How does the temperature-size rule affect consumer-resource dynamics?

MST group project

Recreating Gilbert

Equations 1 and 2 from Gilbert et al 2014 (with potential temperature dependencies added)

```
dRdt[R_, C_, T_] := r[T] R (1 - R/K[T]) - f[R, T] R C;
dCdt[R_, C_, T_] := e f[R, T] R C - m[C, T] C;
```

where R is biomass of resource, C is biomass of consumer, T is temperature, K is resource carrying capacity, f is the functional response, e is the conversion efficiency of resources into new consumers, and m is consumer mortality.

BCR at a given T, as defined by Gilbert (Eqn 5),

```
BCR[T_] := e a[T] K[T] / m[T]
```

Equilibrium biomasses at given temperature assuming a type I functional response and density-independent consumer mortality (as in most of Gilbert)

```
Eq[T_] :=
 Solve[{0 == dRdt[R, C, T], 0 == dCdt[R, C, T]} /. f[R, T] → a[T] /. m[C, T] → m[T], {R, C}]
```

Equilibrium consumer to resource biomass ratio at a given temperature (at the equilibrium where both populations persist)

```
CR[T_] := C/R /. Eq[T][[3]]
```

The Jacobian evaluated at equilibrium (determines stability)

```
Jac = {{D[dRdt[R, C, T] /. f[R, T] → a[T] /. m[C, T] → m[T], R],
     D[dRdt[R, C, T] /. f[R, T] → a[T] /. m[C, T] → m[T], C]},
    {D[dCdt[R, C, T] /. f[R, T] → a[T] /. m[C, T] → m[T], R],
     D[dCdt[R, C, T] /. f[R, T] → a[T] /. m[C, T] → m[T], C]}} /. Eq[T][[3]];
```

The eigenvalues of the Jacobian are

```
lambda = Eigenvalues[Jac];
```

Use Table 1 of Gilbert. Want K=100 at 15 degrees C (Figure 3 of Gilbert), so we need K0 to be

```
K15 = Solve[100 == K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T /. T → 273.15 + 15, K0];
```

Figure 3a of Gilbert (same shape but numbers too large)

```
Show[
 Plot[
  BCR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /. k → 8.62 * 10^-5 /.
       a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.32 /.
   ES → 0.9, {T, 5, 30}, PlotStyle → {Black, Thick}],
 Plot[BCR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
       k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.9 /.
   ES → 0.32, {T, 5, 30}, PlotStyle → {Black, Thick, Dashed}],
 Plot[BCR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
       k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
    EB → 0.9 /. ES → 0.9, {T, 5, 30}, PlotStyle → {Gray, Thick}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "BCR"}
]
```

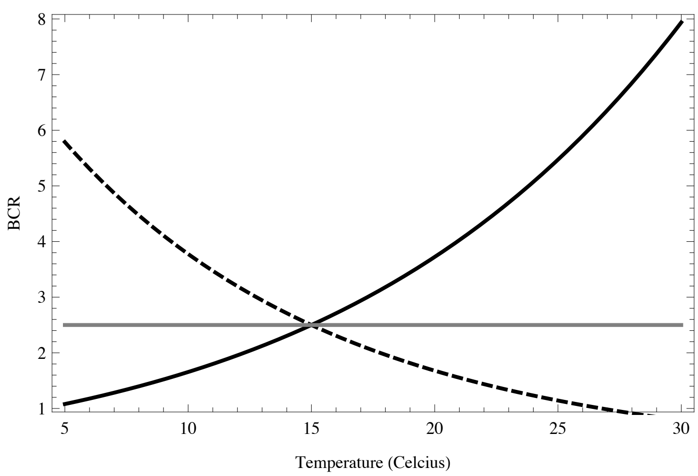


Figure 3b of Gilbert (off by factor of 3)

```
Show[Plot[
  CR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /. k → 8.62 * 10^-5 /.
       a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.32 /.
    ES → 0.9, {T, 5, 30}, PlotStyle → {Black, Thick}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Consumer:resource biomass"}
]
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

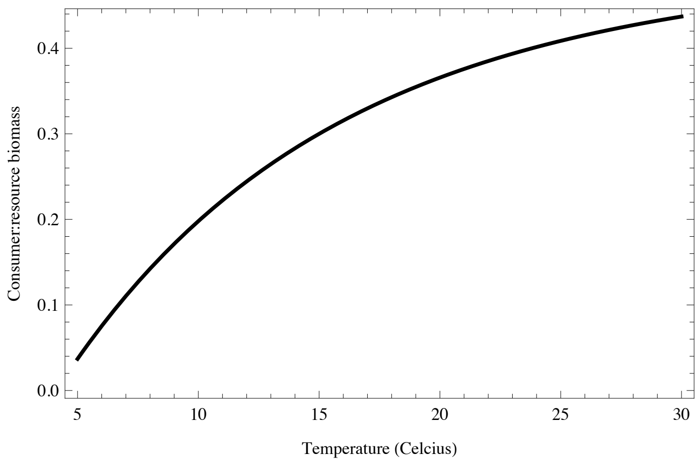
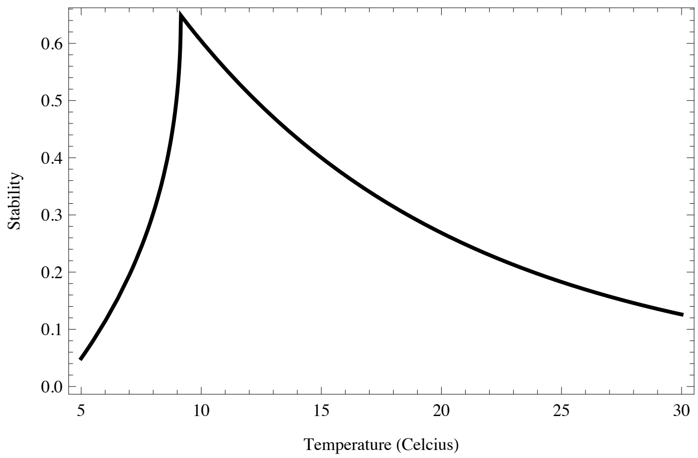


Figure 3c of Gilbert

```
Show[
 Plot[-Max[Re[lambda /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
       k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
    EB → 0.32 /. ES → 0.9]], {T, 5, 30}, PlotStyle → {Black, Thick}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Stability"}
]
```



Adding mass to Gilbert

Now we want to add the body mass relations given in Table 1 of DeLong et al. 2015.

Temperature dependencies of rates given in Table 1 of Gilbert et al. (now letting the constant depend on body mass M)

```
cr = {C, R};
GilbertTable1 = {
   r[T] → r[M] Exp[-EB/(k T[R])],
   K[T] → K[M] Exp[EB/(k T[R]) - ES/(k T[S])],
   m[T] → m[M] Exp[-Em/(k T[C])],
   a[T] → a[M] Sqrt[Sum[(ν0[cr[[i]]] Exp[-Eν[cr[[i]]]/(k T[cr[[i]]])])^2, {i, 1, Length[cr]}]],
   e → e[M]
  };
```

Body mass dependencies from DeLong et al.

```
DeLongTable1 = {
    r[M] → r0 M[R]^ρ,
    K[M] → K0 M[R]^κ,
    a[M] → a0 M[C]^α,
    e[M] → e0 M[C]^ϵ,
    m[M] → m0 M[C]^μ
   };
```

The temperature-size rule (from Forster et al. 2012), for unicells (e.g., algae)

```
TSR = M[i_] → M15 (1 - d (T - 15));
```

where d is the percent decrease in mass with a one degree increase from 15 degrees.

What does mass at 15 C need to be to have K=100 at T=15 C (to stay consistent with Gilbert)

```
m15 = Solve[
  100 == K[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → T + 273.15 /. T → 15,
  M15]
```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. ≫

$$\left\{\left\{M15 \to 1. \times 100.^{\frac{1}{\kappa}} \left(\frac{e^{-\frac{0.00347041\,EB}{k}+\frac{0.00347041\,ES}{k}}}{K0}\right)^{\frac{1}{\kappa}}\right\}\right\}$$

Figure 3a of Gilbert (new predictions in red; new dashed curve and horizontal line are now on the order of 10^{10} and 10^{20} and so do not appear in plot)

```
Show[
 Plot[
  BCR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /. k → 8.62 * 10^-5 /.
        a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.32 /.
   ES → 0.9, {T, 5, 30}, PlotStyle → {Black, Thick}],
 Plot[BCR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
        k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.9 /.
   ES → 0.32, {T, 5, 30}, PlotStyle → {Black, Thick, Dashed}],
 Plot[BCR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
        k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
    EB → 0.9 /. ES → 0.9, {T, 5, 30}, PlotStyle → {Gray, Thick}],
 Plot[Simplify[BCR[T] /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
            GilbertTable1 /. DeLongTable1 /. T[i_] → T + 273.15 /. TSR /. m15 /.
        k → 8.62 * 10^-5 /. κ → -0.81 /. EB → 0.32 /. ES → 0.9 /. d → 0.02,
   K0 > 0], {T, 5, 30}, PlotStyle → {Red, Thick}],
 Plot[Simplify[BCR[T] /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
            GilbertTable1 /. DeLongTable1 /. T[i_] → T + 273.15 /. TSR /. m15 /.
        k → 8.62 * 10^-5 /. κ → -0.81 /. EB → 0.9 /. ES → 0.32 /. d → 0.02,
   K0 > 0], {T, 5, 30}, PlotStyle → {Red, Thick, Dashed}],
 Plot[Simplify[BCR[T] /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
            GilbertTable1 /. DeLongTable1 /. T[i_] → T + 273.15 /. TSR /. m15 /.
        k → 8.62 * 10^-5 /. κ → -0.81 /. EB → 0.9 /. ES → 0.9 /. d → 0.02,
   K0 > 0], {T, 5, 30}, PlotStyle → {Pink, Thick}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "BCR"}
]
```

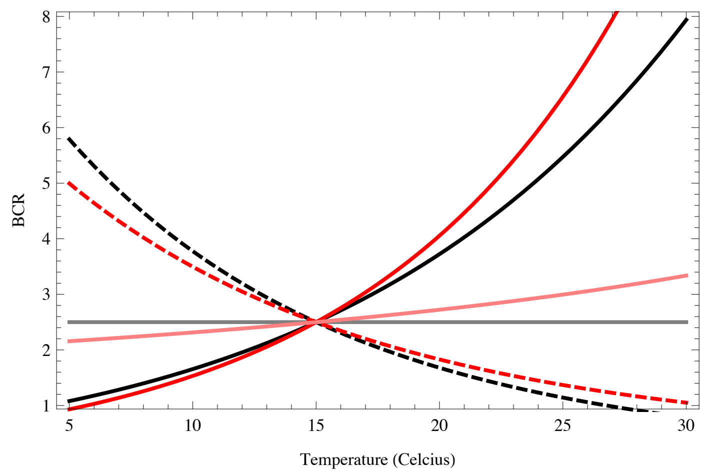


Figure 3b of Gilbert (new prediction in red)

```
Show[
 Plot[
  Simplify[CR[T] /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. GilbertTable1 /.
           DeLongTable1 /. T[i_] → T + 273.15 /. TSR /. m15 /.
       k → 8.62 * 10^-5 /. ϰ → -0.81 /. EB → 0.32 /. ES → 0.9 /. d → 0.02,
    K0 > 0], {T, 5, 30}, PlotStyle → {Red, Thick}],
 Plot[ CR[T] /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
        k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
   EB → 0.32 /. ES → 0.9, {T, 5, 30}, PlotStyle → {Black, Thick}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Consumer:resource biomass"}
]
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

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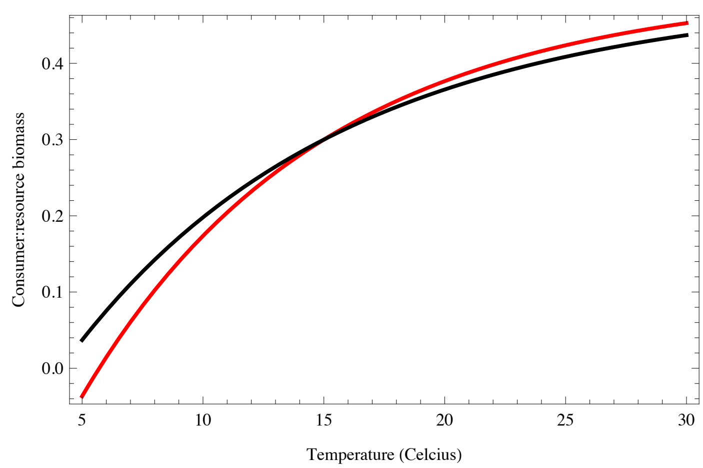
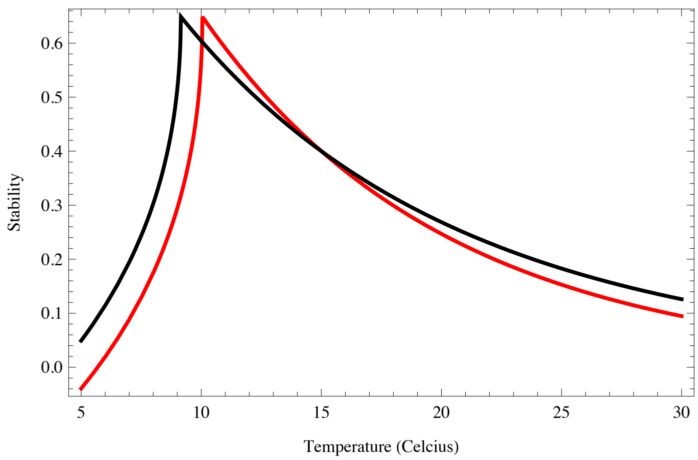


Figure 3c of Gilbert

```
Show[Plot[-Max[Re[Simplify[
        lambda /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. GilbertTable1 /.
              DeLongTable1 /. T[i_] → T + 273.151 /. TSR /. m15 /. k → 8.62 * 10^-5 /.
          κ → -0.8 /. EB → 0.32 /. ES → 0.9 /. d → 0.02, K0 > 0]]] //
   Chop, {T, 5, 30}, PlotStyle → {Red, Thick}], Plot[
  -Max[Re[lambda /. K[T] → K0 Exp[EB/(k T[R]) - ES/(k T[S])] /. T[i_] → T + 273.15 /. K15 /.
          k → 8.62 * 10^-5 /. a[T] → 0.1 /. e → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
       EB → 0.32 /. ES → 0.9]], {T, 5, 30}, PlotStyle → {Black, Thick}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Stability"}
]
```



This only looks at temperature and body mass dependencies in resource carrying capacity K. We next add temperature and mass dependencies to the other rates (and temperature dependence in consumer body mass) to see if this gives bigger discrepancies between the two models.

Introducing all the dependencies

Let the temperature size rule be linear, but potentially different for resource and consumer

```
Clear[TSR]
TSR = M[i_] → M15[i] (1 - β[i] (T[i] - (273.15 + 15)));
```

where M15[i] is the mass of the resource of consumer, i={R,C}, at 15 degrees celcius, β[i] is the percent decline in body size with a degree increase in temperature, and T is the current temperature in Kelvins.

To have the same population dynamics parameter values at 15 degrees as we did above with the TSR only in K, we need

```
a15 =
 Solve[0.1 == a[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, a0] //
  Flatten
```

$$\left\{a0 \to \frac{0.1\,M15[C]^{-1.\,\alpha}}{\sqrt{e^{-\frac{0.00694083\,E\nu[C]}{k}}\,\nu 0[C]^2 + e^{-\frac{0.00694083\,E\nu[R]}{k}}\,\nu 0[R]^2}}\right\}$$

```
e15 = Solve[0.15 == e[M] /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, e0] // Flatten
```

$$\left\{e0 \to 0.15\,M15[C]^{-1.\,\epsilon}\right\}$$

```
k15 =
 Solve[100 == K[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, K0] //
  Flatten
```

$$\left\{K0 \to 100.\,e^{-\frac{0.00347041\,EB}{k}+\frac{0.00347041\,ES}{k}}\,M15[R]^{-1.\,\kappa}\right\}$$

```
r15 =
 Solve[2 == r[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, r0] //
  Flatten
```

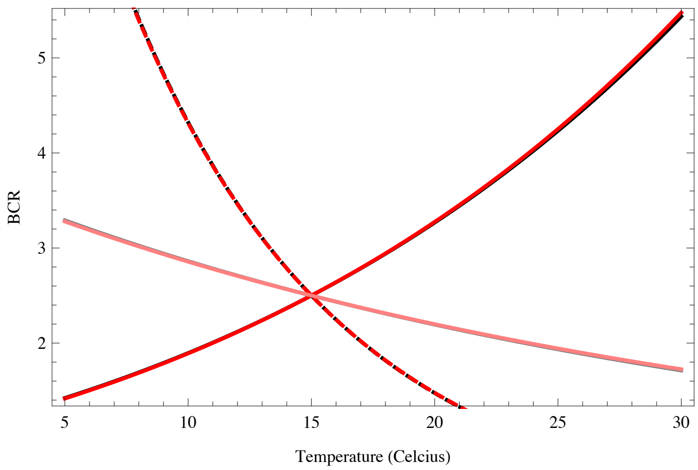
$$\left\{r0 \to 2.\,e^{\frac{0.00347041\,EB}{k}}\,M15[R]^{-1.\,\rho}\right\}$$

```
m15 =
 Solve[0.6 == m[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, m0] //
  Flatten
```

$$\left\{m0 \to 0.6\,e^{\frac{0.00347041\,Em}{k}}\,M15[C]^{-1.\,\mu}\right\}$$

Now, BCR as a function of temperature is

```
Show[
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
        0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
      ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
   β[i_] → 0 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick, Black}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
        EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
      ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Thick, Black, Dashed}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
        EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
      ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Thick, Gray}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
        EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
      ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
   β[i_] → 0.02 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick,
   Red}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
        EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
      ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Thick, Red, Dashed}], Plot[
  BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
        EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
      ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
   β[i_] → 0.02 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick,
   Pink}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "BCR"}
]
```



Nothing much doing here (although notice how the predictions of Gilbert have changed from their Figure 3a, where only K depended on temperature).

```
Show[
 Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                   EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
             ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
       ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Black, Thick},
  Axes →
   False],
 Plot[ CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                   EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
             ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
       ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Red, Thick}, Axes → False],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Consumer:resource biomass"},
 PlotRange → All
]
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

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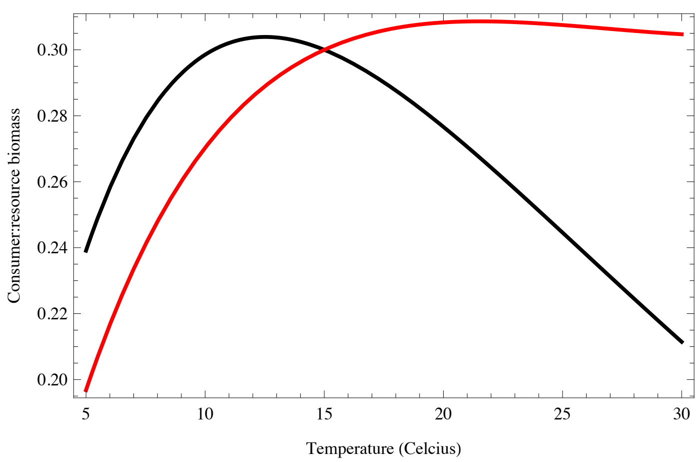
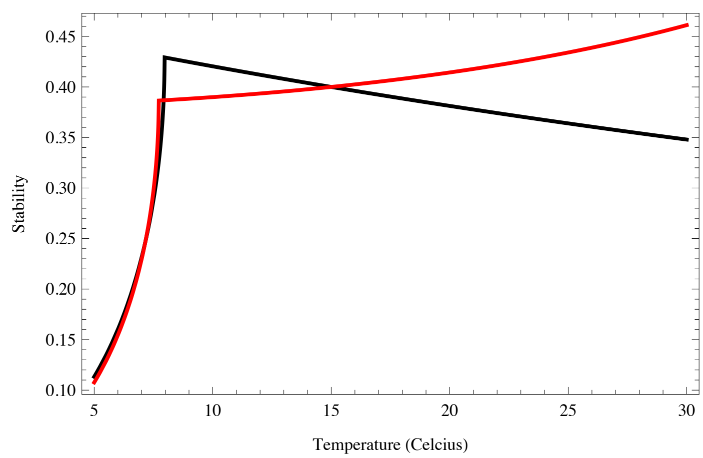


So, the actual prediction of Gilbert et al., when using all the dependencies (not just K, as in Figure 3b), is that the consumer to resource biomass ratio decreases with temperature. However, when we add the TSR we see that the ratio no longer decreases over reasonable temperatures! I.e., temperature directly decreases consumer:resource biomass, but it also decreases body sizes, and decreased body size decreases consumer:resource biomass. In other words, the direct and indirect effects of temperature are of opposite sign.

```
Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                        EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /.
                   Eν[i_] → 0.46 /. ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /.
             μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thick}, Axes →
   False,
  PlotRange →
   {0,
    All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                        EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] →
                    0.46 /. ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
            ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Red, Thick}, PlotRange →
   {0,
    All}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Stability"},
 PlotRange → All
]
```



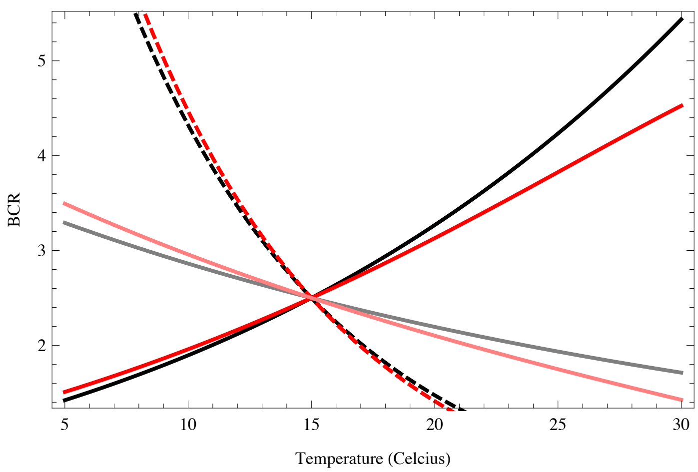
With all the dependencies, stability still decreases with temperature (black), albeit slower than when only K depended on temperature (Gilbert Figure 3c). BUT, with the temperature-size rule, stability increases with increasing temperature! Again, temperature directly destabilizes, but indirectly, through its effect on body size, stabilizes the dynamics.

Exploring asymmetric TSR responses

Until now we've supposed that the consumer and resource have the same TSR response. Lets relax that. According to Forster, if the consumer is larger than the resource, it might also have a larger TSR response, say maybe a 3-4% decrease with degree, rather than the 2% for unicells (we are keeping the assumption that the loss is linear).

With a 3% TSR in consumer and a 2% TSR in resource, our predictions become:

```
Show[
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
                0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
          ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
   β[i_] → 0 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick, Black}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
               EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
          ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
     ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Thick, Black, Dashed}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
               EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
          ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
     ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Thick, Gray}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /.
           Eν[i_] → 0.46 /. ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /.
      μ → -0.29 /. ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.03 /.
   T[i_] → T + 273.15, {T, 5, 30}, PlotStyle →
   {Thick,
    Red}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10^-5 /. Em → 0.65 /.
           Eν[i_] → 0.46 /. ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /.
      μ → -0.29 /. ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.03 /.
   T[i_] → T + 273.15, {T, 5, 30}, PlotStyle →
   {Thick,
    Red,
    Dashed}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /.
           Eν[i_] → 0.46 /. ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /.
      μ → -0.29 /. ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.03 /.
   T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick,
    Pink}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "BCR"}
]
```


BCR
Temperature (Celcius)
5
10
15
20
25
30
2
3
4
5

```
Show[
 Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                 EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
            ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
       ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Black, Thick},
  Axes →
   False],
 Plot[ CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                 EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
            ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
       ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.03 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Red, Thick}, Axes → False],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Consumer:resource biomass"},
 PlotRange → All
]
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

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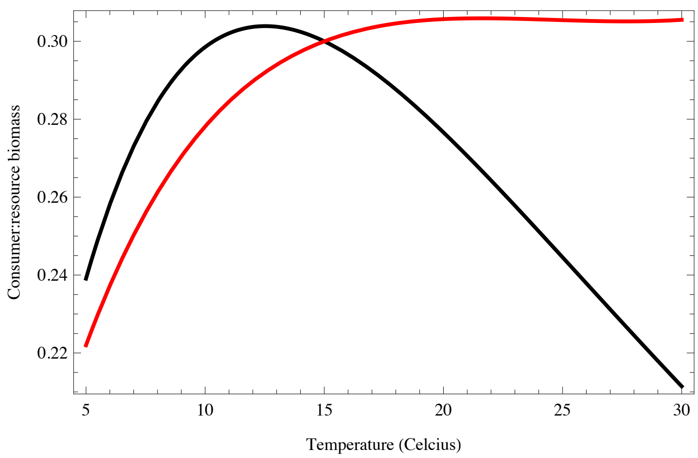
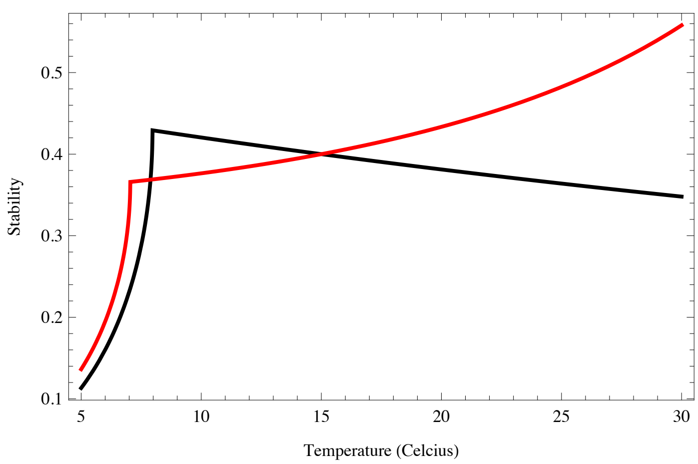


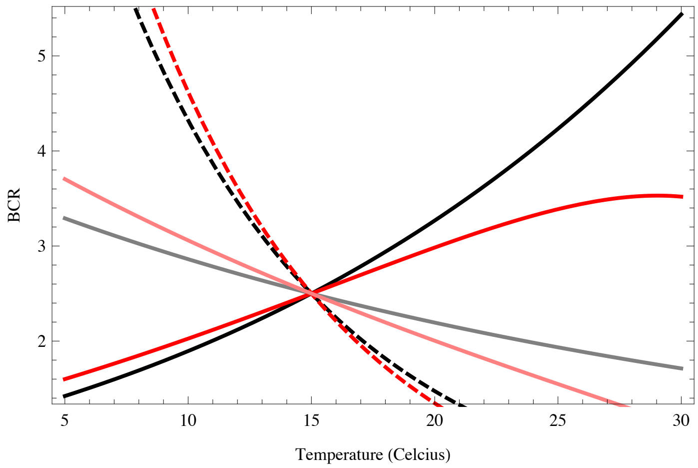
So, the actual prediction of Gilbert et al., when using all the dependencies (not just K, as in Figure 3b), is that the consumer to resource biomass ratio decreases with temperature. However, when we add the TSR we see that the ratio no longer decreases over reasonable temperatures! I.e., temperature directly decreases consumer:resource biomass, but it also decreases body sizes, and decreased body size decreases consumer:resource biomass. In other words, the direct and indirect effects of temperature are of opposite sign.

```
Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /.
              Eν[i_] → 0.46 /. ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /.
         μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thick}, Axes →
   False,
  PlotRange →
   {0,
    All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                 EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] →
               0.46 /. ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
         ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.03 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Red, Thick}, PlotRange →
   {0,
    All}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Stability"},
 PlotRange → All
]
```



With a 4% TSR in consumer and a 2% TSR in resource, our predictions become:

```
Show[
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
                0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
          ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
    β[i_] → 0 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick, Black}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
          ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
     ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Thick, Black, Dashed}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
          ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
     ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Thick, Gray}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /.
           Eν[i_] → 0.46 /. ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /.
      μ → -0.29 /. ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.04 /.
   T[i_] → T + 273.15, {T, 5, 30}, PlotStyle →
   {Thick,
    Red}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10^-5 /. Em → 0.65 /.
           Eν[i_] → 0.46 /. ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /.
      μ → -0.29 /. ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.04 /.
   T[i_] → T + 273.15, {T, 5, 30}, PlotStyle →
   {Thick,
    Red,
    Dashed}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /.
           Eν[i_] → 0.46 /. ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /.
      μ → -0.29 /. ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.04 /.
   T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick,
    Pink}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "BCR"}
]
```


BCR
Temperature (Celcius)
5
10
15
20
25
30
2
3
4
5

```
Show[
 Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                  EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
             ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
       ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Black, Thick},
  Axes →
   False],
 Plot[ CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                  EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
             ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
       ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Red, Thick}, Axes → False],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Consumer:resource biomass"},
 PlotRange → All
]
```

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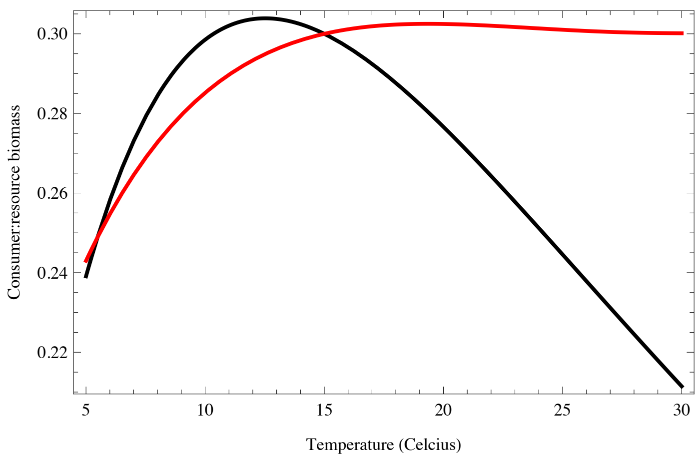
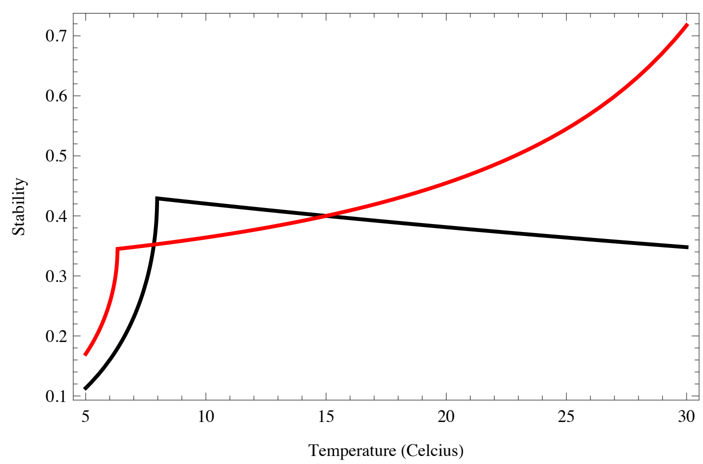


So, the actual prediction of Gilbert et al., when using all the dependencies (not just K, as in Figure 3b), is that the consumer to resource biomass ratio decreases with temperature. However, when we add the TSR we see that the ratio no longer decreases over reasonable temperatures! I.e., temperature directly decreases consumer:resource biomass, but it also decreases body sizes, and decreased body size decreases consumer:resource biomass. In other words, the direct and indirect effects of temperature are of opposite sign.

```
Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /.
               Eν[i_] → 0.46 /. ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /.
         μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thick}, Axes →
   False,
  PlotRange →
   {0,
    All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] →
                0.46 /. ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
         ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Red, Thick}, PlotRange →
   {0,
    All}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Stability"},
 PlotRange → All
]
```



So, with a larger TSR response in consumer (as often expected) the dynamics are even further stabilized!

Making the functional response type-2

BCR at a given T, like that defined by Gilbert (Eqn 5), but with type-II functional response

$$\mathtt{BCR2[T\_]} := \frac{\mathtt{e\,f[R,\,T]\,K[T]}}{\mathtt{m[C,\,T]}} \;/.\; \mathtt{f[R,\,T]} \to \frac{\mathtt{a[T]}}{\mathtt{1 + a[T]\,h[T]\,R}} \;/.\; \mathtt{m[C,\,T]} \to \mathtt{m[T]}$$

Equilibrium biomasses at given temperature assuming a type II functional response and density-independent consumer mortality

```
Eq2[T_] := Solve[{0 == dRdt[R, C, T], 0 == dCdt[R, C, T]} /.
    f[R, T] → a[T]/(1 + a[T] h[T] R) /. m[C, T] → m[T], {R, C}]
```

Equilibrium consumer to resource biomass ratio at a given temperature (at the equilibrium where both populations persist)

```
CR2[T_] := C/R /. Eq2[T][[3]]
```

The Jacobian evaluated at equilibrium (determines stability)

```
Jac2 = {{D[dRdt[R, C, T] /. f[R, T] → a[T]/(1 + a[T] h[T] R) /. m[C, T] → m[T], R],
     D[dRdt[R, C, T] /. f[R, T] → a[T]/(1 + a[T] h[T] R) /. m[C, T] → m[T], C]},
    {D[dCdt[R, C, T] /. f[R, T] → a[T]/(1 + a[T] h[T] R) /. m[C, T] → m[T], R],
     D[dCdt[R, C, T] /. f[R, T] → a[T]/(1 + a[T] h[T] R) /. m[C, T] → m[T], C]}} /. Eq2[T][[3]];
```

The eigenvalues of the Jacobian are

```
lambda2 = Eigenvalues[Jac2];
```

Now the dependencies. We can use the scalings of the other parameters from Gilbert and DeLong:

```
GilbertDeLongT = {r[T] → ⅇ^(-EB/(k T[R])) r[M], K[T] → ⅇ^(EB/(k T[R]) - ES/(k T[S])) K[M], m[T] → ⅇ^(-Em/(k T[C])) m[M], e → e[M]};
GilbertDeLongM = {r[M] → r0 M[R]^ρ, K[M] → K0 M[R]^ϰ, e[M] → e0 M[C]^ϵ, m[M] → m0 M[C]^μ};
```

The attack rate and handling time scalings are (in a 2D environment; Rall et al. 2012)

```
RallT = {h[T] → ⅇ^(0.65/(k T)) h[M], a[T] → ⅇ^(-0.65/(k T)) a[M]};
RallM = {h[M] → h0 M[C]^(-3/4) M[R]^1, a[M] → a0 M[C]^(1/4+1/3) M[R]^(1/3)};
```

To have the same population dynamics parameter values at 15 degrees as Gilbert et al., we need

```
a152 = Solve[
   0.1 == Simplify[a[T]/(1 + a[T] h[T] R) /. Eq2[T][[3]]] /. RallT /. RallM /. GilbertDeLongT /.
      GilbertDeLongM /. TSR /. T[i_] → 273.15 + 15, a0] // Flatten
```

$$\left\{a0 \to \frac{0.1 \times 1.^{-1.58333+1.\,\epsilon}\, e^{\frac{0.65}{k T}}\, e0\, M15[C]^{-0.583333+1.\,\epsilon}}{M15[R]^{1/3}\left(1.\, e0\, M15[C]^{\epsilon} - 1. \times 1.^{-0.75+\mu}\, e^{-\frac{0.00347041\, Em}{k}+\frac{0.65}{k T}}\, h0\, m0\, M15[C]^{-0.75+\mu}\, M15[R]\right)}\right\}$$

```
e15 = Solve[0.15 == e[M] /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, e0] // Flatten
```

$$\left\{e0 \to 0.15\, M15[C]^{-1.\,\epsilon}\right\}$$

```
k15 =
 Solve[100 == K[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, K0] //
  Flatten
```

$$\left\{K0 \to 100.\, e^{-\frac{0.00347041\,EB}{k}+\frac{0.00347041\,ES}{k}}\, M15[R]^{-1.\,\kappa}\right\}$$

```
r15 =
 Solve[2 == r[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, r0] //
  Flatten
```

$$\left\{r0 \to 2.\, e^{\frac{0.00347041\,EB}{k}}\, M15[R]^{-1.\,\rho}\right\}$$

```
m15 =
 Solve[0.6 == m[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, m0] //
  Flatten
```

$$\left\{m0 \to 0.6\, e^{\frac{0.00347041\,Em}{k}}\, M15[C]^{-1.\,\mu}\right\}$$

We can set h0 by asking what it needs to be to have BCR with a type-2 equal the BCR with a type-1 at 5 degrees C (this is rather arbitrary!): (CHANGE THIS, BUT TO WHAT?)

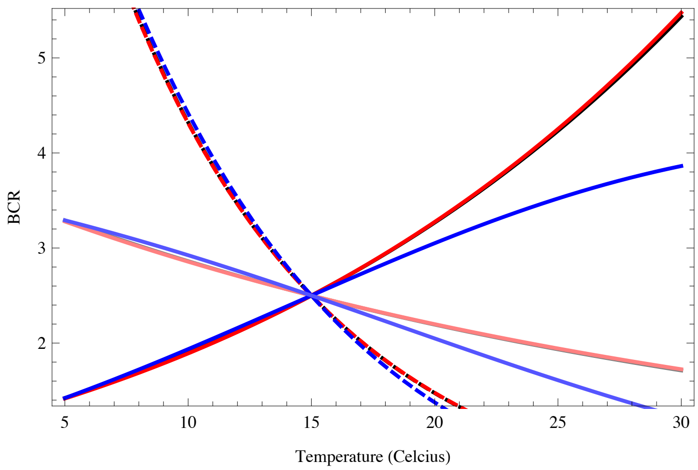
```
h5 = Solve[
   (BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
                    0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
               v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
          ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15 /. T → 5) ==
    (Simplify[BCR2[T] /. Eq2[T][[3]]] /. RallT /. RallM /. GilbertDeLongT /.
                       GilbertDeLongM /. TSR /. a152 /. e15 /. k15 /. r15 /. m15 /.
                T[i_] → T /. k → 8.62 * 10^-5 /. EB → 0.32 /. ES → 0.9 /.
             Em → 0.65 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
         ρ → -0.81 /. β[i_] → 0.02 /. T → 273.15 + T /. T → 5), h0]
```

$$\left\{\left\{h0 \to -\frac{2.0222\times 10^{-12}\, M15[C]^{3/4}}{M15[R]}\right\}\right\}$$

Now, BCR as a function of temperature is

```
Show[
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                 EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
            v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
       ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → 273.15 + t,
  {t, 5, 30}, PlotStyle → {Black, Thick}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                 EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
            v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
       ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → 273.15 + t,
  {t, 5, 30}, PlotStyle → {Black, Thick, Dashed}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                 EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
            v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
```

```
      ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → 273.15 + t,
  {t, 5, 30}, PlotStyle → {Gray, Thick}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
           ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
    β[i_] → 0.02 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick,
    Red}],
 Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
           ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Thick, Red, Dashed}], Plot[
  BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
           ν0[i_] → 1 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
    β[i_] → 0.02 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick,
    Pink}],
 Plot[Simplify[BCR2[T] /. Eq2[T][[3]]] /. RallT /. RallM /. GilbertDeLongT /.
                    GilbertDeLongM /. TSR /. a152 /. e15 /. k15 /. r15 /.
                m15 /. T[i_] → T /. k → 8.62 * 10^-5 /. EB → 0.32 /. ES → 0.9 /.
           Em → 0.65 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. β[i_] → 0.02 /. T → 273.15 + t /. h5,
  {t, 5, 30}, PlotStyle → {Blue, Thick}],
 Plot[Simplify[BCR2[T] /. Eq2[T][[3]]] /. RallT /. RallM /. GilbertDeLongT /.
                    GilbertDeLongM /. TSR /. a152 /. e15 /. k15 /. r15 /.
                m15 /. T[i_] → T /. k → 8.62 * 10^-5 /. EB → 0.9 /. ES → 0.32 /.
           Em → 0.65 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. β[i_] → 0.02 /. T → 273.15 + t /. h5,
  {t, 5, 30}, PlotStyle → {Blue, Thick,
    Dashed}],
 Plot[Simplify[BCR2[T] /. Eq2[T][[3]]] /. RallT /. RallM /. GilbertDeLongT /.
                    GilbertDeLongM /. TSR /. a152 /. e15 /. k15 /. r15 /.
                m15 /. T[i_] → T /. k → 8.62 * 10^-5 /. EB → 0.9 /. ES → 0.9 /.
           Em → 0.65 /. ϰ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. β[i_] → 0.02 /. T → 273.15 + t /. h5,
  {t, 5, 30}, PlotStyle → {Lighter[Blue], Thick}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "BCR"}
]
```

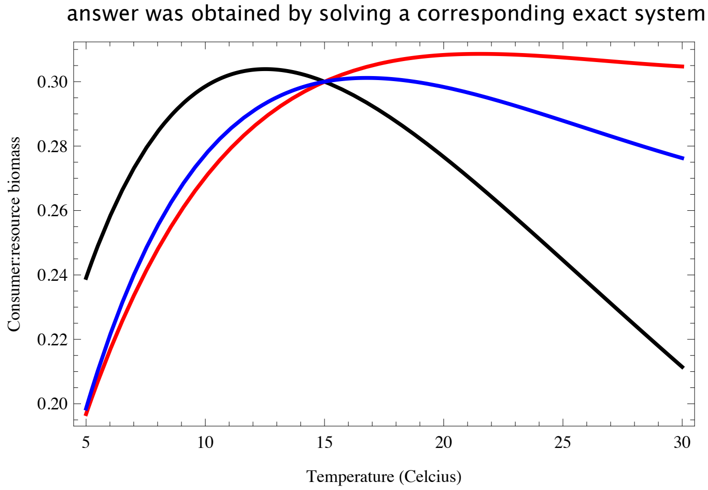


Where all parameters are dependent on temperature and/or mass. Black is Gilbert (only temperature dependencies), Red is mass and temp dependencies with type-1 and Blue is mass and temp dependencies with type-2.

```
Show[
 Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
           ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Black, Thick},
  Axes →
   False, PlotRange →
   {0, All}],
 Plot[ CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Eν[i_] → 0.46 /.
           ν0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Red, Thick}, Axes → False,
  PlotRange →
   {0, All}],
 Plot[Simplify[
   CR2[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. a152 /. e15 /.
                  k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10^-5 /. EB → 0.32 /.
             ES → 0.9 /. Em → 0.65 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /.
        μ → -0.29 /. ρ → -0.81 /. β[i_] → 0.02 /. T → 273.15 + t /. h5],
  {t, 5, 30}, PlotStyle → {Blue, Thick}, PlotRange →
   {0,
    All}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Consumer:resource biomass"},
 PlotRange → All
]
```

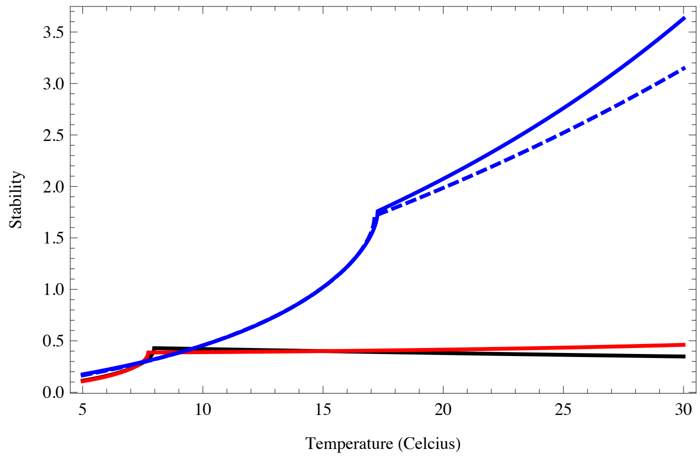
Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫



So, the actual prediction of Gilbert et al., when using all the dependencies (not just K, as in Figure 3b), is that the consumer to resource biomass ratio decreases with temperature. However, when we add the TSR we see that the ratio no longer decreases over reasonable temperatures! I.e., temperature directly decreases consumer:resource biomass, but it also decreases body sizes, and decreased body size decreases consumer:resource biomass. In other words, the direct and indirect effects of temperature are of opposite sign.

```
Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /.
             Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /.
         μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thick}, Axes →
   False,
  PlotRange →
   {0,
    All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
                EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10^-5 /. Em → 0.65 /. Ev[i_] →
              0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /. μ → -0.29 /.
        ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Red, Thick}, PlotRange →
   {0,
    All}],
 Plot[
  -Max[
    Re[
     lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. a152 /.
                  e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10^-5 /.
              EB → 0.32 /. ES → 0.9 /. Em → 0.65 /. κ → -0.81 /. α → 1 /.
          ϵ → -0.5 /. μ → -0.29 /. ρ → -0.81 /. β[i_] → 0.02 /.
       T → 273.15 + t /. h5 /. M15[i_] → 100]],
  {t, 5, 30}, PlotStyle → {Blue, Thick}, PlotRange →
   {0,
    All}],
 Plot[
  -Max[
    Re[
     lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. a152 /.
                  e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10^-5 /. EB →
               0.32 /. ES → 0.9 /. Em → 0.65 /. κ → -0.81 /. α → 1 /. ϵ → -0.5 /.
         μ → -0.29 /. ρ → -0.81 /. β[i_] → 0 /. T → 273.15 + t /. h5 /. M15[i_] → 100]],
  {t, 5, 30}, PlotStyle → {Blue, Thick, Dashed}, PlotRange →
   {0,
    All}],
 Frame → True,
 FrameLabel → {"Temperature (Celcius)", "Stability"},
 PlotRange → All
]
```



Note: even though we have to give M15[i]s to produce blue curve, the result does not (seem to) depend on them.

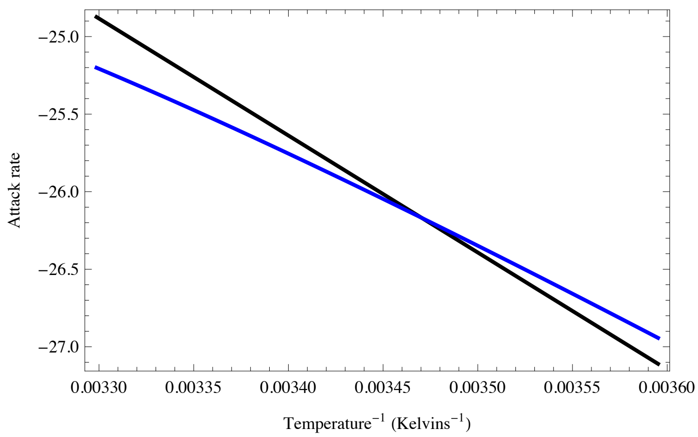
Why dont things line up at 5 and 15 degrees anymore? Maybe because we are only looking at the Real parts?

Anyways, if this is right, then the type-2 with TSR is by far the most stable, especially at high temperatures.
Type-2 without the TSR is blue dashed; we see that things are generally more stable with a type-2 response, and that the TSR adds extra stability at warm temperatures.

Improving Rall's fit

Response of attack rate to temperature, with and without the TSR:

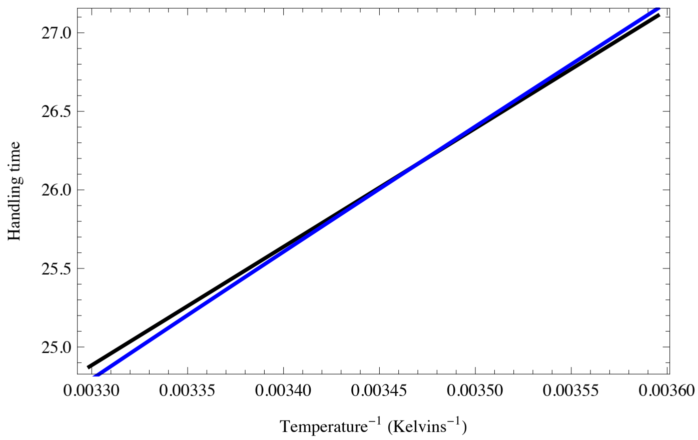
```
Show[
 Plot[Log[a[T] / a0] /. RallT /. RallM /. k → 8.62 * 10^-5 /. M[i_] → 1 /. T → 1 / t,
  {t, 1 / (273.15 + 5), 1 / (273.15 + 30)}, PlotStyle → {Black, Thick}],
 Plot[Log[a[T] / a0] /. RallT /. RallM /. TSR /. T[i_] → T /. k → 8.62 * 10^-5 /.
     M15[i_] → 1 /. β[i_] → 0.02 /. T → 1 / t,
  {t, 1 / (273.15 + 5), 1 / (273.15 + 30)}, PlotStyle → {Blue, Thick}],
 Plot[Log[ⅇ^(-0.4/(k T)) M[C]^(7/12) M[R]^(1/3)] /. k → 8.62 * 10^-5 /. M[i_] → 1 /. T → 1 / t,
  {t, 1 / (273.15 + 5), 1 / (273.15 + 30)}, PlotStyle → Orange],
 Frame → True,
 FrameLabel → {"Temperature^-1  (Kelvins^-1)", "Attack rate"}
]
```



The TSR lowers the activation energy of attack rate, which goes in the same direction as the discrepancy seen in Rall (see Figure 3a in Rall et al 2012).

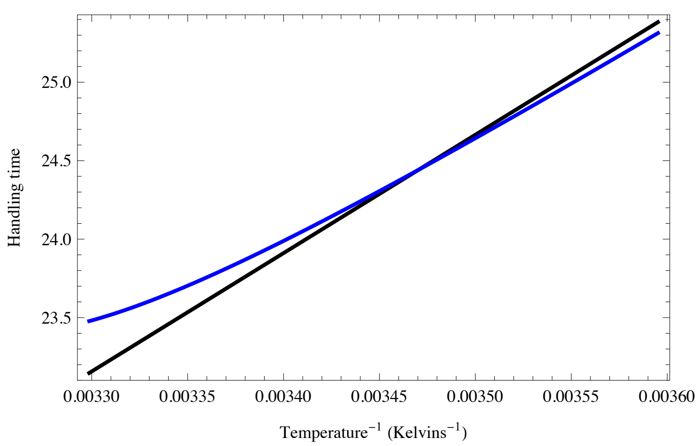
And for handling time:

```
Show[
 Plot[Log[h[T] / h0] /. RallT /. RallM /. k → 8.62 * 10^-5 /. M[i_] → 1 /. T → 1 / t,
  {t, 1 / (273.15 + 5), 1 / (273.15 + 30)}, PlotStyle → {Black, Thick}],
 Plot[Log[h[T] / h0] /. RallT /. RallM /. k → 8.62 * 10^-5 /. TSR /. T[i_] → T /.
     M15[i_] → 1 /. β[i_] → 0.02 /. T → 1 / t,
  {t, 1 / (273.15 + 5), 1 / (273.15 + 30)}, PlotStyle → {Blue, Thick}],
 Frame → True,
 FrameLabel → {"Temperature^-1 (Kelvins^-1)", "Handling time"}
]
```



Here the TSR (slightly) increases the activation energy of handling time. But this is with same TSR in resource and consumer. If we allow the consumer to have a larger TSR, then we have

```
Show[
 Plot[Log[h[T] / h0] /. RallT /. RallM /. k → 8.62 * 10^-5 /. M[R] → 1 /. M[C] → 10 /.
   T → 1 / t, {t, 1 / (273.15 + 5), 1 / (273.15 + 30)}, PlotStyle → {Black, Thick}],
 Plot[Log[h[T] / h0] /. RallT /. RallM /. k → 8.62 * 10^-5 /. TSR /. T[i_] → T /.
       M15[R] → 1 /. M15[C] → 10 /. β[R] → 0.02 /. β[C] → 0.04 /. T → 1 / t,
  {t, 1 / (273.15 + 5), 1 / (273.15 + 30)}, PlotStyle → {Blue, Thick}],
 Frame → True,
 FrameLabel → {"Temperature^-1 (Kelvins^-1)", "Handling time"}
]
```



which makes the activation energy of handling time smaller, as in Rall (see Figure 3d).

Effect of the TSR on the functional response

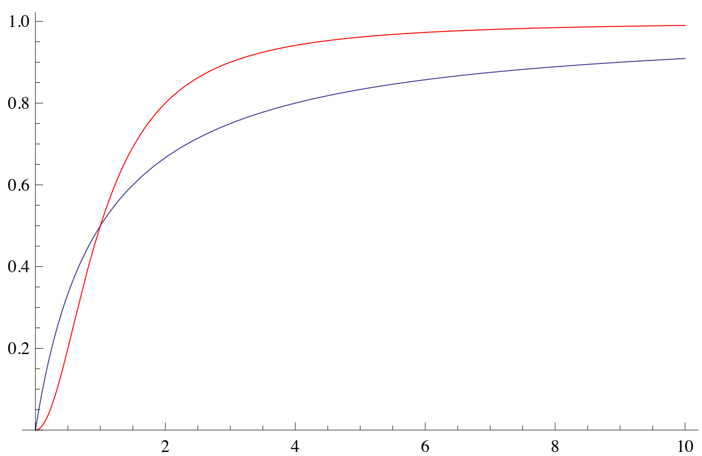
This section is based on Kalinkat et al. 2013.

In particular, we let the functional response be

$$g[R\_, T\_] := \frac{b[T]\, R^{q[M]}}{1 + h[T]\, b[T]\, R^{1+q[M]}}$$

such that when q=0 we have a type-1 functional response and when q>0 we have a type-3 (sigmiodal):

```
Show[
 Plot[g[R, T] R /. b[T] → 1 /. h[T] → 1 /. q[M] → 0, {R, 0, 10}, PlotRange → {0, All}],
 Plot[g[R, T] R /. b[T] → 1 /. h[T] → 1 /. q[M] → 1,
  {R, 0, 10}, PlotRange → {0, All}, PlotStyle → Red],
 PlotRange → {0, All}
]
```

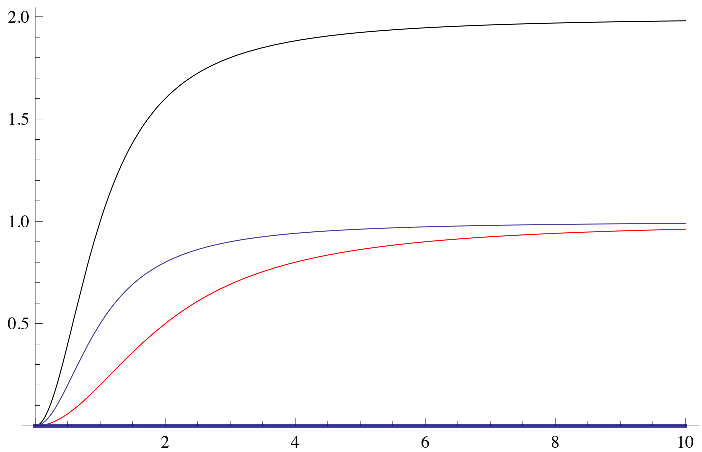


We say q is a function of mass M, because it depends on the ratio of consumer to resource body size

$$q[M\_] := \frac{qmax \left(\frac{M[C]}{M[R]}\right)^2}{q0^2 + \left(\frac{M[C]}{M[R]}\right)^2}$$

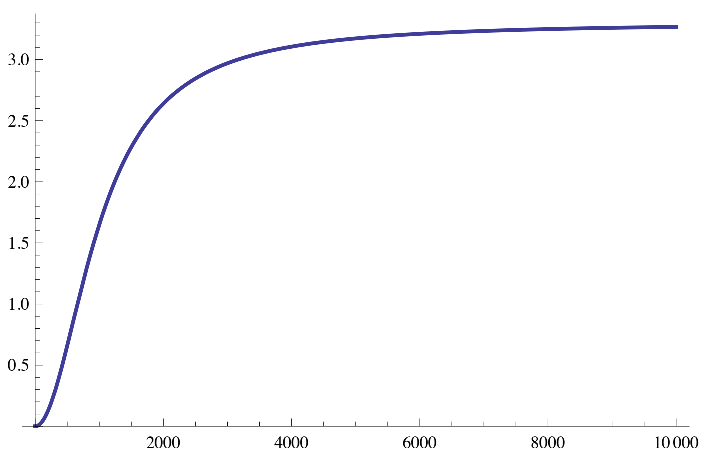
where qmax and q0 are scaling parameters that determine the shape of the sigmoidal response (qmax is the asymptote, $q0^2$ is the half-saturation constant):

```
Show[
 Plot[q[M] /. M[C] → a M[R] /. qmax → 1 /. q0 → 1, {a, 0, 10}, PlotRange → {0, All}],
 Plot[q[M] /. M[C] → a M[R] /. qmax → 1 /. q0 → 2,
  {a, 0, 10}, PlotRange → {0, All}, PlotStyle → Red],
 Plot[q[M] /. M[C] → a M[R] /. qmax → 2 /. q0 → 1, {a, 0, 10},
  PlotRange → {0, All}, PlotStyle → Black],
 Plot[q[M] /. M[C] → a M[R] /. qmax → 3.3 /. q0 → 10^3,
  {a, 0, 10}, PlotRange → {0, All}, PlotStyle → Thick]
]
```



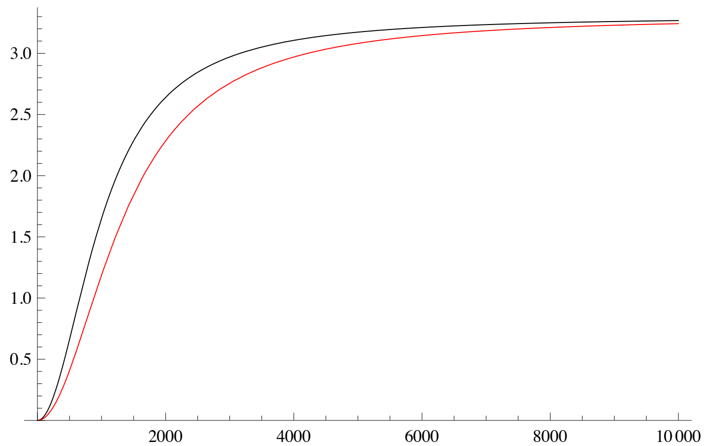
The fitted values in Kalinkat give the following curve

```
Plot[q[M] /. M[C] → a M[R] /. qmax → 3.3 /. q0 → 10^3,
 {a, 0, 10^4}, PlotRange → {0, All}, PlotStyle → Thick]
```



What effect does the TSR have on q?

```
Show[
 Plot[q[M] /. qmax → 3.3 /. q0 → 10^3 /. TSR /. T[i_] → T /. β[R] → 0 /. β[C] → 0 /.
    M15[C] → a M15[R] /. T → 273.15 + 25,
  {a, 0, 10^4}, PlotRange → {0, All}, PlotStyle → Black],
 Plot[q[M] /. qmax → 3.3 /. q0 → 10^3 /. TSR /. T[i_] → T /. β[R] → 0.02 /. β[C] → 0.04 /.
    M15[C] → a M15[R] /. T → 273.15 + 25,
  {a, 0, 10^4}, PlotRange → {0, All}, PlotStyle → Red]
]
```



As we might expect, because q depends on the ratio of body sizes, if both resource and consumer have the same TSR, then the TSR has no affect on q.
However, when the consumer has a larger TSR than the resource, the body size ratio decreases with temperature, decreasing q and therefore slowing the switch from type-2 to type-3.
Because type-3 is more stable, the TSR may be said to reduce stability by this mechanism (as we can see in the above plot, with the TSR (red) q is not reduced by much).