

Ex 7.

$$i\hbar \partial_t \psi = -\frac{\hbar^2}{2m} \partial_x^2 \psi + \frac{1}{2} m \omega^2 x^2 \psi = \hat{H} \psi$$

$$\tilde{\tau} = \frac{\omega}{2}$$

$$\psi(t) = \tilde{\psi}(\underbrace{\tilde{\tau} t}_{=\tau})$$

$$\begin{aligned} \Rightarrow \partial_t \psi(t) &= \tilde{\tau} \partial_{\tau} \tilde{\psi}(\tau) \\ &= \frac{\omega}{2} \partial_{\tau} \tilde{\psi}(\tau) \end{aligned}$$

$$\Rightarrow i\hbar \frac{\omega}{2} \partial_{\tau} \tilde{\psi} = -\frac{\hbar^2}{2m} \partial_x^2 \psi + \frac{1}{2} m \omega^2 x^2 \psi$$

$$\Leftrightarrow i \partial_{\tau} \tilde{\psi} = -\frac{\hbar}{m\omega} \partial_x^2 \psi + \underbrace{\frac{m\omega}{\hbar} x^2}_{=\xi^2} \psi$$

$$\partial_x \psi(x) = \frac{m\omega}{\hbar} \partial_{\xi}^2 \tilde{\psi}(\xi)$$

$$\Rightarrow i \partial_{\tau} \psi = -\partial_{\xi}^2 \psi + \xi^2 \psi = \tilde{\hat{H}} \psi$$

$$\text{with } \tilde{\hat{H}} = \underbrace{\frac{2}{\hbar\omega}}_{=\frac{1}{\beta}} \hat{H}$$