Sheet 4

Ex 1

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

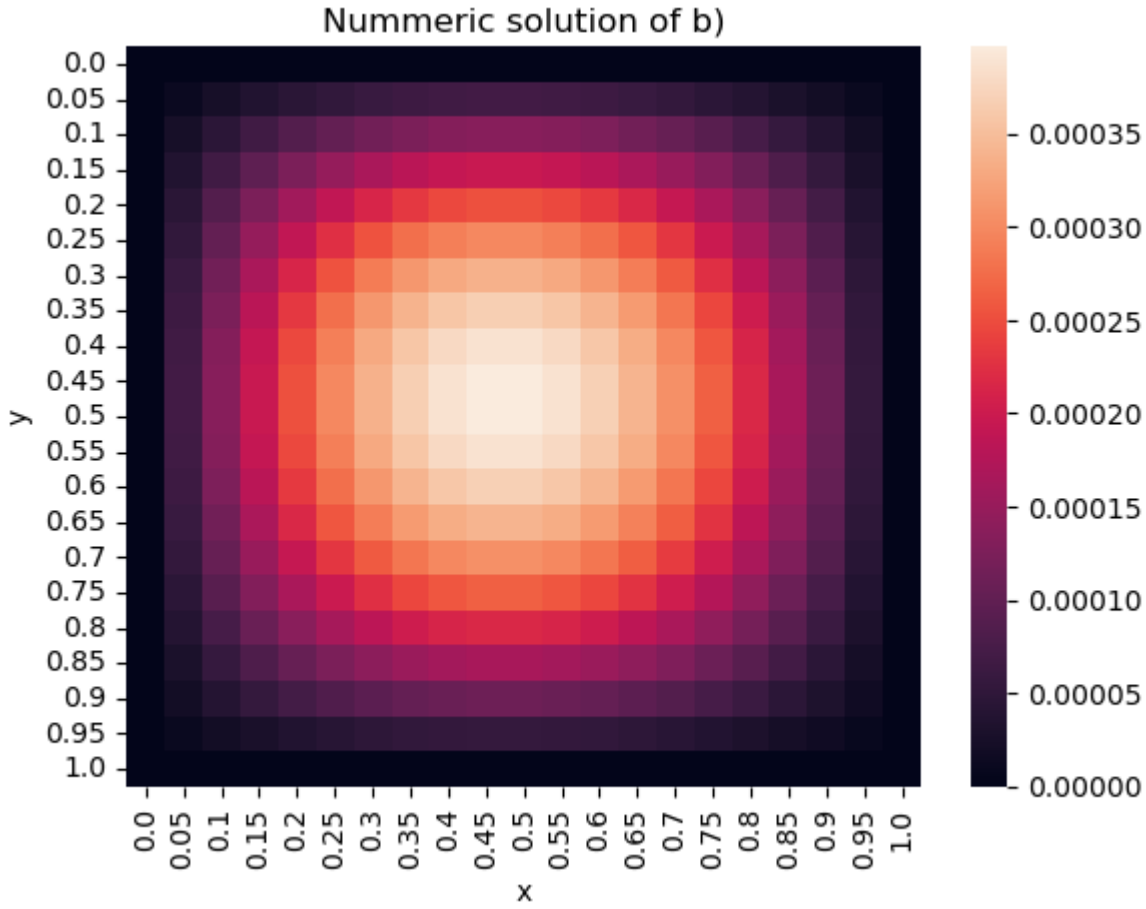
a)

In [ ]:
```
def n_grad(potential):
    # Calculate the gradient using central differences
    gradient_x = np.gradient(potential, axis=1)
    gradient_y = np.gradient(potential, axis=0)

    # Return the negative gradient components
    return -gradient_x, -gradient_y


def Gauss_Seidel(Phi, Delta, epsilon, rho, start_condions):
    J = int(1/Delta)
    L = int(1/Delta)
    #boundry conditions:
    Phi[0,:] = start_condions[0]
    Phi[J,:] = start_condions[1]
    Phi[:,0] = start_condions[2]
    Phi[:,L] = start_condions[3]

    index = np.arange(1,J) # index to go through the grid
    count = 0 # counter for number of iterations
    while True: # do while
        Phi_pre = Phi.copy()
        count += 1
        for j in index:
            for l in index:
                Phi[j,l] = 0.25 * (Phi[j+1,l] + Phi[j-1,l] + Phi[j,l+1] + Phi[j,l-1
        if ((np.abs(Phi-Phi_pre) < epsilon).all()):
            break
    print("Iterations:", count)
    E_x , E_y = n_grad(Phi) # calc E
    return Phi , E_x , E_y
```

b)

In [ ]:
```
Delta = 0.05
epsilon = 1e-5

J = int(1/Delta)
L = int(1/Delta)
x_array = np.arange(0,1+Delta,Delta)
y_array = np.arange(0,1+Delta,Delta)

Phi_b = np.ones((J+1,J+1))
rho_b = np.zeros((J+1,J+1))
start_conditions_b = np.zeros(4)

Phi_solution_b = Gauss_Seidel(Phi_b, Delta, epsilon, rho_b, start_conditions_b)
ax = sns.heatmap(Phi_solution_b[0],xticklabels= x_array.round(2), yticklabels=y_arr
ax.set_title('Nummeric solution of b)')
ax.set_xlabel('x')
ax.set_ylabel('y')
plt.show()
```

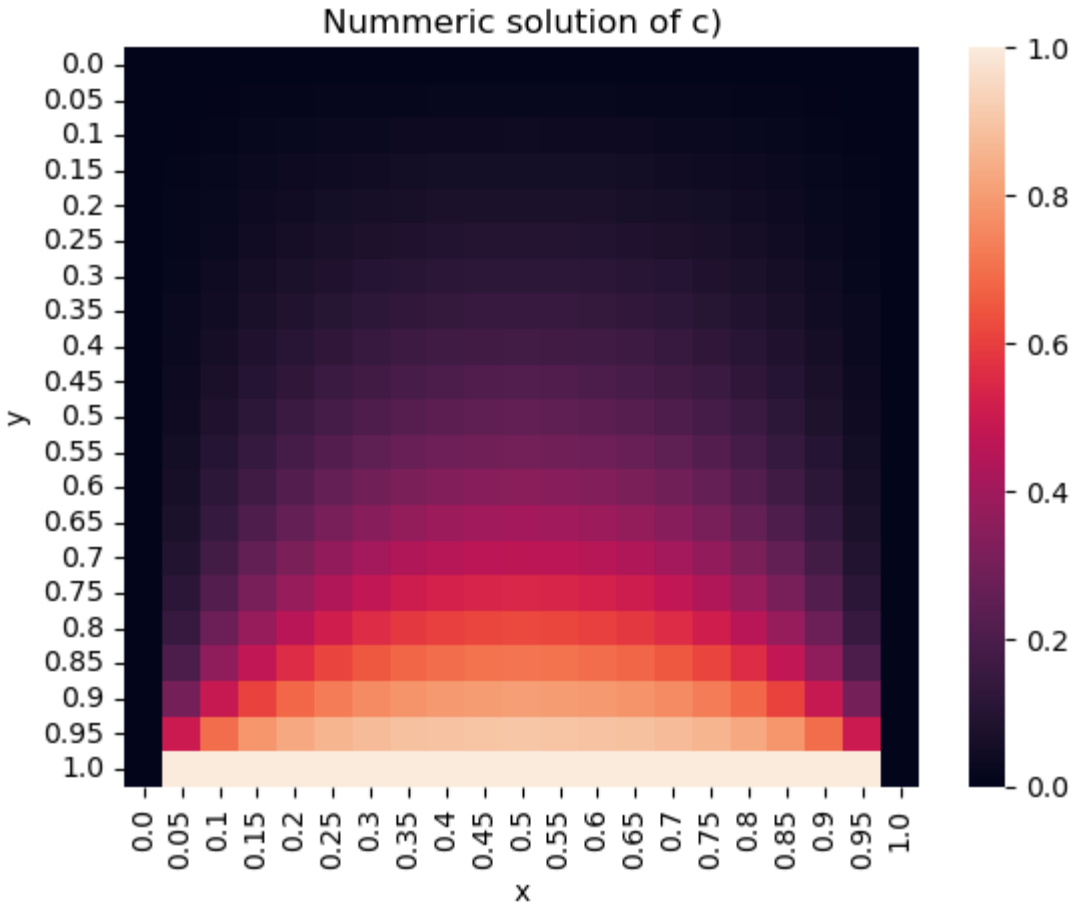
```
Iterations: 336
```

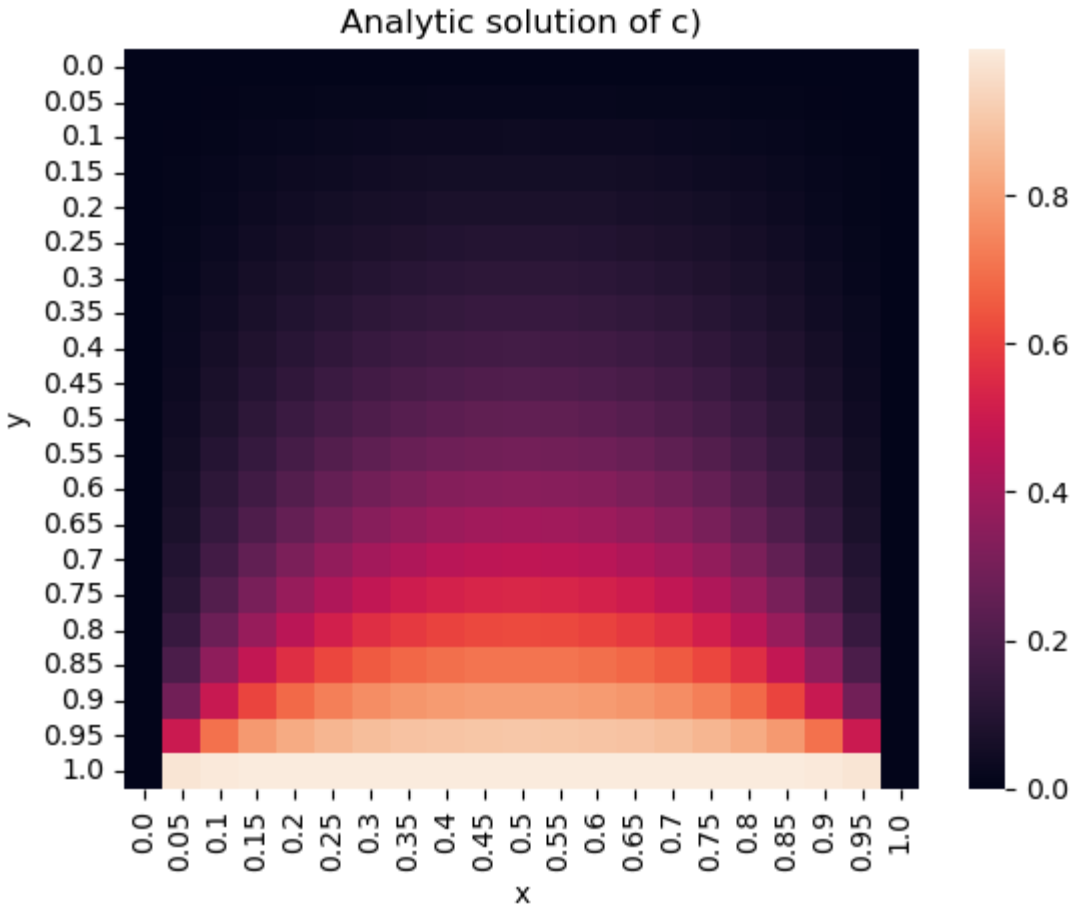


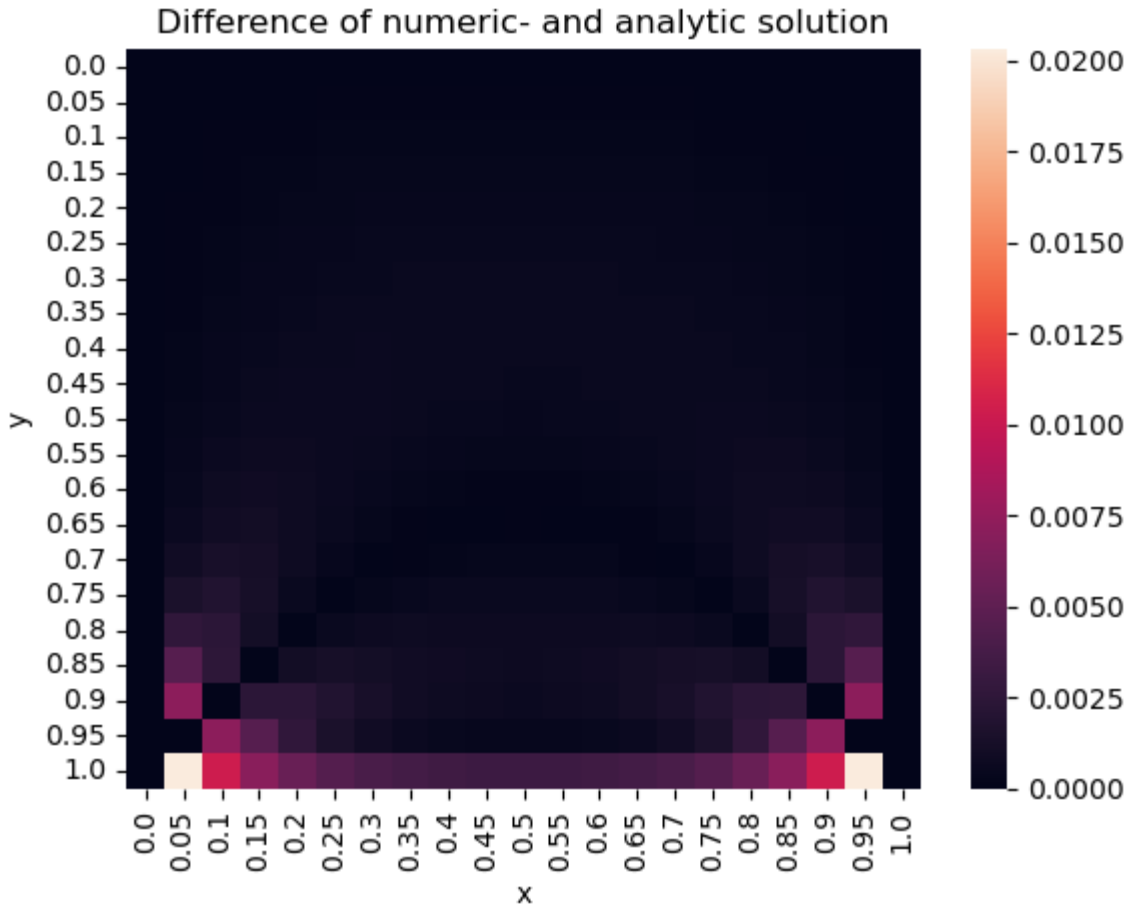
c)

In [ ]:
```
Phi_c = np.ones((J+1,J+1))
rho_c = np.zeros((J+1,J+1))
start_conditions_c = np.array([0,1,0,0])
Phi_solution_c = Gauss_Seidel(Phi_c, Delta, epsilon, rho_c, start_conditions_c)



### analytic solution:
def analytic_solution(x_values, y_values, n_iterations):
    x, y = np.meshgrid(x_values, y_values)
    n_index = np.arange(1, n_iterations)
    result = np.zeros_like(x)
    for n in n_index:
        result += 2 * (1-np.cos(n*np.pi)) / (n*np.pi*np.sinh(n*np.pi)) * np.sin(n*n
    return result

Phi_analytic = analytic_solution(x_array,y_array,200)


ax = sns.heatmap(Phi_solution_c[0],xticklabels= x_array.round(2), yticklabels=y_arr
ax.set_title('Nummeric solution of c)')
ax.set_xlabel('x')
ax.set_ylabel('y')
plt.show()

ax = sns.heatmap(Phi_analytic,xticklabels= x_array.round(2), yticklabels=y_array.ro
ax.set_title('Analytic solution of c)')
ax.set_xlabel('x')
ax.set_ylabel('y')
plt.show()

ax = sns.heatmap(np.abs(Phi_analytic-Phi_solution_c[0]),xticklabels= x_array.round(
ax.set_title('Difference of numeric- and analytic solution')
ax.set_xlabel('x')
ax.set_ylabel('y')
plt.show()
```

```
Iterations: 323
```


Nummeric solution of c)
y
x
0.0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1.0
0.0
0.2
0.4
0.6
0.8
1.0

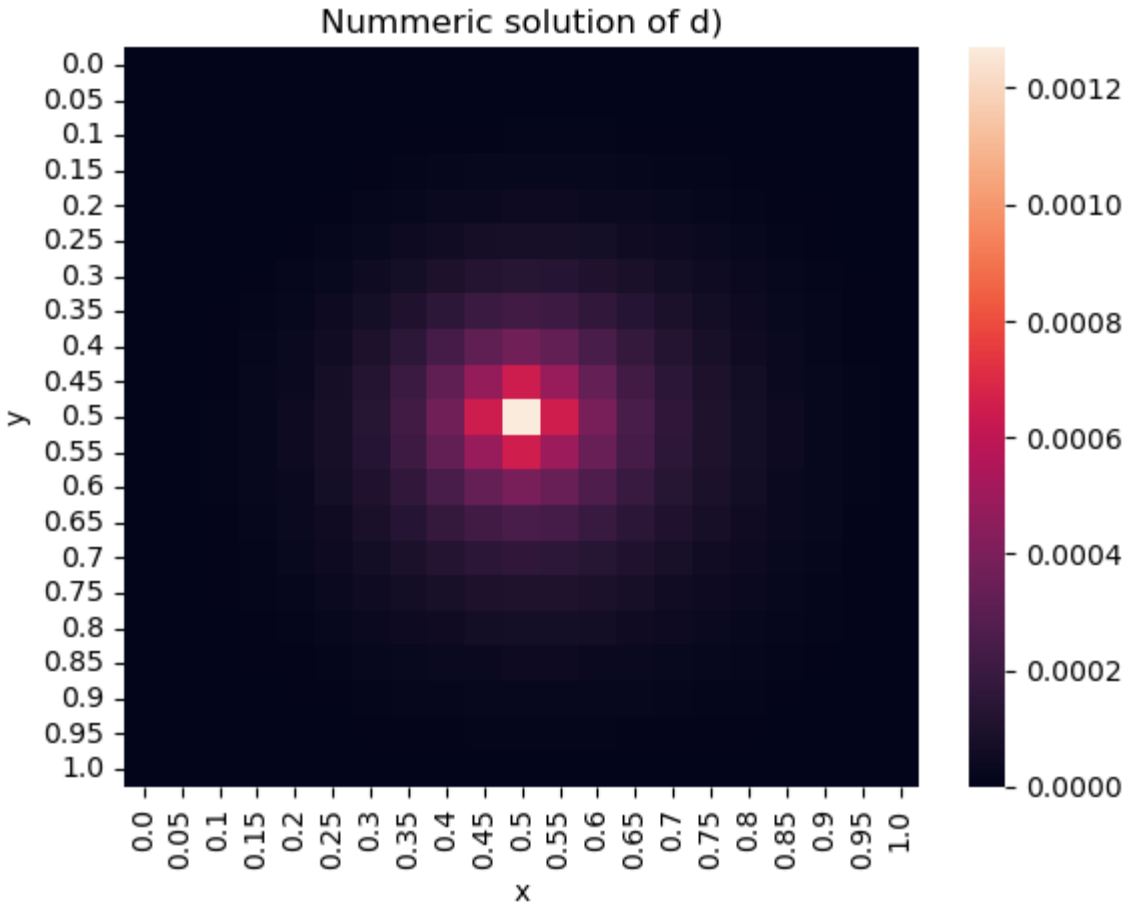

Analytic solution of c)
y
x
0.0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1.0
0.0
0.2
0.4
0.6
0.8

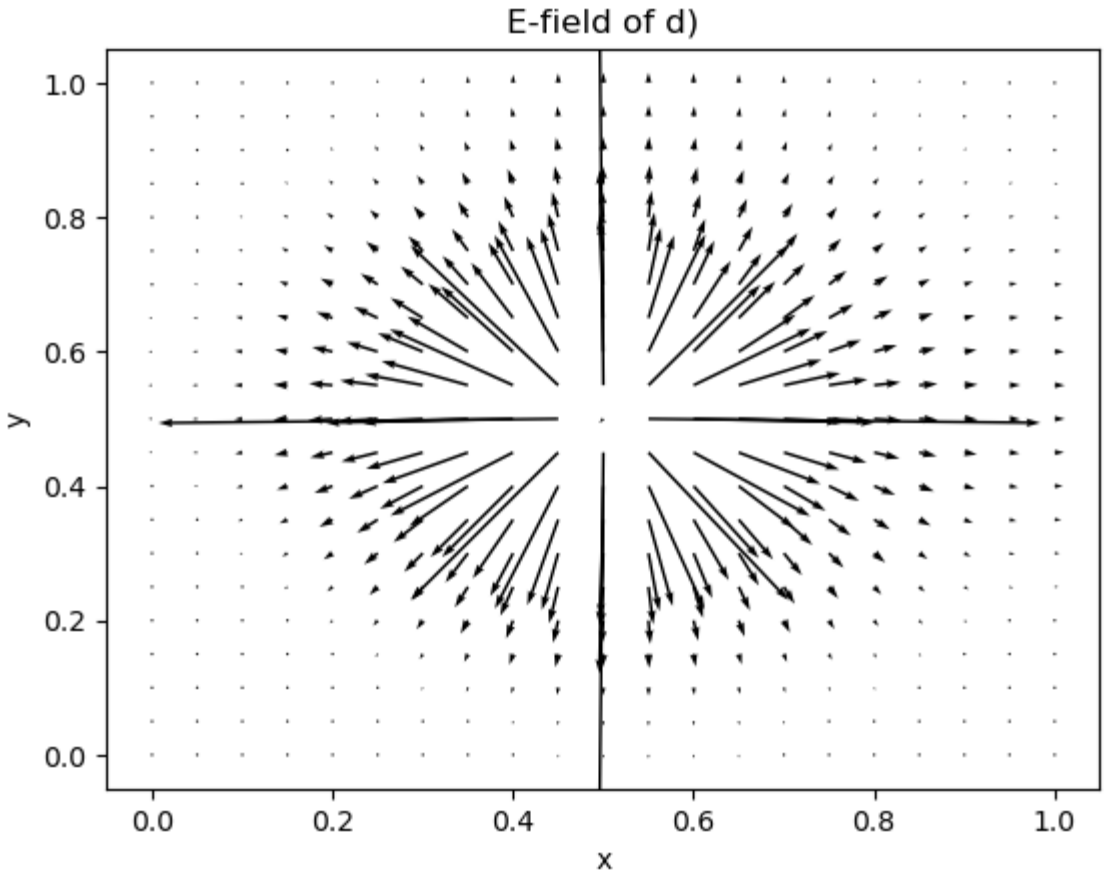


d)

In [ ]:
```
Phi_d = np.zeros((J+1,J+1))
rho_d = np.zeros((J+1,J+1))
rho_d[int(J/2),int(L/2)] = 1
start_conditions_d = np.zeros(4)
Phi_solution_d = Gauss_Seidel(Phi_d, Delta, epsilon, rho_d, start_conditions_d)


ax = sns.heatmap(Phi_solution_d[0],xticklabels= x_array.round(2), yticklabels=y_arr
ax.set_title('Nummeric solution of d)')
ax.set_xlabel('x')
ax.set_ylabel('y')
plt.show()


plt.quiver(x_array, y_array, Phi_solution_d[1], Phi_solution_d[2] )
plt.title('E-field of d)')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

```
Iterations: 21
```

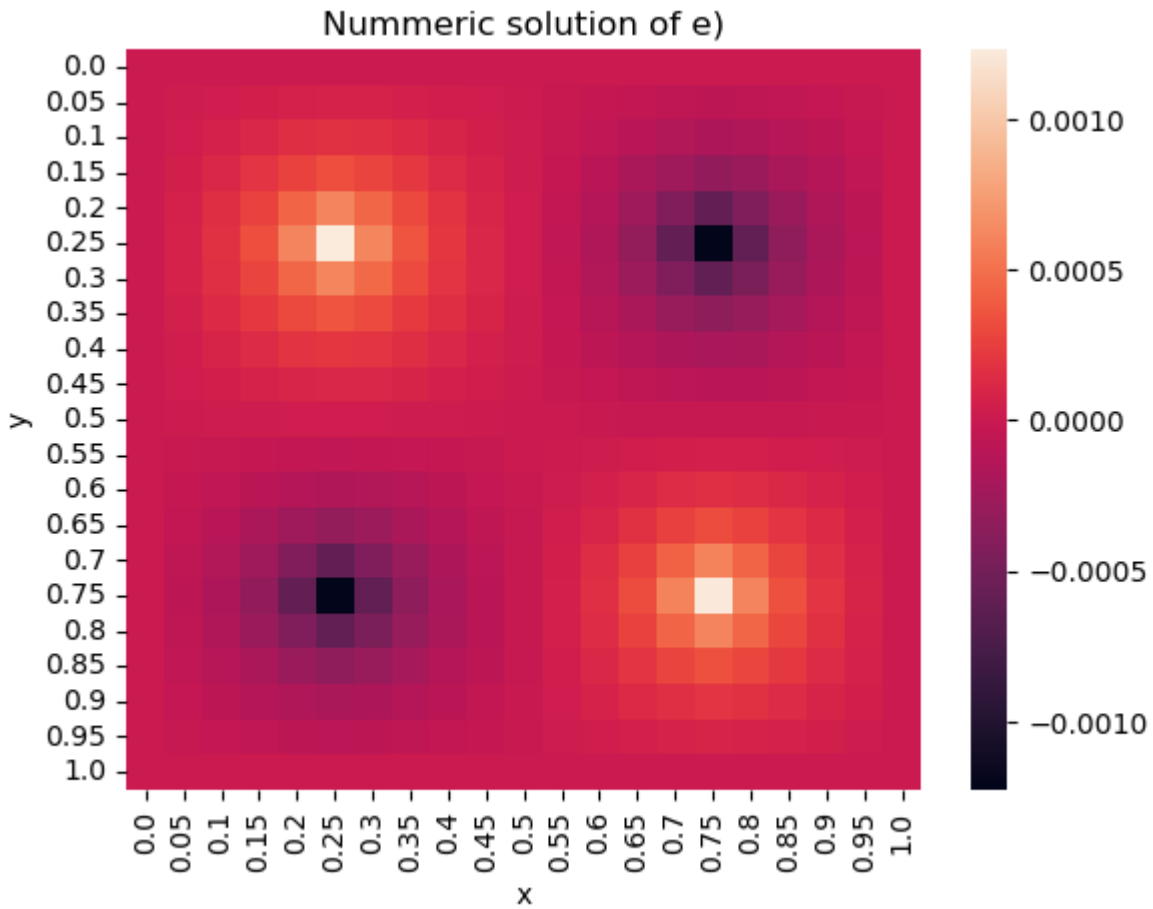

Nummeric solution of d)
y
x
0.0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1.0
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000

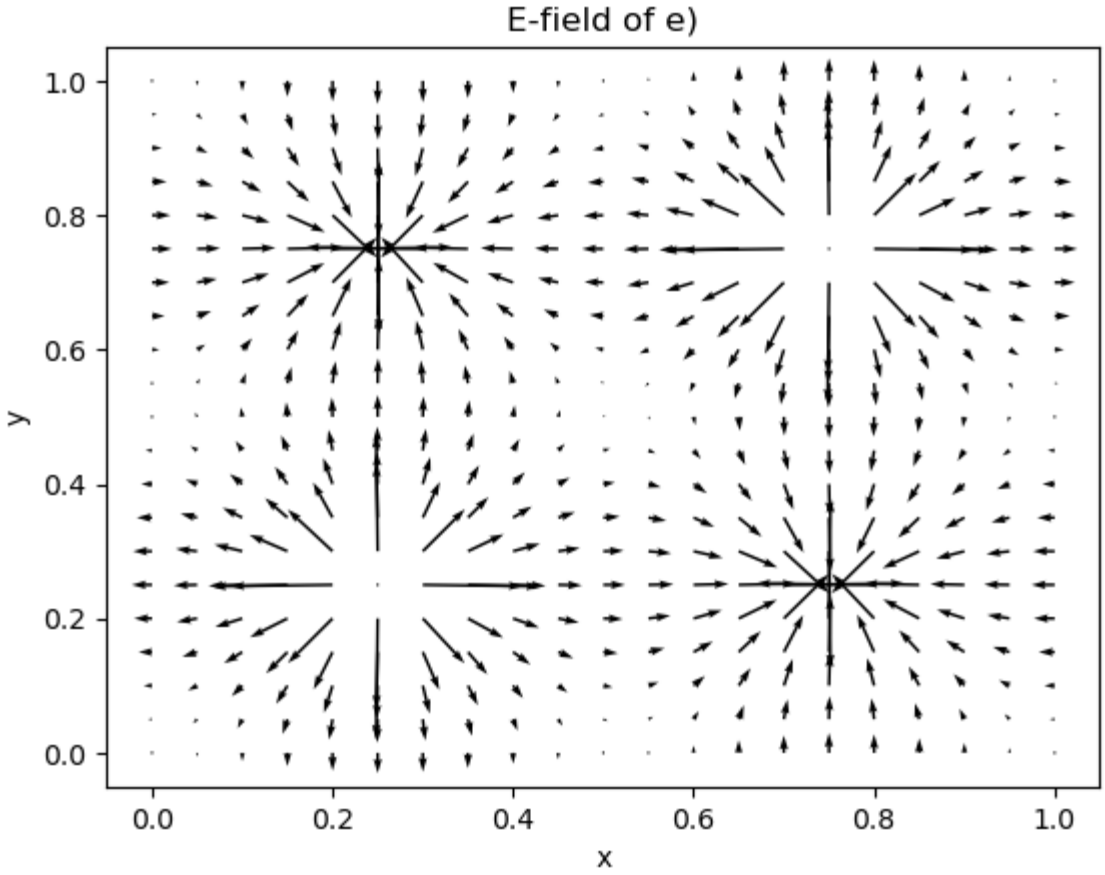

E-field of d)
y
x
0.0
0.2
0.4
0.6
0.8
1.0

e)

In [ ]:
```
Phi_e = np.zeros((J+1,J+1))
rho_e = np.zeros((J+1,J+1))
rho_e[int(J/4),int(L/4)] = 1
rho_e[int(3*J/4),int(3*L/4)] = 1
rho_e[int(J/4),int(3*L/4)] = -1
rho_e[int(3*J/4),int(L/4)] = -1
start_conditions_e = np.zeros(4)
Phi_solution_e = Gauss_Seidel(Phi_e, Delta, epsilon, rho_e, start_conditions_e)


ax = sns.heatmap(Phi_solution_e[0],xticklabels= x_array.round(2), yticklabels=y_arr
ax.set_title('Nummeric solution of e)')
ax.set_xlabel('x')
ax.set_ylabel('y')
plt.show()

plt.quiver(x_array, y_array, Phi_solution_e[1], Phi_solution_e[2] )
plt.title('E-field of e)')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

```
Iterations: 18
```




E-field of e)
1.0
0.8
0.6
0.4
0.2
0.0
y
0.0
0.2
0.4
0.6
0.8
1.0
x