Computational Physics — SuSe 2023
0. Exercise Sheet

Handing out: 3.04.2023 — Prof. H. Jelger Risselada
Submission: 7.04.2023 8 pm

Exercise 0: Comprehension questions — 0 Points

1) What does numerical *stability* mean and why does it occur?

2) Why can higher *accuracy* (for example due to finer discretization) lead to numerical instability?

Exercise 1: Hello World — 4 Points

Install a compiler (e.g. `GCC`) on your system. Test it by writing a program that outputs `Hello World`. Please use only up to date python or C++ Versions for your codes (python3 or C++11 or higher). This holds for this, as well as for all following exercises.
Voluntary bonus task: If you use `GCC`, read up on the `-Ox` compiler flag (with `x` $\in \{1, 2, 3\}$), which optimizes the code during compilation and can reduce the computation time of the program significantly.

Exercise 2: Rounding error — 8 Points

In some cases, numerical stability can be established by cleverly transforming or approximating the critical calculation steps. In the following, you will learn this on the basis of three example calculations in order to be able to work out simple stabilization approaches by yourself in the future.

Write a program that calculates the following expressions:

a) for large $x \gg 1$:

$$\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x+1}}, \tag{1}$$

b) for small $x \ll 1$:

$$\frac{1-\cos x}{\sin x}, \tag{2}$$

c) for small $\delta \ll 1$:

$$\sin(x+\delta) - \sin x. \tag{3}$$

Then look for a numerical calculation path that avoids cancellation. Compare the relative errors between the calculation path with cancellation and without cancellation.

Exercise 3: Stability — 8 Points

Higher accuracy does not always mean higher stability. We will study this by using the Euler method known from the lecture and writing a first numerical integration as preparation for later tasks.

The differential equation

$$\dot{y}(t) = -y(t), \quad y(0) = 1 \tag{4}$$

with the analytical solution

$$y(t) = \exp(-t) \tag{5}$$

shall be solved numerically using the Euler method and the symmetric Euler method with a step size Δt. The Euler method yields the recursion

$$y_{n+1} = y_n(1 - \Delta t) \tag{6}$$

and the symmetric Euler method yields

$$y_{n+1} = -2\Delta t y_n + y_{n-1}. \tag{7}$$

a) Implement both the Euler method and the symmetric Euler method and start with initial values $y = 1$ and for the symmetric Euler method additionally with $y_1 = \exp(-\Delta t)$. Compare your results with the analytical solution in the interval $t \in [0, 10]$.

   Hinweis: Find an appropriate value for Δt.

b) Compare your results from the previous part of the task with the results when you start the Euler method with $y_0 = 1 - \Delta t$ and the symmetric Euler method with $y_0 = 1$ and $y_1 = y_0 - \Delta t$. Interpret your results.