Model-Based Fuzzy Control of an Auto Swing-up Furuta Inverted Pendulum: A Mechatronic Approach

A.H.D Markazi[1], M. Abbasi[2], B. Hoseinkhani[3], M. Hashemi[4], O.Zarei[5]
Digital Control Laboratory, Mechanical Engineering Department,
Iran University of Science and Technology (IUST), Tehran, Iran
Corresponding Author E-mail: markazi@iust.ac.ir

Abstract
In this paper, a report on the IUST Furuta Inverted Pendulum, designed and built at the Mechanical Engineering Department of the Iran University of Science and Technology (IUST), is provided. This work is carried out by a group of students, as part of the *Mechatronics graduate course*. A new fuzzy controller based on the idea of energy control is proposed, to swing up the inverted pendulum to the upright position. Furthermore, a model-based *parallel distributed compensation* (PDC) scheme is employed to stabilize the pendulum at its unstable equilibrium point, while the position of the arm is also controlled. Experimental results show the outstanding stability, performance and robustness of the controlled system.

Keywords*:* Furuta Inverted Pendulum, Fuzzy Control, LMI, PDC, Swing up

1 Introduction
The rotary inverted pendulum known as *Furuta inverted Pendulum* (FIP) is made up of a pendulum rod which has a pivot on an arm which can be rotated horizontally. The control objective in this system is to bring the pendulum to the upper, naturally unstable, position by the controlled rotation of the arm in the horizontal plane. Despite the simplicity of the structure, some specific characteristics of an inverted pendulum such as strong nonlinearity, inherent instability and under-actuation, have motivated many researchers to adopt it as a benchmark to verify the performance and effectiveness of their proposed control strategies.

From an educational point of view, designing, building and controlling of an FIP can be regarded as a comprehensive project for practicing the elements of the, so called, *Mechatronic Approach*, which is the art of developing multi-engineering intelligent systems. The *integration* and *synergistic* combination of mechanical, electrical, computer, and control engineering, through a well-managed team work, can be practiced by students working on a FIP project. Furthermore, by considering the time, cost, and expertise limitations, team members become familiar with the challenges existing in such multi-disciplinary projects.

[1] Associate Professor
[2-5] M.Sc. Students

Various recent works are reported on the swinging up and stabilization of FIP. Astrom and Furuta [1] investigated some properties of the simple strategies for swinging up based on an energy control approach, in which the pendulum was controlled such that the total energy of the pendulum becomes equal to the potential energy at the upright position. Awtar et al. [2] experimentally showed the effectiveness of the energy-based control for swing up of an inverted pendulum, built as part of a *Mechatronic course* at the *Rensselaer* Polytechnic Institute, USA.

The history of the *parallel distributed compensation* (PDC) method begins with the work of Sugeno and Kang [3]; the name PDC, however, was first used by Wang et al. [4]. Wang et al. [4] verified the effectiveness of PDC by stabilization of a moving cart inverted pendulum, where the objective of the control was to stabilize the pendulum, without any specific control on the position of the cart.

In this paper, a report on the IUST FIP, built at the Mechanical Engineering Department of the Iran University of Science and Technology (IUST), is provided. The main contributions of this paper, however, are given in sections 4 and 5. In Section 4, a new simple and effective fuzzy controller, based on the idea of energy control, is proposed to make a smooth swing up of the inverted pendulum possible. Furthermore, in Section 5, a model-based PDC scheme is employed to robustly stabilize the pendulum on its unstable upright position. Unlike the work reported in Wang et al. [4], the position of the arm is also accurately controlled.

2 A Team Work Experience

The inverted pendulum was built with close collaboration among the team members with a background from mechanical engineering discipline, with some additional knowledge from other required disciplines. The design and implementation tasks, namely, simulation, control, actuation, measurement, real-time implementation, and mechanical manufacturing, were distributed among six members of the team. Figure1 illustrates the functional interconnections among the team members.

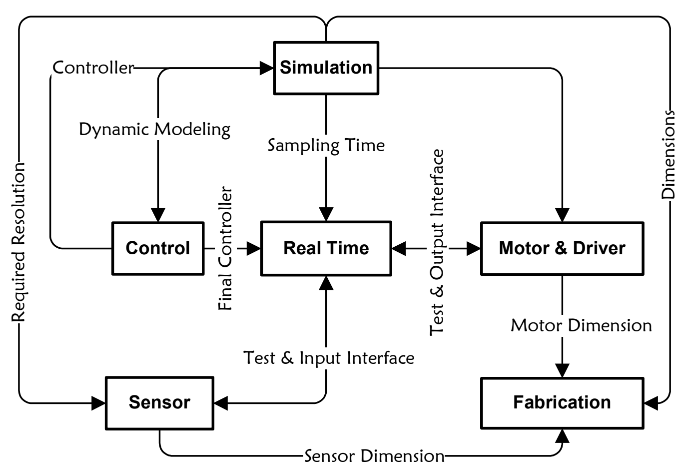

Figure 1: Functional interconnections among team members.

One aim of executing this project was to improve the skills of the team members in modeling/analysis and software/hardware implementation of complex multi-engineering systems. Another important goal of the project was to overcome the possible existing cultural barriers against fruitful and synergistic collaboration among the team members.

3 Physical Modelling

The IUST furuta pendulum is demonsrtated in Figure 2. The arm is driven by a DC motor in the horizontal plane, and the pendulum hinged to this arm, starting from down position, must be forced to swing up and finally be stabilized at the upright position, under the effects of the movements of the arm. Angular positions of the links are measured by two incremental encoders with the resolution of 1024 pulses per revolution.

Figure 2: IUST PC-controlled inverted Pendulum

The control strategy was implemented in the MATLAB/SIMULINK/RTW real-time environment. A data acquisition board from Advantech was used to acquire the position data and send them to the PC through the digital input ports, and send the control outputs to the DC motor through the D/A port.

In order to derive the mathematical representation of the inverted pendulum, some simplifying assumptions are made; in particular, all frictions, gearbox backlash, dynamics of encoders and DC motor driver are neglected. Table 1 shows the abbreviations used in the governing equations of the inverted pendulum and their numerical value in the IUST FIP.

Lagrange method was used to derive the equations of motion. The generalized coordinates for the system are the angular displacements of the pendulum ($q_1 = \theta$) and the arm ($q_2 = \varphi$), respectively. Lagrange equations are given by

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i \tag{1}$$

The potential energy, U, and kinetic energy, T, are as

$$U = \frac{1}{2} m_p g l \cos\theta \tag{2}$$

$$T=\frac{1}{2}(MR'^2+J_A+J_c+J_H+K_g{}^2J_r)\dot{\varphi}^2+\frac{1}{2}((R^2+\frac{1}{3}l_p{}^2\sin^2\theta)\dot{\varphi}^2+Rl_p\dot{\theta}\dot{\varphi}\cos\theta+\frac{1}{3}l_p{}^2\dot{\theta}^2) \quad (3)$$

Table 1: Abreviations used in the equations and their value in the built system.

Notation	Description	Value
M	Encoder mass	0.24 kg
m_c	Connector mass	0.012 kg
m_p	Pendulum mass	0.045 kg
m_A	Arm mass	0.095 kg
l_p	Pendulum Length	0.55 m
l_A	Arm Length	0.4 m
R	Distance of m_p from rotation axis	0.155 m
R'	Distance of *M* from rotation axis	0.325 m
J_p	Pendulum mass moment of inertia	0.0045 kg.m^2
J_A	Arm mass moment of inertia	0.0022 kg.m^2
J_c	Connector mass moment of inertia	0.0012 kg.m^2
J_H	Housing mass moment of inertia	0.00007 kg.m^2
J_r	Motor rotor mass moment of inertia	0.00018 kg.m^2
T	Motor Torque	-

Equations of motion are derived in the following form:

$$\alpha\cos\theta\ddot{\phi}+\beta\ddot{\theta}-\beta\sin\theta\cos\theta\dot{\phi}^2+(\frac{1}{2}m_p g l_p)\sin\theta=0 \quad (4)$$

$$(\beta\sin^2\theta+\lambda)\ddot{\phi}+\alpha\cos\theta\ddot{\theta}+2\beta\sin\theta\cos\theta\dot{\theta}\dot{\phi}-\alpha\sin\theta\dot{\theta}^2=T \quad (5)$$

Define;

$$\alpha=mRl_p+\frac{1}{2}m_pRl_p \quad (6)$$

$$\beta=ml_p{}^2+J_p \quad (7)$$

$$\lambda=(m+m_p)R^2+MR'^2+J_A \quad (8)$$

4 Swing Up Algorithm

The swing up of the inverted pendulum is carried out using a fuzzy controller. The following rules are used to gradually increase the energy of the system:

1. If θ is positive (negative) small and $\dot{\theta}$ is positive (negative) then the system input is positive (negative)

2. If θ is positive (negative) small and $\dot{\theta}$ is negative (positive) then the system input is zero
3. If θ is positive (negative) medium then the system input is zero
4. If θ is positive (negative) big and $\dot{\theta}$ is positive (negative) small then the system input is positive (negative)
5. If θ is positive (negative) big and $\dot{\theta}$ is negative (positive) small then the system input is zero
6. If θ is positive (negative) big and $\dot{\theta}$ is positive (negative) big then the system input is zero

The membership functions of linguistic terms used in above rules are presented in Figure 3.

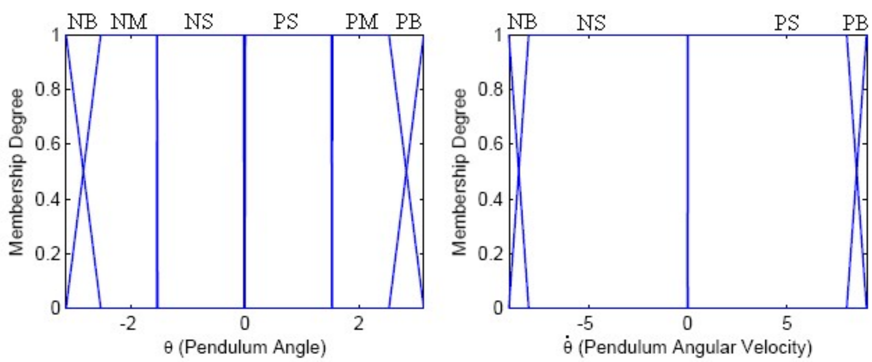

Figure 3: Membership function of linguistic terms used in the swing up controller.

5 Stabilizing

When the pendulum is within $\pm 30^0$ of vertical position the controller switches from swing up controller to balancing controller. Since the analytical model of the system is available, the fuzzy model-based controller, rather than a more conventional fuzzy knowledge-based controller, is employed for stabilizing the pendulum in the upright position. The advantage is that the membership functions of the fuzzy terms are derived directly from the model of the system with a systematic approach, while in the knowledge-based fuzzy method, membership functions need to be determined with trail and error procedure; a matter which is time consuming and difficult to apply for unstable plants like an inverted pendulum. As a first step for designing the model-based fuzzy scheme, it is necessary to obtain a fuzzy model for the plant itself.

5.1 Takagi Sugeno (TS) Fuzzy model of plant

In the TS fuzzy modeling [5], each of the rules representing the local input-output relationships of a nonlinear system is in the form of;

If $z_1(t)$ is M_{i1} and … and $z_p(t)$ is M_{ip} , then $\dot{x}(t) = A_i x(t) + B_i u(t)$

where M_{ij} is a fuzzy set; $z_1(t)$, …, $z_p(t)$ are known premise variables which are usually functions of the state variables, and A_i and B_i are known constant system and input matrices, respectively. The system dynamics is then described as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \tag{9}$$

where r is the number of rules; z(t)=[$z_1(t)$ $z_2(t)$ … $z_p(t)$]; and

$$h_i(z(t)) = \frac{\prod_{j=1}^{p} M_{ij}(z_j(t))}{\sum_{k=1}^{r} \prod_{j=1}^{p} M_{ij}(z_j(t))} \tag{10}$$

while

$$\sum_{i=1}^{r} h_i(z(t)) = 1, \quad h_i(z(t)) \in [0,1] \quad \text{for } i = 1, 2, \ldots, r. \tag{11}$$

To express inverted pendulum nonlinear equations in the form of a TS fuzzy model, three premise variables, which are functions of θ, are to be defined. These premise variables and their corresponding membership functions are analytically found and are shown in Figure 4.

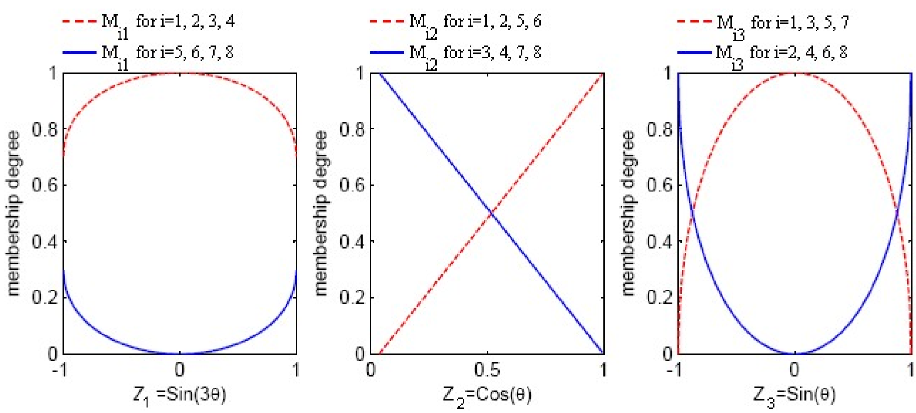

Figure 4: Membership function of the premise variables

5.2 Model Verification

In order to verify the fuzzy model, it is compared with the model obtained by the Lagrange approach, and a similar model obtained using the object-oriented Modelica Language simulated using the Dymola compiler and simulator. The structure of the latter model is shown in Figure 5.

The simulation is performed by releasing the pendulum from the initial position of $\theta = 30^0$ from down position and zero initial velocity. The solution trajectories of θ and $\dot{\theta}$ obtained by mathematical model and that of the Modelica model, shown in Figure 6, are virtually the same, and also close enough to the fuzzy model. As can be seen in the same figure, the agreement between the predicted response and the experimental observation, is quit acceptable. Experiments performed on the real IUST FIP, also proves the whole process of modeling to be accurate enough for control design purpose.

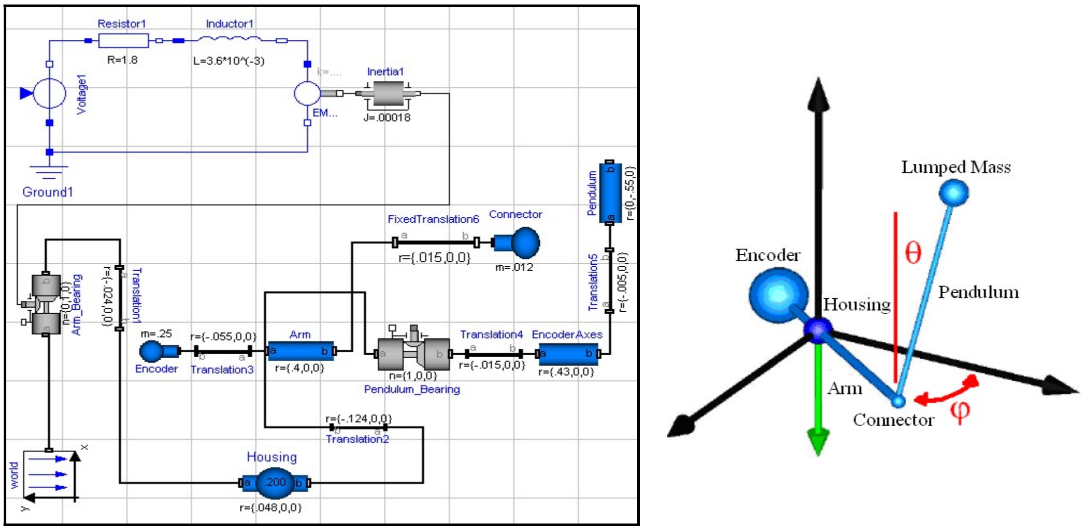

Figure 5: The object based model in Dymola software.

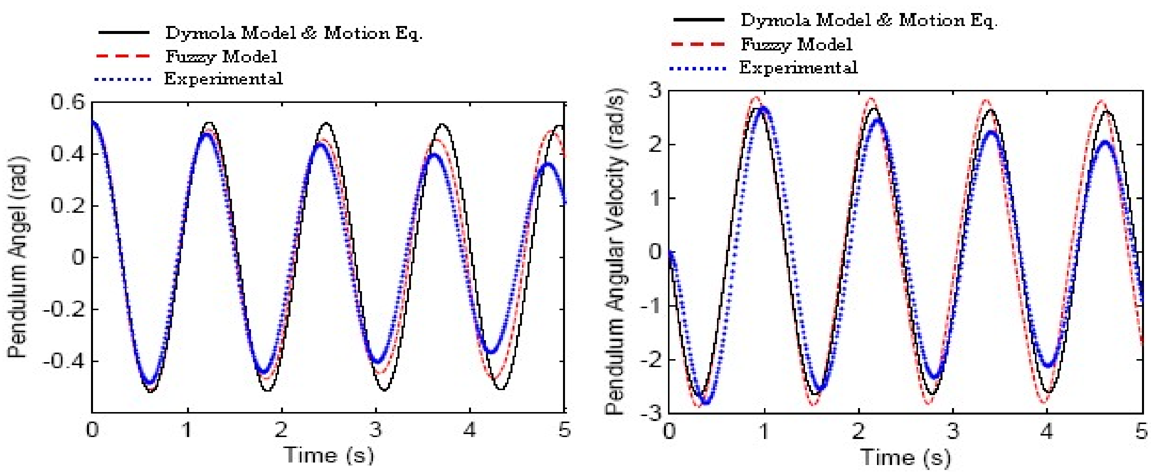

Figure 6: Verification curves.

5.3 PDC Algorithm

In the PDC design, each rule is designed from the corresponding rule of the TS fuzzy model. The designed fuzzy controller shares the same fuzzy sets as the fuzzy model in the premise parts [5]. Each rule of the fuzzy controller can be expressed as;

If $z_1(t)$ is M_{i1} and … and $z_p(t)$ is M_{ip} Then $u(t) = -F_i x(t)$

The overall fuzzy controller is represented by

$$u(t) = -\sum_{i=1}^{r} h_i(z(t)) F_i x(t), \tag{12}$$

where $F_i; i = 1, \ldots, r$, are linear state feedback gains for the subsystems at different regions of θ. These gains are obtained using the LQR approach.

5.4 Fuzzy Observer

In order to implement the fuzzy controller, all states of the system should be available. However, only tow states of the IUST inverted pendulum i.e. arm and pendulum angles are available through shaft encoders. In order to estimate other states (angular velocities and motor current), a fuzzy observer is designed. Similar to the controller design methodology, the PDC concept is employed to design the observer [5]. The observer rules are in the following form:

If $z_1(t)$ is M_{i1} and … and $z_p(t)$ is M_{ip} Then $\dot{\hat{x}}(t) = A_i\hat{x}(t) + B_i u(t) + K_i(y(t) - \hat{y}(t))$

When the premise variables do not depend on the state variables estimated by a fuzzy observer, the fuzzy observer is represented as follows:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i\hat{x}(t) + B_i u(t) + K_i(y(t) - \hat{y}(t))) \tag{13}$$

and it can be proved that in this case the controller gains and the observer gains can be determined separately [5]. This result is similar to the well-known separation principle for linear observed states feedback.

5.5 Stability Analysis

In the previous section, for every subsystem, a stabilizing controller and suitable observers were designed. The following theorem provides sufficient conditions for stability of the overall system, when local controllers/observers are blended in a fuzzy sense.

Theorem [5]: The equilibrium of the augmented system containing both fuzzy controller and observer is globally asymptotically stable if there exists a common positive definite matrix *P* such that the following set of *Linear Matrix Inequalities* (LMI) is satisfied:

$$G_{ii}^T P + P G_{ii} < 0 \tag{14}$$

$$\left(\frac{G_{ij} + G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij} + G_{ji}}{2}\right) < 0 \quad i < j \text{ s.t. } h_i(z(t)) \times h_j(z(t)) \neq 0 \text{ for all } z(t) \tag{15}$$

where,

$$G_{ij} = \begin{bmatrix} A_i - B_i F_j & B_i F_j \\ 0 & A_i - K_i C_j \end{bmatrix} \tag{16}$$

The common stabilizing *P* matrix is determined successfully using the MATLAB LMI toolbox; the details are omitted here. This ensures the global stability of the controlled system.

5.6 Comparative study between PDC and LQR techniques

The response of the pendulum angle using PDC and LQR controllers for different initial conditions of θ and zero initial velocity, is shown in Figure 7. The solid lines show the response of the system with PDC while the dotted lines show the reaponse of the system with LQR controler The simulation indicates that the LQR linear controller fail to balance the pendulum for initial angles 30 degree, while the PDC fuzzy controller can balance the pendulum for initial conditions $\theta \in \left[-45^0 \quad +45^0\right]$. Furthermore the performance of the PDC controller is better than LQR one, a matter that is clear in Figure 7.

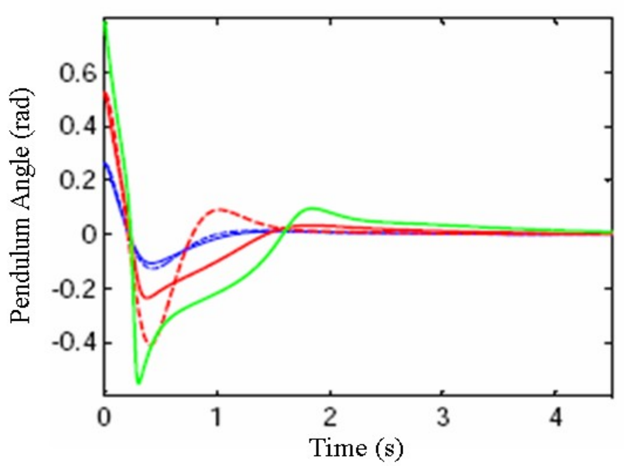

Figure 7: Comparison between PDC and LQR.

6 Results and discussion

Figures 8, Figure 9 and Figure 10 demonstrate the closed-loop response of the simulated system versus the experimental results. Successful results during the swing up and balance phases are clearly observable.

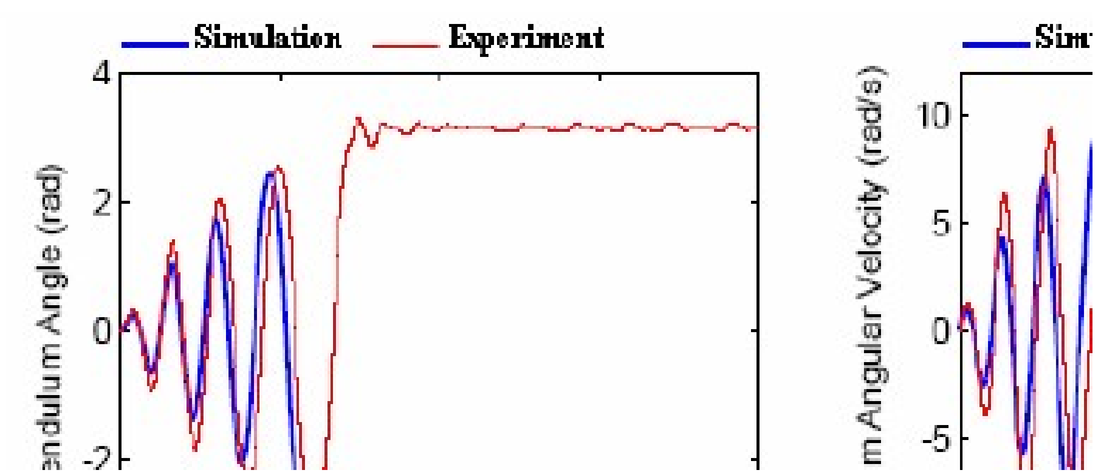

Figure 8: Pendulum angle and velocity vs. time.

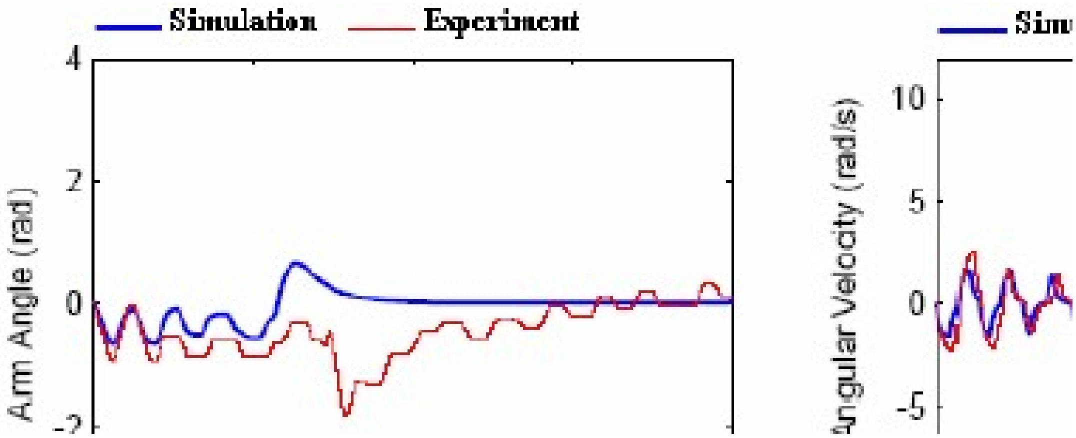

Figure 9: Arm angle and velocity vs. time.

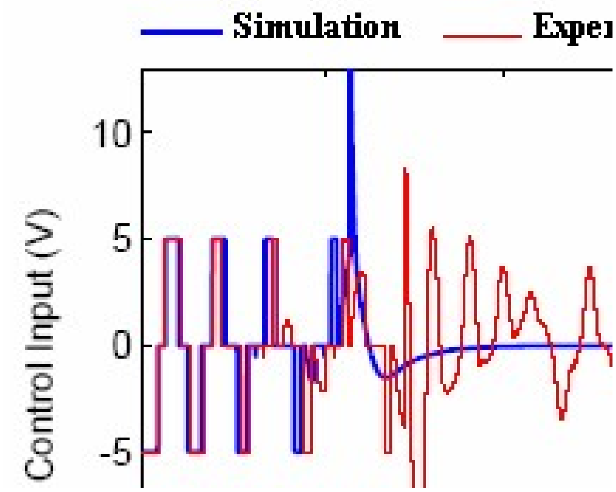

Figure 10: Control input vs. time.

However, the difference between the responses, in particular at the latest part of the response, is due to manufacturing imperfections and simplifying assumption like neglecting the frictions and backlash of the gearbox, and to the less extent to the flexibility of links and joints which was not taken into account during the modeling phase.

Figure 11 shows the resistance of the controlled system against disturbance, known as disturbance rejection capability of the controller. The disturbance is applied to the system in the form of impulse input. This figure, besides Figure 8, Figure 9 and Figure 10, proves the outstanding stability, performance and robustness of the controlled system.

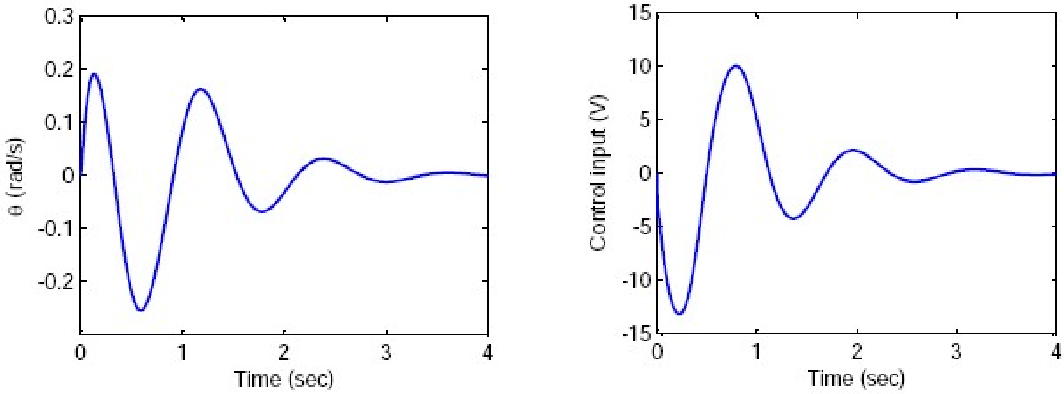

Figure 11: Disturbance rejection curves.

7 Conclusion

An indispensable part of the mechatronics art, is the modeling, simulation, control and manufacturing of multi-disciplinary physical systems, through a team work. The interaction among these actions is crucial. Simulation saves time, reduces costs, and prevents fatal mistakes. In return, the implementation can be used to verify the accuracy of the simulated system and to cope with the existing real world constraints and limitations. In this paper, the mechatronics approach was employed to manufacture a sophisticated PC-controlled Furuta Inverted Pendulum. The swing up phase of the control was performed using an energy based fuzzy technique and the stabilization phase was carried out using the model-based PDC Fuzzy Technique. Experiments on the IUST FIP proved the success of the design.

References

[1] K.J. Astrom and K. Furuta, “Swinging up a pendulum by energy control”, Automatica, Vol. 36 (2000), pp. 287-295.

[2] S. Awtar, N. King, T. Allen, I. Bang, M. Hagan, D. Skidmore and K.Criage, “Inverted Pendulum systems: rotary and arm driven- a mechatronic system design case study”, Mechatronics, Vol. 12 (2002), pp. 357-370.

[3] M. Sugeno and G.T. Kang, “Fuzzy Modeling and Control of Multilayer Incinerator”, Fuzzy Sets Syst., No.18 (1986), pp. 329-346.

[4] H. O. Wang, K. Tanaka, and M.F. Griffin, “Parallel Distributed Compensation of Nonlinear Systems by Takagi-Sugeno Fuzzy Model ”, Proc. FUZZ IEEE/IFES’95 (1995), pp. 531-538,

[5] K. Tanaka, H.O. Wang, “Fuzzy Control Systems Design and analysis: A linear Matrix Inequality Approach.”, Wiley (2001)