Ex 1.

$$i\hbar\,\partial_t\,\psi = -\frac{\hbar^2}{2m}\partial_x^2\psi + \frac{1}{2}m\omega^2 x^2\psi = \hat{H}\psi$$

$$\tilde{\tau} = \frac{\omega}{2}$$

$$\psi(t) = \tilde{\psi}(\underbrace{\tilde{\tau}\,t}_{=\tau})$$

$$\Rightarrow \quad \partial_t\,\psi(t) = \tilde{\tau}\,\partial_\tau\,\tilde{\psi}(\tau) = \frac{\omega}{2}\partial_\tau\,\tilde{\psi}(\tau)$$

$$\Rightarrow \quad i\hbar\frac{\omega}{2}\partial_\tau\,\tilde{\psi} = -\frac{\hbar^2}{2m}\partial_x^2\psi + \frac{1}{2}m\omega^2 x^2\psi$$

$$\Leftrightarrow \quad i\,\partial_\tau\,\tilde{\psi} = -\frac{\hbar}{\omega m}\partial_x^2\psi + \underbrace{\frac{m\omega}{\hbar}}_{=\xi^2}x^2\,\psi$$

$$\partial_x\,\psi(x) = \frac{m\omega}{\hbar}\,\partial_\xi^2\,\tilde{\psi}(\xi)$$

$$\Rightarrow \quad i\,\partial_\tau\,\psi = -\partial_\xi^2\psi + \xi^2\psi = \tilde{\hat{H}}\psi$$

$$\text{with} \quad \tilde{\hat{H}} = \underbrace{\frac{2}{\hbar\omega}}_{=\frac{1}{\beta}}\hat{H}$$