Adding the temperature-size rule to temperature-dependent consumer-resource models

MST group project

Recreating Gilbert

Equations 1 and 2 from Gilbert et al 2014 (with potential temperature dependencies added)

In[1]:=
```
dRdt[R_, C_, T_] := r[T] R (1 - R/K[T]) - f[R, T] R C;
dCdt[R_, C_, T_] := e f[R, T] R C - m[C, T] C;
```

where R is biomass of resource, C is biomass of consumer, T is temperature, K is resource carrying capacity, f is the functional response, e is the conversion efficiency of resources into new consumers, and m is consumer mortality.

BCR at a given T, as defined by Gilbert (Eqn 5),

In[3]:=
```
BCR[T_] := (e a[T] K[T])/m[T]
```

Equilibrium biomasses at given temperature assuming a type I functional response and density-independent consumer mortality (as in most of Gilbert)

In[4]:=
```
Eq[T_] :=
 Solve[{0 == dRdt[R, C, T], 0 == dCdt[R, C, T]} /. f[R, T] → a[T] /. m[C, T] → m[T], {R, C}]
```

Equilibrium consumer to resource biomass ratio at a given temperature (at the equilibrium where both populations persist)

In[5]:=
```
CR[T_] := C/R /. Eq[T][[3]]
```

The Jacobian evaluated at equilibrium (determines stability)

In[6]:=
```
Jac = {{D[dRdt[R, C, T] /. f[R, T] → a[T] /. m[C, T] → m[T], R],
     D[dRdt[R, C, T] /. f[R, T] → a[T] /. m[C, T] → m[T], C]},
    {D[dCdt[R, C, T] /. f[R, T] → a[T] /. m[C, T] → m[T], R],
     D[dCdt[R, C, T] /. f[R, T] → a[T] /. m[C, T] → m[T], C]}} /. Eq[T][[3]];
```

The eigenvalues of the Jacobian are

In[7]:=
```
lambda = Eigenvalues[Jac];
```

Use Table 1 of Gilbert. Want K=100 at 15 degrees C (Figure 3 of Gilbert), so we need K0 to be

```
In[8]:= K15 = Solve[100 == K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T /. T → 273.15 + 15, K0];
```

Figure 3a of Gilbert (same shape but numbers too large)

```
In[9]:= Show[
  Plot[
   BCR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /. k → 8.62 * 10^-5 /.
        a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.32 /.
    ES → 0.9, {T, 5, 30}, PlotStyle → {Black, Thick}],
  Plot[BCR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
         k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.9 /.
    ES → 0.32, {T, 5, 30}, PlotStyle → {Black, Thick, Dashed}],
  Plot[BCR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
         k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
     EB → 0.9 /. ES → 0.9, {T, 5, 30}, PlotStyle → {Gray, Thick}],
  Frame → True,
  FrameLabel → {"Temperature (Celcius)", "BCR"}
 ]
```

Out[9]=

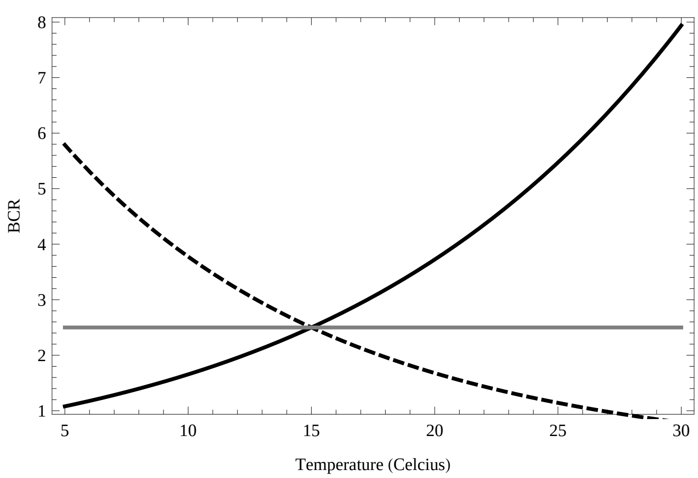


Figure 3b of Gilbert (off by factor of 3)

In[10]:=
```
Show[Plot[
  CR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /. k → 8.62 * 10^-5 /.
        a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.32 /.
    ES → 0.9, {T, 5, 30}, PlotStyle → {Black, Thick}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Consumer:resource biomass"}
]
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

Out[10]=

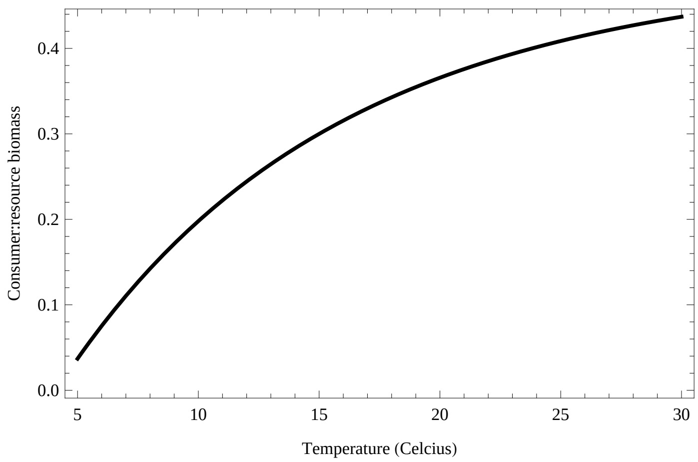
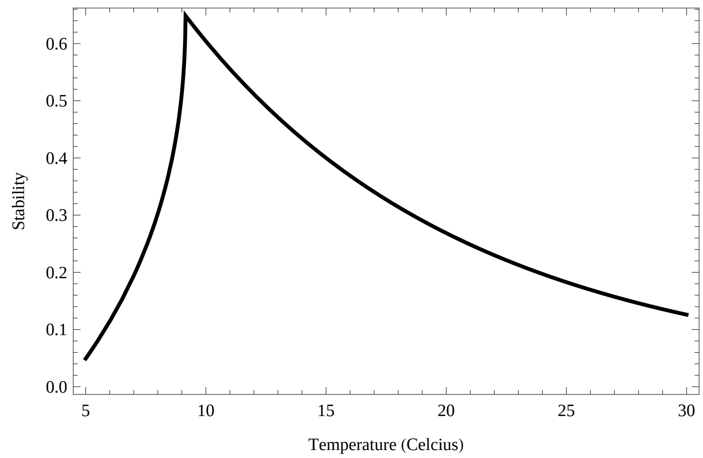


Figure 3c of Gilbert

In[11]:=
```
Show[
 Plot[-Max[Re[lambda /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
        k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
     EB → 0.32 /. ES → 0.9]], {T, 5, 30}, PlotStyle → {Black, Thick}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Stability"}
]
```

Out[11]=



Adding mass to Gilbert

Now we want to add the body mass relations given in Table 1 of DeLong et al. 2015.

Temperature dependencies of rates given in Table 1 of Gilbert et al. (now letting the constant depend on body mass M)

In[12]:=
```
cr = {C, R};
GilbertTable1 = {
   r[T] → r[M] Exp[-EB/(k T[R])],
   K[T] → K[M] Exp[EB/(k T[R]) - ES/(k T[S])],
   m[T] → m[M] Exp[-Em/(k T[C])],
   a[T] → a[M] Sqrt[Sum[(ν0[cr[[i]]] Exp[-Eν[cr[[i]]]/(k T[cr[[i]]])])^2, {i, 1, Length[cr]}]],
   e → e[M]
  };
```

Body mass dependencies from DeLong et al.

```
In[14]:= DeLongTable1 = {
    r[M] → r0 M[R]^ρ,
    K[M] → K0 M[R]^κ,
    a[M] → a0 M[C]^α,
    e[M] → e0 M[C]^ϵ,
    m[M] → m0 M[C]^μ
   };
```

The temperature-size rule (from Forster et al. 2012), for unicells (e.g., algae)

```
In[15]:= TSR = M[i_] → M15 (1 - 0.02 (T - 15));
```

What does mass at 15 C need to be to have K=100 at T=15 C (to stay consistent with Gilbert)

```
In[16]:= m15 = Solve[
   100 == K[T] /. GilbertTable1 /. DeLongTable1 /. T[i_] → T + 273.15 /. k → 8.62 * 10^-5 /.
        κ → -0.81 /. EB → 0.32 /. ES → 0.9 /. TSR /. T → 15]
```

Out[16]= $\{\{M15 \to 1.02553 \times 10^{-15}\, K0^{100/81}\}\}$

Figure 3a of Gilbert (new predictions in red; new dashed curve and horizontal line are now on the order of 10^{10} and 10^{20} and so do not appear in plot)

In[17]:=
```
Show[
 Plot[
  BCR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /. k → 8.62 * 10^-5 /.
        a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.32 /.
   ES → 0.9, {T, 5, 30}, PlotStyle → {Black, Thick}],
 Plot[BCR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
         k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.9 /.
   ES → 0.32, {T, 5, 30}, PlotStyle → {Black, Thick, Dashed}],
 Plot[BCR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
         k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
    EB → 0.9 /. ES → 0.9, {T, 5, 30}, PlotStyle → {Gray, Thick}],
 Plot[Simplify[BCR[T] /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
           GilbertTable1 /. DeLongTable1 /. T[i_] → T + 273.15 /.
         k → 8.62 * 10^-5 /. κ → -0.81 /. TSR /. m15 /. EB → 0.32 /.
    ES → 0.9, K0 > 0], {T, 5, 30}, PlotStyle → {Red, Thick}],
 Plot[Simplify[BCR[T] /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
           GilbertTable1 /. DeLongTable1 /. T[i_] → T + 273.15 /. k → 8.62 * 10^-5 /.
        κ → -0.81 /. TSR /. m15 /. EB → 0.9 /. ES → 0.32, K0 > 0],
  {T, 5, 30}, PlotStyle → {Red, Thick, Dashed}],
 Plot[Simplify[
   BCR[T] /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. GilbertTable1 /.
          DeLongTable1 /. T[i_] → T + 273.15 /. k → 8.62 * 10^-5 /. κ → -0.81 /. TSR /.
      m15 /. EB → 0.9 /. ES → 0.9, K0 > 0], {T, 5, 30}, PlotStyle → {Pink, Thick}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "BCR"}
]
```

Out[17]=

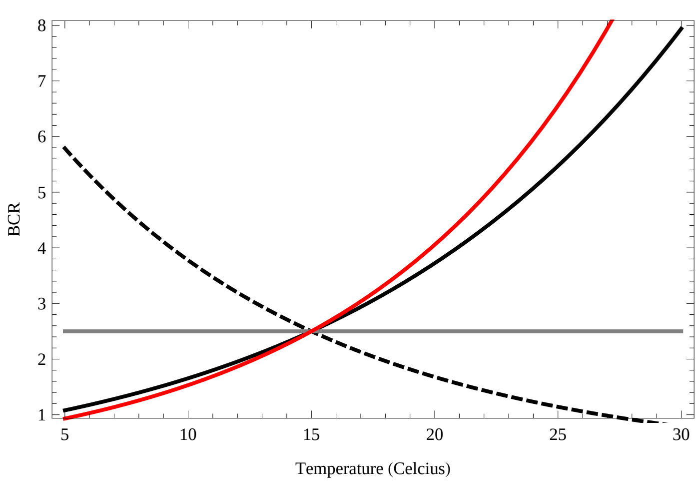


Figure 3b of Gilbert (new prediction in red)

In[18]:=
```
Show[
 Plot[
  Simplify[CR[T] /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. GilbertTable1 /.
          DeLongTable1 /. T[i_] → T + 273.15 /. k → 8.62 * 10^-5 /. κ → -0.81 /. TSR /.
     m15 /. EB → 0.32 /. ES → 0.9, K0 > 0], {T, 5, 30}, PlotStyle → {Red, Thick}],
 Plot[ CR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
        k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
   EB → 0.32 /. ES → 0.9, {T, 5, 30}, PlotStyle → {Black, Thick}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Consumer:resource biomass"}
]
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

Out[18]=

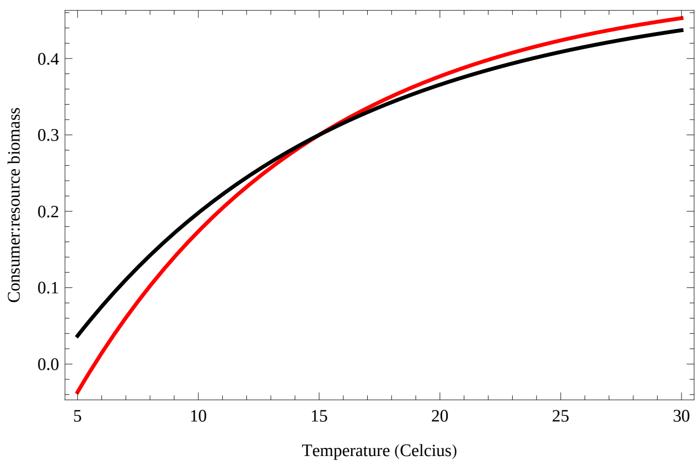
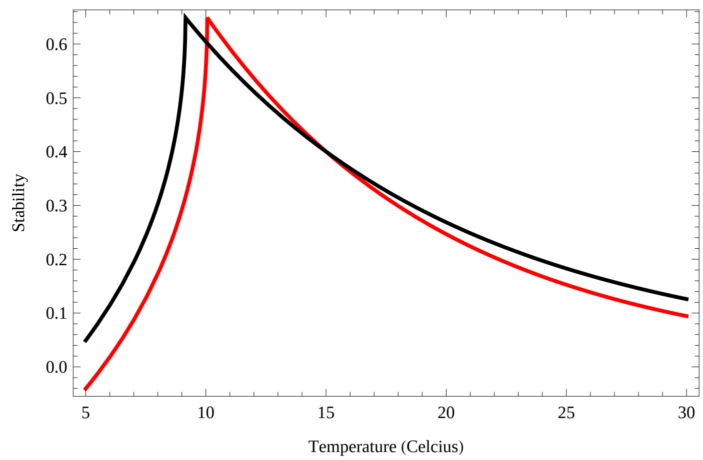


Figure 3c of Gilbert

In[19]:=
```
Show[Plot[-Max[Re[Simplify[
      lambda /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. GilbertTable1 /.
          DeLongTable1 /. T[i_] → T + 273.15 /. k → 8.62 * 10^-5 /. κ → -0.81 /. TSR /.
        m15 /. EB → 0.32 /. ES → 0.9, K0 > 0]]], {T, 5, 30}, PlotStyle → {Red, Thick}],
 Plot[-Max[Re[lambda /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
          k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
       EB → 0.32 /. ES → 0.9]], {T, 5, 30}, PlotStyle → {Black, Thick}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Stability"}
]
```

Out[19]=



This only looks at temperature and body mass dependencies in resource carrying capacity K. We next add temperature and mass dependencies to the other rates (and temperature dependence in consumer body mass) to see if this gives bigger discrepancies between the two models.

All the dependencies

Let the temperature size rule be linear, but potentially different for resource and consumer

In[20]:=
```
Clear[TSR]
TSR = M[i_] → M15[i] (1 - β[i] (T[i] - (273.15 + 15)));
```

where M15[i] is the mass of the resource of consumer, i={R,C}, at 15 degrees celcius, β[i] is the percent decline in body size with a degree increase in temperature, and T is the current temperature in Kelvins.

To have the same population dynamics parameter values at 15 degrees as we did above with the TSR only in K, we need

```
In[22]:= a15 =
          Solve[0.1 == a[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, a0] //
           Flatten
```

Out[22]= $\left\{a0 \to \frac{0.1\, M15[C]^{-1.\,\alpha}}{\sqrt{e^{-\frac{0.00694083\, E\nu[C]}{k}}\, \nu0[C]^2 + e^{-\frac{0.00694083\, E\nu[R]}{k}}\, \nu0[R]^2}}\right\}$

```
In[23]:= e15 = Solve[0.15 == e[M] /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, e0] // Flatten
```

Out[23]= $\left\{e0 \to 0.15\, M15[C]^{-1.\,\epsilon}\right\}$

```
In[24]:= k15 =
          Solve[100 == K[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, K0] //
           Flatten
```

Out[24]= $\left\{K0 \to 100.\, e^{-\frac{0.00347041\, EB}{k}+\frac{0.00347041\, ES}{k}}\, M15[R]^{-1.\,\kappa}\right\}$

```
In[25]:= r15 =
          Solve[2 == r[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, r0] //
           Flatten
```

Out[25]= $\left\{r0 \to 2.\, e^{\frac{0.00347041\, EB}{k}}\, M15[R]^{-1.\,\rho}\right\}$

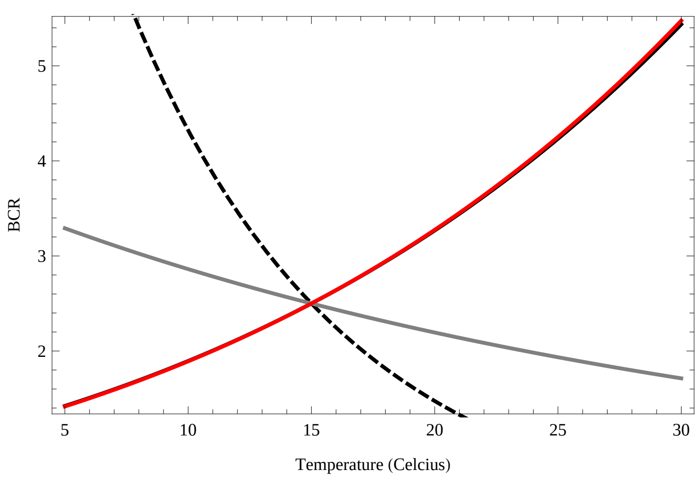
```
In[26]:= m15 =
          Solve[0.6 == m[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, m0] //
           Flatten
```

Out[26]= $\left\{m0 \to 0.6\, e^{\frac{0.00347041\, Em}{k}}\, M15[C]^{-1.\,\mu}\right\}$

Now, BCR as a function of temperature is

In[72]:=
```
Show[
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
       0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
     ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
   ρ → -0.81 /. TSR /. β[i_] → 0 /. M15[R] → 1 /. M15[C] → 2 /.
  T[i_] → T + 273.15, {T, 5, 30}, PlotStyle →
  {Thick,
   Black}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
       0.9 /. ES → 0.32 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
     ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
   ρ → -0.81 /. TSR /. β[i_] → 0 /. M15[R] → 1 /. M15[C] → 2 /.
  T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick,
   Black,
   Dashed}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
     ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
   ρ → -0.81 /. TSR /. β[i_] → 0 /. M15[R] → 1 /. M15[C] → 2 /.
  T[i_] → T + 273.15, {T, 5, 30}, PlotStyle →
  {Thick,
   Gray}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
       0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
     ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
   ρ → -0.81 /. TSR /. β[i_] → 0.02 /. M15[R] → 1 /. M15[C] → 2 /.
  T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick,
   Red}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "BCR"}
]
```

Out[72]=



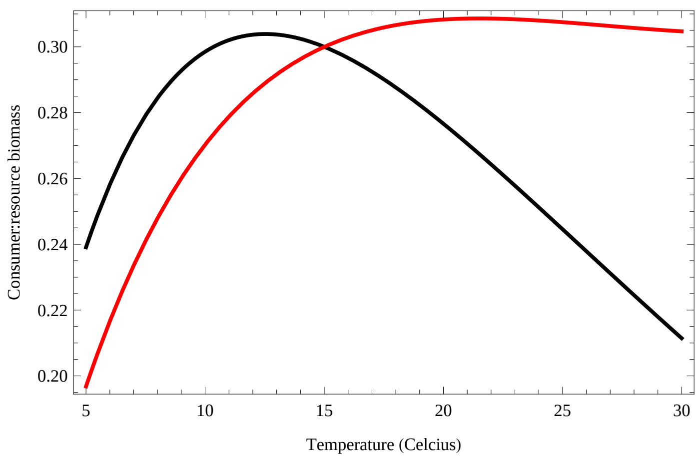
Nothing much doing here.

In[75]:=
```
Show[
 Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
                    0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
               ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
         ρ → -0.81 /. TSR /. β[i_] → 0 /. M15[R] → 1 /. M15[C] → 2 /.
   T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Black,
    Thick}, Axes → False],
 Plot[ CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
                    0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
               ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
         ρ → -0.81 /. TSR /. β[i_] → 0.02 /. M15[R] → 1 /. M15[C] → 2 /.
   T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Red,
    Thick}, Axes → False],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Consumer:resource biomass"},
 PlotRange → All
]
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

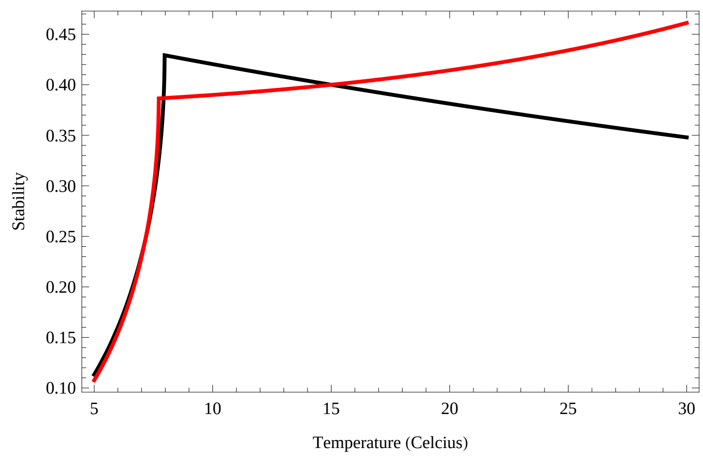
Out[75]=



NOTE: with the TSR, increasing temperature no longer changes the ratio of consumer to resource biomass ratio!!!!!

In[77]:=
```
Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /.
             Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /.
         ϵ → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /.
     M15[R] → 1 /. M15[C] → 2 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thick}, Axes → False,
  PlotRange →
   {0,
    All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /.
             Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /.
         ϵ → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0.02 /.
     M15[R] → 1 /. M15[C] → 2 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Red, Thick}, PlotRange → {0,
    All}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Stability"},
 PlotRange → All
]
```

Out[77]=



NOTE: with the TSR, increasing temperature no destabilizes coexistence, instead stability increases!!!