ACADEMY OF ENGINEERING AND AUTOMOTIVE
ELECTRICAL AND ELECTRONIC ENGINEERING

NETWORKS 4_

Week A_

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April 2023

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OF APPLIED SCIENCES

NET4 IN THE NET SERIES

Net1: constant voltage & current (DC)
Net2: sinusoidal voltage & current (AC)
Net3: sinusoidal voltage & current (AC), variation in frequency

Net4: transient (switching) behavior of systems

- 1st order networks, DC source
- 1st order networks, switching DC source
- Solving differential equations
  - Homogeneous solution
  - Complementary solution
- Steady-state situation
- Complementary solution for sinusoidal & exponential sources

SWITCHING BEHAVIOUR

- NET4: solve in time domain

- Next year: solve in frequency domain
  - Laplace transformation

NET4 PROGRAM BY WEEK

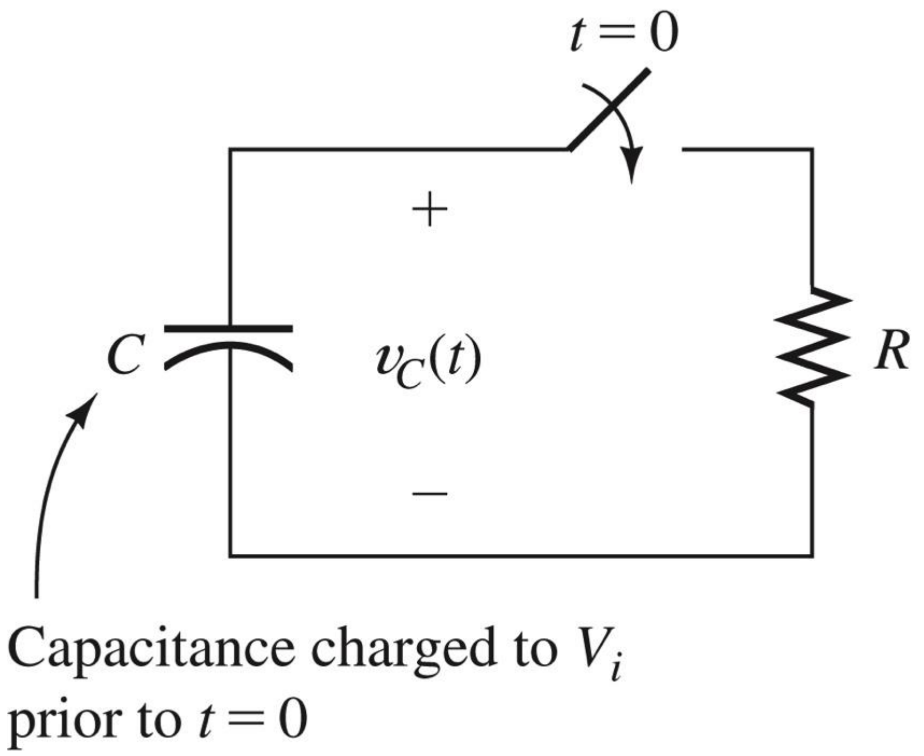
1. 1^{st} order RC networks – (dis)charging – DC source
2. 1^{st} order RC networks – (dis)charging – switched DC source
3. RL networks – Steady-state DC
4. RL networks – Switched DC source
5. RC & RL networks – complementary solution
6. <spare week>
7. Sample Exam

REMEMBER NET2: CAPACITOR & INDUCTOR

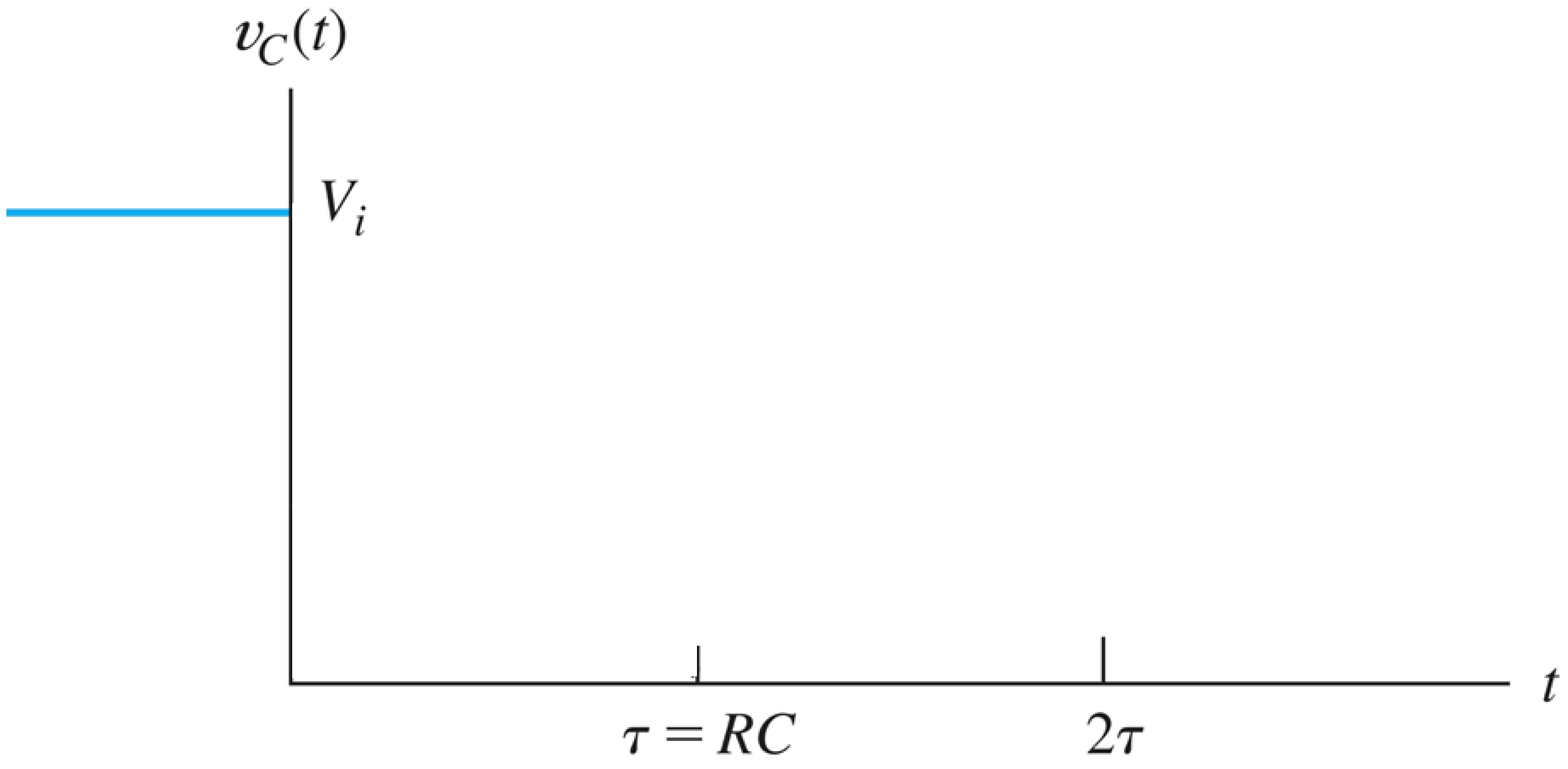
	Capacitor	Inductor
Voltage	$v = \frac{1}{C}\int i(t)dt$	$v = L\frac{di}{dt}$
Current	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int v(t)dt$
Power	$P(t) = v(t) \cdot i(t)$	
Energy	$W(t) = \frac{1}{2}Cv^2(t)$	$W(t) = \frac{1}{2}Li^2(t)$
Energy stored in:	Electric field	Magnetic field

DISCHARGING A CAPACITOR: ESTIMATIONS

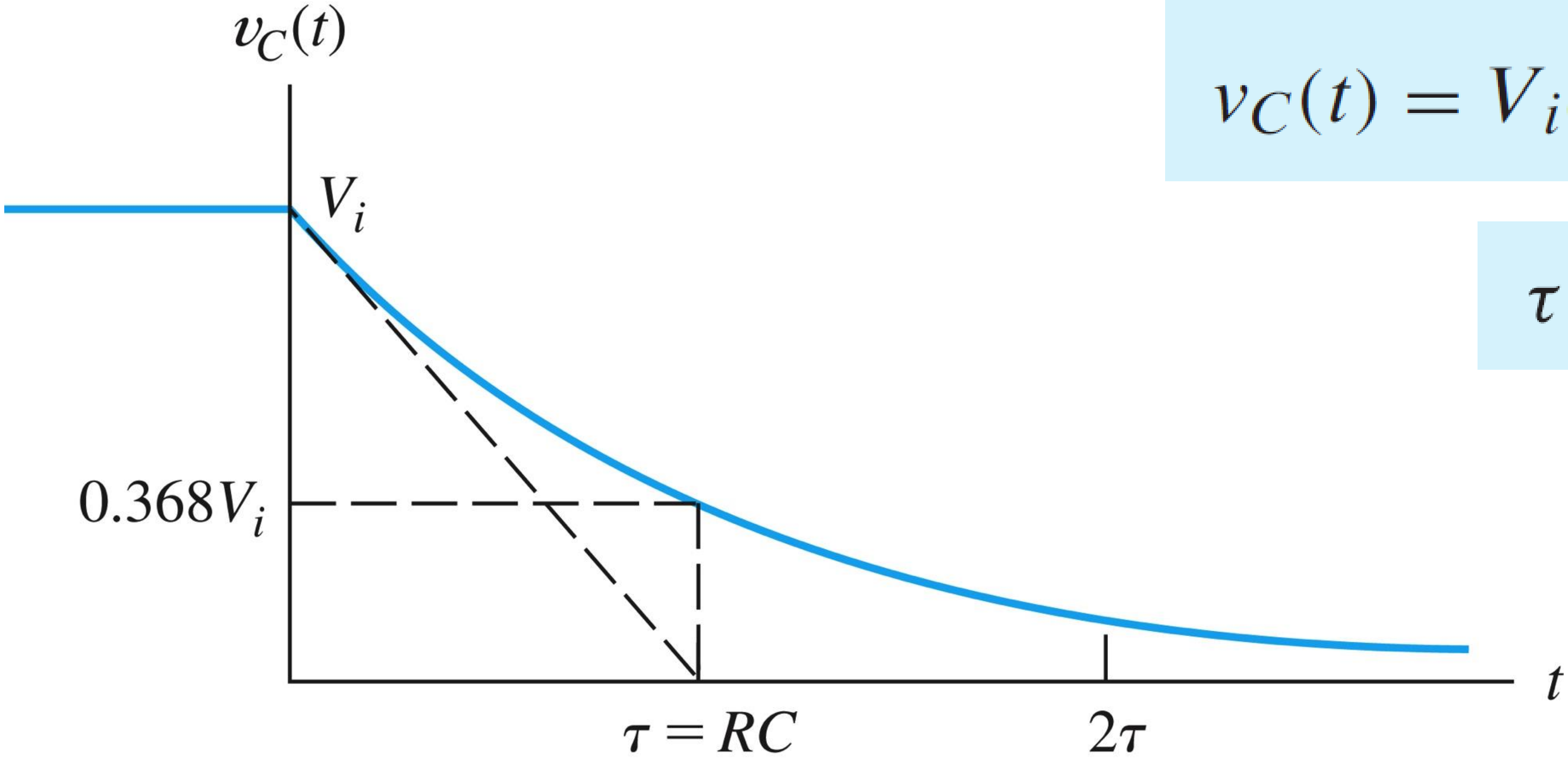
$V_i = 100$ V
$C = 10$ µF
$R = 1$ MΩ



WHAT WILL HAPPEN AFTER TIME=0?



SPOILER: IT'S AN EXPONENTIAL DECAY

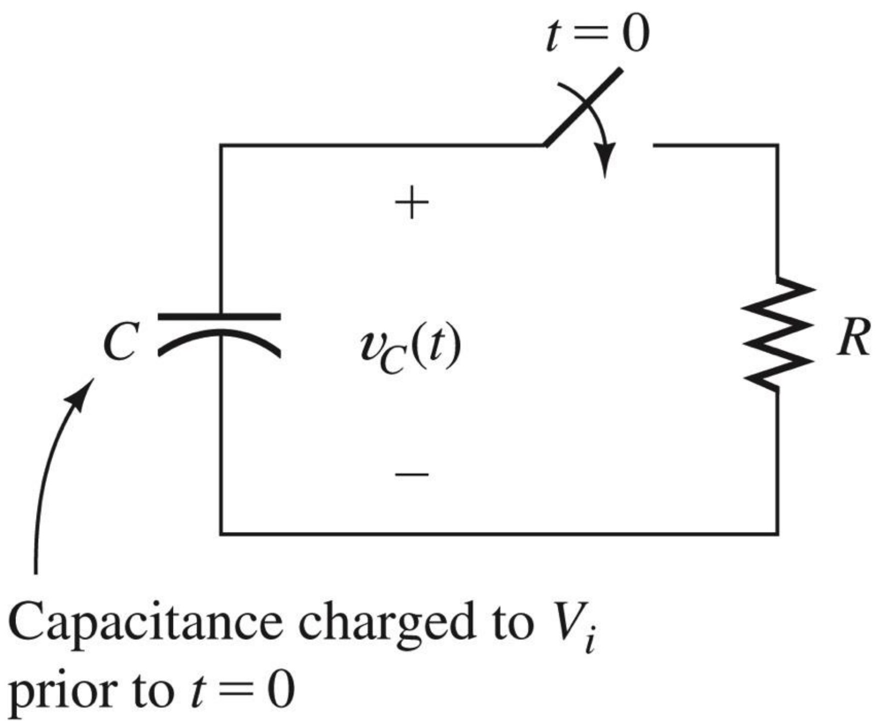


$$v_C(t) = V_i e^{-t/RC}$$

$$\tau = RC$$

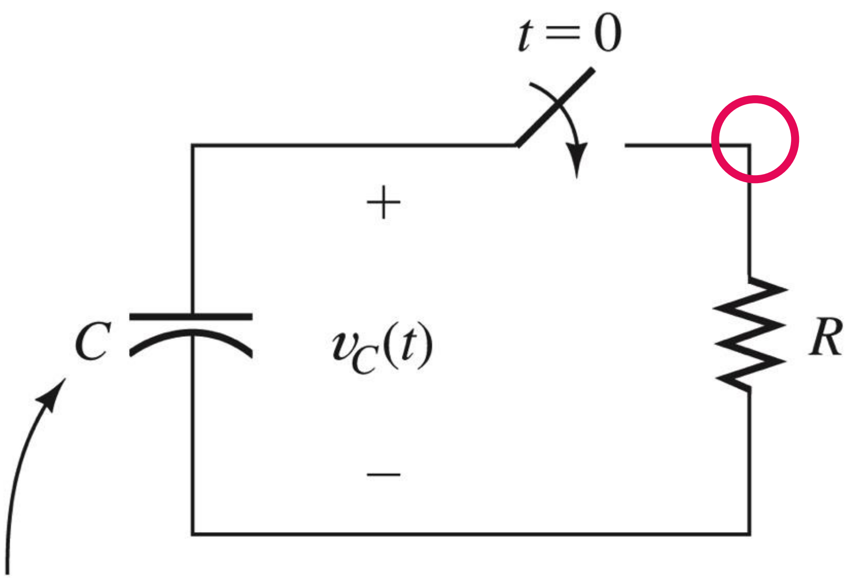
DISCHARGING A CAPACITOR: ANALYTIC SOLUTION

$V_i = 100$ V
$C = 10\ \mu F$
$R = 1\ M\Omega$



DISCHARGING A CAPACITOR: KCL

Vi = 100 V
C = 10 µF
R = 1 MΩ



Capacitance charged to V_i
prior to $t = 0$

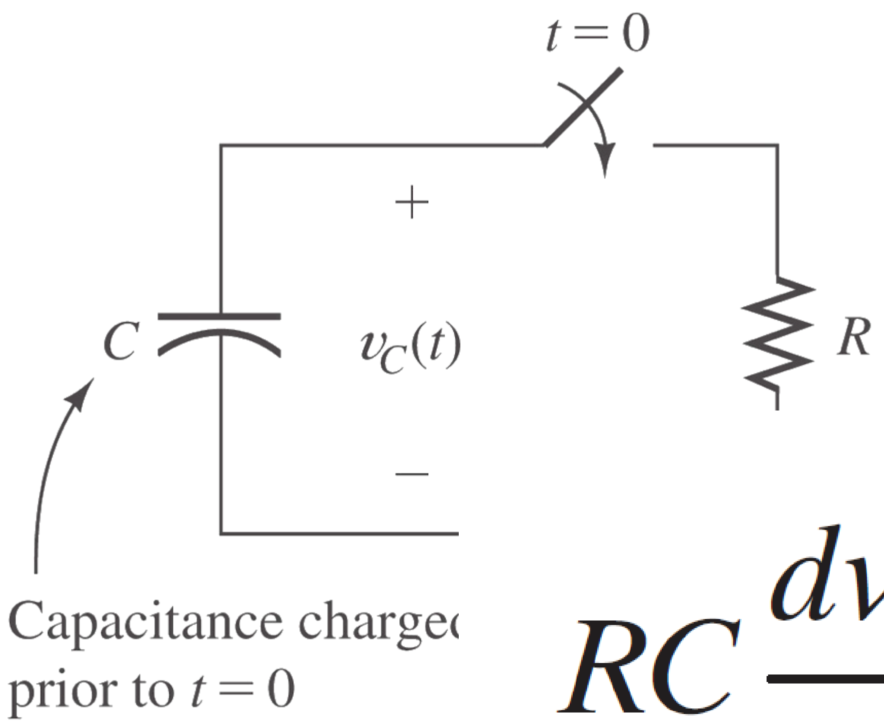
KCL:

- sum of currents = 0

Capacitor
$v = \frac{1}{C}\int i(t)dt$
$i = C\frac{dv}{dt}$
$P(t) =$

$$C\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R} = 0$$

DIFFERENTIAL EQUATION



$$C\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R} = 0$$

$$RC\frac{dv_C(t)}{dt} + v_C(t) = 0$$

DIFFERENTIAL EQUATION: HOW TO SOLVE

Educated guess $v_C = Ke^{st}$

$$RC\frac{dv_C(t)}{dt} + v_C(t) = 0$$

$$\frac{dv_C(t)}{dt} = Kse^{st}$$

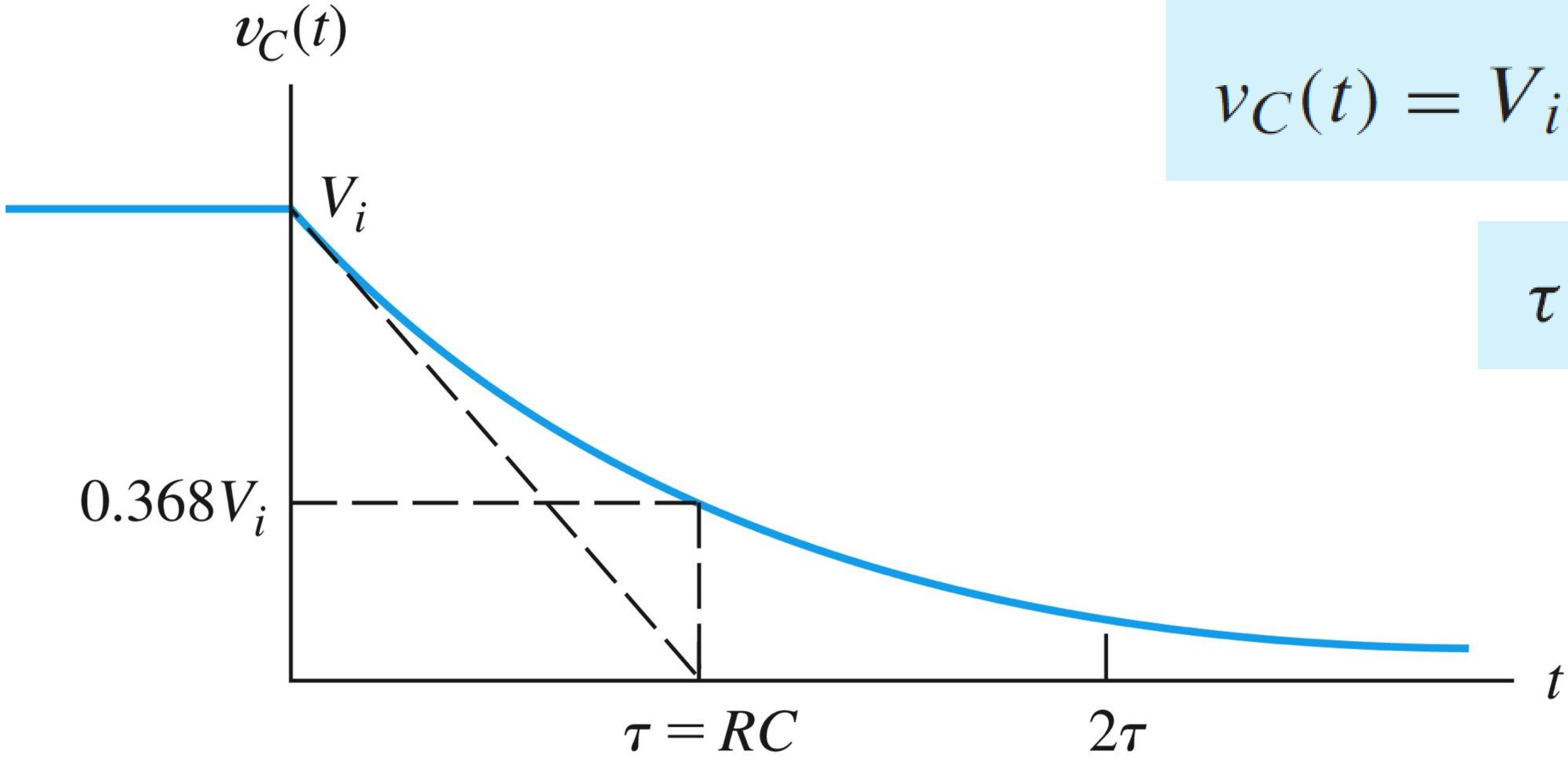
$$v_C(t) = Ke^{st} \qquad (4.2)$$

$$RCKse^{st} + Ke^{st} = 0 \qquad (4.3)$$

$$s = \frac{-1}{RC} \qquad (4.4)$$

Forget about K=0 ☺

INDEED IT'S AN EXPONENTIAL DECAY



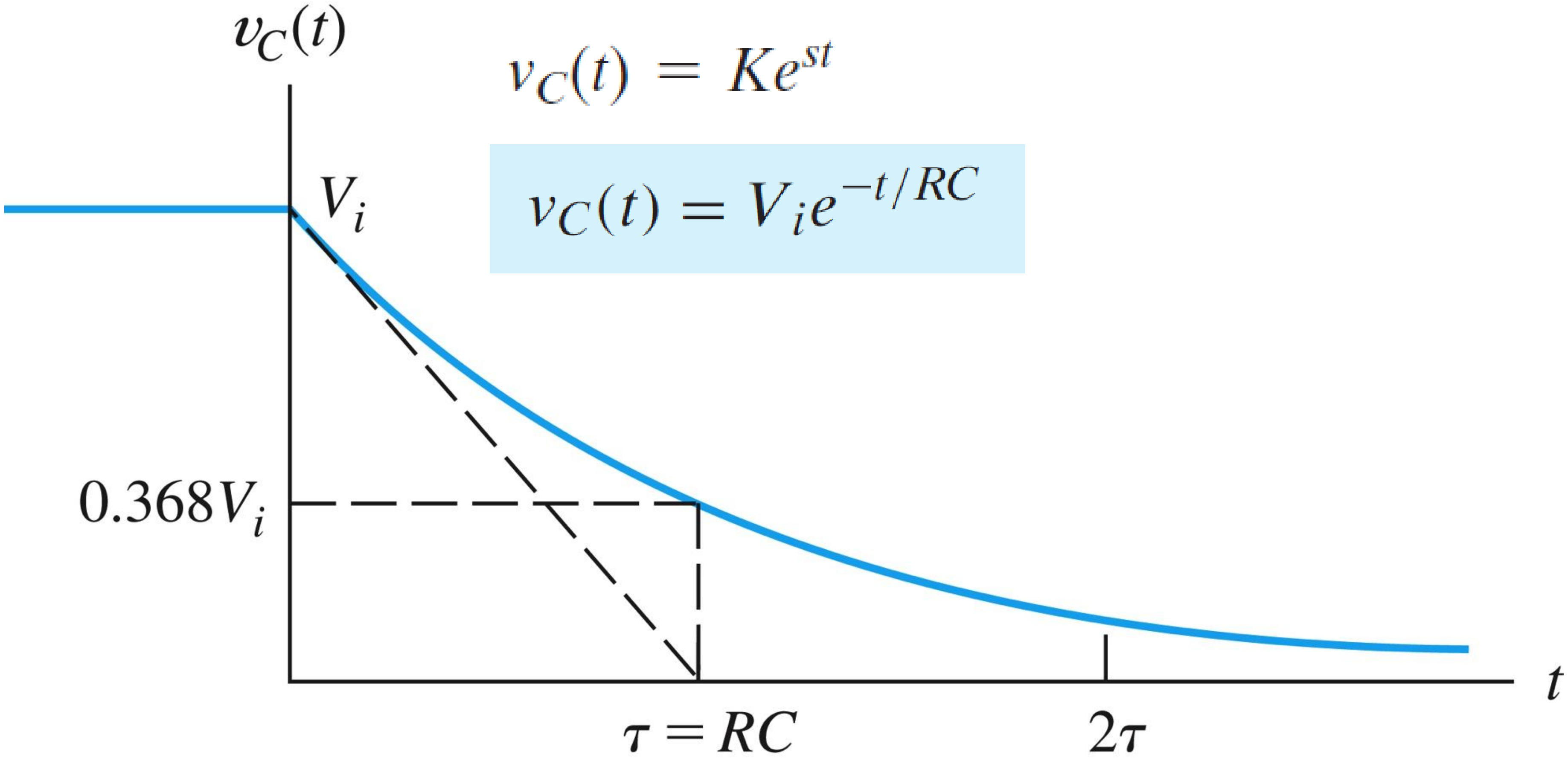
$$v_C(t) = V_i e^{-t/RC}$$

$$\tau = RC$$

HOW DID YOU FIND K=V$_I$?

$$v_C(t) = Ke^{st} \qquad (4.2)$$

$$\boxed{v_C(t) = V_i e^{-t/RC}}$$



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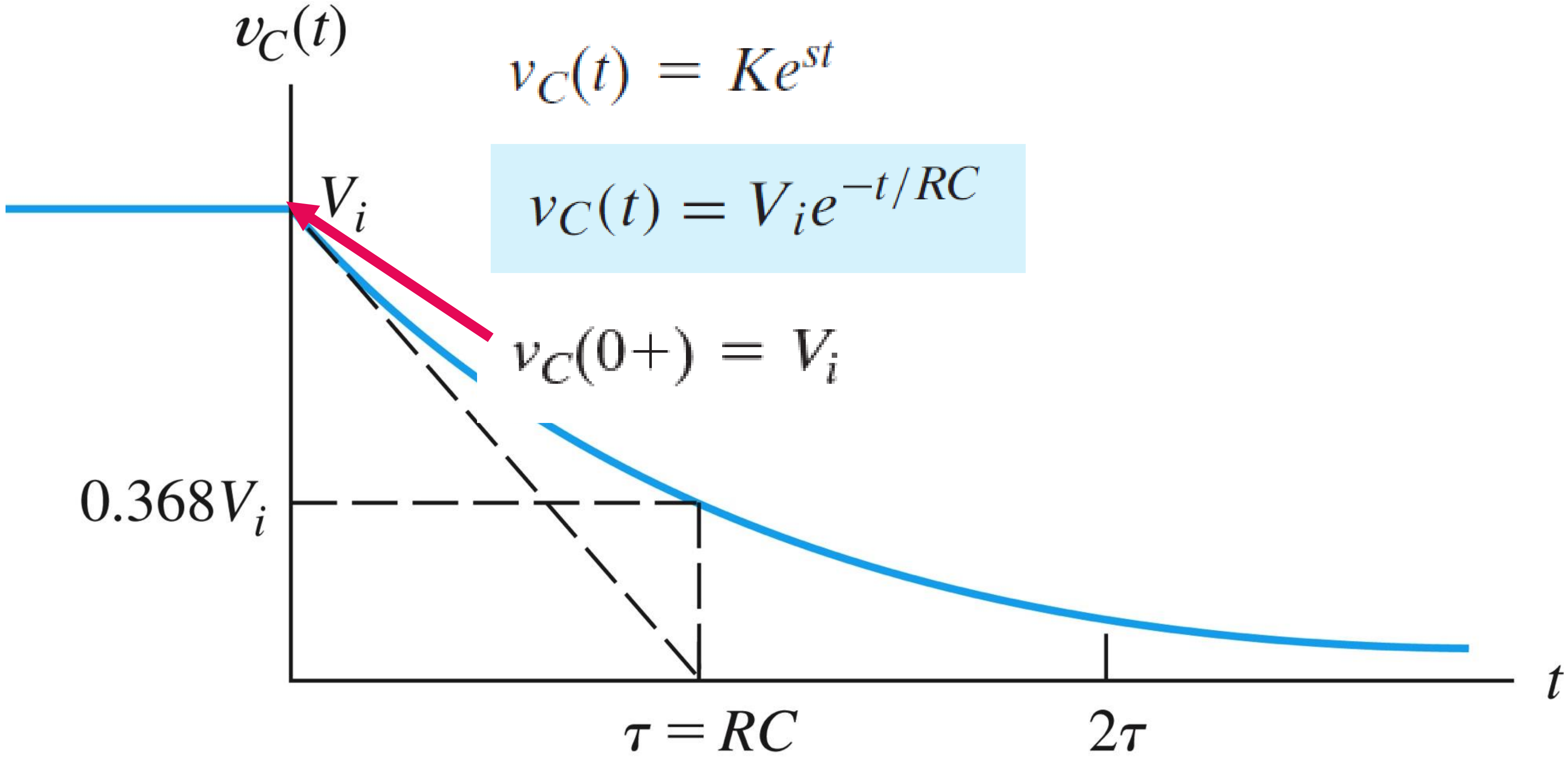
LOOK AT T=0 S: MUST BE CONTINUOUS

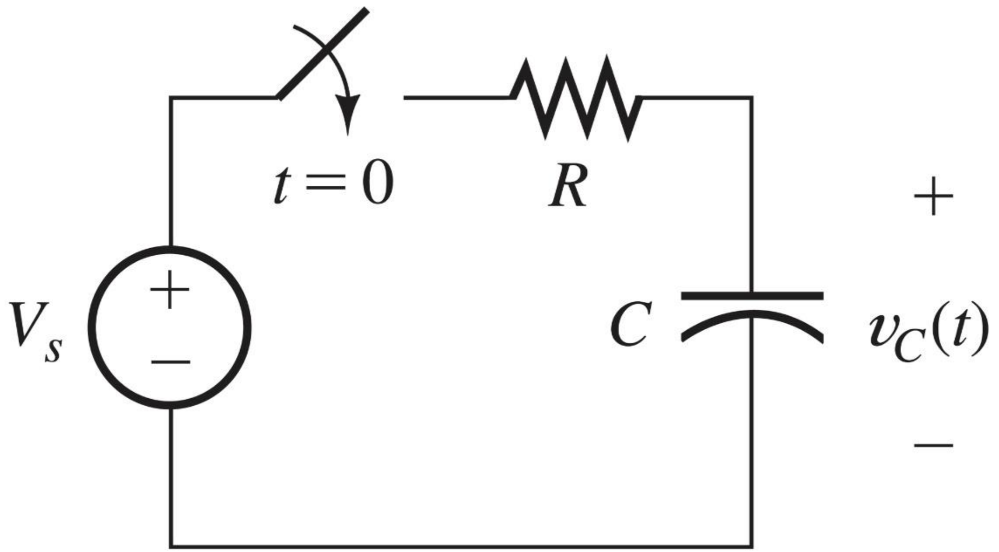
$$v_C(t) = Ke^{st} \qquad (4.2)$$

$$\boxed{v_C(t) = V_i e^{-t/RC}}$$

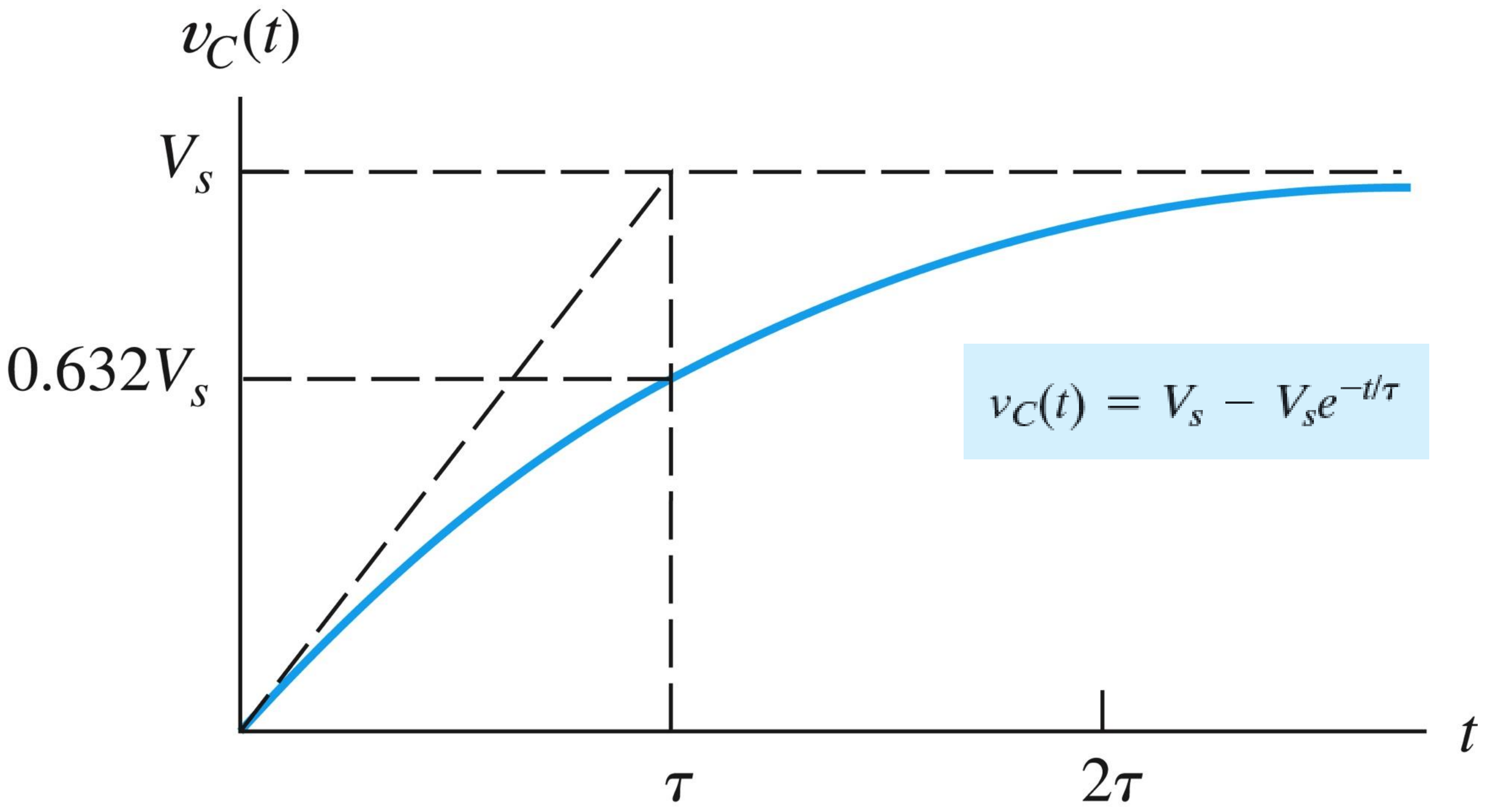
$$v_C(0+) = V_i \qquad (4.6)$$



CHARGING A CAPACITOR



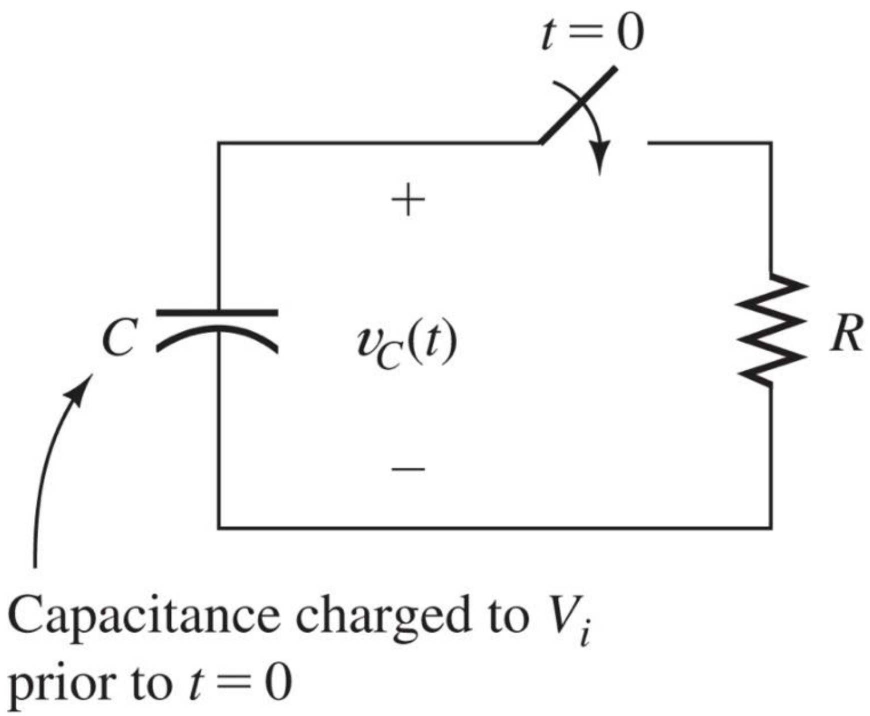
CHARGING A CAPACITOR



$$v_C(t) = V_s - V_s e^{-t/\tau} \quad (4.20)$$

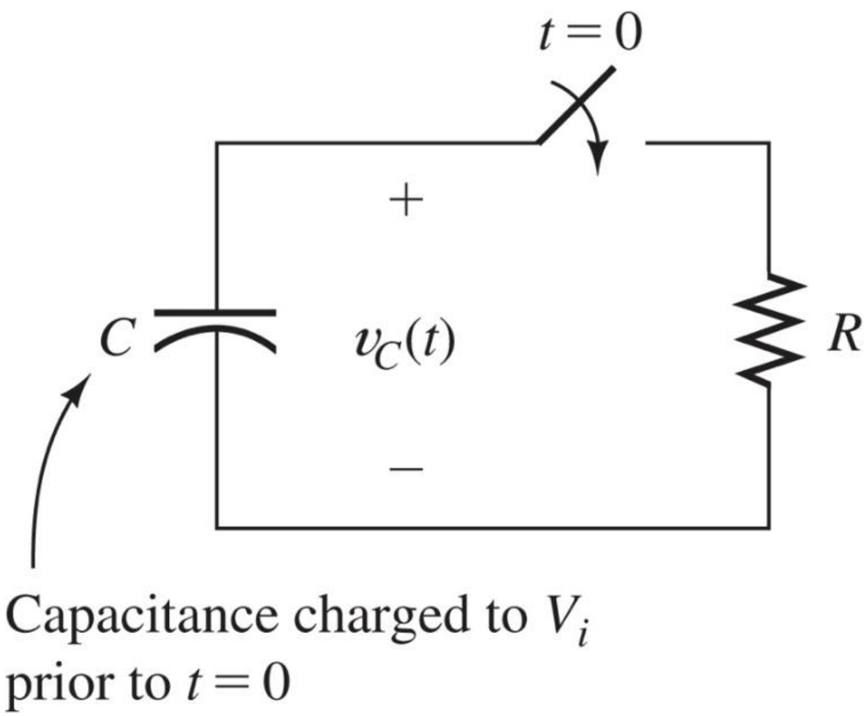
EXERCISE 4.1

Exercise 4.1 Suppose that $R = 5000\ \Omega$ and $C = 1\ \mu\text{F}$ in the circuit of Figure 4.1(a). Find the time at which the voltage across the capacitor reaches 1 percent of its initial value.



EXERCISE 4.1 - SOLUTION

Exercise 4.1 Suppose that $R = 5000\ \Omega$ and $C = 1\ \mu F$ in the circuit of Figure 4.1(a). Find the time at which the voltage across the capacitor reaches 1 percent of its initial value.



$$v_C(t) = V_i e^{-t/RC}$$

$$\tau = RC$$

$$\frac{v_C(t)}{V_i} = 1\% = 0.01$$

$$e^{\frac{-t}{RC}} = 0.01$$

$$\frac{-t}{RC} = \ln(0.01)$$

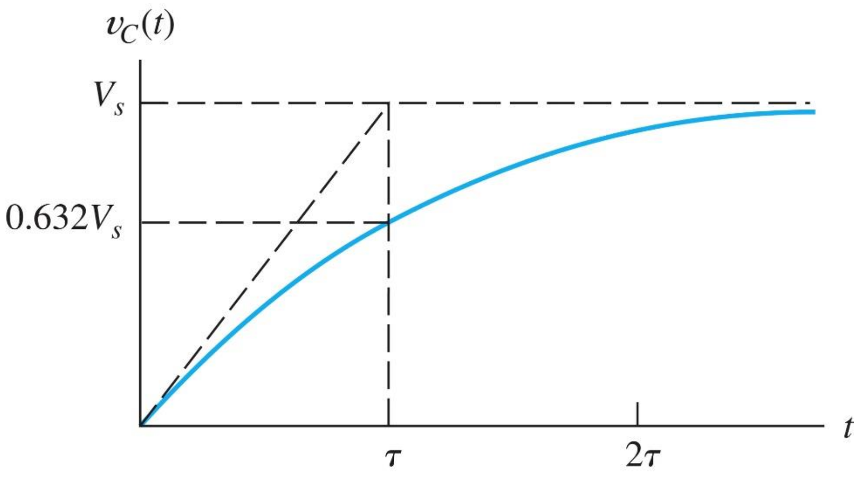
$$t = -RC \cdot \ln(0.01)$$

$$t \cong 23\ \text{ms}$$

EXERCISE 4.2

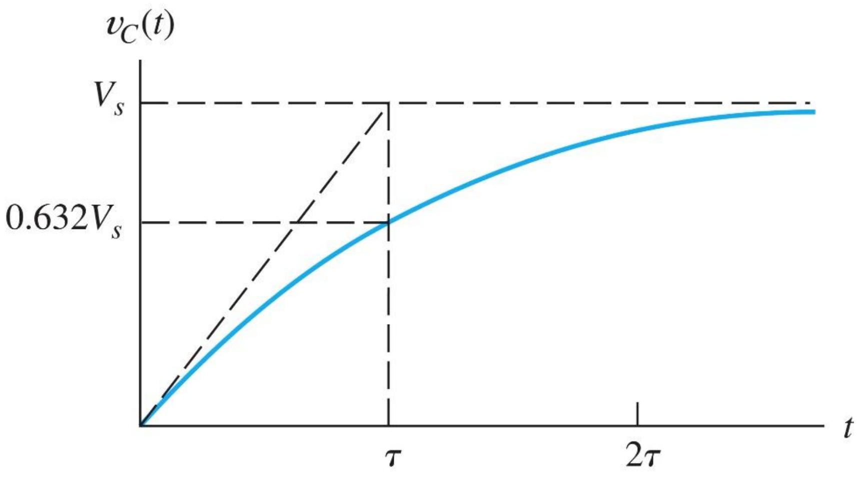
Exercise 4.2 Show that if the initial slope of $v_C(t)$ is extended, it intersects the final value at one time constant, as shown in Figure 4.4. [The expression for $v_C(t)$ is given in Equation 4.20.] □

$$v_C(t) = V_s - V_s e^{-t/\tau}$$



EXERCISE 4.2 – SOLUTION STEP 1

Exercise 4.2 Show that if the initial slope of $v_C(t)$ is extended, it intersects the final value at one time constant, as shown in Figure 4.4. [The expression for $v_C(t)$ is given in Equation 4.20.]



$$v_C(t) = V_s - V_s e^{-t/\tau}$$

$$v_C(t) = V_S\left(1 - e^{-t/\tau}\right)$$

slope: $$\frac{dv_C}{dt} = V_S \frac{d\left(1 - e^{-t/\tau}\right)}{dt} = V_S \frac{1}{\tau} e^{-t/\tau}$$

so, at t=0: slope = $\frac{V_S}{\tau}$

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PROBLEMS TO FINISH BEFORE NEXT CLASS

- P4.2
- P4.5
- P4.7
- P4.10
- P4.11
- P4.15