Jee linalg 2d Problem 35

EE17BTECH11017 EE17BTECH11020

Geometry Question

A straight line through the origin **O** meets the lines

$4\mathbf{x} + 3\mathbf{y} = 10$

$8\mathbf{x} + 6\mathbf{y} + 5 = 0$

at **A** and **B** respectively. Find the ratio in which **O** divides **AB**.

Matrix transformation of the question

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at **A** and **B** respectively. Find the ratio in which **O** divides **A**B.

Method to find Point of Intersection of 2 lines

Let the respective equations be

$\mathbf{n}_1^T\mathbf{x} = p_1$ and

$\mathbf{n}_2^T\mathbf{x} = p_2$

This can be written as the matrix equation

$$\begin{bmatrix}\mathbf{n}_1^T \\ \mathbf{n}_2^T\end{bmatrix} \mathbf{x} = \begin{bmatrix}\mathbf{p}_1 \\ \mathbf{p}_2\end{bmatrix}$$

$$\mathbf{N}^T\mathbf{x} = p$$

where $\mathbf{N} = (\mathbf{n}_1\ \mathbf{n}_2)$ and p$=\begin{bmatrix}\mathbf{p}_1 \\ \mathbf{p}_2\end{bmatrix}$

The point of intersection is then obtained as

$$\mathbf{x} = (\mathbf{N}^T)^{-1}\mathbf{p}$$

Solution approach

Equation of line through **O** origin is $m\mathbf{x} + \mathbf{y} = 0$.
Let the points of intersection of this line with
$4\mathbf{x} + 3\mathbf{y} = 10$ and
$8\mathbf{x} + 6\mathbf{y} + 5 = 0$
be **A** and **B** respectively. Then we find the lengths **OA** and **OB** to get the RATIO = **OA/OB**.

Solution in form of matrix

Equation of line through $\mathbf{O}$ origin is (m -1) x = 0.
$\mathbf{n}^T = (m \quad -1)$, p = 0.
For $(4\ 3)\ \mathbf{x} = 10$
$\mathbf{n}_1^T = (4 \quad 3)$, $p_1 = 10$.
For $(8\ 6)\ \mathbf{x} + 5 = 0$
$\mathbf{n}_2^T = (8 \quad 6)$, $p_2 = -5$.
Let $\mathbf{N}_1 = (\mathbf{n}\ \mathbf{n}_1)$ and $\mathbf{N}_2 = (\mathbf{n}\ \mathbf{n}_2)$.
$\mathbf{P}_1 = \begin{bmatrix} \mathbf{p} \\ \mathbf{p}_1 \end{bmatrix}$ and $\mathbf{P}_2 = \begin{bmatrix} \mathbf{p} \\ \mathbf{p}_2 \end{bmatrix}$

$$\mathbf{N}_1^T = \begin{bmatrix} m & -1 \\ 4 & 3 \end{bmatrix} \text{ and } \mathbf{N}_2^T = \begin{bmatrix} m & -1 \\ 8 & 6 \end{bmatrix}.$$

$$\mathbf{P}_1 = \begin{bmatrix} 0 \\ 10 \end{bmatrix} \quad \text{and} \quad \mathbf{P}_2 = \begin{bmatrix} 0 \\ -5 \end{bmatrix}$$

$$(\mathbf{N}_1^T)^{-1} = \begin{bmatrix} \frac{3}{3m+4} & \frac{1}{3m+4} \\ \frac{-4}{3m+4} & \frac{m}{3m+4} \end{bmatrix} \text{ and } (\mathbf{N}_2^T)^{-1} = \begin{bmatrix} \frac{6}{6m+8} & \frac{1}{6m+8} \\ \frac{-8}{6m+8} & \frac{m}{6m+8} \end{bmatrix}.$$

$$\mathbf{A} = (\mathbf{N}_1^T)^{-1}\mathbf{P}_1$$

$$\mathbf{B} = (\mathbf{N}_2^T)^{-1}\mathbf{P}_2$$

$$\mathbf{A} = \begin{bmatrix} \frac{10}{3m+4} \\ \\ \frac{10m}{3m+4} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \frac{-5}{6m+8} \\ \\ \frac{-5m}{6m+8} \end{bmatrix}$$

$$\mathbf{OA} = \frac{10\sqrt{m^2+1}}{3m+4}$$

$$\mathbf{OB} = \frac{5\sqrt{m^2+1}}{6m+8}$$

$$Ratio = \frac{OA}{OB} = 4$$

Figure