SCHOOL OF ENGINEERING, ELECTRICAL AND ELECTRONIC ENGINEERING

NETWORKS 4_

Week D_

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HAN_UNIVERSITY OF APPLIED SCIENCES

NET4 PROGRAM BY WEEK

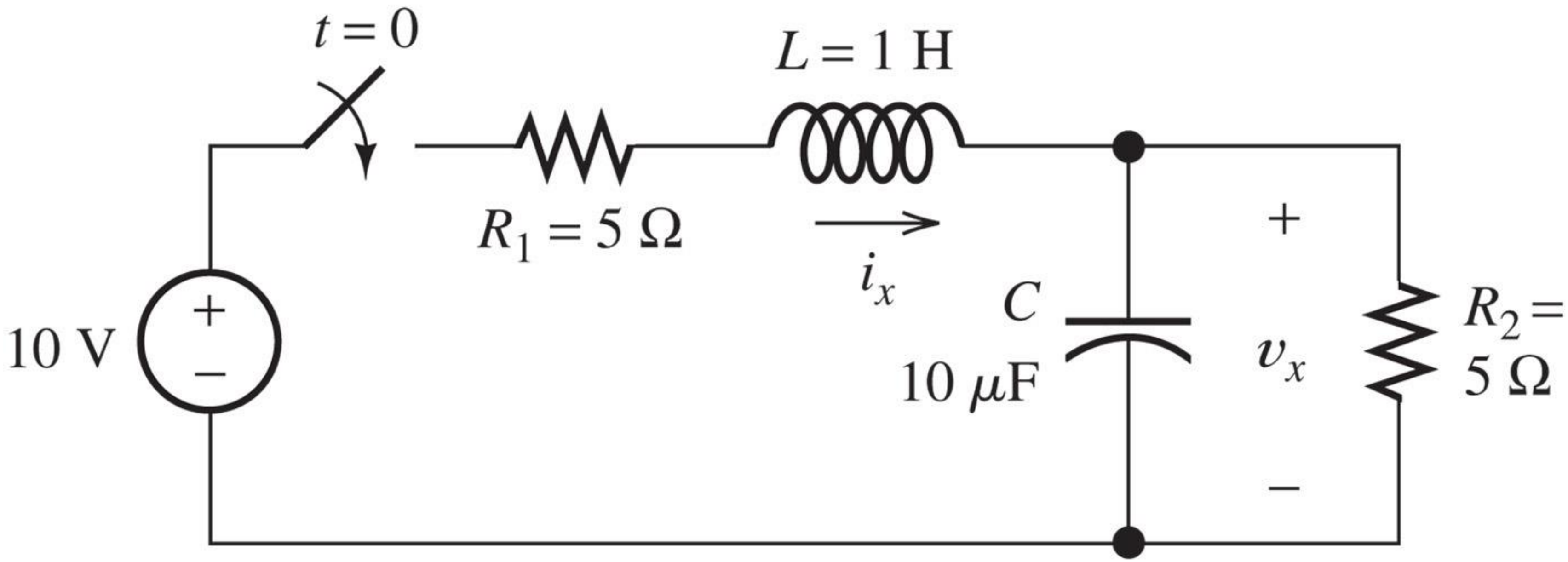
1. 1^{st} order RC networks – discharging – DC source
2. 1^{st} order RC networks – charging –DC source
3. RL networks – Steady-state DC
4. RL networks – Switched DC source
5. RC & RL networks – complementary solution
6. <spare week>
7. Sample Exam

CAPACITOR & INDUCTOR RELATIONS

	Capacitor	Inductor
Voltage	$v = \frac{1}{C}\int i(t)dt$	$v = L\frac{di}{dt}$
Current	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int v(t)dt$
Power	$P(t) = v(t) \cdot i(t)$	
Energy	$W(t) = \frac{1}{2}Cv^2(t)$	$W(t) = \frac{1}{2}Li^2(t)$
Energy stored in:	Electric field	Magnetic field

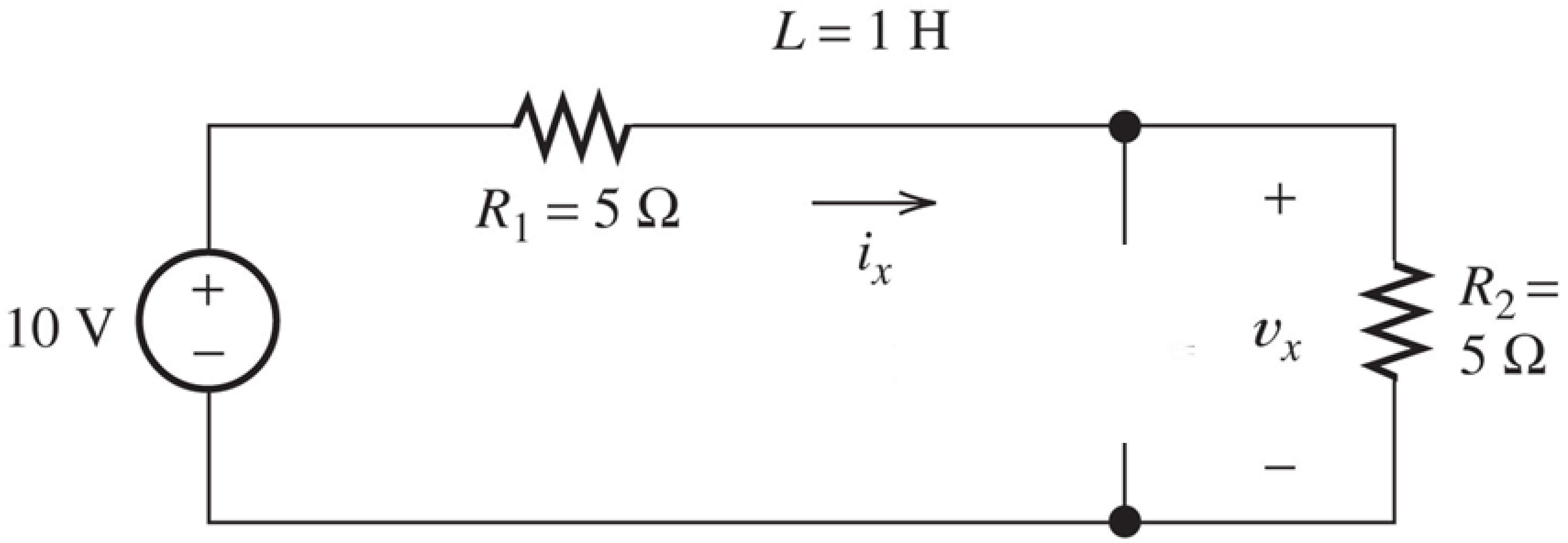
WEEK C: DC STEADY STATE

At $t = 0$ we close the switch



WEEK C: DC STEADY STATE

Equivalent circuit at $t = \infty$



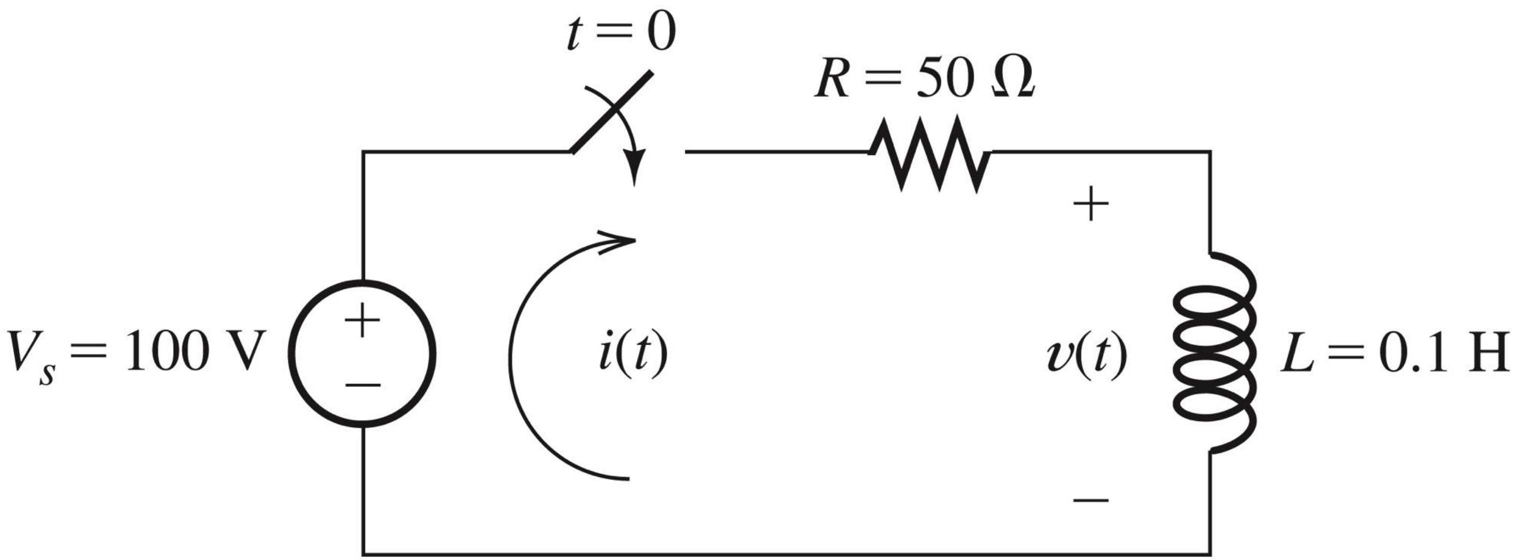
$i_x = V\ /\ (R_1 + R_2) = 1$ A

$v_x = R_2\ /\ (R_1 + R_2) \times V = 5$ V (voltage division)

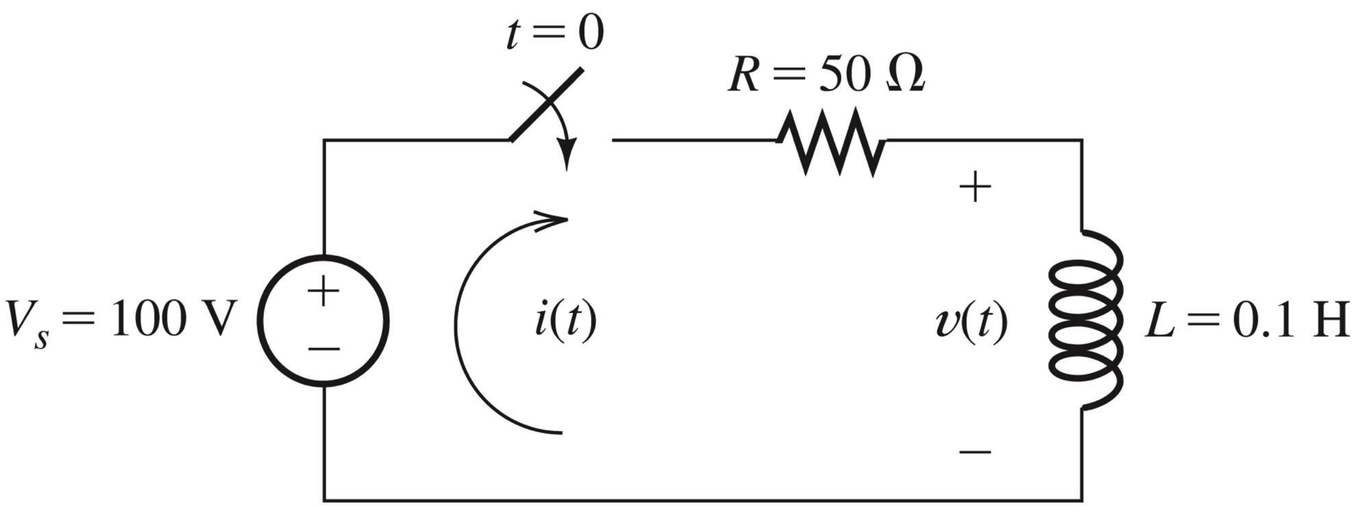
RL CIRCUITS

Find expressions for $i(t)$ and $v(t)$ for $t > 0$

- probably a differential equation?



RL CIRCUITS - USING KVL



1. KVL: Write individual voltages *with polarities*
   a) $+V_S$
   b) $v_R = -i(t) \cdot R$
   c) $v_L = -L\mathrm{d}i/\mathrm{d}t$
2. Write KVL:

$V_S - i(t) \cdot R - L\mathrm{d}i/\mathrm{d}t = 0$

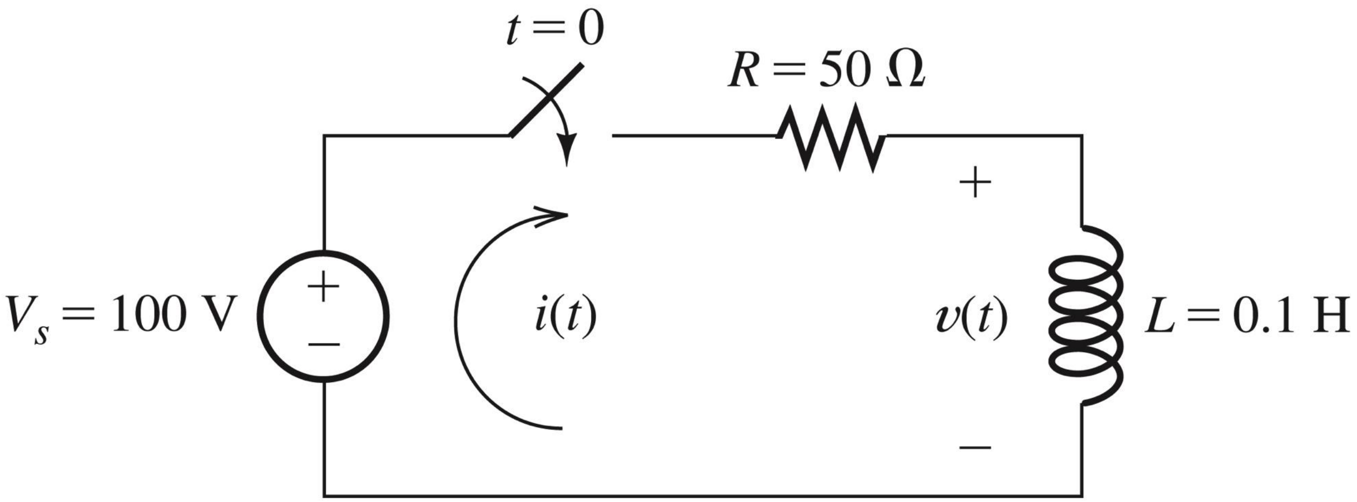
3. Re-arrange:

$$i(t) + \frac{L}{R}\frac{di(t)}{dt} = \frac{V_S}{R}$$

Inductor
$v = L\dfrac{di}{dt}$

Haven't we seen this before?

RL CIRCUITS - USING KVL



$$i(t) + \frac{L}{R}\frac{di(t)}{dt} = \frac{V_S}{R}$$

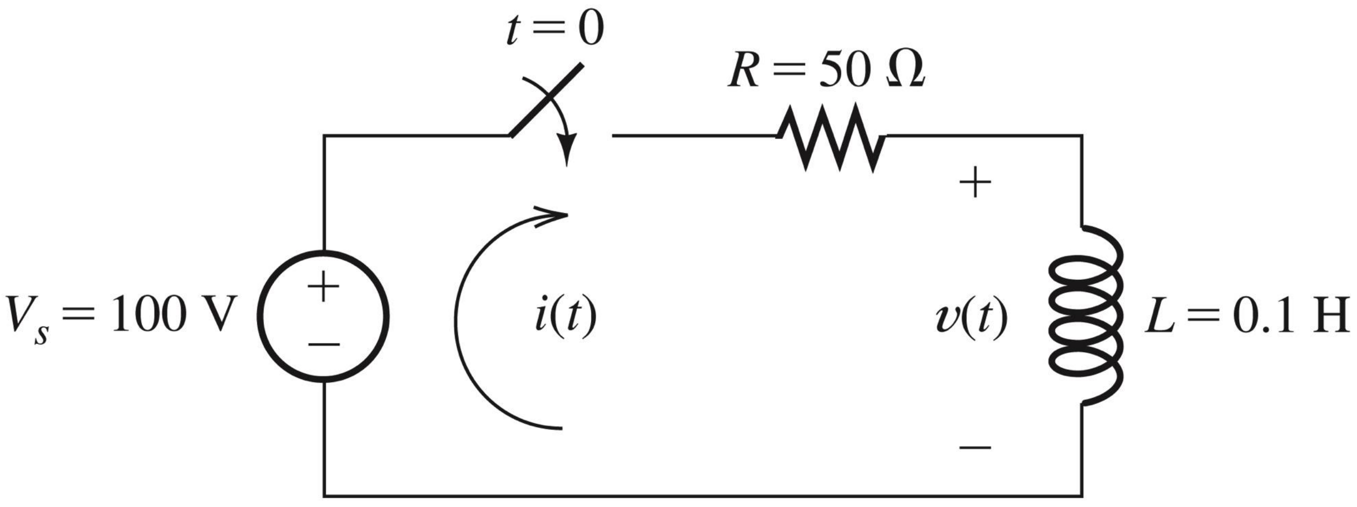
Inductor
$v = L\dfrac{di}{dt}$

Remember week B, charging a capacitor (and K**C**L):

$$C\frac{dv_C(t)}{dt} + \frac{v_C(t) - V_s}{R} = 0$$

Educated guess $i = K_1 + K_2 e^{st}$

RL CIRCUITS - USING KVL

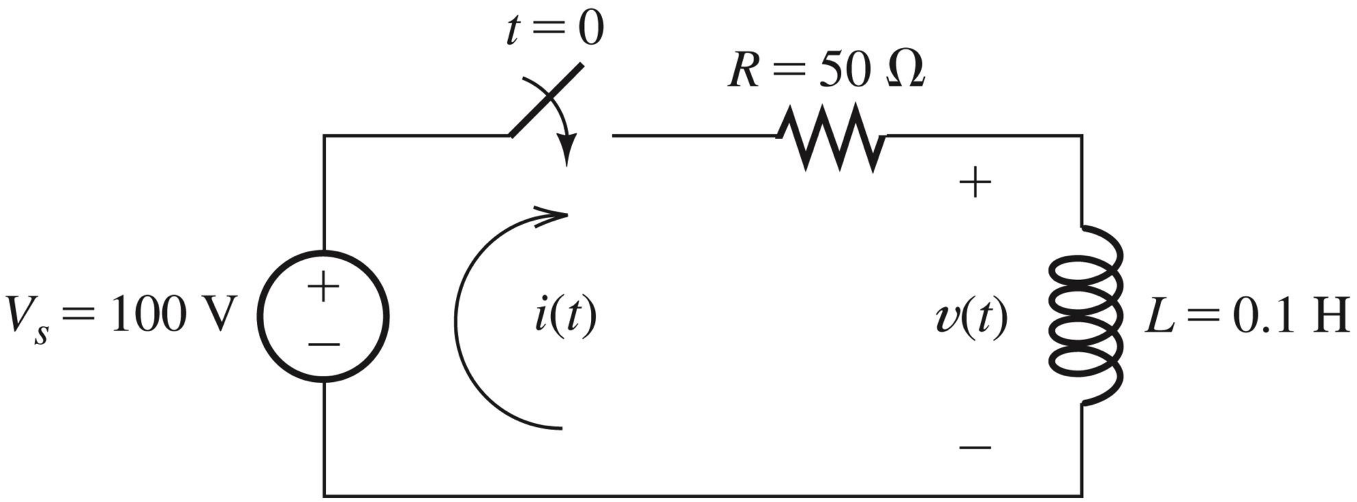


$$i(t) + \frac{L}{R}\frac{di(t)}{dt} = \frac{V_S}{R}$$

insert $i(t) = K_1 + K_2 e^{st}$:

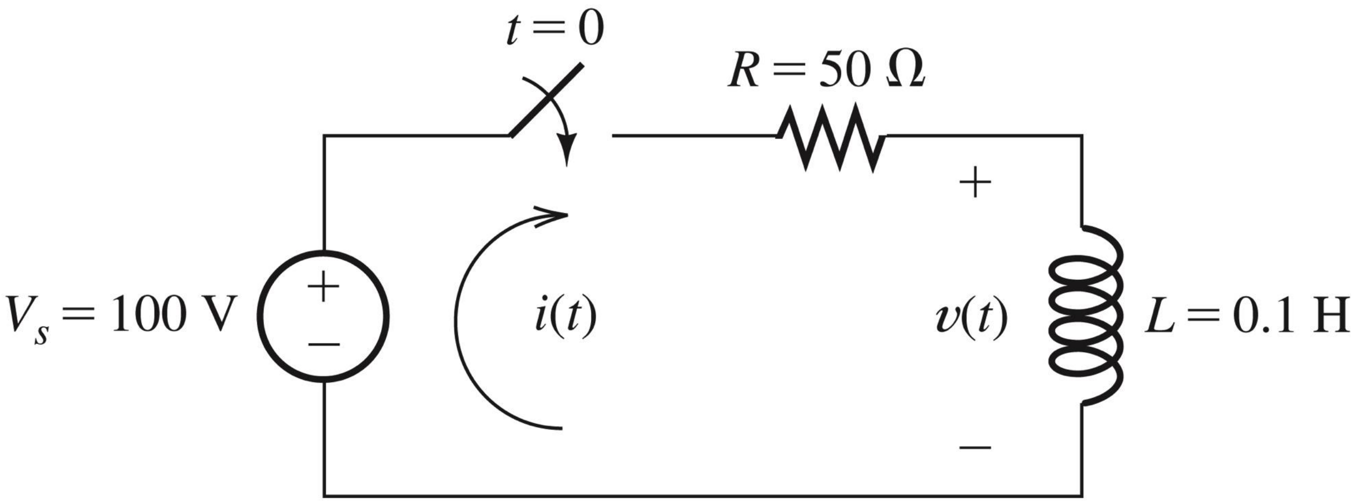
$$K_1 + K_2 e^{st} + \frac{L}{R} sK_2 e^{st} = \frac{V_S}{R}$$

RL CIRCUITS - USING KVL



$$K_1 + K_2 e^{st} + \frac{L}{R} s K_2 e^{st} = \frac{V_S}{R}$$

RL CIRCUITS - USING KVL



$$K_1 + K_2 e^{st} + \frac{L}{R} s K_2 e^{st} = \frac{V_S}{R}$$

$$K_1 = \frac{V_S}{R} = 2 \text{ A}$$

$$s = -\frac{R}{L} = -500 \text{ s}^{-1}$$

$$\tau = \frac{L}{R}$$

K_2?

$i(t) = K_1 + K_2 e^{st}$

Aha, *initial condition around t=0*

i. $i(t=0-) = 0$
ii. $i(t=0+) = 0$ (i must be continuous)
iii. So, $K_2 = -K_1 = -2$ A

$i(t) = 2 - 2e^{-500t}$ A

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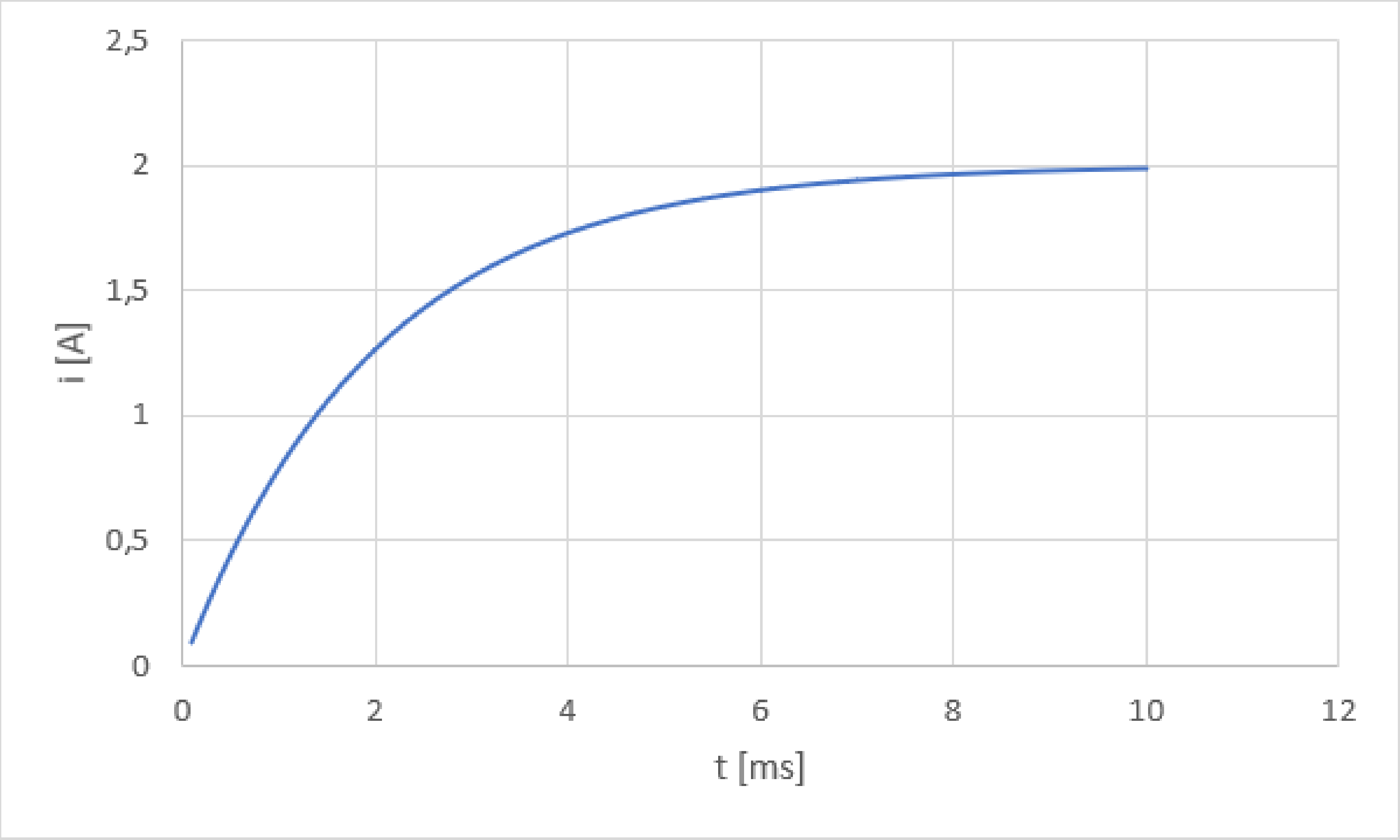
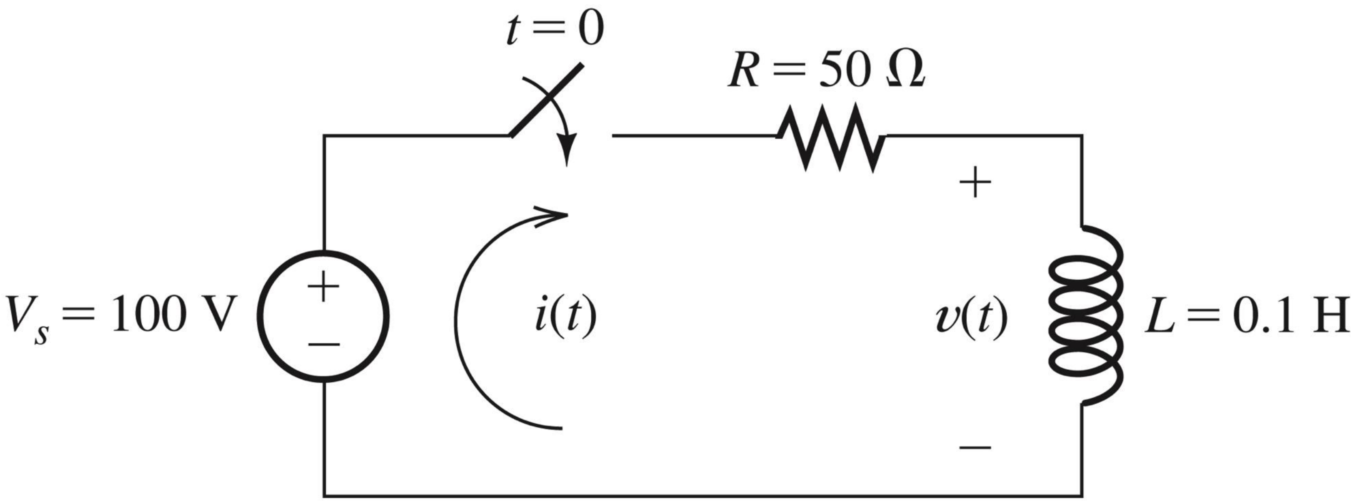
$$i(t) = 2\text{-}2e^{-500t}\ \text{A}$$

t [ms]	i(t)
0,1	0,097541
0,2	0,190325
0,3	0,278584
0,4	0,362538
0,5	0,442398
0,6	0,518364
0,7	0,590624
0,8	0,65936
0,9	0,724744
1	0,786939
1,1	0,8461
1,2	0,902377
1,3	0,955908
1,4	1,006829
1,5	1,055267
1,6	1,101342



RL CIRCUITS - USING KVL



Final step: find $v(t)$

$i(t) = 2\text{-}2e^{-500t}$ A

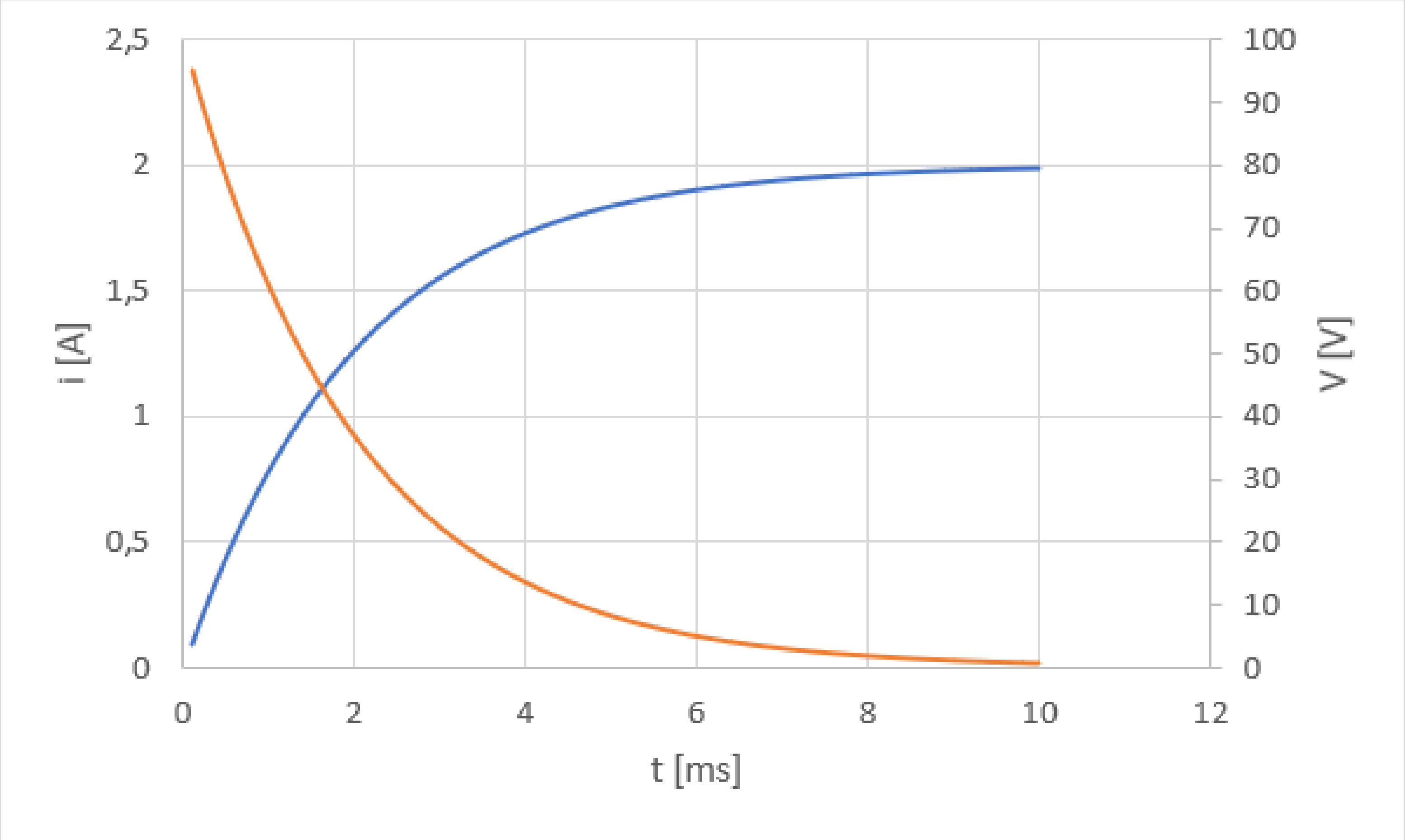
so (how's your Math?)

Inductor
$v = L\dfrac{di}{dt}$

$0.1 * d(2-2e^{-500t})/dt$

$v(t) = 100e^{-500t}$ V

t [ms]	i(t)	v(t)
0,1	0,097541	95,12294245
0,2	0,190325	90,4837418
0,3	0,278584	86,07079764
0,4	0,362538	81,87307531
0,5	0,442398	77,88007831
0,6	0,518364	74,08182207
0,7	0,590624	70,46880897
0,8	0,65936	67,0320046
0,9	0,724744	63,76281516
1	0,786939	60,65306597
1,1	0,8461	57,69498104
1,2	0,902377	54,88116361
1,3	0,955908	52,20457768
1,4	1,006829	49,65853038
1,5	1,055267	47,23665527
1,6	1,101342	44,93289641
1,7	1,14517	42,74149319
1,8	1,186861	40,65696597



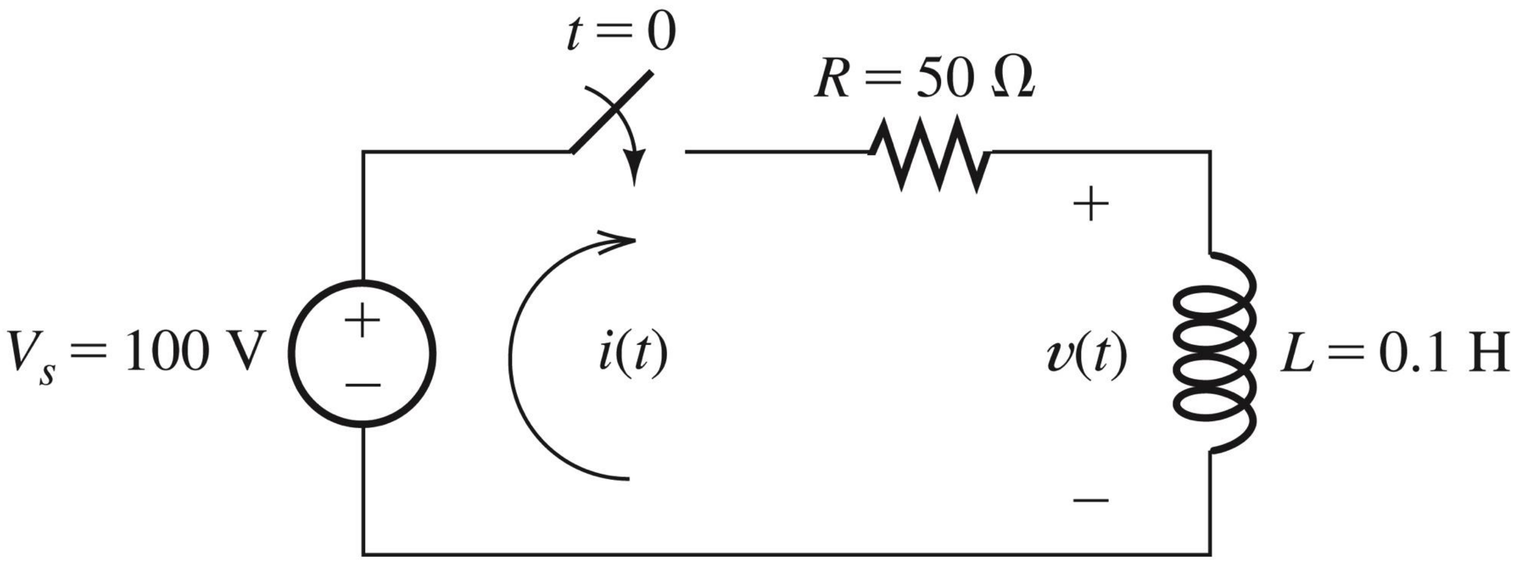
RECIPE FOR SIMPLE CIRCUITS (RC & RL)

1. Use one of Kirchhoff's laws to get a circuit equation
2. If the equation has integrals, differentiate all terms to get a pure differential equation
3. Try a solution of the type $K_1 + K_2 e^{st}$
    1. Enter it into the equation
    2. Solve for K_1& s
    3. Use starting conditions (*t*=0-, *t*=0+) to get K_2
4. Write the now fully known equation
    1. i.e. using *R*, *L*, *C*, τ

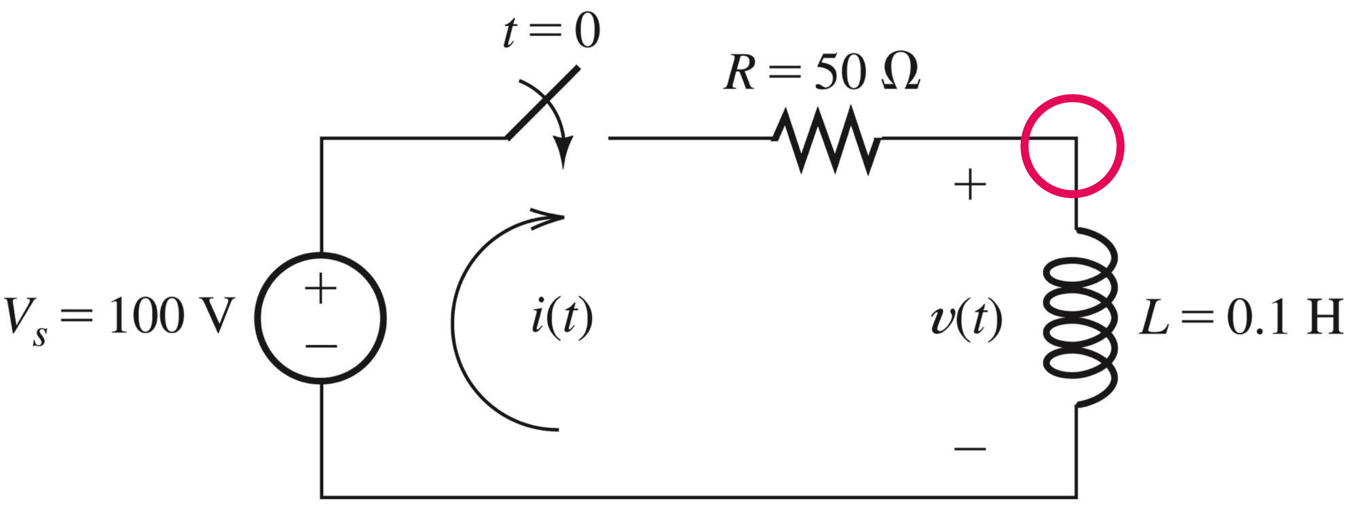
RL CIRCUITS – NOW VIA KCL

Find expressions for $i(t)$ and $v(t)$ for $t > 0$

- surely another differential equation



RL CIRCUITS – NOW VIA KCL



1. Write individual currents *with polarities*
   a) $i_R = (V_S - v(t)) / R$
   b) $i_L = -\frac{1}{L}\int v(t)dt$

$$i = \frac{1}{L}\int v(t)dt$$

2. Apply KCL: sum = 0 @ node

$$[V_S - v(t)]/R - \frac{1}{L}\int v(t)dt = 0$$

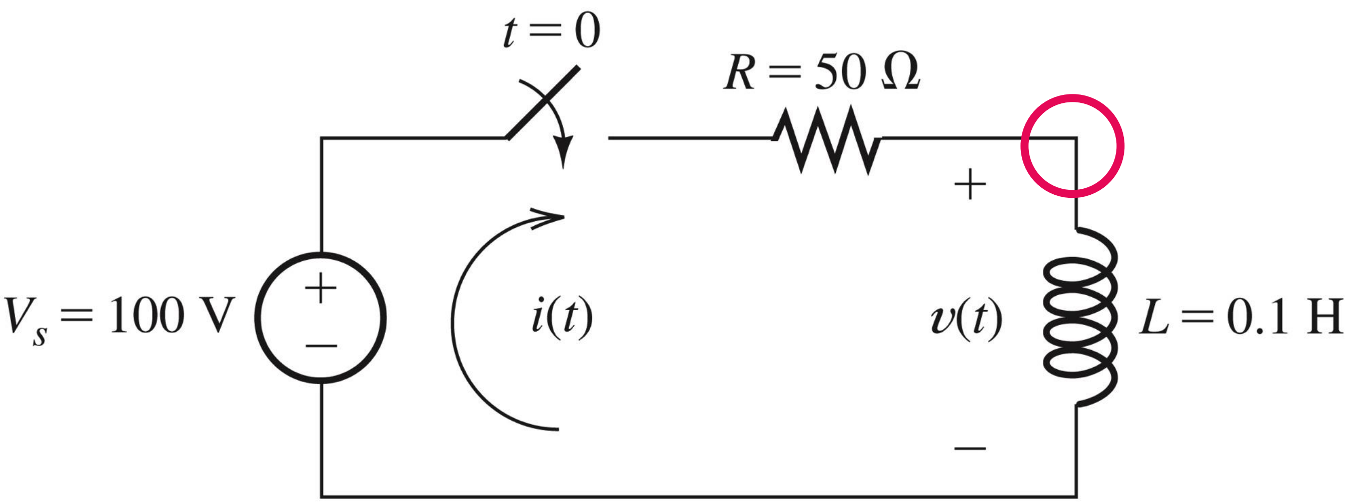
3. Differentiate to remove integral:

$$0 - \frac{1}{R}\frac{dv(t)}{dt} - \frac{1}{L}v(t) = 0$$

4. Rearrange:

$$v(t) + \frac{L}{R}\frac{dv(t)}{dt} = 0$$

RL CIRCUITS – NOW VIA KCL



$$v(t) + \frac{L}{R}\frac{dv(t)}{dt} = 0$$

$\boxed{\text{Try } v = K_1 + K_2 e^{st}}$

$$K_1 + K_2 e^{st} + \frac{L}{R} s K_2 e^{st} = 0$$

$K_1 = 0$

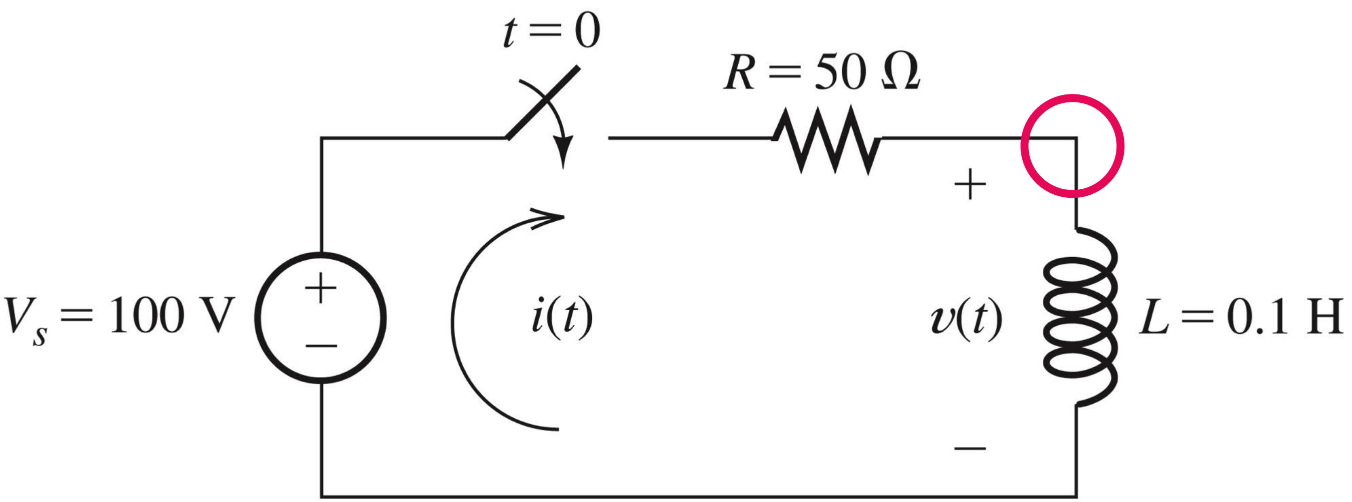
$s = -\frac{R}{L}$

Initial: $v(0+) = V_S = 100$ V

$K_2 = 100$ V

$v(t) = 100e^{-500t}$ V

RL CIRCUITS – NOW VIA KCL



Final step: find $i(t)$

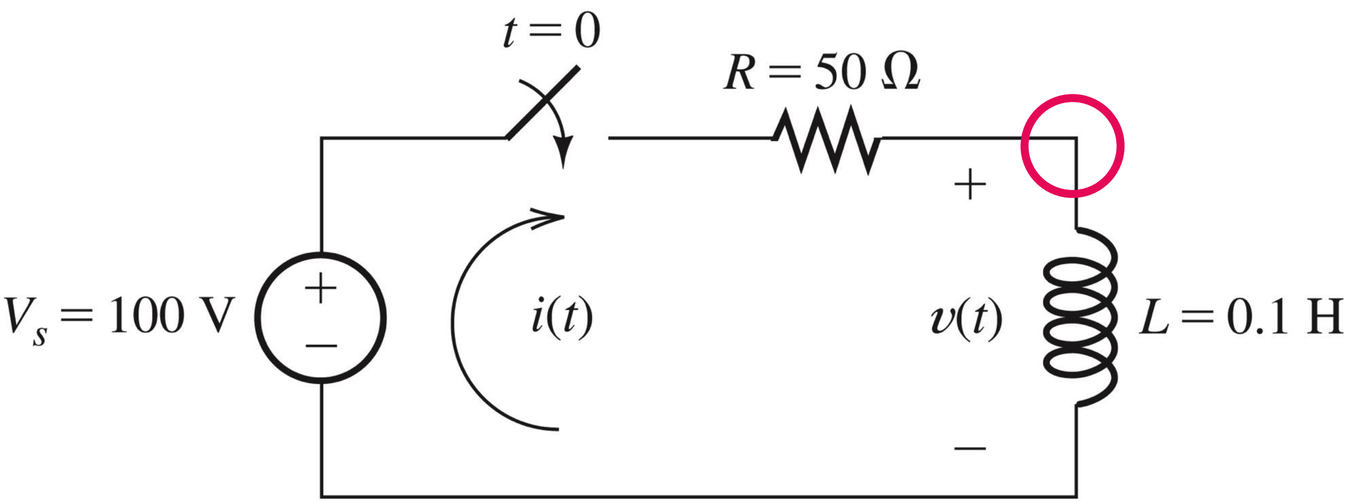
i(t) is the same everywhere, so choose *R* to get it using Ohm's law

$i(t) = (Vs - v(t)) / R$
$\quad = (100 - 100e^{-500t}) / 50$

$i(t) = 2 - 2e^{-500t}$ A

$v(t) = 100e^{-500t}$ V

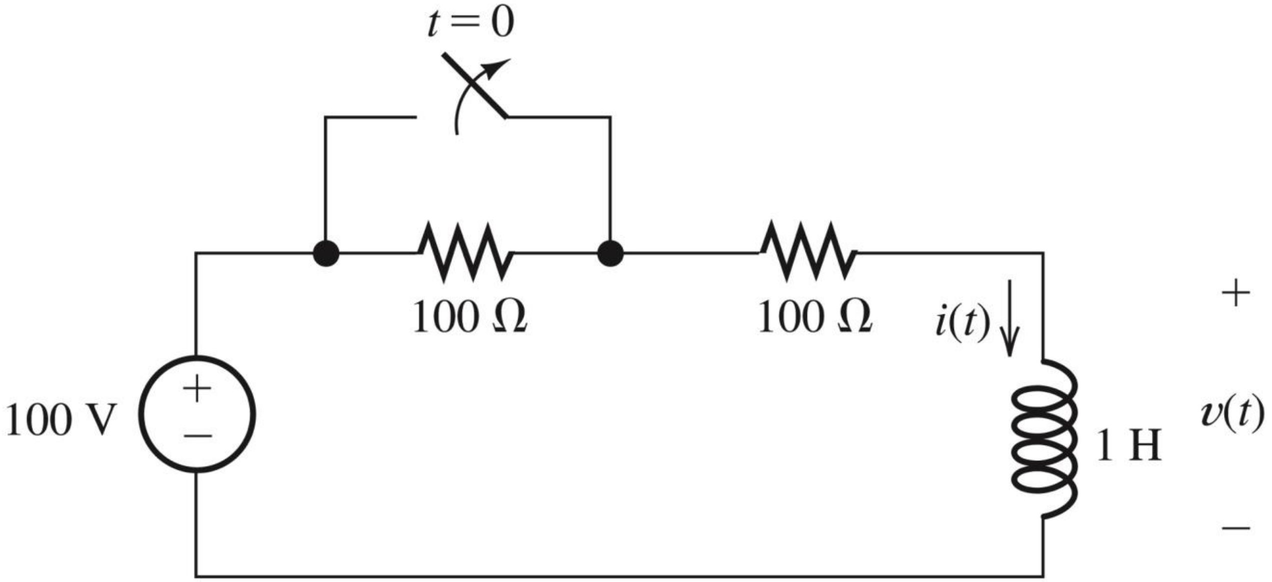
RL CIRCUITS – VIA DC STEADY STATE



will follow later

EXERCISE

Find i(t) and v(t) for t>0



PROBLEMS FOR THE EXERCISES SESSION

- P4.31
- P4.33
- P4.34
- P4.37
- P4.39
- P4.42