Data Transparency in Disaster Risk Management: Estimating Actual COVID-19 Cases Amid Underestimation Challenges

M. Aykut Attar* Ayça Tekin-Koru†

October 17, 2024

Supplementary Appendix

Appendix A. Calibration Results

The seven tables presented below collect the detailed calibration results for seven different values of δ, i.e., the fixed death rate among the resolving cases. The algorithm minimizes the quadratic form $Q(\zeta, \gamma)$ by choosing (ζ, γ) from a compact set where

$$\zeta \in [0, 2] \quad \text{and} \quad \gamma \in [0, 0.005]. \tag{A.1}$$

For all the runs, the initial guesses are

$$\zeta = 0.4 \quad \text{and} \quad \gamma = 0.001. \tag{A.2}$$

The tables below have three columns documenting the results, and each column corresponds to a particular run of the calibration algorithm with a given IFR target for December 10th, 2020:

$$\text{IFR}_T^{\text{data}} \in \{0.39\%, 0.66\%, 1.33\%\}. \tag{A.3}$$

*Dept. of Economics, FEAS, Hacettepe University, Beytepe Campus, 06800 Cankaya/Ankara, TÜRKİYE, Tel.: +90-535-0359086, maattar@hacettepe.edu.tr.

† ***Corresponding Author***, Dept. of Economics, TED University, Ziya Gökalp Caddesi, No.47, 06420 Cankaya/Ankara, TÜRKİYE, Tel.: +90-312-5850034, ayca.tekinkoru@tedu.edu.tr.

The other data moment that is common across all specifications is the case-patient ratio $\mathrm{CP}_\tau^{\mathrm{data}}$ on October 3rd, 2020 and is set to 6.87 (Uçar et al., 2020).

The algorithm uses MATLAB's `fmincon` routine. By default, this routine searches for an interior optimum within a compact set. The Hessian is approximated through the Broyden–Fletcher–Goldfarb–Shanno algorithm. In all of the runs, the algorithm has converged to a solution by hitting the step-size tolerance that is equal to 10^{-20}.

Table A.1: Calibration Results for $\delta = 1\%$

	$\mathrm{IFR}_T^{\mathrm{data}} = 0.66\%$	$\mathrm{IFR}_T^{\mathrm{data}} = 0.39\%$	$\mathrm{IFR}_T^{\mathrm{data}} = 1.33\%$
ζ	0.41311650	0.31833305	1.79925404
γ	0.00015896	0.00004590	0.00163752
$\mathrm{CP}_\tau^{\mathrm{model}}$	6.86969279	6.86983096	6.86470456
$\mathrm{IFR}_T^{\mathrm{model}}$	0.65195260	0.38507261	1.15113794
$Q(\zeta, \gamma)$	0.00006486	0.00002431	0.03201968
C_T^{model}	1,995,980	2,113,317	1,645,057
D_T^{model}	13,013	8,138	18,937

Table A.2: Calibration Results for $\delta = 1.2\%$

	$\mathrm{IFR}_T^{\mathrm{data}} = 0.66\%$	$\mathrm{IFR}_T^{\mathrm{data}} = 0.39\%$	$\mathrm{IFR}_T^{\mathrm{data}} = 1.33\%$
ζ	0.38363504	0.31214880	1.62699469
γ	0.00014924	0.00004596	0.00175058
$\mathrm{CP}_\tau^{\mathrm{model}}$	6.86984582	6.86992727	6.86965623
$\mathrm{IFR}_T^{\mathrm{model}}$	0.65610866	0.38776070	1.31213495
$Q(\zeta, \gamma)$	0.00001517	0.00000502	0.00031928
C_T^{model}	2,027,229	2,123,047	1,657,509
D_T^{model}	13,301	8,232	21,749

Table A.3: Calibration Results for $\delta = 1.5\%$

	$\mathrm{IFR}_T^{\mathrm{data}} = 0.66\%$	$\mathrm{IFR}_T^{\mathrm{data}} = 0.39\%$	$\mathrm{IFR}_T^{\mathrm{data}} = 1.33\%$
ζ	0.35713181	0.30584194	0.71127614
γ	0.00013930	0.00004578	0.00073892
$\mathrm{CP}_\tau^{\mathrm{model}}$	6.86988344	6.86999244	6.86960653
$\mathrm{IFR}_T^{\mathrm{model}}$	0.65682194	0.38975343	1.31854187
$Q(\zeta,\gamma)$	0.00001011	0.00000006	0.00013144
C_T^{model}	2,058,983	2,133,294	1,812,919
D_T^{model}	13,524	8,315	23,904

Table A.4: Calibration Results for $\delta = 2\%$

	$\mathrm{IFR}_T^{\mathrm{data}} = 0.66\%$	$\mathrm{IFR}_T^{\mathrm{data}} = 0.39\%$	$\mathrm{IFR}_T^{\mathrm{data}} = 1.33\%$
ζ	0.33452902	0.29939993	0.49581989
γ	0.00013128	0.00004508	0.00050789
$\mathrm{CP}_\tau^{\mathrm{model}}$	6.86997581	6.87000084	6.86983760
$\mathrm{IFR}_T^{\mathrm{model}}$	0.65905167	0.38996712	1.32581496
$Q(\zeta,\gamma)$	0.00000090	0.00000000	0.00001754
C_T^{model}	2,089,396	2,144,107	1,925,872
D_T^{model}	13,770	8,361	25,533

Table A.5: Calibration Results for $\delta = 3\%$

	$\mathrm{IFR}_T^{\mathrm{data}} = 0.66\%$	$\mathrm{IFR}_T^{\mathrm{data}} = 0.39\%$	$\mathrm{IFR}_T^{\mathrm{data}} = 1.33\%$
ζ	0.31453646	0.29310936	0.38985490
γ	0.00012372	0.00004412	0.00039510
$\mathrm{CP}_\tau^{\mathrm{model}}$	6.86999531	6.86996764	6.87001764
$\mathrm{IFR}_T^{\mathrm{model}}$	0.65970339	0.38917236	1.32986580
$Q(\zeta,\gamma)$	0.00000009	0.00000069	0.00000002
C_T^{model}	2,119,290	2,155,029	2,020,368
D_T^{model}	13,981	8,387	26,868

Table A.6: Calibration Results for $\delta = 4\%$

	$\mathrm{IFR}_T^{\mathrm{data}} = 0.66\%$	$\mathrm{IFR}_T^{\mathrm{data}} = 0.39\%$	$\mathrm{IFR}_T^{\mathrm{data}} = 1.33\%$
ζ	0.30544199	0.29011661	0.35406967
γ	0.00012012	0.00004386	0.00035678
$\mathrm{CP}_\tau^{\mathrm{model}}$	6.86997594	6.86998216	6.87001287
$\mathrm{IFR}_T^{\mathrm{model}}$	0.65945713	0.38955008	1.32998076
$Q(\zeta,\gamma)$	0.00000030	0.00000020	0.00000000
C_T^{model}	2,133,948	2,160,373	2,062,951
D_T^{model}	14,072	8,416	27,437

Table A.7: Calibration Results for $\delta = 5\%$

	$\text{IFR}_T^{\text{data}} = 0.66\%$	$\text{IFR}_T^{\text{data}} = 0.39\%$	$\text{IFR}_T^{\text{data}} = 1.33\%$
ζ	0.30028009	0.28833673	0.33608492
γ	0.00011816	0.00004368	0.00033762
$\text{CP}_\tau^{\text{model}}$	6.86998412	6.86998724	6.87000500
$\text{IFR}_T^{\text{model}}$	0.65965149	0.38967514	1.33007881
$Q(\zeta, \gamma)$	0.00000012	0.00000011	0.00000001
C_T^{model}	2,142,601	2,163,593	2,087,205
D_T^{model}	14,134	8,431	27,761

Appendix B. The Evolution of Cumulative Case Count

Our purpose in this appendix is to clarify why the SEIRD model bounds seem to be converging to each other some time in mid-November.

In the middle of November 2020, the total number C_t of confirmed cases under different specifications get extremely close to each other. There is no particular day on which such a convergence occurs, and the convergence is not absolute. Yet, on different days around November 13th, cumulative case count is close to 1.3 million people under different specifications.

Since the SEIRD model is a system of nonlinear difference equations, it is not feasible to provide a formal proof of this outcome. However, as shown by Li (2020), for instance, the solution of the model for C_t can be approximated by

$$C_t = C_0 \mathrm{e}^{\Theta t} + \Psi(\mathrm{e}^{\Omega t} - \mathrm{e}^{\Theta t}) \tag{B.1}$$

where $\Theta, \Psi, \Omega \in \mathbb{R}$ are meta-parameters that depend on the structural parameters of the SEIRD model, and Θ and Ω have switching signs by construction, i.e., depending on sign and magnitude restrictions on structural parameters. Furthermore, such time paths are strictly convex and strictly increasing for the beginning of a single wave of a pandemic, and the growth rate of C_t is not fixed. Hence, generally and purely from a mathematical point of view, such paths that start from the same initial value C_0 intersect once for some $t > 0$.

Our calibration algorithm chooses different (ζ, γ) pairs for different runs, and we thus obtain a particular (Θ, Ψ, Ω) tuple for each of them. Importantly, the specification that attains the lowest growth rate near $t = 0$ also attains the highest growth rate near $t = T$ and *vice versa*. With the additional restriction that the absolute increase in C_t on October 3rd $(t = \tau)$ is targeted, the two paths imply similar C_t levels on a particular day between τ and T.

We should also note the following remarks regarding this feature of the SEIRD model. First, the date on which different specifications imply $C_t \approx 1.3$ million lies between τ and T since the absolute increase in C_t is targeted for τ. Second, once we change the initial values $(S_0, E_0, I_0, R_0, D_0)$ of the model in alternative runs, this particular feature vanishes. Third, the proximity of C_t to 1.3 million under different specifications is closely related with the value attained by $Q(\zeta, \gamma)$; in specifications where the algorithm is less successful in matching the targets, the SEIRD model bounds gets larger for the mid-November as well.

References

- Li K (2020) Transmission dynamics of the global covid-19 epidemic: Analytical modeling and future prediction. Research Square Preprint.

- Uçar A, Arslan S¸, Balcı Yapalak AN (2020, November 23) Türkiye covid-19 pandemisinde resmi ve tahmini sayılar [sunum]. Bilim Akademisi COVID-19 Modelleme Çalıştayı.