Sheet06

Computational Physics

1. Juni 2023

1 Solution 1

1. Zum Warmlaufen wird die Abbildung $N = 30$ mal durchiteriert. Die Zahl wird relativ willkürlich gewählt, solle aber nicht zu niedrig sein.

   Überlegungen für das Maximale r: Werden die möglichen Werte $f(x) \equiv x_{n+1}$ gegen x_n aufgetragen entstehen umgedrehte Parabeln, wobei die x-position des Maximums/Scheitelpunkts r-unabhägig ist:

$$
\begin{aligned}
f'(x) &= r(1-2x) \overset{!}{=} 0 && (1)\\
\Rightarrow x_0 &= 0.5 && (2)\\
\Rightarrow f(x_0) &= r(x_0 - x_0^2) \overset{!}{=} 1 && (3)\\
\Rightarrow r &= 4 && (4)
\end{aligned}
$$

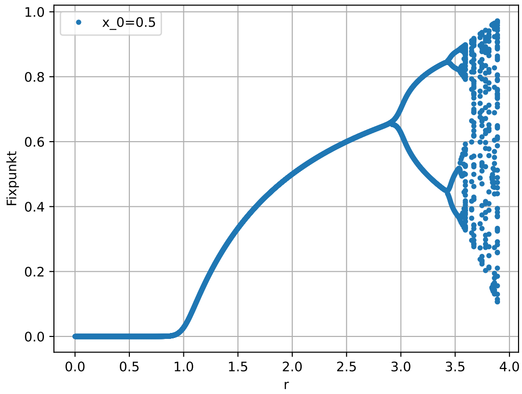
   r ist also Mximal 4.

b) Es wird grafisch herausgefunden, dass $r \leq 3$ sein darf.

2./3. r wird systematisch in Schritten $\Delta r = 0.001$ vatiiert. Jeder r- Wert erzeugt jeweilige Fixpunkte, welche dann in r-Abhängigkeit in ein Diagramm aufgetragen werden. Es werden verschiedene Startwerte x_0 verwendet.

b)

(a) $x_0 = 0.5$



(b) $x_0 = 0.1$

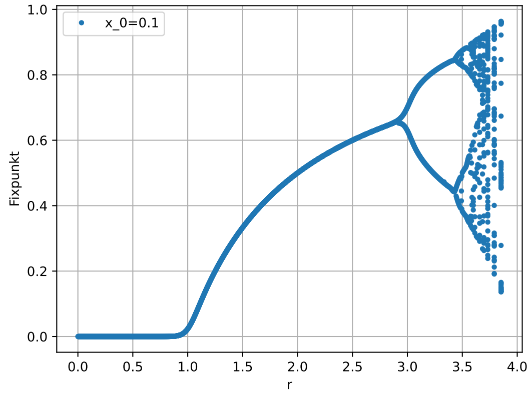
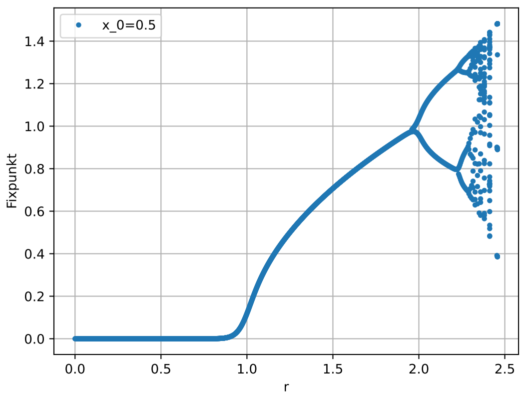


Abbildung 1: Bifurkationsdiagramm der logistischen Abbildung.

(a) $x_0 = 0.5$



(b) $x_0 = 0.1$

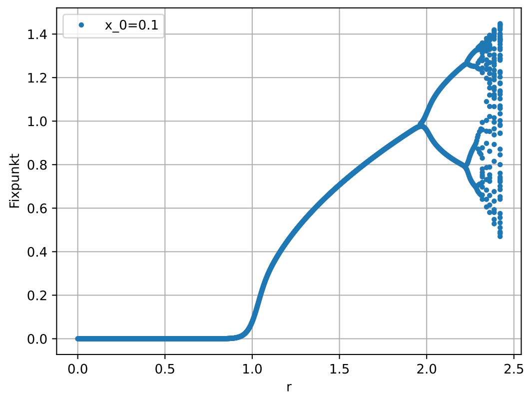


Abbildung 2: Bifurkationsdiagramm der kubischen Abbildung.

Tabelle 1: Werte r_∞ für die logistische Abbildung.

x_0	r_∞
0.1	3.853
0.3	3.632
0.5	3.889
0.7	3.632
0.9	3.732

Tabelle 2: Werte r_∞ für die kubische Abbildung.

x_0	r_∞
0.1	2.421
0.3	2.102
0.5	2.457
0.7	2.346
0.9	2.323

2 Solution 2

a) In the second exercise we were asked to solve the Lorentz equations

$$\begin{aligned}\dot{X} &= -\sigma X + \sigma Y \\ \dot{Y} &= -XZ + rX - Y \\ \dot{Z} &= XY - bZ\end{aligned}$$

with the fourth order Runge-Kutta scheme. The implementation can be found in file "2.cpp". The parameters r, σ and b were given as:

$$\begin{aligned}r &= 20 \,\mathrm{or}\, 28 \\ \sigma &= 10 \\ b &= 8/3\end{aligned}$$

In the following plots one can clearly see that a change in the starting parameter such as r results in a different behaviour. One gives a stable orbit around an attractor, the other falls into the attractor. Other parameters show even different behaviour. However, the starting position does not influecs the trajectory.

b) After the implementation we were asked to visualize our solution. This should be done in three ways

1. First as a projection of the trajectory on the xy-plane.

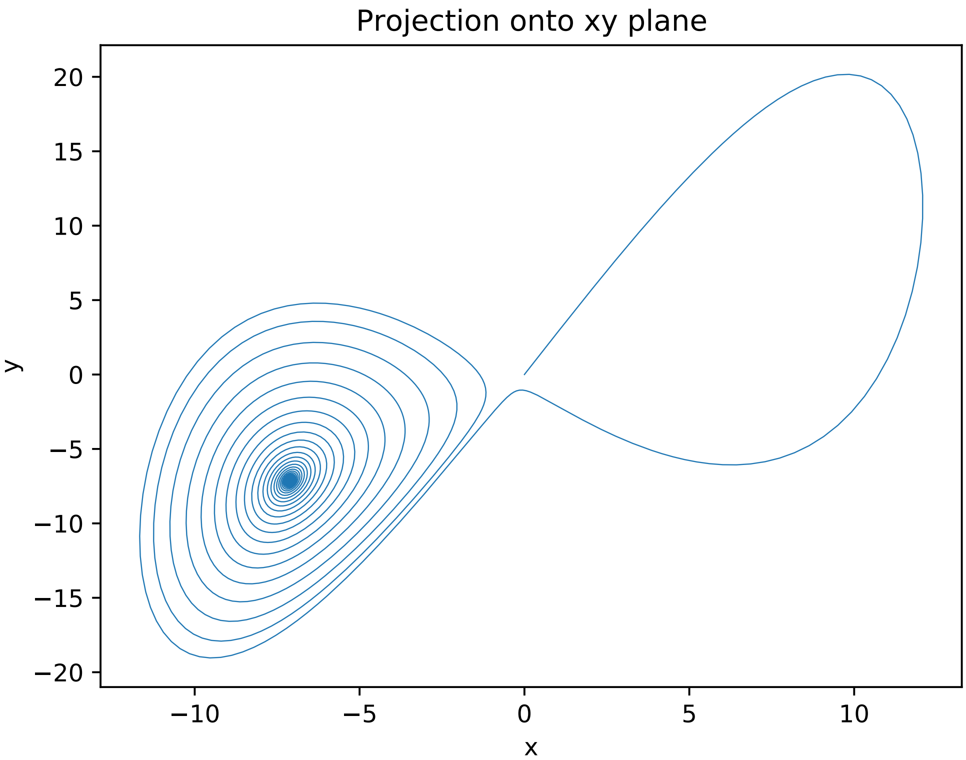


Abbildung 3: The projection of the trajectory onto the xy-plane with the starting parameter $r = 20$.

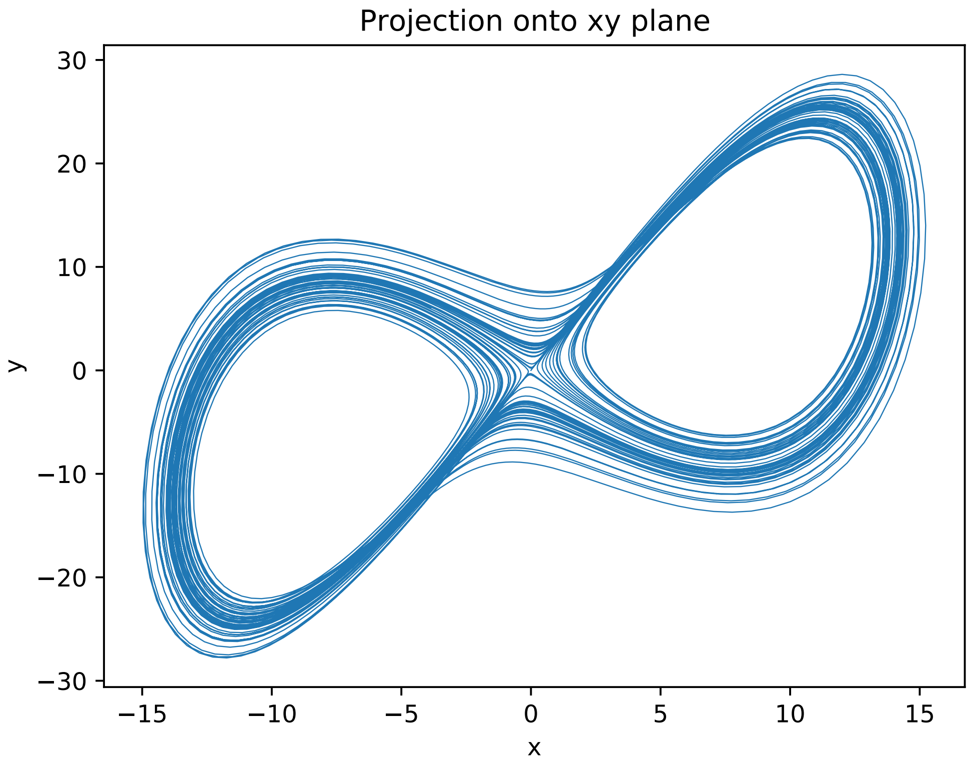


Abbildung 4: The projection of the trajectory onto the xy-plane with the starting parameter $r = 28$.

2. As a Poincare slice at $Z = 20$ with the condition that $\dot{Z} < 0$. We tested for this condition by subtracting the $\vec{f}(t_i)$ with $\vec{f}(t_{i+1})$. If the result is basically a linear interpolation between point $\vec{f}(t_i)$ and $\vec{f}(t_{i+1})$. We then plotted all points that forfill the two conditions onto a xy-plane.

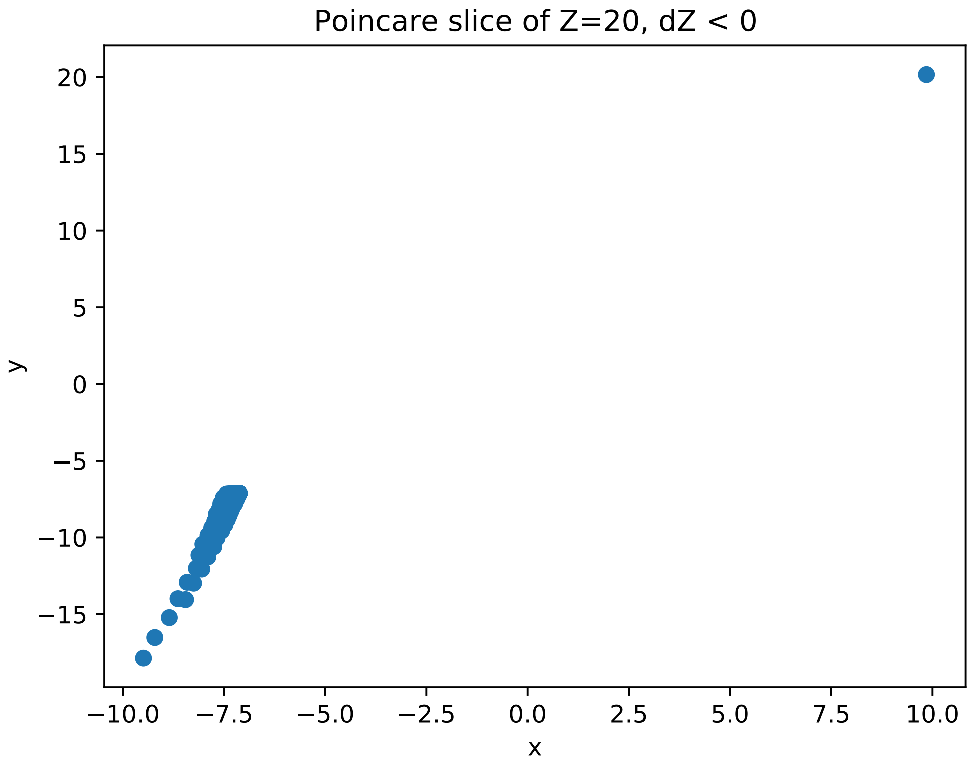


Abbildung 5: The Poincare slice of the trajectory onto the xy-plane with the starting parameter $r = 20$.

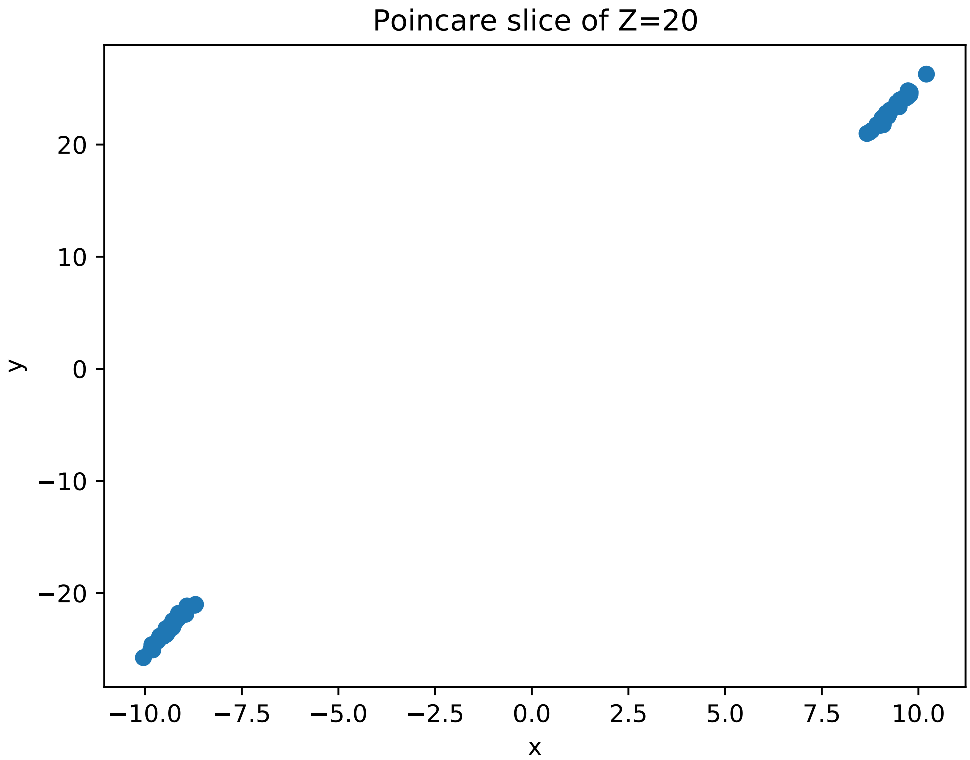


Abbildung 6: The Poincare slice of the trajectory onto the xy-plane with the starting parameter $r = 28$.

3. For the last part we plotted the whole xyz-trajectory onto a 3d-plot.

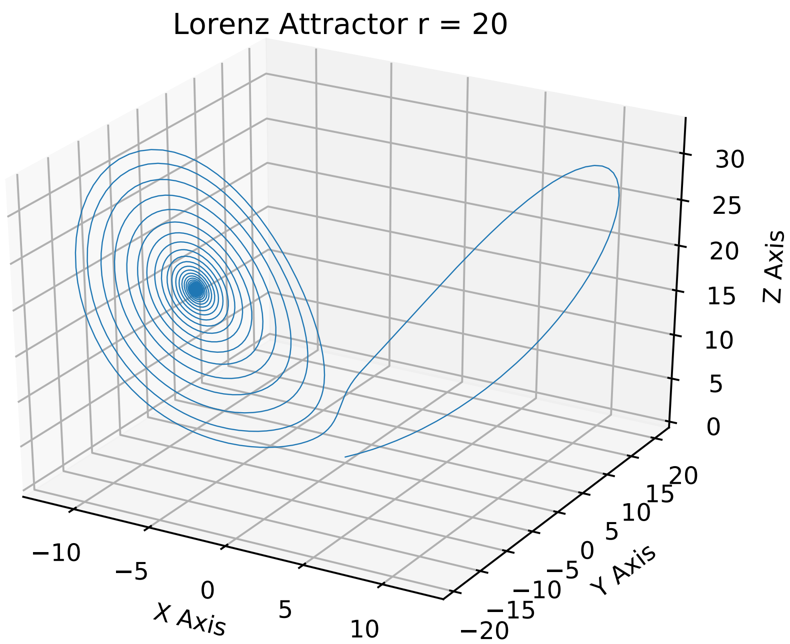


Abbildung 7: The 3d-Plot of the trajectory with the starting parameter $r = 20$.

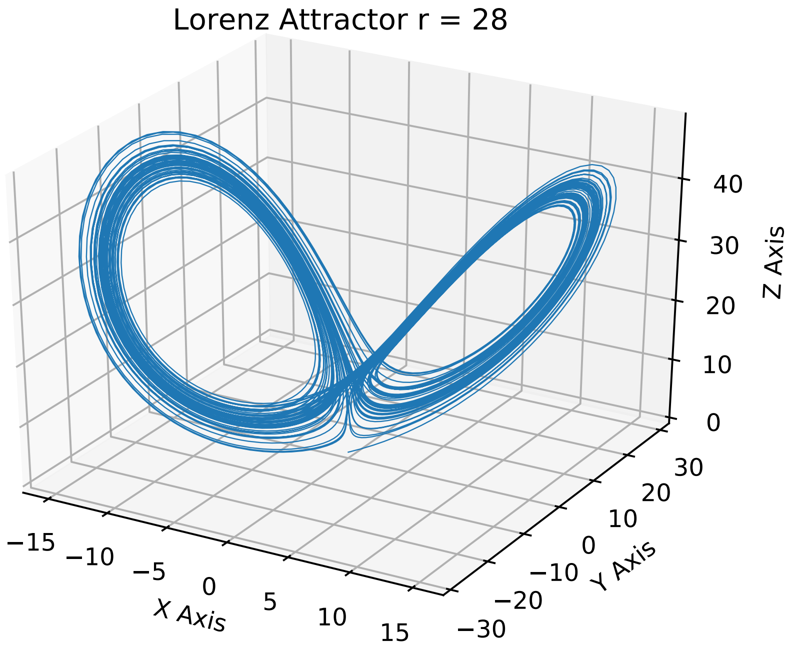


Abbildung 8: The 3d-Plot of the trajectory with the starting parameter $r = 28$.