Continuous Hopfield

$$E = \frac{-1}{2}\sum_i\sum_j W_{ij} y_i y_j - \sum_i b_i y_i + \lambda_x \sum_j \int_0^{y_j} \sigma^{-1}(u)\,du + \frac{\lambda_w}{2}\|W\|_F^2 + \frac{\lambda_b}{2}\|b\|_2^2$$

a) $$\frac{\partial E}{\partial y_i} = -\sum_j W_{ij} y_j - b_i + \lambda_x \underbrace{\frac{\partial}{\partial y_i}\sum_j \int_0^{y_j} \sigma^{-1}(u)\,du}_{} + 0 + 0$$

$$= \frac{\partial}{\partial y_i}\int_0^{y_i} \sigma^{-1}(u)\,du$$

$= \sigma^{-1}(y_i)$ by the fundamental theorem of calculus.

$= x_i$ since $y_i = \sigma(x_i)$

$$\therefore \boxed{\frac{\partial E}{\partial y_i} = -\sum_j W_{ij} y_j - b_i + \lambda_x x_i}$$

b) $$\frac{\partial E}{\partial x_i} = \frac{\partial E}{\partial y_i}\frac{dy_i}{dx_i}$$

But $y_i = \tanh(x_i)$

and $\frac{dy_i}{dx_i} = 1 - \tanh^2(x_i) = 1 - y_i^2$

$$\boxed{\therefore \frac{\partial E}{\partial x_i} = \left[-\sum_j W_{ij} y_j - b_i + \lambda_x x_i\right](1 - y_i^2)}$$

c) $$\frac{\partial E}{\partial W_{ij}} = -\frac{1}{2}\sum_k\sum_l \frac{\partial}{\partial W_{ij}} W_{kl} y_k y_l + 0 + 0 + \frac{\lambda_w}{2}\frac{\partial}{\partial W_{ij}}\|W\|_F^2 + 0$$

$$= -\frac{1}{2} y_i y_j + \frac{\lambda_w}{2} 2 W_{ij}$$

$$\boxed{\therefore \frac{\partial E}{\partial W_{ij}} = -\frac{1}{2} y_i y_j + \lambda_w W_{ij}}$$

$$\boxed{\therefore \frac{\partial E}{\partial w_{ij}} = -\frac{1}{2} y_i y_j + \lambda_w W_{ij}}$$

d) $$\frac{\partial E}{\partial b_i} = 0 - \frac{\partial}{\partial b_i} \sum_j b_j y_j + 0 + 0 + \frac{\lambda_b}{2} \frac{\partial}{\partial b_i} \|b\|_2^2$$

$$= -y_i + \frac{\lambda_b}{2} 2 b_i$$

$$\boxed{\therefore \frac{\partial E}{\partial b_i} = -y_i + \lambda_b b_i}$$