

Non-ambiguous CFG for $\{a^n b^n \mid n \geq 0\}$

Write a **non-ambiguous** CFG generating the language over $\{a,b\}$ where the first half of each word only contains a's and the second half only contains b's.

$S \rightarrow aSb$
 $S \rightarrow \epsilon$

Non-ambiguous CFG for $\{a^n cb^n \mid n > 0\}$

Write a **non-ambiguous** CFG generating the words over $\{a,b,c\}$ such that there is an occurrence of c exactly at the middle, to its left there are only a's (and there is at least one a), and to its right there are only b's (and there is at least one b). Note that, since the only occurrence of c must be exactly at the middle, the number of a's must be equal to the number of b's.

$S \rightarrow acb$
 $S \rightarrow aSb$

Non-ambiguous CFG for $\{a^i b^j \mid i \geq j\}$

Write a **non-ambiguous** CFG generating the words of the form $a^i b^j$ where the number of a's is at least the number of b's.

$S \rightarrow aSb \mid X$
 $X \rightarrow aX \mid \epsilon$

Non-ambiguous CFG for $\{a^i b^j \mid i \leq j\}$

Write a **non-ambiguous** CFG generating the words of the form $a^i b^j$ where the number of a's is at most the number of b's.

$S \rightarrow aSb \mid X$
 $X \rightarrow Xb \mid \epsilon$

o també es valid

$S \rightarrow AB$
 $B \rightarrow Bb \mid \epsilon$
 $A \rightarrow aAb \mid \epsilon$

Non-ambiguous CFG for $\{a^i b^j \mid 2i \leq j\}$

Write a **non-ambiguous** CFG generating the words of the form $a^i b^j$ where the number of b's is at least twice the number of a's.

$S \rightarrow S b \mid X$

$X \rightarrow aXbb \mid$

CFG for $\{a^i b^j \mid 2i \geq j\}$

Write a CFG generating the words of the form $a^i b^j$ where the number of b's is at most twice the number of a's.

$S \rightarrow aS \mid aSbb \mid ab \mid$

Non-ambiguous CFG for $\{a^i b^j \mid 2i \geq j\}$

Write a **non-ambiguous** CFG generating the words of the form $a^i b^j$ where the number of b's is at most twice the number of a's.

$S \rightarrow aS \mid X$

$X \rightarrow aXbb \mid ab \mid$

Non-ambiguous CFG for $\{a^i b^j \mid j \leq i \leq 2j\}$

Write a non-ambiguous CFG generating the words of the form $a^i b^j$ where the number of a's is at least the number of b's, but at most twice the number of b's.

$S \rightarrow aaSb \mid A$

$A \rightarrow aAb \mid$

Non-ambiguous CFG for $\{a^i b^j \mid i \geq j \vee i \leq 2j\}$

Write a **non-ambiguous** CFG generating the words of the form $a^i b^j$ where the number of a's is at least the number of b's, or it is at most twice the number of b's.

$S \rightarrow AB$

$A \rightarrow aA \mid$

$B \rightarrow bB \mid$

Non-ambiguous CFG for $\{a^i b^j c^k \mid i = j + k\}$

Write a **non-ambiguous** CFG generating the words of the form $a^i b^j c^k$ such that the number of a's coincides with the number of b's plus the number of c's.

$K \rightarrow aKc \mid J$

$J \rightarrow aJb \mid$

Non-ambiguous CFG for $\{a^i b^j c^k \mid j=i+k\}$

Write a **non-ambiguous** CFG generating the words of the form $a^i b^j c^k$ such that the number of b's coincides with the number of a's plus the number of c's.

$S \rightarrow XY$
 $X \rightarrow aXb \mid$
 $Y \rightarrow bYc \mid$

CFG for $\{a^i b^j c^k \mid i=j \vee j=k \vee i=k\}$

Write a CFG (**which will be ambiguous**) generating the words of the form $a^i b^j c^k$ where the number of aaa's equals the number of bbb's, or the number of bbb's equals the number of ccc's, or the number of ccc's equals the number of aaa's.

$S \rightarrow AB \mid CD \mid E$

$A \rightarrow aAb \mid$
 $B \rightarrow Bc \mid$

$C \rightarrow aC \mid$
 $D \rightarrow bDc \mid$

$E \rightarrow aEc \mid F$
 $F \rightarrow Fb \mid$

CFG for $\{a^{n_0} b a^{n_1} b \dots a^{n_{m-1}} b a^{n_m} \mid m \geq 1 \wedge \exists i \in \{1, \dots, m\} : (n_0 = n_i)\}$

Write a CFG (**which will be ambiguous**) generating the words of the form

$a^{n_0} b a^{n_1} b \dots a^{n_{m-1}} b a^{n_m}$ for which there exists an $i \in \{1, \dots, m\}$ such that $n_0 = n_i$.

$S \rightarrow A \mid AbC$

$A \rightarrow aAa \mid B$
 $B \rightarrow b \mid bCb$
 $C \rightarrow aC \mid bC \mid$

Non-ambiguous CFG for $\{a^{n_0} b a^{n_1} b \dots a^{n_{m-1}} b a^{n_m} \mid m \geq 1 \wedge (n_0 = \sum_{1 \leq i \leq m} n_i)\}$

Write a **non-ambiguous** CFG generating the words of the form $a^{n_0} b a^{n_1} b \dots a^{n_{m-1}} b a^{n_m}$, with $m \geq 1$, such that n_0 is equal to the sum $n_1 + n_2 + \dots + n_m$.

$S \rightarrow aSa \mid Sb \mid b$

CFG for $\{ a^{n_0} b a^{n_1} b \dots a^{n_{m-1}} b a^{n_m} \mid m \geq 1 \wedge \exists I \subseteq \{1, \dots, m\} : (n_0 = \sum_{i \in I} n_i) \}$

Write a CFG (**which will be ambiguous**) generating the words of the form

$a^{n_0} b a^{n_1} b \dots a^{n_{m-1}} b a^{n_m}$, with $m \geq 1$, for which n_0 is equal to the sum of a selection of naturals from n_1, n_2, \dots, n_m , i.e. $n_0 = \sum_{i \in I} n_i$ where $I \subseteq \{1, \dots, m\}$. Note that, in particular, the selection might be empty, and therefore a word where n_0 is 0 is necessarily correct.

$S \rightarrow R \mid T$
 $R \rightarrow a R a \mid S b \mid b$
 $T \rightarrow T a \mid S b \mid b$

Non-ambiguous CFG for $\{ w \in \{a,b\}^* \mid w = w^R \}$

Write a **non-ambiguous** CFG generating the palindromic words over $\{a,b\}$.

$S \rightarrow a S a \mid b S b \mid a \mid b \mid$

Non-ambiguous CFG for $\{ w \in \{a,b\}^* \mid w = w^R \wedge |w|_{aba} = 0 \}$

Write a **non-ambiguous** CFG generating the palindromic words over $\{a,b\}$ with no occurrence of aba.

$S \rightarrow a A a \mid b S b \mid a \mid b \mid$
 $A \rightarrow b B b \mid a A a \mid a \mid$
 $B \rightarrow b S b \mid b \mid$

Non-ambiguous CFG for $\{ w \in \{a,b\}^* \mid w = w^R \wedge |w|_a > 0 \wedge |w|_b > 0 \}$

$S \rightarrow a B a \mid b A b$
 $A \rightarrow a C a \mid b A b \mid a$
 $B \rightarrow b C b \mid a B a \mid b$
 $C \rightarrow a C a \mid b C b \mid a \mid b \mid$

Non-ambiguous CFG for $\{ w \in \{a,b\}^* \mid w = w^R \wedge |w|_{aba} > 0 \}$

$S \rightarrow a A a \mid b S b$
 $A \rightarrow a A a \mid b B b \mid b$
 $B \rightarrow a R a \mid b S b \mid a$
 $R \rightarrow a R a \mid b R b \mid a \mid b \mid$

CFG for the well-parenthesized words over $\{ (,) \}$

$S \rightarrow (S) \mid () \mid S S \mid$

CFG for the well-parenthesized words over $\{[,], (,)\}$

$S \rightarrow (S) \mid \emptyset \mid SS \mid [S] \mid \square \mid$

Non-ambiguous CFG for the well-parenthesized words over $\{ (,) \}$

$S \rightarrow (XS \mid$
 $X \rightarrow (XX \mid)$

Non-ambiguous CFG for the well-parenthesized words over $\{[,], (,)\}$

$S \rightarrow (S)S \mid [S]S \mid$

CFG for $\{w \in \{a, b\}^* \mid |w|_a = |w|_b\}$

$S \rightarrow aSb \mid bSa \mid SS \mid$

CFG for $\{w \in \{a, b\}^* \mid |w|_a = |w|_b\}$

$S \rightarrow aSb \mid bSa \mid SS \mid c \mid$

CFG for $\{w \in \{a, b, c\}^* \mid |w|_a + |w|_b = |w|_c\}$

$S \rightarrow cX \mid Xc \mid SS \mid$
 $X \rightarrow aS \mid bS \mid Sa \mid Sb$

CFG for $\{w \in \{a, b\}^* \mid 2|w|_a = |w|_b\}$

$S \rightarrow aSbSb \mid bSaSb \mid bSbSa \mid SS \mid$

Non-ambiguous CFG for $\{w \in \{a, b\}^* \mid 2|w|_a = |w|_b\}$

$S \rightarrow aBS \mid bAS \mid$
 $A \rightarrow a \mid bAA$
 $B \rightarrow b \mid aBB$

Non-ambiguous CFG for $\{w \in \{a, b, c\}^* \mid 2|w|_a = |w|_b\}$

$S \rightarrow aBS \mid bAS \mid cS \mid$
 $A \rightarrow a \mid bAA \mid cA$
 $B \rightarrow b \mid aBB \mid cB$