Primer trabajo teoría de la información y la comunicación

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1. Desarrollar en series de Fourier

$$f(t) = t^2,\ -\pi \le t \le \pi$$

R/ Se plantea la serie de fourier de la siguiente manera:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n\, cos(n\omega_0 t) + b_n\, sin(n\omega_0 t))$$

Por lo que los coeficientes a_x se pueden definir de la siguiente forma:

$$\begin{aligned} a_0 &= \frac{2}{T}\int_{-T/2}^{T/2} f(t)dt \\ a_n &= \frac{2}{T}\int_{-T/2}^{T/2} f(t)cos(n\omega_0 t)dt \\ b_n &= \frac{2}{T}\int_{-T/2}^{T/2} f(t)sin(n\omega_0 t)dt \end{aligned}$$

Reemplazando por las variables del enunciado se llega a:

$$\begin{aligned} a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} t^2 dt \\ a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} t^2 cos(n\,t)dt \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} t^2 sin(n\,t)dt \end{aligned}$$

Resolviendo para a_0:

$$\begin{aligned} a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} t^2 dt \\ &= \frac{1}{\pi}\frac{t^3}{3}\bigg|_{t=-\pi}^{\pi} \\ &= \frac{1}{3\pi}(\pi^3 - (-\pi)^3) \\ &= \frac{2}{3}\pi^2 \end{aligned}$$

Resolviendo para a_n:

$$\begin{aligned} a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} t^2 cos(n\,t)dt \\ &= \frac{1}{\pi}\left(\frac{t^2 sin(n\,t)}{n} + \frac{2t\,cos(n\,t)}{n^2} - \frac{2\,sin(n\,t)}{n^3}\right)\bigg|_{t=-\pi}^{\pi} \\ &= \frac{\pi^2 sin(n\,\pi)}{n\pi} + \frac{2\pi\,cos(n\,\pi)}{n^2\pi} - \frac{2\,sin(n\,\pi)}{n^3\pi} \\ &\quad -\frac{\pi^2 sin(n(-\pi))}{n\pi} + \frac{2\pi\,cos(n(-\pi))}{n^2\pi} + \frac{2\,sin(n(-\pi))}{n^3\pi} \\ &= 2\pi^2 sinc(n\,\pi) + \frac{4\pi}{n}cosc(n\,\pi) - \frac{4}{n^2}sinc(n\,\pi) \end{aligned}$$

Resolviendo para b_n:

$$\begin{aligned} b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} t^2 sin(n\,t)dt \\ &= \frac{1}{\pi}\left(-\frac{t^2 cos(n\,t)}{n} - \frac{2t\,sin(n\,t)}{n^2} + \frac{2\,cos(n\,t)}{n^3}\right)\bigg|_{t=-\pi}^{\pi} \\ &= -\frac{\pi^2 cos(n\,\pi)}{n\pi} - \frac{2\pi\,sin(n\,\pi)}{n^2\pi} + \frac{2\,cos(n\,\pi)}{n^3\pi} \\ &\quad +\frac{\pi^2 cos(n(-\pi))}{n\pi} - \frac{2\pi\,sin(n(-\pi))}{n^2\pi} - \frac{2\,cos(n(-\pi))}{n^3\pi} \\ &= 0 \end{aligned}$$

De manera que la serie de Fourier de $f(t)$ queda de la siguiente forma:

$$\begin{aligned} f(t) &= \frac{\pi^2}{3} + \sum_{n=1}^{\infty} (a_n\, cos(n\, t)) \\ a_n &= \left(2\pi^2 - \frac{4}{n^2}\right) sinc(n\,\pi) + \frac{4\pi}{n} cosc(n\,\pi) \end{aligned}$$

2. Desarrollar en series de Fourier

$$f(t) = t\, sin(t),\ -\pi \leq t \leq \pi$$

3. Desarrollar en series de Fourier

$$f(t) = t,\ -\pi \leq t \leq \pi$$

R/ Se plantea la serie de fourier de la siguiente manera:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n\, cos(n\omega_0 t) + b_n\, sin(n\omega_0 t))$$

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Reemplazando por las variables del enunciado se llega a:

$$\begin{aligned} a_0 &= \frac{1}{\pi} \int_{-\pi}^{\pi} t\, dt \\ a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} t\, cos(n\, t) dt \\ b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} t\, sin(n\, t) dt \end{aligned}$$

Resolviendo para a_0:

$$\begin{aligned} a_0 &= \frac{1}{\pi} \int_{-\pi}^{\pi} t\, dt \\ &= \frac{1}{\pi} \frac{t^2}{2} \bigg|_{t=-\pi}^{\pi} \\ &= \frac{1}{2\pi} (\pi^2 - (-\pi)^2) \\ &= 0 \end{aligned}$$

Resolviendo para a_n:

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} t\, cos(n\, t) dt \\ &= \frac{1}{\pi} \left(\frac{t\, sin(n\, t)}{n} + \frac{cos(n\, t)}{n^2} \right) \bigg|_{t=-\pi}^{\pi} \\ &= \frac{\pi\, sin(n\,\pi)}{n\pi} + \frac{cos(n\,\pi)}{n^2\pi} - \frac{(-\pi) sin(n(-\pi))}{n\pi} - \frac{cos(n(-\pi))}{n^2\pi} \\ &= 0 \end{aligned}$$

Resolviendo para b_n:

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} t\, sin(n\, t) dt \\ &= \frac{1}{\pi} \left(-\frac{t\, cos(n\, t)}{n} + \frac{sin(n\, t)}{n^2} \right) \bigg|_{t=-\pi}^{\pi} \\ &= -\frac{\pi\, cos(n\,\pi)}{n\pi} + \frac{sin(n\,\pi)}{n^2\pi} + \frac{(-\pi) cos(n(-\pi))}{n\pi} - \frac{sin(n(-\pi))}{n^2\pi} \\ &= \frac{2}{n} sinc(n\,\pi) - 2\pi\, cosc(n\,\pi) \end{aligned}$$

De manera que la serie de Fourier de $f(t)$ queda de la siguiente forma:

$$f(t) = \sum_{n=1}^{\infty} \left(\left(\frac{2}{n} sinc(n\,\pi) - 2\pi\, cosc(n\,\pi) \right) sin(n\, t) \right)$$

4. Desarrollar en series de Fourier

$$f(t) = \begin{cases} \pi + t, & -\pi \leq t \leq 0 \\ t, & 0 \leq t \leq \pi \end{cases}$$

5. Hallar el periodo

a) $f(t) = sin(\frac{2\pi}{b-a})t$

b) $f(t) = sin(t) + \frac{1}{3} sin(3t) + \frac{1}{5} sin(5t)$

c) $f(t) = cos(10t) + cos((10 + \pi)t)$