VI

Image-forming instruments

THE three preceding chapters give an account of the geometrical theory of optical imaging, using for the main part the predictions of Gaussian optics of the Seidel theory. An outstanding instance of the invaluable service rendered by this branch of optics lies in its ability to present the working principles of optical instruments in an easily visualized form. Although the quality of optical systems cannot be estimated by means of Gaussian theory alone, the purpose served by the separate optical elements can be indicated in this way, so that a simple, though somewhat approximate, picture of the action of the system can often be obtained without entering into the full intricacy of the techniques of optical design.

The development of optical instruments in the past has proceeded just as fast as technical difficulties have been overcome. It is hardly possible to give a step-by-step account of the design of optical systems, for two reasons. Firstly, the limitations of a given arrangement are not indicated by the predictions of the simple theory; in particular cases this needs to be supplemented by a fuller analysis often involving tedious calculations. Secondly, difficulties of a practical nature may prevent an otherwise praiseworthy arrangement from being used. It is not intended in this account to discuss the theoretical and practical limitations in individual cases; only the basic principles underlying the arrangement of some of the more important optical instruments will be given, in order to provide a framework for some of the later chapters which deal with the more detailed theories of optical image formation.*

6.1 The eye

Perhaps the simplest of optical instruments is that consisting of a single convergent lens forming a real image of an object upon a light-sensitive surface. Examples of such an optical system are found in the photographic camera and in the eye (Fig. 6.1). The back inner surface of the eye,† called the *retina*, consists of a layer of light-sensitive

* For fuller accounts of optical instruments see, for example, R. Glazebrook (ed.), *Dictionary of Applied Physics*, Vol. IV (London, Macmillan, 1923); L. C. Martin, *An Introduction to Applied Optics*, Vol. II (London, Pitman, 1950); G. A. Boutry, *Optique Instrumentale* (Paris, Masson, 1946). References to books and articles dealing with particular types of instruments will be found in D. A. Jacobs, *Fundamentals of Optical Engineering* (New York, McGraw-Hill, 1943).

† For a detailed account of physiological optics the reader is referred to the standard work of H. Helmholtz, *Handbuch der Physiologischen Optik* (Hamburg and Leipzig); L. Voss, 3 Aufl., I (1909), II (1911), III (1910). English translation edited by J. P. C. Southall (published by the Optical Society of America, 1924); and to J. P. C. Southall, *An Introduction to Physiological Optics* (Oxford, Oxford University Press, 1937).

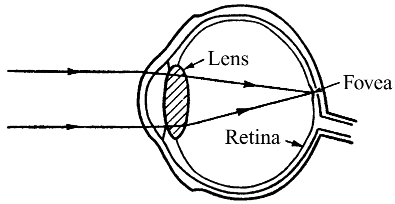

Fig. 6.1 The human eye.

cells. Inasmuch as the eye forms an integral part of many optical systems, an understanding of its characteristics is an essential part of instrumental optics. The present section will include a description of a few of its features.

Refraction in the eye takes place mostly at the first surface or *cornea* of the eye, but also at the surfaces of the *lens* of the eye situated behind a contractible *iris*. The latter gives an aperture diameter which may be varied by unconscious muscular action, according to the total flux of light entering the eye, between approximately 1.5 mm and 6 mm. The lens is doubly convex, and its density, and hence its refractive index, is not uniform but decreases towards the outer zones. This serves partly to correct for spherical aberration (see §5.3). The average value of the refractive index of the lens is 1.44, while that of the *vitreous humour* between the lens and the retina is approximately 1.34, near to that of water.

The radius of curvature of the lens surfaces may be altered by muscular contraction, to serve the purpose of focusing. This power of adjustment is called *accommodation*, and the least distance of distinct vision of a normal or *emmetropic* eye is around 25 cm. The accommodation of the eye varies greatly from one person to another and, in each person, with age.* The correction of excessive departures from normality by the introduction of supplementary eye-lenses forms the subject of *ophthalmics*. There are three principal abnormalities which occur in the eye:

(a) *Myopia* or short sight, in which the rays from an infinitely distant object-point reach a focus in front of the retina. This is corrected by means of a divergent lens placed in front of the eye.
(b) *Hypermetropia* or long sight, in which the true focus of an infinitely distant object lies behind the retina. Correction is obtained by means of a convergent lens.
(c) *Astigmatism*, in which the power of the eye differs in different planes containing its optical axis. This defect is corrected by means of a cylindrical or toroidal surface on the lens of the eye-glass.

The retina contains two principal types of light-sensitive cells, the *rods* and the *cones*. The latter, according to the usual theory of colour vision, may be subdivided into the three types of colour sensitivity. They predominate and are of greatest concentration in the *fovea*, the central part of the retina. The field of most distinct vision defined by this region and determining the *fixation*, or direction of 'seeing', is slightly less than 1° in angular extent. The diameter of the cones in the fovea varies

* At 10 years the power of the eye can cover 14 dioptres, at 25 years 10 dioptres, and at 50 years 2.5 dioptres.

between 0.0015 mm and 0.005 mm. The rods, which are not so highly selective in colour range, predominate in the outer regions and are the most responsive to low-intensity sources. This is the cause of the phenomenon of 'averted vision' in scanning, for instance, the night sky.

There is an ultimate limit to the clarity of detail which one can attain with any given optical system. In the last analysis this limit arises, as will be seen in §8.6.2, from the wave nature of light. The practical limit down to which two object points separated by a small distance may be distinguished by the eye, its *visual acuity*, is clearly a quantity of the greatest interest to the optical designer, for upon it depend the tolerances, both optical and mechanical, to which a visual instrument ought to be made. For a normal eye, this limiting angular separation is about one minute of arc in angular measure, corresponding to separation of about 0.0045 mm on the retina, if a focal length of 15 mm is assumed. When the aperture of the eye is increased beyond 5 mm, the effect of chromatic aberration and spherical aberration will decrease the resolving power. In designing a visual instrument one therefore usually assumes that a pencil not more than 4–5 mm in diameter is to enter the eye. Visual instruments are seldom, if ever, designed with a view to compensating the defects of the eye, because the amounts present in the eye vary from person to person.

6.2 The camera

In the *photographic camera* (Fig. 6.2), a real inverted image of an object is formed by a lens or combination of lenses upon the surface of a photographic film or plate. The object may be stationary or in motion relative to the camera. In the latter case short exposures are necessary, and the aperture must therefore be as large as possible, in order to collect sufficient light. It follows from §4.8 (24) that the light reaching the image per unit area (the illumination E) from an extended object is proportional to the ratio d^2/f^2, d being the diameter of the entrance pupil and f the focal length, i.e. it is inversely proportional to the square of the F number (nominal focal ratio). This may usually be varied by means of a variable *diaphragm*.

In the absence of aberrations, the image of a distant object which subtends an angle $\delta\theta$ at the first nodal points is of linear dimensions $f\delta\theta$. Hence, in order to produce a large image, the focal length must be large. Thus the two main characteristics of a camera are the focal length of its optical system and the range of the focal ratios at which it can operate. Another possible requirement of a camera is that it should cover

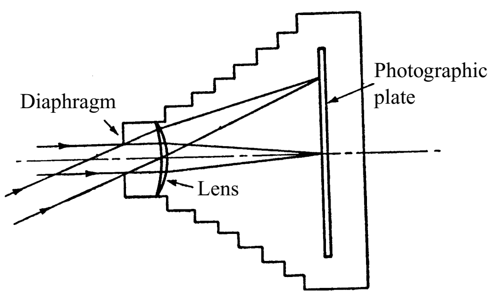

Fig. 6.2 The photographic camera.

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a large angular field. Cameras are often designed to cover a field between 10° and 90°, using a focal length in the range from 1 in to 6 ft. Greater focal lengths (6–60 ft) are needed in astronomical photography, where fields of much smaller angular extent ($\sim 1°$) containing effective point sources must often be reproduced on a large scale.

The simplest of small cameras use a single convergent lens, usually of meniscus form (concavo-convex) with a stop somewhat displaced from the lens, as shown in Fig. 6.2. No correction is made for chromatic effects and therefore such lenses were more satisfactory when photographic emulsions were generally sensitive to only a small region at the blue end of the spectrum (4000–4800 Å). Such lenses are often used in cheaper types of camera at a focal ratio $f/11$ over a 45° field.

The '*landscape lens*', as this meniscus form was once called, originated about 1812 and was followed, soon after the introduction of photography, by the achromatic doublet of Chevalier [Fig. 6.3(a)]. The aim of the achromatic combination was to make the focus of the photographically active light coincident with that of the visual light (brightest around $\lambda = 5800$ Å); good images could not be obtained at focal ratios lower than $f/16$. The need for a much 'faster' lens (i.e. one with a lower focal ratio) for portrait photography resulted in the widespread use of the *Petzval portrait lens* invented in about 1840 [Fig. 6.3(b)] and comprising two separated doublets. This arrangement gave a field surface with considerable curvature, but good definition was obtainable over a field of a few degrees at focal ratios down to $f/4$. A development of the Petzval lens by Dallmeyer has been used extensively in cine projectors.

It has been known since about 1841 that a symmetrical lens would automatically be free from distortion. This is strictly true only for the case of unit magnification, but is substantially true for other working distances. This principle has been the basis of design of several long lines of successful photographic lenses. The *Rapid-Rectilinear* of Dallmeyer [Fig. 6.3(c)] dates from 1866. It was used in many cameras and eventually at focal ratios down to $f/8$ (as late as 1920), over fields of about 45°. Many firms manufactured such lenses and they became known under the general term of '*aplanat*'. The same principle of symmetry, leading to freedom from distortion, has been continuously followed in the design of lenses intended to cover very wide fields.

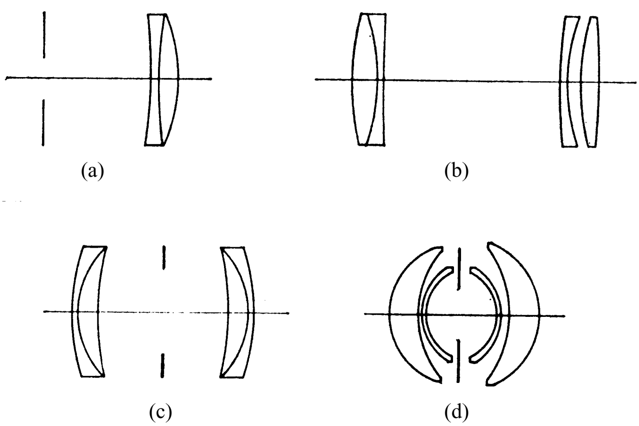

Fig. 6.3(a) The Chevalier lens; (b) the Petzval lens; (c) the Rapid-Rectilinear lens; (d) the Topogon lens.

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The original lens of this kind, the *Goerz Hypergon* of 1900, could, at a focal ratio around 25, cover a flat field up to 150°, although it has more often been sufficient to cover 90° at $f/16$. The four-element *Topogon* [Fig. 6.3(d)] of 1933 which will cover a field of 90° at $f/6.3$, represents the basic design for modern wide-angle lenses.

During most of the nineteenth century, lens designers were restricted to the use of two main types of glass: boro-silicate crown, and dense flint. Around 1890 the Jena Company introduced a radically new type of glass – the barium crown – having a high refractive index but low dispersive power. Incorporating this additional type of glass into lens systems permitted higher degrees of correction to be obtained. Rudolph of Jena, whose name is associated with the origin of many types of photographic lenses, coined the word '*anastigmatic*' to describe a lens for which the astigmatism at one off-axis angle could be reduced to zero. Using the older types of glass only, Rudolph had produced an anastigmat which later became known as the *Zeiss Protar* [Fig. 6.4(a)], but using three types of glass he was able to make an anastigmatic triplet. Two such lenses used symmetrically constituted a highly corrected combination. Such a lens was produced by Zeiss under the name *Triple Protar* and by Goerz as the *Dagor* or *double anastigmat* [Fig. 6.4(b)]. A typical lens of this type might work over a 50° field at $f/4.5$. Rudolph found further that if each half were formed of four cemented components the system could be made 'convertible', i.e. the halves, having different focal lengths, could be used separately or in combination. This was produced as the *Double Protar* of 1894.

A lens consisting of four separate elements was developed by Goerz in 1898 and called *Celor* [Fig. 6.4(c)]. This became the basis of design of several important lenses at focal ratios down to $f/4.5$, such as the *Unifocal* of Steinheil and the *Cooke Aviar lens* for aerial photography. Rudolph was working along similar lines and his *Tessar* of 1902 [Fig. 6.4(d)] is still well known as a relatively inexpensive lens giving excellent images over fields up to 60° and at focal ratios as low as $f/3.5$. It has been constructed with a great variety of focal lengths.

An earlier symmetrical design by Rudolph, the *Planar* of 1895, incorporated two meniscus-type lenses as the first and last elements [Fig. 6.5(a)]. Since 1920 many very

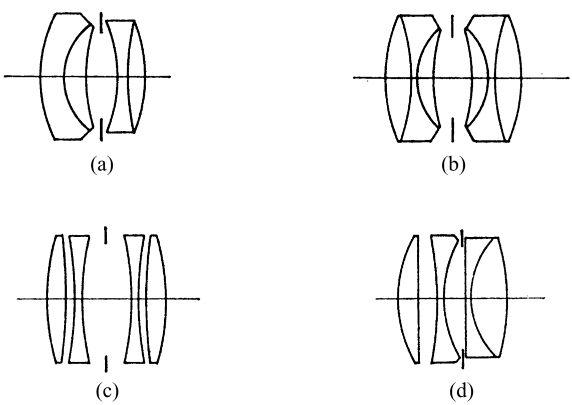

Fig. 6.4(a) The Protar lens; (b) the Dagor lens; (c) the Celor lens; (d) the Tessar lens.

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fast lenses have been developed in which a meniscus lens is used as the first element, although in most cases the last element is a cemented doublet as for example in Merté's *Zeiss Biotar* [Fig. 6.5(b)].

A separate stream in lens design was initiated by the development of the three-lens anastigmat, otherwise known as the *Cooke Triplet* [Fig. 6.5(c)], by H. D. Taylor in 1895. Full advantage was taken of the various parameters which can be altered in a system of three separated elements. This 'anastigmatic' design largely replaced the older 'aplanatic' lenses of the rapid-rectilinear-type in low- to medium-priced cameras. Types of high-speed lens whose design developed from the Cooke Triplet [e.g. the *Sonnar*, $f/1.5$, Fig. 6.5(d)], have been used widely in cine cameras and miniature cameras. In the Sonnar the central negative component and the second positive component are both triplets.

In cameras used for certain purposes which require long effective focal lengths and correspondingly small fields an optical arrangement consisting essentially of an objective and an enlarging lens may be used. In the *telephoto lens*, the enlarging lens is a divergent element placed anterior to the primary image [Fig. 6.6(a)]. In the *photo-heliograph*, an instrument used for photographing the sun on a large scale, the

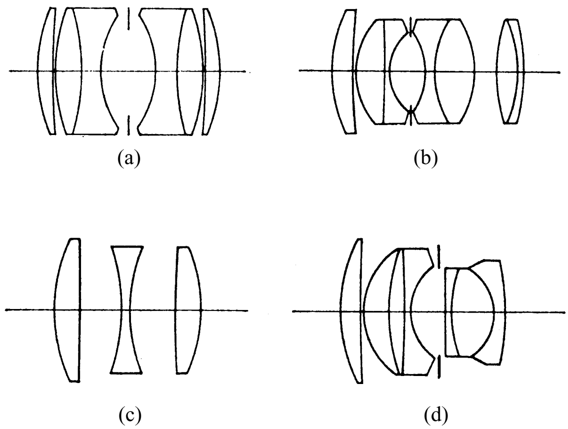

Fig. 6.5(a) The Planar lens; (b) the Zeiss Biotar lens; (c) the Cooke Triplet; (d) the Sonnar lens.

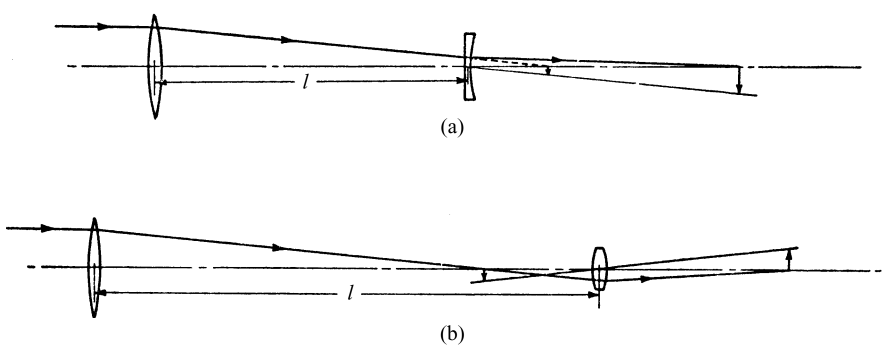

Fig. 6.6(a) The telephoto lens; (b) the photoheliograph.

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enlarging lens is usually a convergent system [Fig. 6.6(b)]. In these systems the effective focal length f is approximately given by the expression §4.7 (3):

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{l}{f_1 f_2},$$

where f_1 and f_2 are respectively the focal lengths of the objective and the enlarging lens, and l is their separation. The effective focal length may be greater than the overall length from the objective to the focal plane by a factor up to 15.*

Finally it is to be noted that the F number of a photograpic objective is an approximate measure of the 'speed of action' of the system only where the object is an extended one. With a point-source object (as is effectively the case in astronomical photography), the light reaching the image plane would, under ideal conditions, be concentrated in a vanishingly small area, so that the square of the aperture diameter, rather than F number, would characterize its light-gathering power. In reality the situation is more complicated, as several factors contribute to a spreading of the light over a finite (though often a very small) area. Chief amongst these are diffraction (see Chapter VIII), the granularity of the photographic emulsion, and the unsteadiness of the atmosphere.

6.3 The refracting telescope†

The *telescope* is an optical system by means of which an enlarged image of a distant object may be viewed. The principle of the *astronomical refracting telescope* is shown in Fig. 6.7. It consists of two convergent lenses the first of which, the *objective*, is usually an achromatic doublet forming a real inverted image, I, which is examined using the second lens, the *eyepiece*. In the normal state of adjustment the second focal plane of the objective coincides with the first focal plane of the eyepiece, so that an

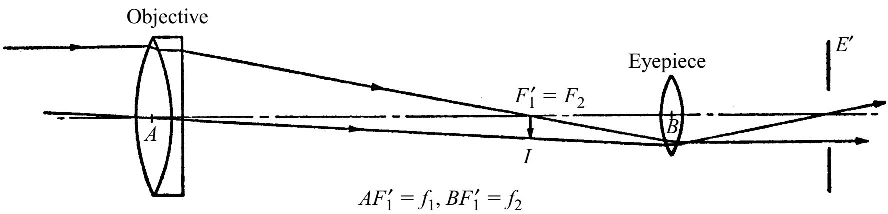


Fig. 6.7 The astronomical refracting telescope.

* Fuller discussions of photographic lenses will be found in W. Merté, R. Richter and M. Von Rohr, *Das Photographische Objectiv* (Vienna, J. Springer, 1932). More recent designs are described in articles by W. Taylor and H. W. Lee, *Proc. Phys. Soc.*, **47** (1935), 502; H. W. Lee, *Rep. Progr. Phys.* (London, Physical Society), **7** (1940), 130; R. Kingslake, *Proceedings of the London Conference on Optical Instruments* (London, Chapman & Hall, 1951), p.1 and C. G. Wynne, *Rep. Progr. Phys.* (London, Physical Society), **19** (1956), 298. For a historical account, see R. Kingslake, *A History of the Photographic Lens* (Academic Press, Boston, 1989).

† For a fuller discussion of such telescopes see, for example, A. Danjon and A. Couder, *Lunettes et Télescopes* (Paris, Revue d'Optique, 1935).

incident pencil of parallel rays emerges as a parallel pencil. The image may be erected by the use of an auxiliary lens.

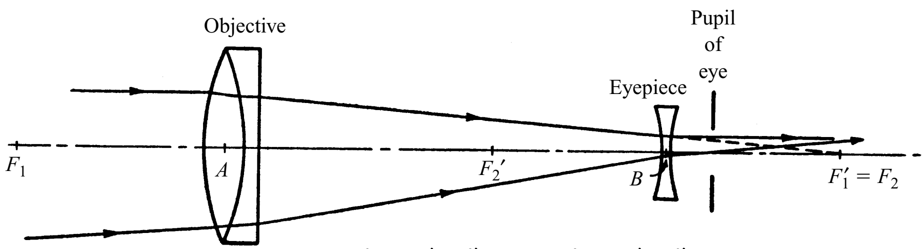
The entrance pupil of the system coincides with the objective; its image by the remainder of the system, namely the exit pupil, is denoted by E' in Fig. 6.7. The eye should be so placed that its entrance pupil coincides with E'; then all the light entering the objective at different off-axis angles will reach the eye.

The *magnifying power* of an instrument used for examination of objects at infinity is defined as the angular magnification at the pupils. By the Smith–Helmholtz formula §4.4 (49), this is the reciprocal of the linear magnification at the pupil planes, so that the magnifying power is equal to the ratio of the radius of the entrance pupil to that of the exit pupil. An expression for the magnifying power is immediately obtained from §4.3 (31) where, because the angles are assumed to be small, γ and γ' may be written in place of $\tan\gamma$ and $\tan\gamma'$. This gives

$$\frac{\gamma'}{\gamma} = \frac{f_1}{f_2'}.$$

Now the eye may be accommodated to distances other than infinity and the telescope may easily be adjusted to form a virtual image at, say, a distance D from the eye.* D should, however, not be decreased below the value of about 25 cm which, as already mentioned, is the closest distance at which an object can be viewed distinctly without eye strain with a normal eye.

The earliest telescope of which there exists definite knowledge is that of Galileo, constructed in 1609. In this system (Fig. 6.8) the objective was a convergent lens, but the eyepiece was a divergent lens, placed anterior to the primary image in such a way that the focal points of the two lenses coincided beyond the eyepiece. An erect image was then obtained at infinity, there being no intermediate image. In this type of telescope, known as the *Galilean telescope*, the aperture stop is not in the plane of the objective, for the image of the objective formed by the eyepiece lies between the two lenses and is therefore not accessible to the eye. The eye is placed close behind the eyepiece, so that the pupil of the eye becomes the aperture stop and the exit pupil. The



$AF_1 = f_1, \quad AF_1' = f_1', \quad BF_2 = f_2, \quad BF_2' = f_2'$

Fig. 6.8 The Galilean telescope.

* The magnification is then changed to the value:

$$\frac{\gamma'}{\gamma} = \frac{f_1}{f_2'} + \frac{f_1}{D}.$$

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principal ray for points at the limit of the field will pass very near to the edge of the objective, so that the objective serves as a field stop rather than as an aperture stop.

A disadvantage of the Galilean telescope is that it is limited to small fields and that it does not possess a real image where cross-wires, or a graticule, may be placed. On the other hand, its small overall length and the fact that it gives an erect image make the system very suitable for use in opera glasses, usually with a magnification of two to three times.

In the *binocular*, which uses an optical arrangement similar to that of an astronomical telescope, an erect image is obtained by means of four reflections, as shown in Fig. 6.9. These reflections take place at an incidence angle of 45° at the glass–air surfaces of so-called *Porro prisms*; this angle being, of course, beyond the critical angle. A reflection of this kind forms a convenient and efficient device often used in small optical instruments. An alternative form of binocular uses the *König erecting prism*. This incorporates the device of the 'roof' where two adjacent reflecting surfaces, at right angles to each other within a second of arc, are placed athwart the optical beam, as shown in Fig. 6.10. The larger spacing in the more usual type of binocular of the two objectives is of advantage, however, since it serves to enhance the stereoscopic effect.*

Terrestrial telescopes are generally similar to the astronomical telescope; but an erect image must be obtained, either with the help of erecting prisms, as in the binocular, or with the use of additional lenses. A typical system is shown in Fig. 6.11.

The telescope objective is usually an achromatic combination of two lenses in which the radii of curvature are such as to give, with spherical surfaces, little spherical aberration. If a cemented doublet is to be used, a design in which the first (crown) component is equi-convex and the second (flint) component is plano-concave is often a suitable solution [Fig. 6.12(a)]. If freedom from coma is desired as well as freedom from spherical aberration, as in a high-quality astronomical telescope, it is necessary to utilize the extra flexibility in design of two noncemented components, and to choose

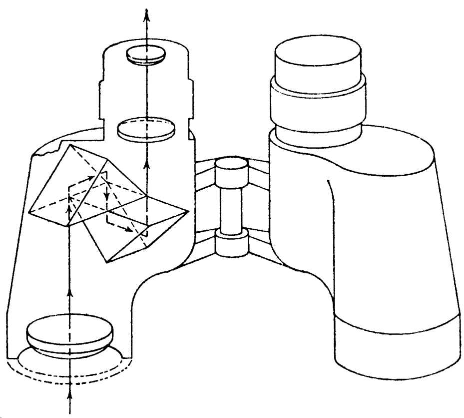

Fig. 6.9 The binocular telescope.

* For a fuller account of binoculars see M. von Rohr, *Die Binokularen Instrumente* (Berlin, J. Springer, 1920).

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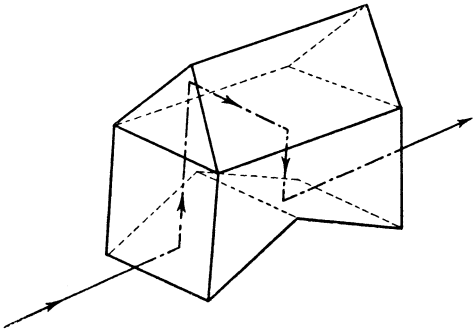

Fig. 6.10 The König erecting prism.

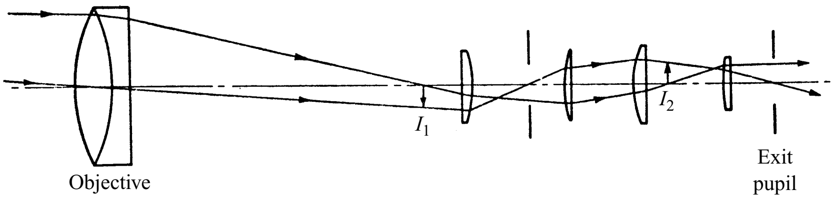

Fig. 6.11 The terrestrial telescope.

the radii to give an aplanatic design. The design may then be approximately that shown in Fig. 6.12(b). This includes a concavo-convex flint component. The steep concave curve of the first surface of this component can be reduced if a coma-free design with aspherical figuring on one surface to give axial stigmatism is adopted.

An objective with the crown lens in front is the usual form, originally known as the *Fraunhofer objective*. The *Steinheil*, or '*flint-in-front*', *objective* is less used because of the greater susceptibility of the flint glass (less stable than crown glass) to atmospheric degradation.

The *photovisual objective*, shown in Fig. 6.12(c), is the third type of objective commonly used in refracting telescopes. The use of three different glasses permits a considerable reduction of chromatic difference of focus over a given spectral range to be obtained. This objective is almost equally good for 'photographic' and 'visual' wavelengths, and at one time was popular for astronomical use at apertures up to about 12 in. The steep curves of the centre component, making centring difficult, and the early use of unstable varieties of flint glass, have detracted, however, from the value of these lenses.

The eyepiece of a visual telescope must, broadly, satisfy two requirements. Firstly, it must have a focal length which will give the required degree of magnification. Secondly, it must have sufficient aperture to collect the light from an extended object. These requirements can be achieved by means of a single lens, as shown in Fig. 6.13(a), but more conveniently two smaller lenses can be used as in Fig. 6.13(b), which shows the optical arrangement of the *Ramsden eyepiece*.

The eyepiece should form an image of the aperture stop at a position convenient for placing the eye. The distance of this image behind the last surface of the eyepiece is called the *eye-relief*.

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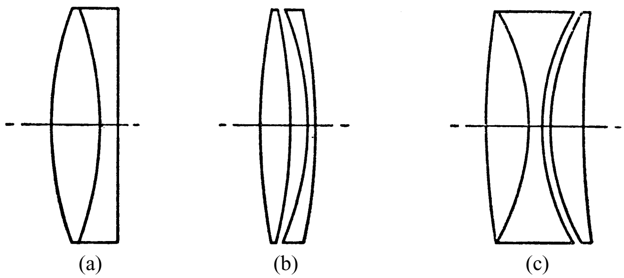

Fig. 6.12(a) Cemented achromat; (b) aplanatic achromat; (c) photovisual objective.

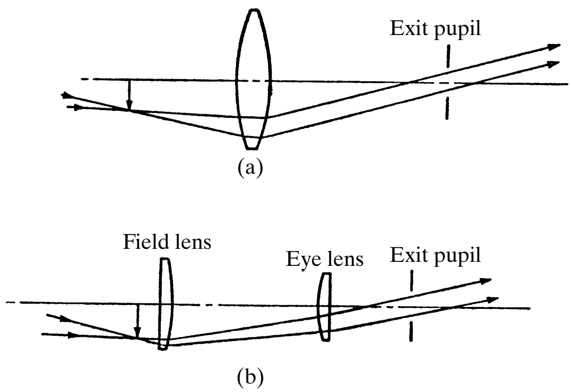

Fig. 6.13(a) Single-lens eyepiece; (b) the Ramsden eyepiece.

The first component of a two-component eyepiece is called the *field lens* because it is often very near to the primary image. It is separated from this image partly to prevent any dust or imperfection on the lens being visible, and partly to render the plane of the image accessible to cross-wires, or a graticule. The second component is called the *eye lens*.

The most common eyepiece in use is the Ramsden eyepiece, and the other chief type is the *Huygenian eyepiece*, shown in Fig. 6.14(a), which has the field lens anterior to the primary image. The disadvantages of the Huygenian eyepiece are firstly the rather short eye-relief obtainable, and secondly the fact that it cannot be used with cross-wires external to itself. It cannot give such a high correction for spherical aberration as the Ramsden eyepiece, but is free from lateral chromatic aberration and off-axis coma.

The use of two lenses (see Fig. 6.14) instead of one aids the removal of harmful chromatic aberrations, and correction in the Ramsden eyepiece can be greatly improved by using a doublet as the eye lens. More complicated are the 'orthoscopic' and 'symmetrical' eyepieces, each using four components but generally similar to the Ramsden eyepiece. A higher degree of correction for aberrations is obtained, especially with regard to distortion, and longer eye-reliefs are available for a given power.

The principle of the *periscope*, i.e. an instrument used for viewing objects which are situated so that direct viewing is precluded by an obstacle, is the same as that of

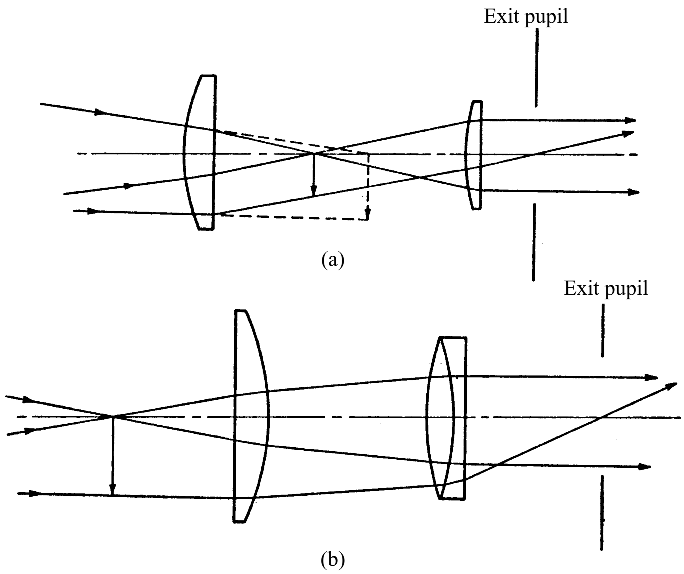

Fig. 6.14(a) The Huygenian eyepiece; (b) the Kellner eyepiece.

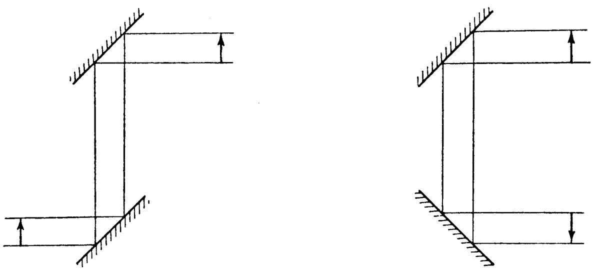

Fig. 6.15 Periscope mirrors.

the telescope, but two requirements make the designing of such an instrument more complicated. In the first place, the periscope must operate with a bent ray-path and give an erect nonreversed image. An even number of reflections, or none, are necessary to give a nonreversed image, and two simple arrangements are shown in Fig. 6.15, the first producing parallel and the second antiparallel directions of incident and emergent beams. The first must be used with an erecting telescopic system and the second with an inverting telescope. Another arrangement is shown in Fig. 6.16, where the direction of the emergent beam is fixed while the first prism rotates to give different directions of view. The *Dove prism D*, which incorporates a single reflection, rotates at half the speed of the top element in order to maintain an erect image. The fourth prism is of the 'roof' type (Amici) so as to provide, in all, four reflections.

The second complication which exists in periscopes arises from the desire for a large angular field of view, even though the telescope is contained within a narrow tube. A tube 8 inches in diameter and 40 ft long permits, at maximum, a total field of

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view of only 2°. The solution to this problem which is adopted makes use of a series of lenses passing the light down the tube. Different spacings of the lenses are possible. In Fig. 6.17 three arrangements are shown of a system of unit magnification which employs three lenses.

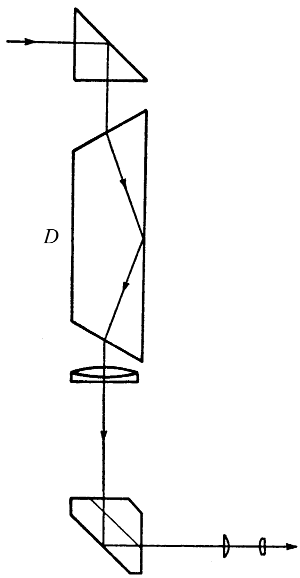

Fig. 6.16 Periscope with Dove prism.

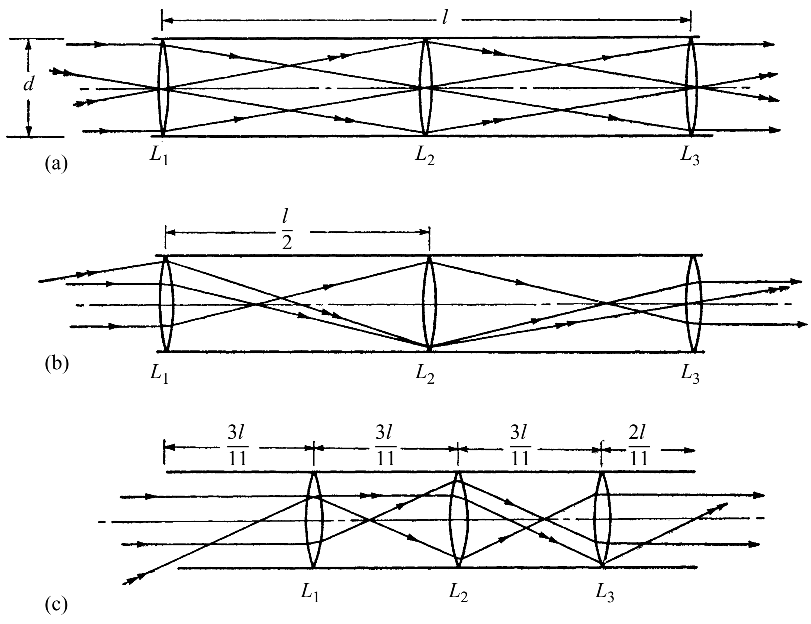

Fig. 6.17 Three-lens system; unit magnification.

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Table 6.1. *Some properties of three-lens systems*

	Focal lengths of lenses			Image	Aperture on-axis	Field (radians)	Vignetting at limit of field
	L_1	L_2	L_3				
(a)	$l/2$	$l/4$	$l/2$	Inverted	d	$2d/l$	0
(b)	$l/6$	$l/6$	$l/6$	Erect	$d/2$	$2d/l$	100%
(c)	$l/11$	$l/11$	$l/11$	Erect	$d/2$	$11d/2l$	$\sim$ 86%

The range of fields of view of these three systems and their effective apertures are shown in Table 6.1. It will be clear that, with a larger number of lenses, a wider range of possibilities exists.*

6.4 The reflecting telescope†

In the *astronomical reflecting telescope*, the light from a celestial objective is received on a concave primary mirror. The mirror serves the same purpose as the objective in a refracting telescope, namely to form a real image of the object in its focal plane. This image is either received directly on a photographic plate, or is examined visually by an eyepiece. This type of arrangement is the most common form of astronomical telescope used today. As in the case of the refracting telescope, the magnifying power is equal to the ratio of the focal length of the objective to that of the eyepiece.

The first telescope of this type was made in 1668 by Newton. In the arrangement which he used a small plane mirror was placed in the path of the rays reflected by the primary mirror, so as to divert the rays to one side of the tube, where the image could be conveniently examined (Fig. 6.18). This arrangement is known as the *Newtonian telescope*.

In order to have a stigmatic axial image, the figure of the mirror must be a paraboloid. The fact that off-axis images formed by a parabolic mirror suffer strongly from coma, together with the circumstances that large relative apertures are employed, makes the usable field very small. In the case of a 36 in $f/6$ mirror the total field which can be covered is of the order of 20 minutes of arc, while in the case of the Hale telescope, which has a primary mirror of diameter 200 in, the field is about 45 minutes in angular diameter.‡

Other types of reflecting telescope employ two principal mirrors, a primary concave mirror and a secondary mirror. In the *Cassegrain telescope* (Fig. 6.19) the secondary mirror is convex. In a large astronomical telescope, such an arrangement is often an alternative to the Newtonian arrangement. Different mirrors can be utilized to provide

* Further details will be found in an article by T. Smith in the *Dictionary of Applied Physics*, ed. R. Glazebrook, Vol. IV, (London, Macmillan, 1923), p. 350.

† A thorough treatment of reflecting telescopes, including the history of their development is given in R. N. Wilson, *Reflecting Telescope Optics*, Vol. I (1996), Vol. II (1999), (Berlin, Springer).

‡ F. E. Ross, *Publ. Astr. Soc. Pac.*, **46** (1934), 342. For a general account of the 200 in telescope see articles by J. A. Anderson, *Publ. Astr. Soc. Pac.*, **60** (1948), 221; and B. Rule, *ibidem*, (1948), 225.

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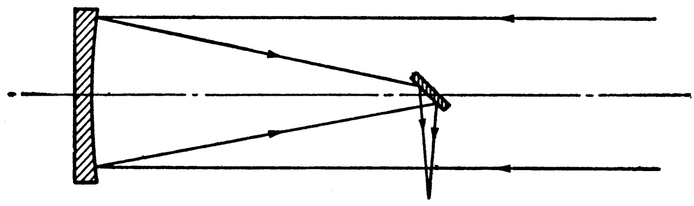

Fig. 6.18 The Newtonian telescope.

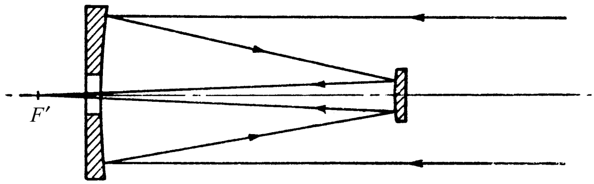

Fig. 6.19 The Cassegrain telescope.

a variety of focal lengths. If the figure of the primary mirror is a paraboloid, the secondary mirror must have a hyperbolic figure. Other figures for the two mirrors can be chosen which give a stigmatic axial image and different amounts of off-axis coma. In §4.10 we mentioned a two-mirror system proposed by Schwarzschild, which is completely aplanatic; the figures of the two mirrors are, however, more complicated. The Cassegrain system is often employed in terrestrial telescopes used for viewing landscapes as it provides a compact and inexpensive instrument with a long focal length, and a high degree of enlargement can be obtained without the need for a short eye-relief.

The *Gregorian telescope*, less common than the Cassegrain, employs a concave secondary mirror placed beyond the prime focus of the primary mirror.

In the largest reflecting telescopes, very great focal lengths, greater than that of the largest refractor, may be obtained by the use of a secondary mirror. In the *Coudé arrangement* the beam is reflected by a third (plane) mirror down the polar axis (about which the telescope rotates), so that it can be made to feed a fixed spectrograph.

One of the principal advantages of the reflecting telescope is its complete freedom from chromatic aberration. This, as well as the fact that the curvature required in the mirror surface is much less than that in a lens, enables reflectors to be built which have a smaller focal ratio than refractors. As well as giving brighter images, this results in a much more compact optical system. Moreover, a mirror can be built in larger sizes than a lens, because optical inhomogeneities in a mirror block are of no significance.

The disadvantages of the reflecting telescope for astronomical use lie principally in its sensitivity to thermal effects in the mirror surface and in the telescope tube, in its inability to cover more than a small angular field of view, and in the mechanical difficulties of making a sufficiently rigid mounting. It is this last aspect, in fact, which produced the most difficult of the problems which confronted the builders of the Hale telescope.

The successful use of the reflecting telescope with photographic plates has produced the desire for covering larger fields, and one of the methods available for increasing the usable field consists in employing field lenses just in front of the photographic plate. These lenses are designed to introduce off-axis coma such as to compensate that of the main mirror. Fields of the order of $1\frac{1}{2}°$ may be obtained with a 36 in telescope.*

Much more successful in an attempt to photograph wide angular fields with large reflecting systems has been the optical arrangement already referred to in §4.10, invented by B. Schmidt around 1930.† While a parabolic mirror gives perfect imaging for the axial rays and badly comatic images a short distance off-axis, a spherical mirror with an aperture stop at its centre of curvature C would give uniform images over a wide spherical field surface concentric with itself, each image suffering from a large amount of spherical aberration. Schmidt introduced into the aperture stop a thin, nearly plane-parallel plate, called the *corrector plate*, one face of which was plane, whilst the other was figured (Figs. 6.20, 6.21). The purpose of this plate was to 'pre-correct' the

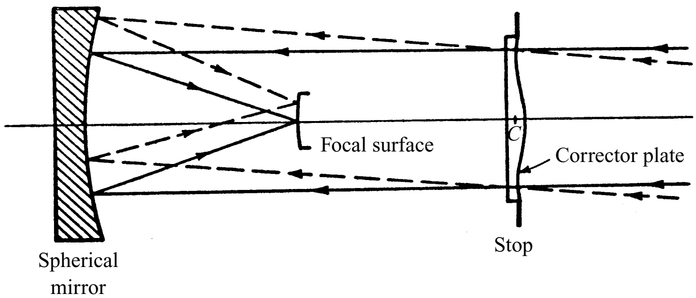

Fig. 6.20 The Schmidt camera. (The figuring of the corrector plate is greatly exaggerated.)

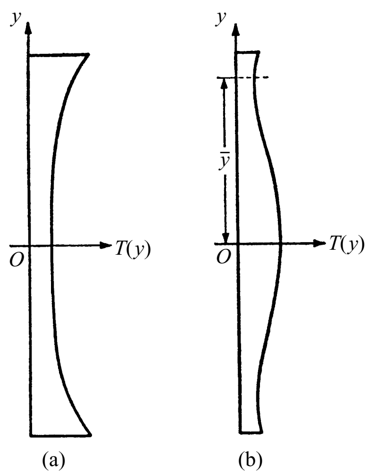

Fig. 6.21 Profiles of the Schmidt corrector plates. (The figurings are greatly exaggerated.)

* See F. E. Ross, *Astrophys. J.*, **81** (1935), 156.

† B. Schmidt, *Central-Zeitung f. Optik u. Mechanik*, **52** (1931), Heft 2, and *Mitt. Hamburg. Sternw. Bergedorf*, **7** (1932), No. 36, 15. See also R. Schorr, *Zeitschr. f. Instrumkde*, **56** (1936), 336, *Mitt. Hamburg. Sternw. Bergedorf*, **7** (1936), Nr. 42, 175 and *Astr. Nachr.*, **258** (1936), 45.

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plane wave-fronts entering the system in such a way as to compensate exactly the spherical aberration introduced by the spherical mirror.

We can easily derive an expression for the aspheric profile of the corrector plate of the *Schmidt camera*. For this purpose let us compare the expressions for a spherical and a paraboloidal mirror. The equation of a spherical mirror of radius $R = 2f$ is given by (see Fig. 6.22)

$$(z^{(s)} - 2f)^2 + y^2 = (2f)^2,$$

i.e.

$$z^{(s)} = \frac{y^2}{4f} + \frac{y^4}{64f^3} + \cdots,$$

whilst for a paraboloid with the same paraxial radius ($R = 2f$),

$$z^{(p)} = \frac{y^2}{4f}.$$

Now a pencil of parallel rays incident in the direction of the axis would be rendered stigmatic by the paraboloid; hence at the zone of radius y, the amount of pre-correction which has to be introduced in the waves incident upon the spherical mirror is given by

$$2(z^{(s)} - z^{(p)}) = \frac{y^4}{32f^3}$$

approximately, terms involving sixth and higher powers of y being neglected.* After passage through the plate, the rays are still nearly parallel, so that it makes no appreciable difference whether y is measured at the mirror or at the plate. Hence if n is the refractive index of the plate, its thickness $T(y)$ for a zone of radius y must

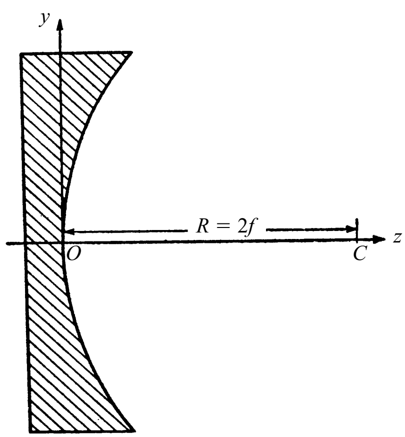

Fig. 6.22 Spherical mirror.

* This may be shown to be a permissible approximation if the focal ratio of the Schmidt camera is not lower than about $f/3$.

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exceed its axial thickness $T(0)$ by an amount given by

$$(n-1)[T(y)-T(0)] = 2(z^{(s)} - z^{(p)}),$$

i.e.

$$T(y) - T(0) = \frac{y^4}{32(n-1)f^3}. \tag{1}$$

The profile of such a corrector plate is shown in Fig. 6.21(a).

The asphericities required are very small. Consider, for example, an $f/3.5$ Schmidt camera, with a corrector plate of aperture diameter $2y_0 = 40$ cm. Then $f = 140$ cm. With $n = 1.5$ the maximum asphericity $[T(y) - T(0)]_{\max}$ is, according to (1), equal to $y_0{}^4/32(n-1)f^3 = 0.0036$ cm.

A further improvement may be obtained by comparing the spherical mirror with a somewhat different 'reference paraboloid'. If f' is the focal length of the paraboloid, then $z^{(p)} = y^2/4f'$, and one obtains in place of (1) the following expression for the plate profile:

$$T(y) - T(0) = \frac{y^4 - Ay^2}{32(n+1)f^3}, \tag{2}$$

where

$$A = 16f^3\left(\frac{1}{f'} - \frac{1}{f}\right).$$

With a corrector plate given by (2), rays incident parallel to the axis will converge, after passing through the system, to a point at a distance f' from the axial point of the mirror. This additional degree of freedom (choice of f') may be used to minimize the chromatic aberration introduced by the plate. Since

$$\frac{\mathrm{d}T}{\mathrm{d}y} = 0 \qquad \text{when} \qquad y(2y^2 - A) = 0,$$

it follows that a ray parallel to the axis and incident at height $\overline{y} = \sqrt{A/2}$ is not deflected by the plate. The zone of radius $\overline{y} = \sqrt{A/2}$ is known as the *neutral zone*. In order to minimize the chromatic aberration, the neutral zone must be fairly close to the edge of the plate,* as shown in Fig. 6.21(b).

It is evident that the corrector plate does not impart to rays which pass obliquely through the plate precisely the same deviations as to those passing normally. In consequence, the system will not be free from off-axis aberrations. Judged by ordinary standards, the images formed on the spherical receiving surface concentric with the mirror are, however, of excellent quality;† this is so even in a Schmidt camera of low

* More precisely, the diameter of the colour confusion circle may be shown to be least when the neutral zone is at about 0.87 of the full radius y_0 of the plate. This corresponds to a choice $A = \frac{3}{2}y_0{}^2$. For a discussion of this point see B. Strömgren, *Vierteljahrsschrift der Astr. Ges.*, **70** (1935), 82 and E. H. Linfoot, *Mon. Not. Roy. Astr. Soc.*, **109** (1949), 279. (Also his *Recent Advances in Optics* (Oxford, Clarendon Press, 1955), pp. 182, 192.)

† This is partly due to the fact that field curvature may be shown to be the only primary aberration of the system. (B. Strömgren, *loc. cit.*) The fifth-order aberrations of the Schmidt camera were investigated by C. Carathéodory, *Elementare Theorie des Spiegelteleskops von B. Schmidt* (Leipzig, Teubner, 1940), and E. H. Linfoot, *loc. cit*.

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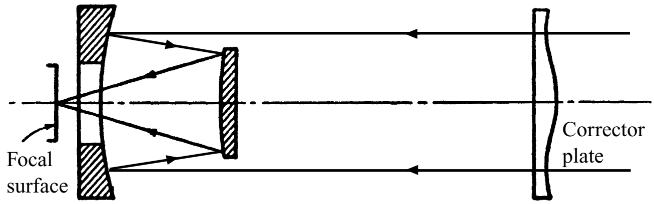
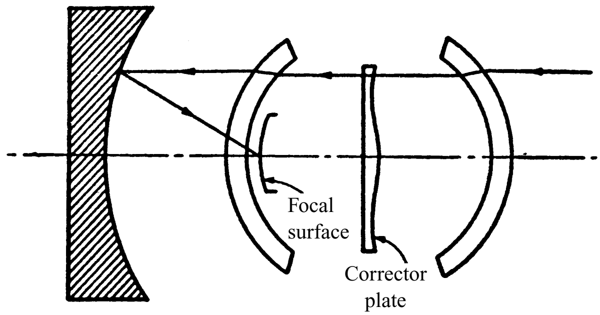


(a)



(b)

Fig. 6.23(a) The Schmidt–Cassegrain camera; (b) the Baker Super-Schmidt camera.

focal ratio, working over a field many times larger than is possible with systems of a more conventional design.

More complex systems using the principle of the Schmidt camera have been considered* and in some cases built. Most important amongst these are the *Schmidt-Cassegrain camera* which employs two (spherical or aspheric) mirrors together with a corrector plate [Fig. 6.23(a)] and the *Baker Super-Schmidt camera*, consisting of a spherical mirror, two menisci, and a corrector plate [Fig. 6.23(b)]. The former possesses the advantage that it operates with a flat field surface, which can be made more accessible than in the Schmidt camera; also, the overall length of this system is shorter than that of the Schmidt camera with the same focal length. The Baker Super-Schmidt camera can operate with extremely low focal ratios.

6.5 Instruments of illumination

The function of collecting as much light as possible from an artificial source and of deflecting it through the entrance pupil of an optical system, covering specified angles of acceptance at that entrance pupil, is carried out by an *instrument of illumination*. Light can only be obtained from a source of finite size and so the design of the

* For a review of such systems see H. Slevogt, *Zeitschr. f. Instrumkde*, **62** (1942), 312; H. Kohler, *Astr. Nachr.*, **278** (1949), 1; E. H. Linfoot, *Mon. Not. Roy. Astr. Soc.*, **108** (1948), 81. (Also his *Recent Advances in Optics* (Oxford, Clarendon Press, 1955) Chapter IV.) For a fuller description of the Baker Super-Schmidt camera see F. L. Whipple, *Sky and Telescope*, **8** (1949), 90.

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instrument depends upon the nature of the source as well as upon the specification of the illumination required.

In Fig. 6.24 is shown the entrance pupil of diameter r of an optical system requiring illumination over directions making angles with the axis up to a value ϕ. Let d be the diameter of the source, supposed circular. For greatest efficiency the illuminating system must work with magnification r/d, while its aperture must subtend an angle 2ϕ from the entrance pupil. The working distance w must be greater than a certain prescribed value. It is clear from a consideration of Fig. 6.24 that for lower values of d the *angle of collection*, 2ψ, will be greater, hence less light will be wasted. Also, large angles of illumination (large ϕ) will often make possible smaller values of w/r, so that larger sources can then be used with reasonable efficiency. Thus it is clear that in any particular case there is a restriction on the largest source which will give reasonable efficiency and that this restriction is less stringent when large angles of illumination are required. In a photographic enlarger ($\phi \sim 20°$), large sources (frosted bulbs or cold-cathode tubes) often provide reasonably efficient illumination. In a cinematograph projector ($\phi \sim 5°$) requiring maximum brightness, an electric filament is suitable. With a searchlight, where a strong beam over only 1° or 2° is required, especially small bright sources are the only possible ones, unless the size of the instrument is to be unduly great.

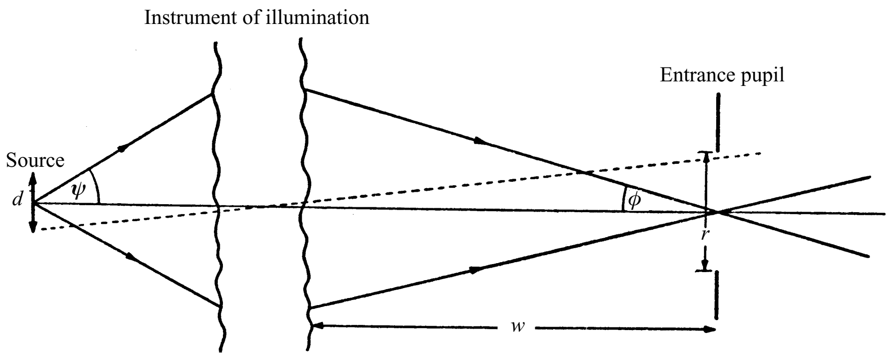

Fig. 6.24 Notation relating to an instrument of illumination.

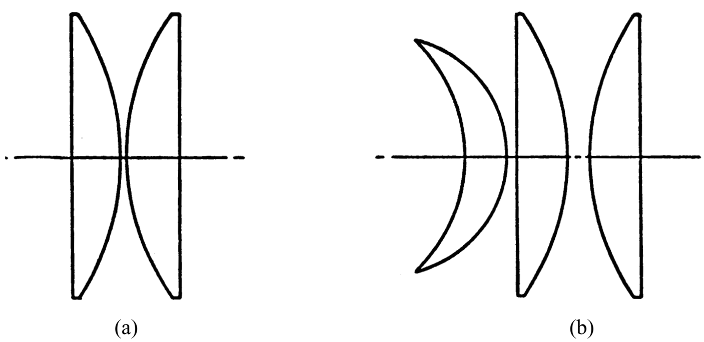

Fig. 6.25 Condenser lenses.

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In many instruments, the illumination can be provided by means of a suitable source and a condenser lens. This lens must be sufficiently free from aberrations to be able to utilize the smallness of the source. Every condenser may be said to possess a 'focal sphere', which represents the minimum source size with which it can efficiently be used. A design which is usually sufficiently good to give satisfactory results is that of the double plano-convex condenser [Fig. 6.25(a)], though sometimes a triple condenser is used [Fig. 6.25(b)]. In other cases a single lens may be adequate.

The illumination required by microscopes involves more complex consideration and will be briefly discussed later (§8.6.3 and §10.6.2).

6.6 The microscope

The apparent size of an object is determined by the size of its retinal image. If the eye is unaided, this apparent size depends on the angle which the object subtends at the eye. For a normal eye, the least distance of distinct vision is, as mentioned in §6.1, about 25 cm. This is the most favourable distance at which to examine the detail of any object. If a convergent lens is placed before the eye, the object may be brought much closer; for the lens will form an enlarged virtual image at a distance greater than the object distance (see Fig. 6.26), and it is the virtual image rather than the object itself, which is then being viewed.

The *magnifying power* of a visual instrument used to examine nearby objects is defined as the ratio of the angle subtended at the eye by the image of an object when the object is so placed that the image is at a standard distance (usually 25 cm) from the eye, to the angle subtended by the object when placed at the standard distance from the eye and viewed directly. This is equivalent to defining the magnifying power M as the ordinary linear magnification when the image is situated at the standard distance in front of the exit pupil of the instrument. For a simple lens, we have (see Fig. 6.26)

$$M = \frac{Y'}{Y} = \frac{\zeta'}{\zeta},$$

where $\zeta' = 25$ cm is the standard distance. By §4.4 (31),

$$\frac{1}{\zeta} - \frac{1}{\zeta'} = \frac{1}{|f|}, \tag{2}$$

so that

$$M = 1 + \frac{\zeta'}{|f|} = 1 + \frac{25}{|f|}, \tag{3}$$

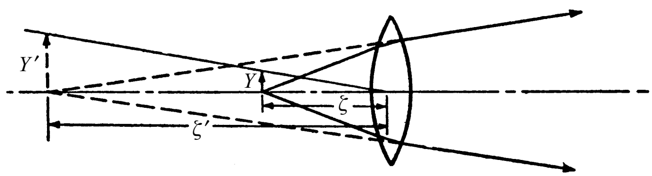

Fig. 6.26 The magnifier.

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the focal length f being measured in centimetres in the last expression. Since f is usually small compared to 25 cm, the magnifying power may simply be written as

$$M \sim \frac{25}{|f|}. \tag{4}$$

Various forms of magnifiers are used; the simple double-convex lens, or an achromatic doublet, is the most usual in pocket magnifiers or watchmaker's eyeglasses. A separated pair of plano-convex lenses similar to the Ramsden eyepiece [Fig. 6.13(b)] is often used. It is evident that the distance between the object and the eye will become inconveniently small if larger apparent fields of view are required with large magnification. For this reason the optical system of the *microscope* (invented by Galileo about 1610), employing an objective of short focal length and a magnifying eyepiece, was evolved, the magnification being achieved in two stages (Fig. 6.27).

The *objective* of the microscope forms an enlarged image of the object in a position suitable for viewing through the *eyepiece*. The magnification at the objective is given by $-\zeta_0'/\zeta_0$, where ζ_0 and ζ_0' are the working distances of the objective, satisfying a relation of the form (2). If f_1 is the focal length of the eyepiece, measured in centimetres, the magnification at the eyepiece may, according to (4), be written as $25/|f_1|$. Hence the magnification of the system working as a whole is

$$M = -\frac{25}{|f_1|}\frac{\zeta_0'}{\zeta_0},$$

the negative value indicating that the image is inverted. The magnifications of the objective and eyepiece are usually stated separately in microscope specifications.

The other quantity of interest in the general specification of a microscope objective is its *numerical aperture* (NA) already defined in §4.8.2 as the product $n \sin\theta$, where θ is the angular semiaperture on the object side (i.e. the semiangle of the cone of rays from the axial object point which is received by the objective) and n is the refractive index of the medium of the object space. This quantity is a measure not only of the light-gathering power of the objective, but also, as will be shown in §8.6.3, of its resolving power, i.e. of the ultimate limit to the clarity of detail which can be obtained with it. Here we shall describe how the highest numerical apertures are attained.

Microscope objectives must, in general, be highly corrected for spherical aberration and coma and for chromatic aberration, since they are to receive rays over as large an

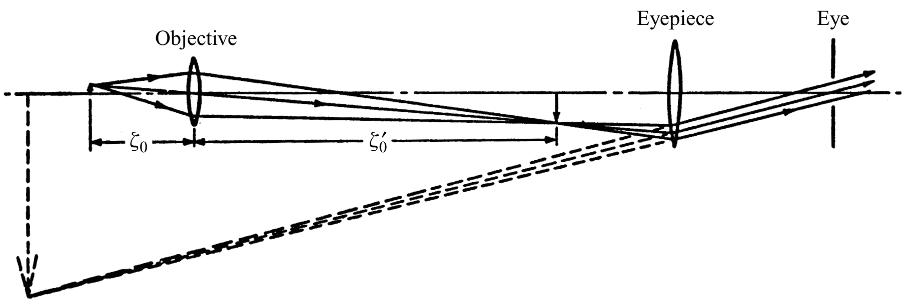

Fig. 6.27 Illustrating the principle of the microscope.

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aperture as possible. In objectives of low magnification (∼ 10×) it is possible to employ two separated cemented achromats [Fig. 6.29(a)], the combination being corrected for spherical aberration and coma. Such lens combinations are, however, not suitable for objectives of high magnification (∼ 50× or more). One then uses different lens systems, which utilize the existence of aplanatic points of a spherical surface (see §4.2.3) in the following way:

The object is placed at a point P close to the plano-convex lens [Fig. 6.28(a)] and the space between the object and the lens is filled with oil, the refractive index n of which is nearly equal to that of the lens. If C is the centre of curvature of the curved portion of the lens, and r the radius of curvature, then a point at distance r_1/n_1 from C will give rise to a virtual image P_1 at distance $n_1 r_1$ from C, the imaging between the two points being aplanatic. In this way a higher numerical aperture may be employed, the angular divergence of the rays from P being reduced within the system, without the introduction of monochromatic aberrations. Chromatic aberration is, however, introduced and this must be compensated by the rest of the system.

The angular divergence of the rays may be reduced still further by the use of a convergent meniscus* [Fig. 6.28(b)]. The front surface of the meniscus has its centre of curvature at P_1, and the radius of its rear surface is such that P is an aplanatic point with respect to it. The rays refracted from this surface form a virtual image at the other aplanatic point P_2. Adding further menisci, it is possible to produce successive virtual images P_3, P_4, ... lying further and further away from P, thus reducing the angular

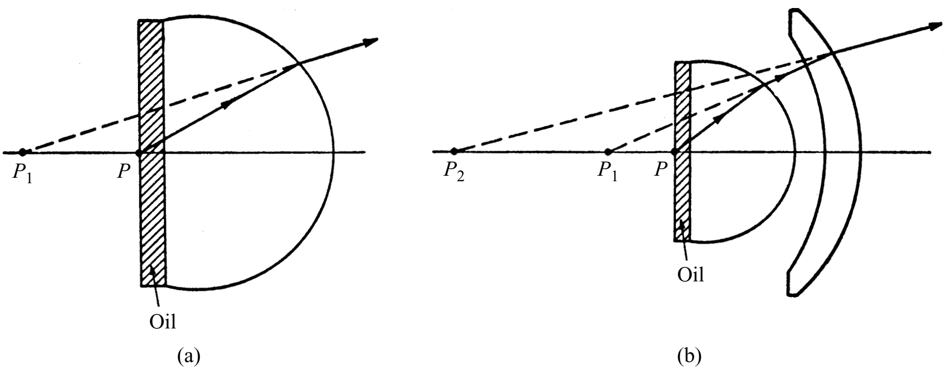

Fig. 6.28 Reduction of the angular divergence of rays in microscopes.

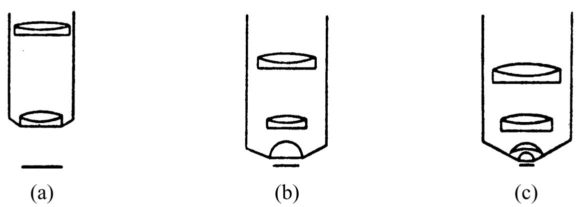

Fig. 6.29 Microscope objectives: (a) low power; (b) medium power; (c) high power.

* This method is due to G. B. Amici, *Ann. de chim. et phys.* (3), **12** (1844), 117.

divergence of the rays more and more. However, more than two lenses are seldom employed as otherwise the chromatic aberration cannot be adequately compensated.

The *oil-immersion objectives* just described admit a wider cone of rays than a *dry objective* of the same diameter would. For in a dry objective, the cone of rays emerging from the cover glass which protects the object passes into air, and consequently the rays are bent outwards by the refraction, the angular divergence being thus increased; in an oil-immersion objective, on the other hand, they are not refracted on emergence from the cover glass. It is only with oil-immersion objectives that the highest numerical apertures (~ 1.4) can be attained.

Demand has also arisen for microscope objectives which can operate with light which includes the near ultra-violet wavelengths ($\lambda \sim 2500$ Å) and wavelengths of the visible and the near infra-red regions. For example, if a microscope is to be used in conjunction with a spectrograph and a photometric instrument, a high degree of achromatism is necessary.

The design of achromatic microscope objectives was first systematically investigated by Burch,* who took as the starting point Schwarzschild's analytical solution of the aplanatic two-mirror systems mentioned in §4.10. Burch found that, if the numerical aperture is to exceed 0.5, at least one of the mirrors must be aspherical. Numerical apertures as high as those of the best conventional oil-immersion objectives cannot, however, be realized in a reflecting system. A typical Burch achromatic objective is shown diagrammatically in Fig. 6.30. Apart from achromatism, its chief advantage lies in its great working distance (distance between the object and the nearest surface of the objective), which may be as great as the focal length of the objective. A disadvantage is the effect of the obstruction due to the secondary mirror; in systems of lower numerical aperture which employ spherical surfaces, the obstruction may exceed 45 per cent.

Microscope objectives which combine reflection and refraction have also been

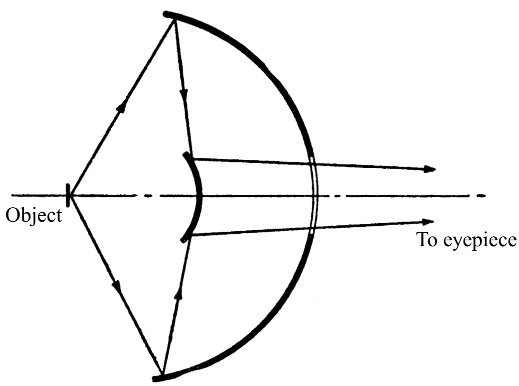

Fig. 6.30 The Burch reflecting microscope objective.

* C. R. Burch, *Proc. Phys. Soc.*, **59** (1947), 41; *ibid.*, **59** (1947), 47. The first paper also contains a review of earlier researches on achromatic microscope objectives.

considered. Thus Bouwers* has constructed a system in which the reflections take place at concentric spherical silvered surfaces within glass, the silvering on the second surface being so distributed that the effective obstruction is reduced to 20 per cent. A two-mirror objective with low obstruction ratio was also described by Grey.† It used quartz and fluorite components of considerable power which, whilst maintaining adequate colour correction throughout the ultra-violet and visible wavelengths, effectively replace the figuring which would otherwise be necessary on the mirrors.

The objects examined under a microscope are not usually luminous and have therefore to be illuminated. In most cases the object has the form of a thin slice of transparent material. It is then illuminated from behind, or *transilluminated*. In systems of low numerical aperture (NA up to 0.25), diffuse ambient light reflected at oblique incidence from a concave mirror will suffice, but otherwise illumination from an artificial source is necessary. To obtain sufficient concentration of light, an auxiliary lens system, a condenser, is used. There are various methods of illumination, two of which will be described in Chapter X.

The present section has included only the basic principles of a microscope. An adequate discussion of the image formation in a microscope needs more refined methods (see §8.6.3, §9.5 and §10.6).

* A. Bouwers, *Achievements in Optics* (Amsterdam, Elsevier, 1946).
† D. S. Grey, *Proceedings of the London Conference on Optical Instruments* (London, Chapman & Hall, 1951), p. 65.