\alpha k_1 = k_1$, and thus x will not be a Line.

Lemma 4. *A 2-blade of one* Line *and a non-*Line $\ell \wedge k$ *contains at least two* Lines.

This will be similar to the previous lemma:

$$\begin{aligned} & x \wedge (\ell \wedge k) = 0 && \text{and} \quad x^2 = 0 \\ \Leftrightarrow \quad & x = \alpha\ell + \beta k && \text{and} \quad 2\alpha\beta\ell \cdot k = 0 \\ \Leftrightarrow \quad & x = \alpha\ell \text{ or } x = \beta k && \\ \Leftrightarrow \quad & x = \ell \text{ or } x = k && \end{aligned}$$

This blade seems to contain just one line, being ℓ.