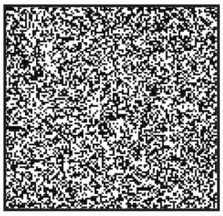
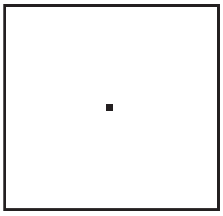
Common interpretations of Entropy in the urban context

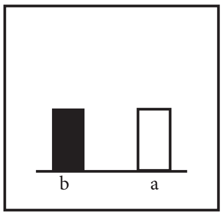
In thermodynamics the phase space dimensions are clear (usually the location and velocity of particles). In cities on the other hand microstates, macrostates and phase space dimensions are open to interpretation; The literature shows a variety of different definitions, often without discussing the conceptual implications.
Here is an overview over the most common ways the phase space is defined for cities:

A) Geographical space as phase space
<u>Source:</u> Batty et. al.
<u>phase space:</u> x-y location
<u>measures:</u> evenness of spatial distribution
<u>max. entropy:</u> uniform spatial distribution
<u>min. entropy:</u> all occurences in the same location


maximum entropy pattern


minimum entropy pattern

B) Aspatial characteristical phase space
<u>Source:</u> A. Gudmundsson et al. etc.
<u>phase space:</u> a characteristic of elements
<u>measures:</u> global proportion of elements of different types
<u>max. entropy:</u> even dist. of elements across a characteristics
<u>min. entropy:</u> all elements have the same characteristic



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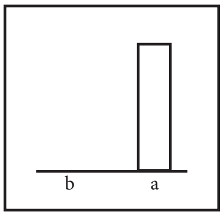
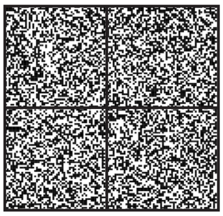


minimum entropy pattern

C) Aspatial categorical phase space, spatially disaggregated
<u>Source:</u> Theil et. al.
<u>phase space:</u> phase space is categories of elements
<u>measures:</u> local divergence from global proportions
<u>max. entropy:</u> uniform spatial distribution
<u>min entropy:</u> all elements of each category in one zone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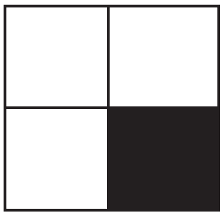
minimum entropy pattern

D) Cooccurence matrix phase space
<u>Source</u>: Leibovici et al.
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Reasons & implications of these definitions:

A) Geographical space as phase space:
- intuitively the most obvious one - it's both called "space".
- location is a dimension in the most common thermodynamical phase space. Nonetheless, so is "velocity", which is quietly ignored; because velocity makes no sense. In T.D. spatial distributions are reached because molecules actually fly around randomly and interact weakly. Buildings, roads etc. are different.
-assumes all places were the same and nothing would interact with anything, and also that the observed things fly around.

B) Aspatial characteristical phase space:
- intuitively the most obvious one when coming from a Shannon perspective
- global measure of variety using entropy as a proxy
- assumes observed elements randomly change characteristic and don't iteract (streets rotate)
- a proxy for

C) Aspatial categorical phase space, spatially disaggregated:
- a pure proxy to measure spatial evenness as an index of segregation. Not initially concerned with entropy, just uses same formula
- unevenness on larger scale considered 'error' that is accounted for on the more local scale

D) Cooccurence matrix phase space:
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Summary of common phase space definitions

All based on the assumptions that:

- The geographical space would somehow directly correspond to the phase space (except B)
- elements do not react to each other
- elements do not react to any characteristics of the different places
- The GPS coordinate / zone ID itself is the defining variable that is expected to be random.

Expect uniform spatial distributions; any conglomeration in space deviating from uniform spatial distribution is decreased entropy. But cities themselve *are* conglomerations. These measures are clearly not concerned with how urban patterns are produced, but serve as a proxy for evenness of some sort.
That's all fine and ok; the formula of the form sum(-p log p) is useful to measure the evenness of some distribution; but that doesn't automatically mean one is receiving a meaningful measure of *entropy* of a system. If a conceptually relevant meaning of microstates, macrostates and the phase space dimensions is not discussed, and the translation of entropy to cities is not rooted in the processes that generate spatial structures, one can not make conclusions based on any of the attributes ascribed to entropy.

Another interpretation

In the reviewed literature, entropy is always used as a proxy to measure evenness of some sort. The interest of this paper is not to discuss how good these applications of the boltzmann formula are at measuring what each of them was desinged for. We simply recognise that none of the reviewed literature made an attempt to define entropy for urban structures in a meaningful way in terms of statistical mechanics, based on reasonable assumptions about the relevant factors in the formation of urban structures.

Entropy in statistical mechanics

Roughly speaking, the entropy of a system's state is given by (the logarithm of) the number of ways that state can occur. That means that we can expect a system to be more likely in a state that can occur in a large number of ways rather than in a state that can occur in only very few ways.
If the system's macroscopic state relies on the states of sufficiently many smaller parts, some macroscopic states are overwhelmingly more likely than others, and the system is expected to continiously converge to the state with the highest entropy.

Important aspects of the evolution of cities

For this statistical understanding of entropy, we begin our interpretation of entropy for cities with a set of general assumptions about how cities evolve and work:

1. Over long periods of time, the way people use the city has an impact on its morphology; urban structures are partly a result of the individual lifes and decisions of its inhabitants.
2. Urban morphology and the different places available in a city are the framework that enables and limits the different ways in which people can use the city.
3. Places in the city are not independent ("Everything is related to everything else"). The surroundings on different scales of a place are a crucial aspect of its features that changes how that place is and can be used.

Linking entropy to spatial structures

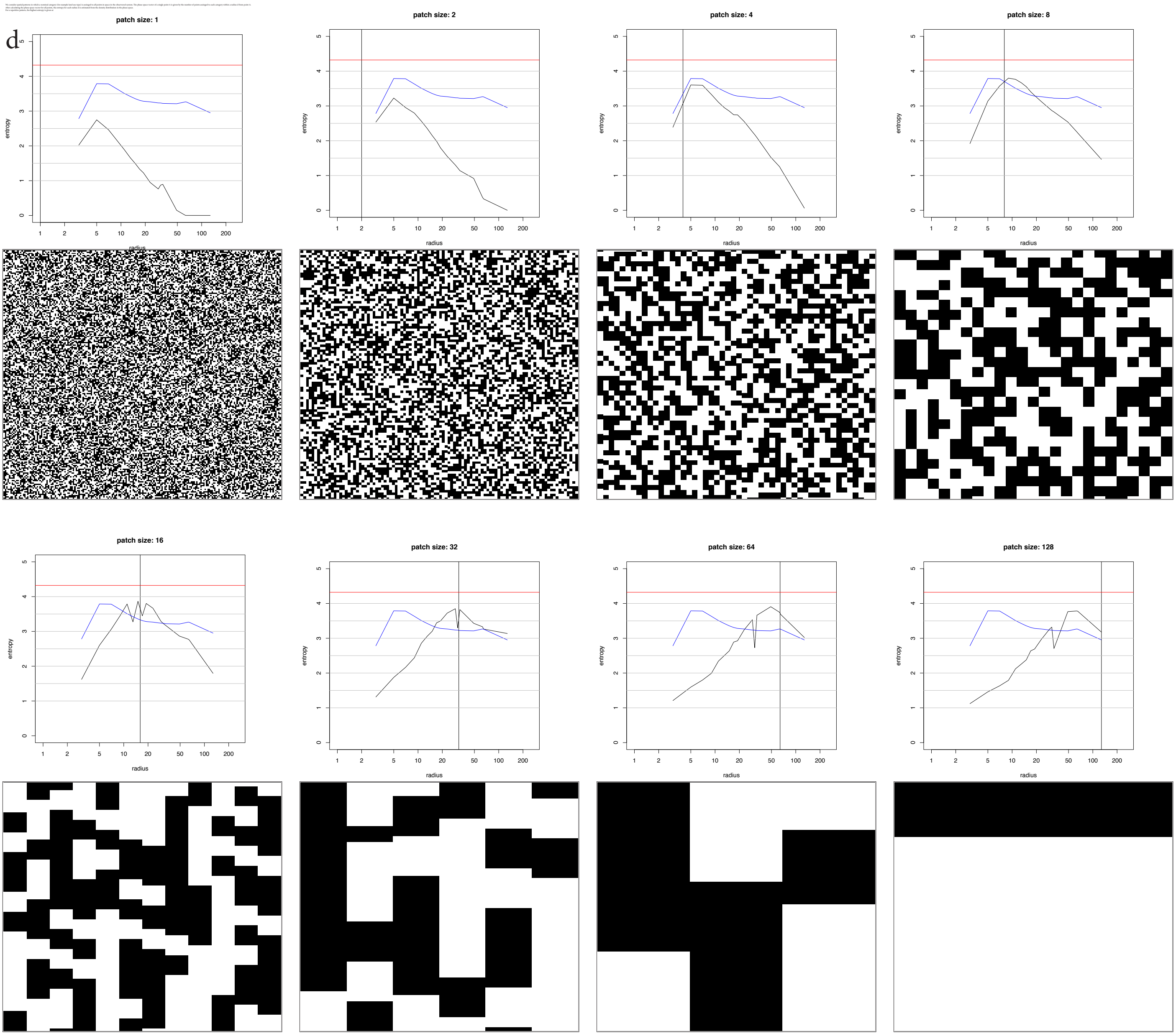
A city with very low variety in its structure - for example a city in which all residential buildings and residential areas are exactly identical - can only emerge from a small set of combinations of individual choices. There are much more possible combinations for people to choose to live in a large variety of different ways, and in conclusion we would expect the monotonous city to acquire a larger variety in its structure over time.

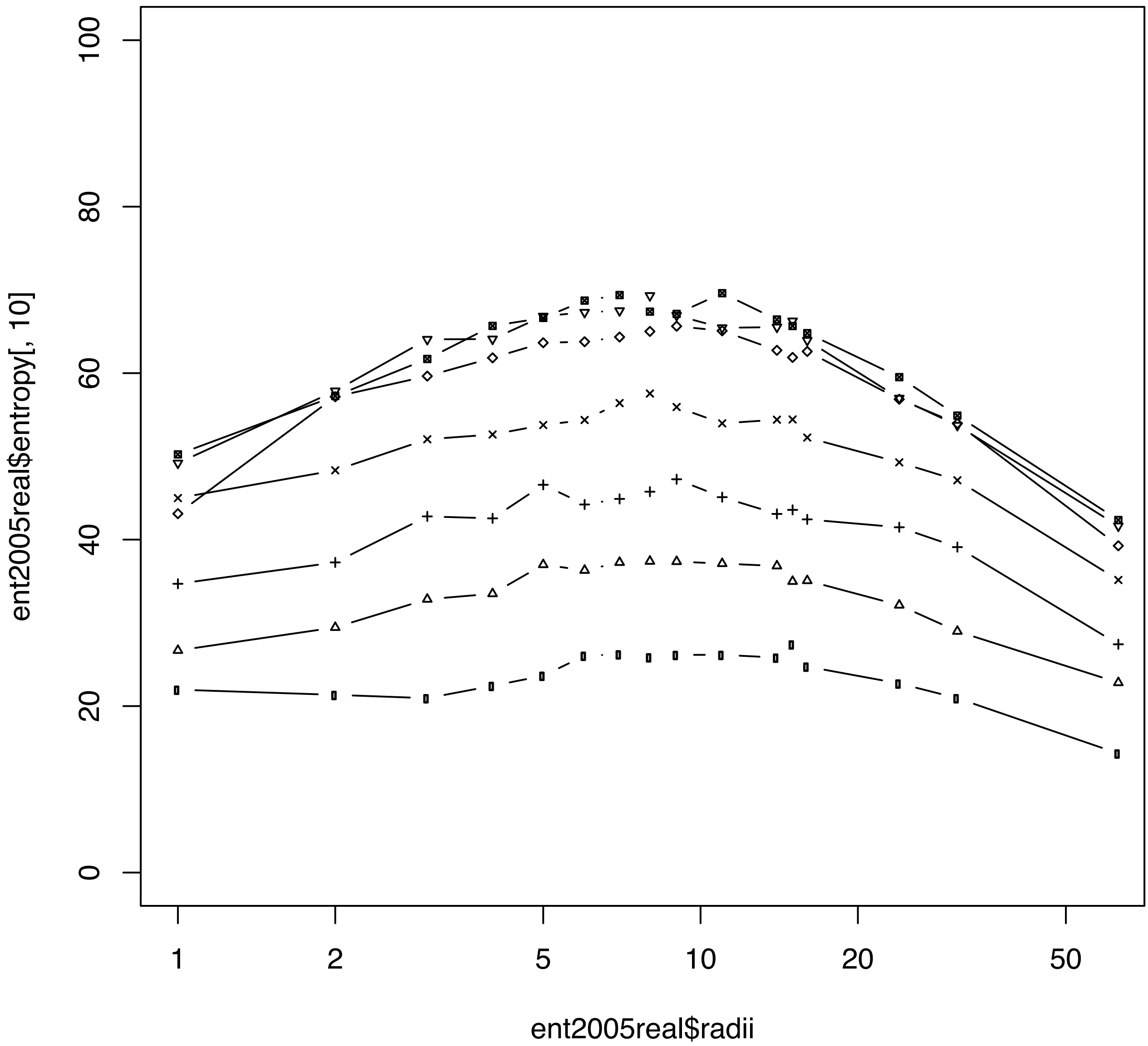
This is in fact what happend to the Hippodamian city layouts that were initially built to provide an identical living situation to each settler based on the ideals of isonomia.

Based on the general assumptions about how morphology relates to individual choices, how places are used and how places relate to each other, our interpretation of Entropy should fulfill the following:

- observations correspond to places - points in space
- the phase space dimensions are given by some characteristic of these places that have an effect on how they can be used
- Places interact and are not independent from one another. The measure should consider the environment of each place on multiple scales as a fundamental aspect of its characteristics

A multiscale approach to entropy in cities

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