SCHOOL OF ENGINEERING, ELECTRICAL AND ELECTRONIC ENGINEERING

NETWORKS 4_

Week E_

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May 2023

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NET4 PROGRAM BY WEEK

1. 1^{st} order RC networks – discharging – DC source
2. 1^{st} order RC networks – charging –DC source
3. RL networks – Steady-state DC
4. RL networks – Switched DC source
5. RC & RL networks – complementary solution
6. <spare week>
7. Exam Prep

CAPACITOR & INDUCTOR RELATIONS

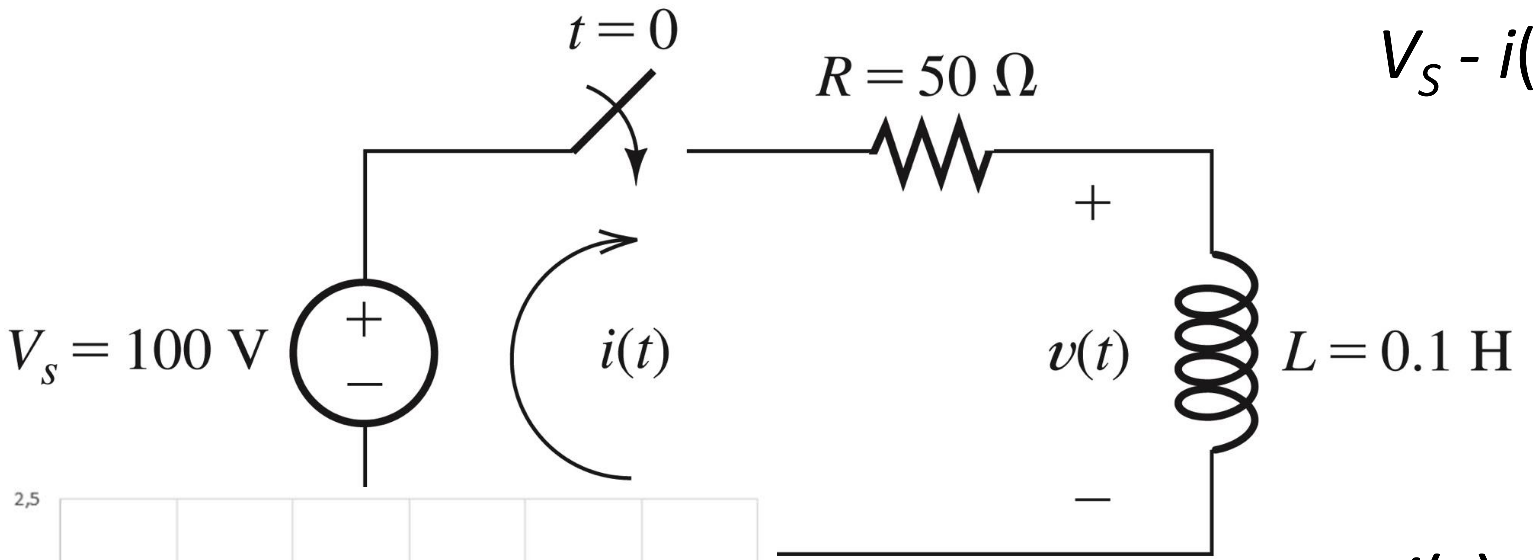
	Capacitor	Inductor
Voltage	$v = \frac{1}{C}\int i(t)dt$	$v = L\frac{di}{dt}$
Current	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int v(t)dt$
Power	$P(t) = v(t) \cdot i(t)$	
Energy	$W(t) = \frac{1}{2}Cv^2(t)$	$W(t) = \frac{1}{2}Li^2(t)$
Energy stored in:	Electric field	Magnetic field

WEEK D: SWITCHED DC SOURCE, RL CIRCUITS

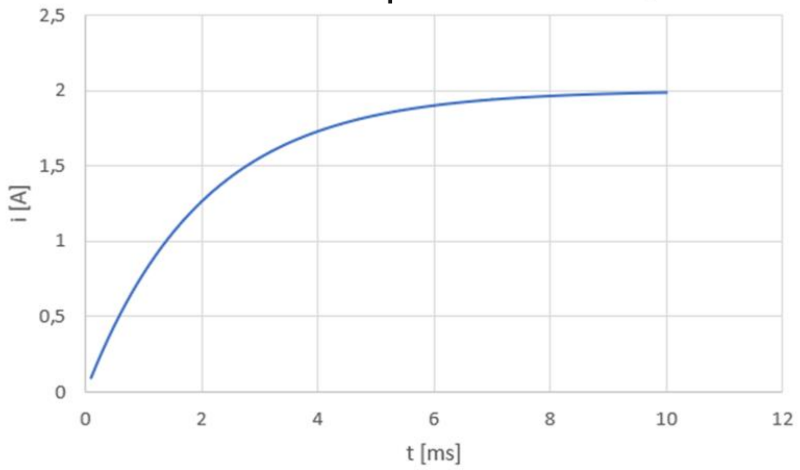
At $t = 0$ we close the switch

Inductor
$v = L\dfrac{di}{dt}$

V_S - $i(t)\cdot R$ - Ldi/dt = 0



$$i(t) + \frac{L}{R}\frac{di(t)}{dt} = \frac{V_S}{R}$$



$i(t) = 2\text{-}2e^{-500t}$ A

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THIS WEEK

- Find expressions for $i(t)$ and $v(t)$ using *just steady-state values*
  - *and smart reasoning* ☺

- Switching on AC sources

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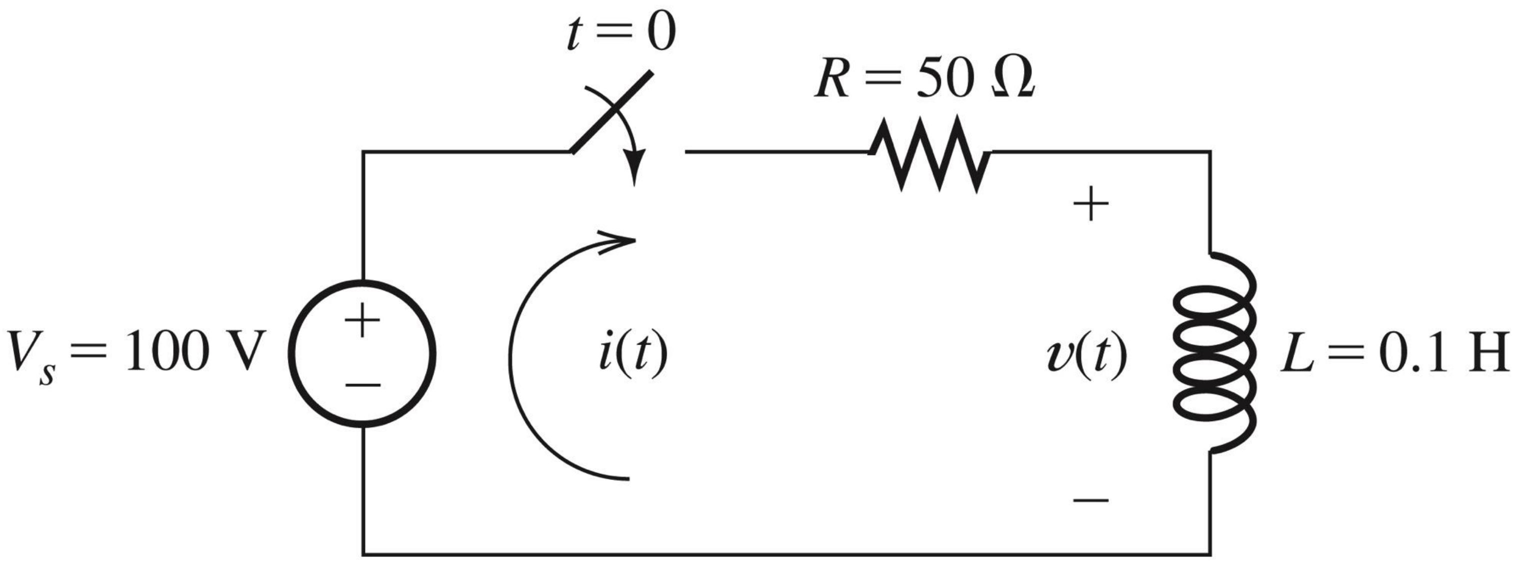
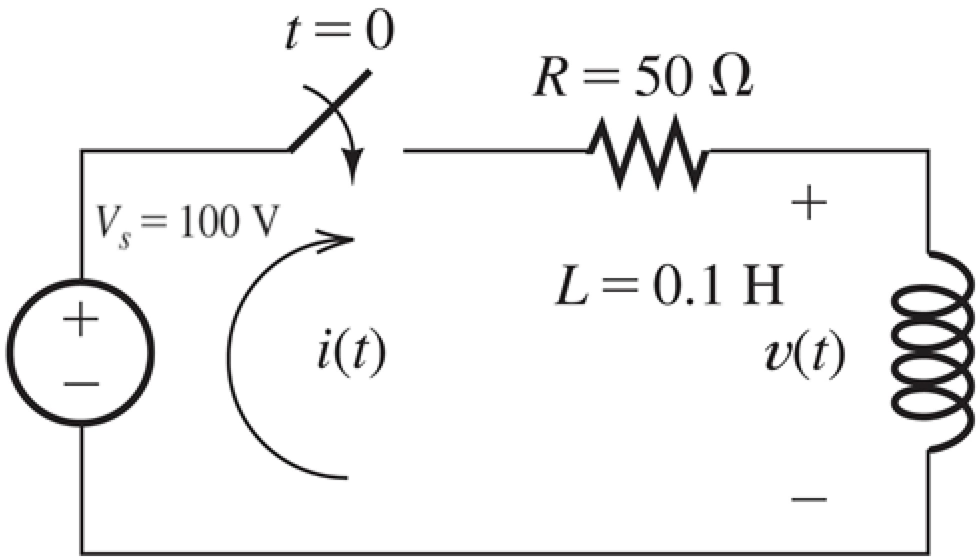
RL CIRCUITS – VIA DC STEADY STATE

Find expressions for $i(t)$ and $v(t)$ for $t > 0$



RL CIRCUITS – VIA DC STEADY STATE



- at t = 0- s, $i(t)$ = 0 A
- at $t = \infty$, $i(t)$ = 100/50 = 2 A
- time constant $\tau = L/R$ = 0.002 s = 2 ms

- general form $i(t) = K_1 + K_2 e^{-t/\tau}$

- we know all we need to know:
  - for $t = \infty$, we get K_1 = 2 A
  - for t = 0- s: $K_1 + K_2 = 0 \rightarrow K_2 = -2$ A

- $i(t) = 2 - 2e^{-t/0.002} = 2 - 2e^{-500t}$

GENERAL (AC) SOURCES

$$i(t) + \frac{L}{R}\frac{di(t)}{dt} = \frac{V_S}{R}$$

$$RC\frac{dv_C(t)}{dt} + v_C(t) = V_s$$

General form:

$$\boldsymbol{\tau}\frac{d\boldsymbol{x}(\boldsymbol{t})}{d\boldsymbol{t}} + \boldsymbol{x}(\boldsymbol{t}) = \boldsymbol{f}(\boldsymbol{t})$$

τ	depends only on component values ($\tau = RC$, $\tau = L/R$)
x(t)	*i*(*t*), *v*(*t*) from circuit equations (KVL, KCL)
f(t)	*source(s)*

COMPLEMENTARY SOLUTION

$$\boldsymbol{\tau}\frac{d\boldsymbol{x}(\boldsymbol{t})}{d\boldsymbol{t}} + \boldsymbol{x}(\boldsymbol{t}) = \boldsymbol{f}(\boldsymbol{t})$$

COMPLEMENTARY SOLUTION

$$\boldsymbol{\tau}\frac{d\boldsymbol{x}(\boldsymbol{t})}{d\boldsymbol{t}} + \boldsymbol{x}(\boldsymbol{t}) = \mathbf{0}$$

- Set the sources to 0: *homogeneous equation*
- Complementary Solution $\boldsymbol{x_c}(\boldsymbol{t}) = \boldsymbol{K}\boldsymbol{e}^{-\frac{\boldsymbol{t}}{\boldsymbol{\tau}}}$
- To be done: find *K*
  - *from initial conditions (typically, at t = 0 s)*

PARTICULAR SOLUTION

Original (full) equation $$\boldsymbol{\tau \frac{dx(t)}{dt} + x(t) = f(t)}$$

- Do an intelligent guess that fits: $\boldsymbol{x_p(t)}$
- Be inspired by $\boldsymbol{f(t)}$ and its time derivatives
  - $\boldsymbol{f(t) = constant}$ -> $\boldsymbol{x_p(t) = A}$
  - $\boldsymbol{f(t) = linear}$ -> $\boldsymbol{x_p(t) = A.t + B}$
  - $\boldsymbol{f(t) = quadratic}$ -> $\boldsymbol{x_p(t) = A.t^2 + B.t + C}$
  - $\boldsymbol{f(t) = sinusoidal}$ -> $\boldsymbol{x_p(t) = A \sin \omega t + B \cos \omega t}$

COMBINE COMPLEMENTARY & PARTICULAR SOLUTIONS

Original (full) equation $\boldsymbol{\tau \frac{dx(t)}{dt} + x(t) = f(t)}$

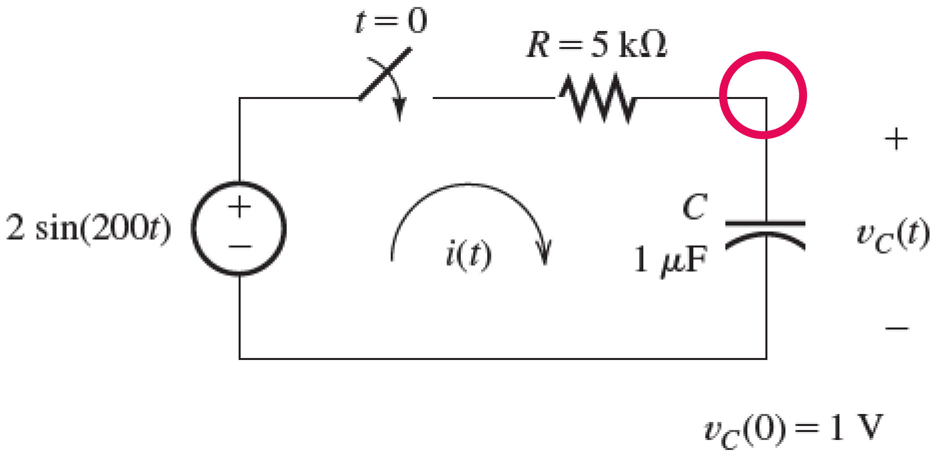
- Sum complementary & particular solutions:
  - $\boldsymbol{x(t) = x_c(t) + x_p(t)}$

- Now remember: $\boldsymbol{x_c(t) = Ke^{-\frac{t}{\tau}}}$
  - Need to find *K*
  - *Use initial conditions*

NEW RECIPE FOR SIMPLE CIRCUITS (RC & RL)

1. Use one of Kirchhoff's laws to get a circuit equation
2. If the equation has integrals, differentiate all terms to get a pure differential equation
   1. This will get you τ
3. Set the equation equal to 0 (homogeneous equation)
   1. *This will get you the Complementary Solution* $Ke^{-t/\tau}$
4. Do an intelligent guess to solve the full equation
   1. *This will get you the Particular Solution*
5. Write the now fully known solution – Complementary + Particular
   1. using *R*, *L*, *C*, τ, …
6. Use starting conditions (*t*=0-, *t*=0+) to get K

EXAMPLE 4.6



Flashback from NET4 week B

1. KCL at top-right node: sum = 0
   1. incoming: $(v_C(t) - V_S)/R$
   2. outgoing: $C \cdot dv_C/dt$

Capacitor
$v = \frac{1}{C}\int i(t)dt$
$i = C\frac{dv}{dt}$
$P(t) =$

$$C\frac{dv_C(t)}{dt} + \frac{v_C(t) - V_s}{R} = 0$$

$$RC\frac{dv_C(t)}{dt} + v_C(t) = 2\sin(200t)$$

3. Set the equation equal to 0 (homogeneous equation)
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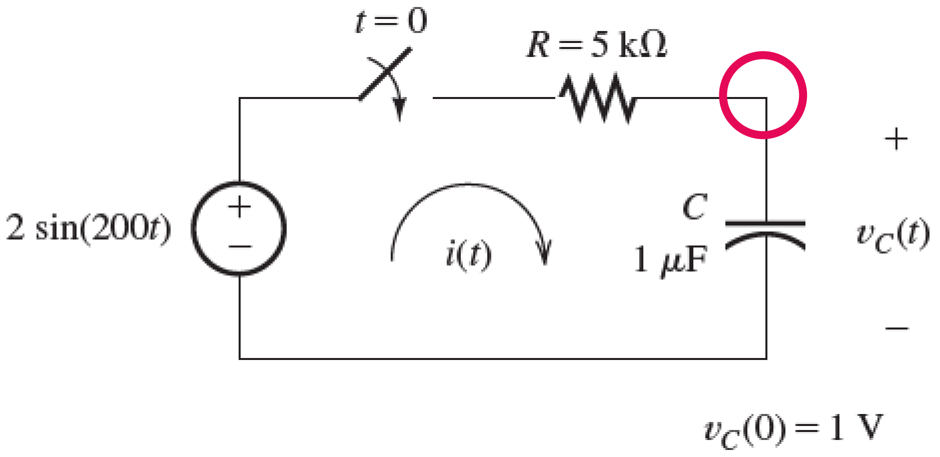

Apply the recipe step 3

$$RC\frac{dv_C(t)}{dt} + v_C(t) = 0$$

Compl. Sol. $v_{CC}(t) = Ke^{-t/RC}$

$$RC\frac{dv_C(t)}{dt} + v_C(t) = 2\sin(200t)$$

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3. Set the equation equal to 0 (homogeneous equation)
    1. *This will get you the Complementary Solution $Ke^{-t/\tau}$*
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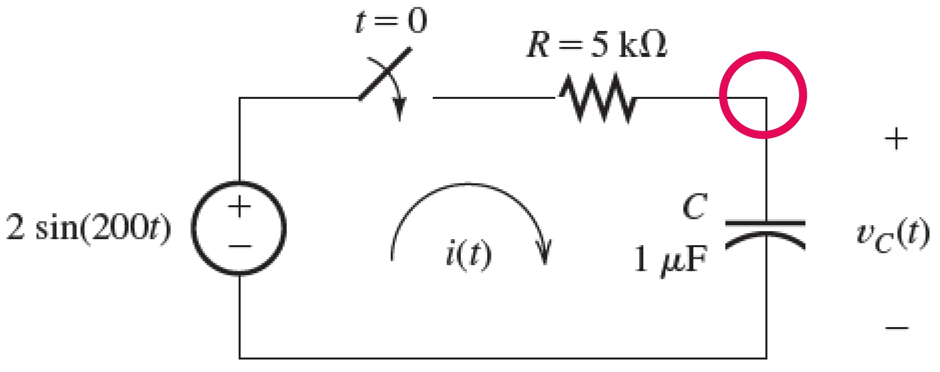

$v_C(0) = 1$ V

Apply the recipe step 4

$$RC\frac{dv_C(t)}{dt} + v_C(t) = 2\sin(200t)$$

Partic. Sol. $v_{CP}(t) = \boldsymbol{A}\sin 200\boldsymbol{t} + \boldsymbol{B}\cos 200\boldsymbol{t}$

1. *substitute into equation*
2. *turns out: A = 1   B = -1*

So: $v_{CP}(t) = \sin 200\boldsymbol{t} - \cos 200\boldsymbol{t}$

$$RC\frac{dv_C(t)}{dt} + v_C(t) = 2\sin(200t)$$

3. Set the equation equal to 0 (homogeneous equation)
   1. *This will get you the Complementary Solution* $Ke^{-t/\tau}$
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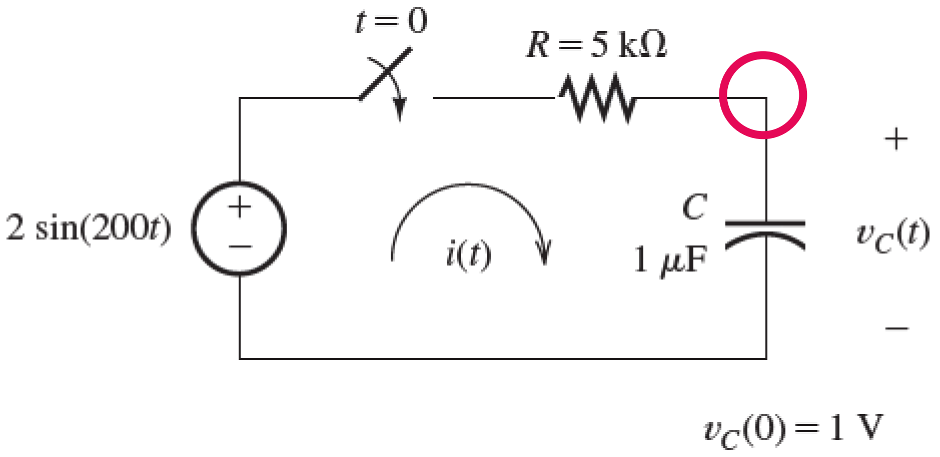

Apply the recipe step 5

Compl. Sol. $v_{CC}(t) = Ke^{-t/RC}$

Partic. Sol. $v_{CP}(t) = \sin 200\boldsymbol{t} - \cos 200\boldsymbol{t}$

$v_C(t) = Ke^{-t/RC} + \sin 200\boldsymbol{t} - \cos 200\boldsymbol{t}$

$$RC\frac{dv_C(t)}{dt} + v_C(t) = 2\sin(200t)$$

3. Set the equation equal to 0 (homogeneous equation)
    1. *This will get you the Complementary Solution* $Ke^{-t/\tau}$
4. Do an intelligent guess to solve the full equation
    1. *This will get you the Particular Solution*
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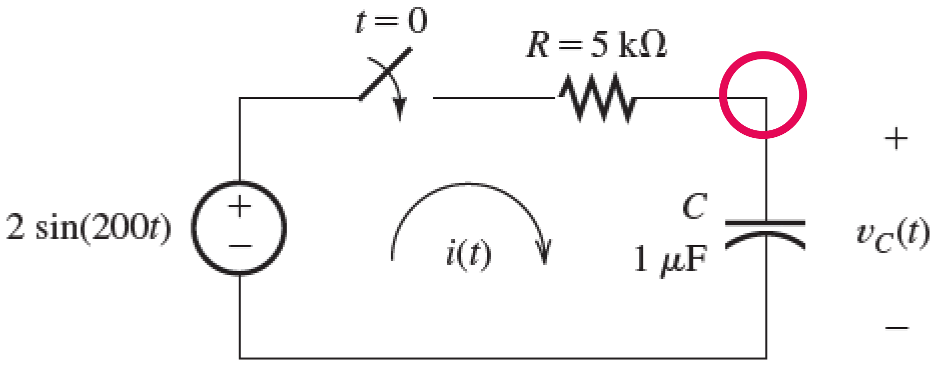
    1. using R, L, C, τ, ...
6. Use starting conditions (t=0-, t=0+) to get K

EXAMPLE 4.6



$v_C(0) = 1$ V

Apply the recipe step 6

$v_C(t) = Ke^{-t/RC} + \sin 200\boldsymbol{t} - \cos 200\boldsymbol{t}$

At t = 0 s, $v_C(t)$ = 1 V

- $K + 0 - 1 = 1$
- $\rightarrow K = 2$

$$\begin{aligned} v_C(t) &= 2e^{-t/RC} + \sin 200\boldsymbol{t} - \cos 200\boldsymbol{t} \\ &= 2e^{-200t} + \sin 200\boldsymbol{t} - \cos 200\boldsymbol{t} \end{aligned}$$

$$RC\frac{dv_C(t)}{dt} + v_C(t) = 2\sin(200t)$$

3. Set the equation equal to 0 (homogeneous equation)
   1. *This will get you the Complementary Solution* $Ke^{-t/\tau}$
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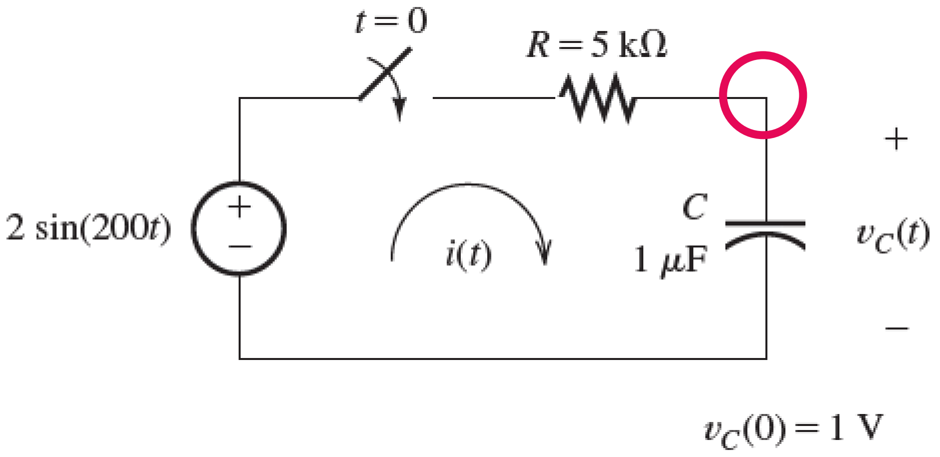

Check Hambley's answer for *i*(*t*)

$v_C(t) = 2e^{-200t} + \sin 200t - \cos 200t$

$i(t) = C \cdot dv_C(t)/dt$

$= C \cdot [-400e^{-200t} + 200\cos 200t + 200 \sin 200t]$ A

$= -400e^{-200t} + 200\cos 200t + 200 \sin 200t$ μA

$$i(t) = 200\cos(200t) + 200\sin(200t) - 400e^{-t/RC}\ \mu\text{A} \qquad (4.56)$$

$$RC\frac{dv_C(t)}{dt} + v_C(t) = 2\sin(200t)$$

PROBLEMS FOR THE EXERCISES SESSION

- P4.45
- P4.47
- P4.49
- P4.50
- P4.51
- P4.54