

1. Bayesian Inference

a) $P(\text{smart}) = 0.3$, $P(\text{study}|\text{smart}) = 0.4$, $P(\text{pass}|\text{smart},\text{study}) = 0.95$

$$P(\text{smart}, \text{study}, \text{pass}) = P(\text{smart}) * P(\text{studying}|\text{smart}) * P(\text{pass}|\text{smart}, \text{study}) = 0.3 * 0.4 * 0.95 = 0.114$$

b) $P(-\text{smart}, -\text{study}, -\text{pass}) = (1-0.3) * (1-0.4) * (1-0.95) = 0.021$

$$P(-\text{smart}, \text{study}, -\text{pass}) = (1-0.3) * (0.4) * (1-0.95) = 0.014$$

$$P(\text{smart}, -\text{study}, -\text{pass}) = (0.3) * (1-0.4) * (1-0.95) = 0.009$$

$$P(\text{smart}, \text{study}, -\text{pass}) = 0.3 * 0.4 * (1-0.95) = 0.006$$

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$$P(\text{smart}, \text{study}, \text{pass}) = 0.3 * 0.4 * 0.95 = 0.114$$

$$P(\text{smart}, -\text{study}, \text{pass}) = 0.3 * (1-0.4) * 0.95 = 0.171$$

$$P(-\text{smart}, \text{study}, \text{pass}) = (1-0.3) * 0.4 * 0.95 = 0.266$$

$$P(-\text{smart}, -\text{study}, \text{pass}) = (1-0.3) * (1-0.4) * 0.95 = 0.399$$

	smart		-smart	
	pass	-pass	pass	-pass
study	0.114	0.006	0.266	0.014
-study	0.171	0.009	0.399	0.021

c) $P(\text{smart}|\text{pass}, \text{study}) = (P(\text{pass}|\text{smart}, -\text{study}) * P(\text{smart}|- \text{study})) / P(\text{pass}|- \text{study})$

$$P(\text{pass}|- \text{study}) = P(\text{pass}|\text{smart}, -\text{study}) * P(\text{smart}) + P(\text{pass}|- \text{smart}, -\text{study}) * P(-\text{smart}) = 0.7 * 0.3 + 0.2 * 0.7 = 0.35$$

= $(0.7 * 0.3) / 0.35 = \mathbf{0.6}$, or **60% chance** that a student is smart given that they pass but did not study.

d) $P(-\text{study}|\text{smart}, -\text{pass}) = ((P(-\text{pass}|\text{smart}, -\text{study}) * P(-\text{study}, \text{smart})) / P(-\text{pass}|\text{smart})$

$$P(-\text{pass}|\text{smart}) = P(-\text{pass}|\text{smart}, -\text{study}) * P(-\text{study}) + P(-\text{pass}|\text{smart}, \text{study}) * P(\text{study}) = 0.3 * 0.6 + 0.05 * 0.4 = 0.2$$

= $((1-0.7) * (1-0.6)) / 0.2 = \mathbf{0.6}$, or **60% chance** that a student did not study, given that they are smart but did not pass

e) $P(\text{pass}|\text{smart}) = P(\text{pass} \wedge \text{smart}) / P(\text{smart}) = (0.114 + 0.171) / (0.006 + 0.009 + 0.114 + 0.171) = \mathbf{0.95}$

f) $P(\text{pass}|\text{study}) = P(\text{pass} \wedge \text{study}) / P(\text{study}) = (0.114 + 0.266) / (0.4) = \mathbf{0.95}$

2. Bayesian Networks

- a) $P(\text{cold}, \text{sneeze}, \text{allergic}, \text{scratches}, \text{cat}) = P(\text{cold}) * P(\text{sneeze}|\text{cold}, \text{allergic}) * P(\text{allergic}|\text{cat}) * P(\text{scratches}|\text{cat}) * P(\text{cat})$
- b) $P(-\text{cold}, \text{sneeze}, \text{allergic}, \text{scratches}, \text{cat}) = P(-\text{cold}) * P(\text{sneeze}|- \text{cold}, \text{allergic}) * P(\text{allergic}|\text{cat}) * P(\text{scratches}|\text{cat}) * P(\text{cat}) = 0.95 * 0.7 * 0.75 * 0.5 * 0.02 = \mathbf{0.005}$
- c) $P(-\text{cold}, \text{sneeze}, \text{allergic}, \text{scratches}, \text{cat}) / \sum_{\text{cat}} (P(-\text{cold}, \text{sneeze}, \text{allergic}, \text{scratches}, \text{cat})) = P(-\text{cold}) \cdot P(\text{sneeze} | -\text{cold}) \cdot P(\text{allergic} | -\text{cold}) \cdot P(\text{scratches} | \text{sneeze}, \text{cat}) \cdot P(\text{cat}) / (P(-\text{cold}, \text{sneeze}, \text{allergic}, \text{scratches}, \text{cat}=\text{T}) + P(-\text{cold}, \text{sneeze}, \text{allergic}, \text{scratches}, \text{cat}=\text{F})) = 0.005 / (0.95 * 0.7 * 0.75 * 0.5 * 0.02 + 0.95 * 0.7 * 0.75 * 0.5 * 0.98) = \mathbf{0.020}$
- d) $P(\text{cat}|\text{scratches}) = (P(\text{scratches}|\text{cat}) * P(\text{cat})) / P(\text{scratches}) = \mathbf{0.5 * 0.2 / P(\text{scratches})}$
- e) $\mathbf{P(\text{scratches}) = P(\text{cat}) * P(\text{scratches}|\text{cat}) + P(-\text{cat}) * P(\text{scratches}|- \text{cat})}$

3. PDDL

- a) $\text{Init}(\text{At}(\text{Person}, \text{Car}) \wedge \text{HasKey}(\text{Person}, \text{Car}) \wedge \text{ChargedBattery}(\text{Car}) \wedge \text{GasInTank}(\text{Car}))$
 $\text{Goal}(\text{Running}(\text{Car}) \wedge \text{At}(\text{Person}, \text{Car}) \wedge \text{HasKey}(\text{Person}, \text{Car}))$
 $\text{Action}(\text{StartCar},$
 $\text{PRECOND: At}(\text{Person}, \text{Car}) \wedge \text{HasKey}(\text{Person}, \text{Car}) \wedge \text{ChargedBattery}(\text{Car}) \wedge \text{GasInTank}(\text{Car})$
 $\text{EFFECT: Running}(\text{Car}) \wedge \text{At}(\text{Person}, \text{Car}) \wedge \text{HasKey}(\text{Person}, \text{Car}))$
- b) **Fluents:**
 $\text{At}(x, l, s)$: Object x is at location l in situation s .
 $\text{HasKey}(x, y, s)$: Person x has the key for car y in situation s .
 $\text{ChargedBattery}(x, s)$: The battery of car x is charged in situation s .
 $\text{GasInTank}(x, s)$: There is gas in the tank of car x in situation s .
 $\text{Running}(x, s)$: Car x is running in situation s .

Possibility Axiom:

$\text{StartCar}(x, s)$: Try to start car x in situation s .

Preconditions:

$\text{At}(\text{Person}, \text{Car}, s) \wedge \text{HasKey}(\text{Person}, \text{Car}, s) \wedge \text{ChargedBattery}(\text{Car}, s) \wedge \text{GasInTank}(\text{Car}, s) \Rightarrow \text{Poss}(\text{StartCar}(\text{Car}, s))$

Successor State Axiom:

$$\text{Poss}(\text{StartCar}(\text{Car},s)) \Rightarrow (\text{Running}(\text{Car},\text{Result}(\text{StartCar}(\text{Car},s),s)) \Leftrightarrow \text{Running}(\text{Car},s))$$

Unique Action Axiom

$$\text{StartCar}(\dots) \neq \text{AnyOtherAction}(\dots):$$

$$\text{StartCar}(x\dots x_n) = \text{StartCar}(y\dots y_n) \Leftrightarrow x=y, \dots, x_n=y_n$$

Solution:

$$\text{Running}(\text{Car},s) \wedge \text{At}(\text{Person},\text{Car},s) \wedge \text{HasKey}(\text{Person},\text{Car},s)$$

c) Frame Axiom:

$$\text{Poss}(\text{StartCar}(\text{Car1},s)) \Rightarrow \forall \text{Car2} =$$

$$\text{Car1}(\text{GasInTank}(\text{Car2},\text{Result}(\text{StartCar}(\text{Car1},s),s)) \Leftrightarrow \text{GasInTank}(\text{Car2},s))$$