



OXFORD JOURNALS
OXFORD UNIVERSITY PRESS

Biometrika Trust

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Source: *Biometrika*, Vol. 1, No. 3 (Apr., 1902), pp. 375-383

Published by: Oxford University Press on behalf of Biometrika Trust

Stable URL: <http://www.jstor.org/stable/2331550>

Accessed: 07-04-2016 11:11 UTC

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ON THE INFLUENCE OF PREVIOUS VACCINATION IN CASES OF SMALLPOX.

BY W. R. MACDONELL, LL.D.

IN *Biometrika*, Vol. I. Part II. p. 177 *et seq.*, Professor Karl Pearson's method of finding the correlation coefficients and other constants of characters not quantitatively measurable* was extensively applied to the case of characters quantitatively measurable, in order to avoid the very considerable labour involved in forming correlation tables of the usual detailed kind. I have since used the method in an investigation in which quantitative scales are unobtainable, and to which therefore it is peculiarly applicable, viz., the degree of effectiveness of vaccination in smallpox, and the object of this note is to give my results. I propose to show the correlation, first, between degree of effective vaccination and (1) strength to resist smallpox and (2) type of disease; and secondly, between type of disease and (1) degree of foveation, (2) scar area and (3) number of scars. The data have been extracted from the First Report of the Vaccination Commission, 1896, from a Report by Dr R. S. Thomson and Dr E. L. Marsh on the cases admitted to the City of Glasgow Smallpox Hospital, Belvidere, during the epidemic outbreak in 1892-5, and from the *Times* newspaper of November 30 and January 13 last.

1. The Commissioners' Report, pp. 55-58, gives statistics of the following epidemics: Sheffield 1887-8, London 1892-3, Dewsbury 1891-2, Warrington 1892-3, Leicester 1892-3 and Gloucester 1895-6; the facts were obtained from the local reports upon the epidemics in the six towns, and with regard to these reports the Commissioners write as follows (§ 212): "It is quite possible that the "classification" (vaccinated and unvaccinated) "may not be strictly accurate, "though great pains appear to have been taken to make it so. Doubtful cases "were in general included amongst the vaccinated class, and care was taken to see "that none should be included in the unvaccinated class except those who properly "came within it. Where the doubtful cases were separately stated in the reports 'we have added them to the vaccinated class for the purpose of our calculations."

* *Phil. Trans.* Vol. 195, pp. 1-47.

In Sheffield, Warrington, Leicester and Gloucester the doubtful cases do not appear to be stated separately; in London there were 191 doubtful cases, of whom 44 died, and in Dewsbury 24, of whom 2 died.

The figures for the six towns can then be arranged in the following table:—

TABLE I.
Epidemics for Six Towns.

| | Recoveries | Deaths | Totals |
|---------------|------------|--------|--------|
| Vaccinated... | 8283 | 461 | 8744 |
| Unvaccinated | 1499 | 822 | 2321 |
| Totals | 9782 | 1283 | 11065 |

The constants h and k were calculated, and the equation for r , the coefficient of correlation between degree of effective vaccination and strength to resist the disease, found in the usual way.

$$h = 1.19554, \quad k = .80726,$$

$$.032834r^6 + .014289r^5 + .148325r^4 - .024924r^3 + .482556r^2 + r = .888664,$$

whence

$$r = .6561 \pm .0092.$$

On account of the magnitude of the epidemic in Sheffield, I have calculated the result for that town separately.

TABLE II.
Sheffield.

| | Recoveries | Deaths | Totals |
|---------------|------------|--------|--------|
| Vaccinated... | 3951 | 200 | 4151 |
| Unvaccinated | 278 | 274 | 552 |
| Totals | 4229 | 474 | 4703 |

$$h = 1.27716, \quad k = 1.18833,$$

$$.097083r^7 + .008170r^6 + .119614r^5 + .137450r^4 + .043352r^3 \\ + .758844r^2 + r = 1.336056,$$

whence

$$r = .7694 \pm .0124.$$

The Leicester and Gloucester epidemics are of special interest owing to the practice of vaccination having fallen into disuse in these towns for some years prior to the epidemic. They are therefore shown separately in the two following tables.

TABLE III.

Leicester.

| | Recoveries | Deaths | Totals |
|----------------|------------|--------|--------|
| Vaccinated ... | 197 | 2 | 199 |
| Unvaccinated | 139 | 19 | 158 |
| Totals | 336 | 21 | 357 |

$$h = 1.56497, \quad k = .14444,$$

$$.055622r^7 + .016221r^6 + .136492r^5 - .015457r^4 + .236484r^3 \\ - .113022r^2 - r + .587355 = 0,$$

whence

$$r = .6112 \pm .0728.$$

TABLE IV.

Gloucester.

| | Recoveries | Deaths | Totals |
|----------------|------------|--------|--------|
| Vaccinated ... | 1091 | 120 | 1211 |
| Unvaccinated | 454 | 314 | 768 |
| Totals | 1545 | 434 | 1979 |

$$h = .77455, \quad k = .28434,$$

$$.040653r^6 - .005035r^5 + .064292r^4 + .061288r^3 + .110118r^2 + r = .649608,$$

whence

$$r = .5897 \pm .0198.$$

The Commissioners' Report, p. 59, also gives the results of an examination of 10403 cases at the Homerton Hospital between the years 1873 and 1884, and of 2584 cases at the Fulham Hospital between the years 1880 and 1885; these are exhibited in the following Table.

TABLE V.

Homerton and Fulham Hospitals.

| | Recoveries | Deaths | Totals |
|----------------|------------|--------|--------|
| Vaccinated ... | 9328 | 1132 | 10460 |
| Unvaccinated | 1424 | 1103 | 2527 |
| Totals | 10752 | 2235 | 12987 |

$$h = \cdot 94596, \quad k = \cdot 86115,$$

$$\cdot 063062r^6 + \cdot 011756r^5 + \cdot 161372r^4 + \cdot 004529r^3 + \cdot 407310r^2 + r = \cdot 732600,$$

whence

$$r = \cdot 5760 \pm \cdot 0089.$$

These figures include among the vaccinated 1561 doubtful cases, of whom 440 died; if these are excluded altogether the table becomes

TABLE VI.

Homerton and Fulham (doubtful cases excluded).

| | Recoveries | Deaths | Totals |
|---------------|------------|--------|--------|
| Vaccinated... | 8207 | 692 | 8899 |
| Unvaccinated | 1424 | 1103 | 2527 |
| Totals | 9631 | 1795 | 11426 |

$$h = 1\cdot 00650, \quad k = \cdot 76829,$$

$$\cdot 059812r^6 + \cdot 003304r^5 + \cdot 154271r^4 - \cdot 0008905r^3 + \cdot 386642r^2 + r = \cdot 865473,$$

whence

$$r = \cdot 6615 \pm \cdot 0083.$$

The Glasgow statistics have now to be dealt with; they are given on p. 10 of the Report referred to above. The doubtful cases, 20, of whom 5 died, are not included.

TABLE VII.

Glasgow.

| | Recoveries | Deaths | Totals |
|---------------|------------|--------|--------|
| Vaccinated... | 622 | 21 | 643 |
| Unvaccinated | 31 | 26 | 57 |
| Totals | 653 | 47 | 700 |

$$h = 1\cdot 49766, \quad k = 1\cdot 39567,$$

$$\cdot 051044r^8 + \cdot 099843r^7 + \cdot 004768r^6 + \cdot 221286r^5 + \cdot 069366r^4 + \cdot 196370r^3 \\ + 1\cdot 045120r^2 + r = 1\cdot 617863,$$

whence

$$r = \cdot 7783 \pm \cdot 0365.$$

Finally I give the result of an examination of the statistics published in the *Times* of January 13 last, which give particulars of 1017 cases of smallpox in

London during the present epidemic. These figures deal only with the cases that were completed in 1901; I regret that I have not seen later figures than these. The doubtful cases, 63, of whom 41 died, have been excluded.

TABLE VIII.

London, for the year 1901.

| | Recoveries | Deaths | Totals |
|---------------|------------|--------|--------|
| Vaccinated... | 652 | 108 | 760 |
| Unvaccinated | 96 | 98 | 194 |
| Totals | 748 | 206 | 954 |

$$h = \cdot78603, \quad k = \cdot82972,$$

$$\cdot071607r^6 + \cdot001780r^5 + \cdot149636r^4 + \cdot019844r^3 + \cdot326092r^2 + r = \cdot7101,$$

whence

$$r = \cdot5779 \pm \cdot0311.$$

On November 30 last the *Times* gave similar particulars for 330 cases completed up to that date, which are shown in

TABLE IX.

London, to November 30, 1901.

| | Recoveries | Deaths | Totals |
|---------------|------------|--------|--------|
| Vaccinated... | 195 | 45 | 240 |
| Unvaccinated | 30 | 60 | 90 |
| Totals | 225 | 105 | 330 |

$$h = \cdot47281, \quad k = \cdot51572,$$

$$\cdot053861r^6 + \cdot021001r^5 + \cdot077122r^4 + \cdot094990r^3 + \cdot121918r^2 + r = \cdot762763,$$

whence

$$r = \cdot6605 \pm \cdot0406.$$

It will be noticed on comparing this result with the previous one that the correlation diminished as the epidemic progressed; this will be an interesting point to investigate again when later figures are available.

The foregoing results may now be collected in the following Table.

TABLE X.

Coefficient of Correlation, r , between effectiveness of vaccination and strength to resist the disease.

| r | Doubtful cases |
|--|------------------------|
| For the 6 towns enumerated... .. '6561 \pm '0092 | Included in vaccinated |
| „ Sheffield '7694 \pm '0124 | „ |
| „ Leicester '6112 \pm '0728 | „ |
| „ Gloucester '5897 \pm '0198 | „ |
| „ Homerton and Fulham Hospitals '5760 \pm '0089 | „ |
| „ „ „ „ '6615 \pm '0083 | Excluded |
| „ Glasgow '7783 \pm '0365 | „ |
| „ London, 1901 Epidemic ... '5779 \pm '0311 | „ |
| „ London, 1892-3 Epidemic* ... '5954 \pm '0272 | „ |

In Sheffield and Glasgow the correlation is nearly the same, and considerably higher than elsewhere; in the other towns it is remarkably uniform, the coefficient approximating to '6. It will also be noted that the correlation in the present epidemic is nearly the same as that in the epidemic of 1892-3. We have clearly in this coefficient a fairly stable statistical constant for smallpox epidemics.

2. Coming next to the correlation between degree of effective vaccination and type of disease, I divide the types into two classes, (1) Mild, = mild, varioloid, and discrete, and (2) Severe, = coherent and confluent, and exhibit in Table XI. the statistics of the cases whose types were observed in the Sheffield, Dewsbury, Leicester and Warrington epidemics. The London figures for 1892-3 are excluded because a somewhat different classification was adopted there. No figures appear to be available for Gloucester. (See Report of Commission, pp. 66-69.)

TABLE XI.

Sheffield, Dewsbury, Leicester and Warrington.

| | Mild | Severe | Totals |
|---------------|------|--------|--------|
| Vaccinated... | 2229 | 505 | 2734 |
| Unvaccinated | 229 | 804 | 1033 |
| Totals | 2458 | 1309 | 3767 |

$$h = \cdot 39212, \quad k = \cdot 60009,$$

$$\cdot 001160r^7 + \cdot 050798r^6 + \cdot 016967r^5 + \cdot 073650r^4 + \cdot 090250r^3 \\ + \cdot 117624r^2 + r = \cdot 959775,$$

whence

$$r = \cdot 7935 \pm \cdot 0093.$$

* Pearson: *Phil. Trans.* Vol. 195, p. 43.

In Glasgow, the classification is (i), Mild = discrete, and (ii), Severe = confluent and haemorrhagic, which appears to be practically the same as in the above four towns. The figures from which Table XII. is formed are taken from the Report of Drs Thomson and Marsh, p. 11.

TABLE XII.

Glasgow.

| | Mild | Severe | Totals |
|---------------|------|--------|--------|
| Vaccinated... | 608 | 45 | 653 |
| Unvaccinated | 9 | 48 | 57 |
| Totals | 617 | 93 | 710 |

$$h = 1.12179, \quad k = 1.40323,$$

$$\begin{aligned} & \cdot 090996r^7 - \cdot 007112r^6 + \cdot 122066r^5 + \cdot 117763r^4 + \cdot 041736r^3 \\ & + \cdot 787065r^2 + r = 1.801254, \end{aligned}$$

whence

$$r = \cdot 9123 \pm \cdot 0181.$$

This high correlation between vaccination and type is in agreement with the comparatively high correlation between vaccination and strength of resistance in Glasgow.

3. Table XIII. is formed to show the correlation between degree of foveation and type in 631 cases of vaccinated persons in Glasgow who took smallpox. (See Report, p. 13.)

TABLE XIII.

Glasgow.

| Scars | Mild | Severe | Totals |
|---------------|------|--------|--------|
| Foveated ... | 479 | 24 | 503 |
| Unfoveated... | 107 | 21 | 128 |
| Totals | 586 | 45 | 631 |

$$h = 1.46625, \quad k = \cdot 83150,$$

$$\cdot 029479r^5 + \cdot 099698r^4 - \cdot 059144r^3 + \cdot 609594r^2 + r = \cdot 489370,$$

whence

$$r = \cdot 3951 \pm \cdot 0594.$$

4. Table XIV. gives the facts as to scar area in the same 631 cases.

TABLE XIV.

Glasgow.

| Area of Scar | Mild | Severe | Totals |
|------------------------------|------|--------|--------|
| Over half square inch | 379 | 16 | 395 |
| Half square inch and under | 207 | 29 | 236 |
| Totals | 586 | 45 | 631 |

$$h = 1.46625, \quad k = .32125,$$

$$.017164r^6 + .105170r^5 - .048332r^4 + .171870r^3 - .235517r^2 - r + .373833 = 0,$$

whence

$$r = .3520 \pm .0584.$$

5. Table XV. gives the facts as to number of scars in these 631 cases.

TABLE XV.

Glasgow.

| Scars | Mild | Severe | Totals |
|-----------------|------|--------|--------|
| Two and upwards | 320 | 16 | 336 |
| One | 266 | 29 | 295 |
| Totals | 586 | 45 | 631 |

$$h = 1.46625, \quad k = .08153,$$

$$.012675r^4 - .190374r^3 + .059772r^2 + r = .233054,$$

whence

$$r = .2323 \pm .0616.$$

This value of r is unexpectedly small, but it is confirmed by the facts observed in Sheffield in 1887-8 and London in 1892-3; these are given in Table XVI., from which doubtful cases, where the records with respect to the nature of the vaccination were incomplete, are excluded. (See Report of the Commission, pp. 71-74.)

TABLE XVI.

Sheffield and London.

| Scars | Mild | Severe | Totals |
|-----------------|------|--------|--------|
| Two and upwards | 1855 | 161 | 2016 |
| One or none ... | 325 | 64 | 389 |
| Totals | 2180 | 225 | 2405 |

$$h = 1.31930, \quad k = .98736,$$

$$.069843r^5 + .138433r^4 - .0031005r^3 + .651312r^2 + r = .280372,$$

whence

$$r = .2418 \pm .0325.$$

It is obvious that in dealing with the last four tables we have descended to a much lower plane of correlation, and the results may possibly somewhat modify medical opinion as to the degree of significance of foveation, number of scars and scar area.

I understand that the figures relating to the recent smallpox epidemic in Glasgow will soon be available, and no doubt more statistics of the present London epidemic will be issued shortly; their publication will furnish a mass of extremely interesting and valuable material for statistical work. It is to be hoped that information will soon be given regarding the social rank and occupation of the patients, as an investigation of the type and mortality of the disease in the different classes of the community seems to me a very important line of statistical inquiry, having regard to the state of the controversy at the present time.

Our numbers demonstrate that high correlation exists between the presence of the vaccination scar and both the recovery from and the mildness of the attack. To complete a logical demonstration, however, of the effectiveness of prior vaccination in cases of smallpox we at least require to determine the correlation between the physique and nourishment of the attacked—to some extent indicated by their social class—and the presence or absence of the scar.