Application of Fourier Transform in Hydrology

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Introduction

- *Fourier Transform* - a mathematical technique that decomposes a function into its constituent frequencies, allowing for the analysis of signals in the frequency domain (Sneddon 1951).
- *Fast Fourier Transform* - an algorithm computes the Discrete Fourier Transform (DFT) of a sequence.
- FT is useful in signal processing, but not enough literature explores its application in hydrology.

Objective

This exercise aims to apply FFT to the long-term hydrologic flow time series to see if any underlying signals depict long-term trends or any variation on certain timescales (decadal, yearly, seasonal, monthly, etc.) which can provide meaningful insights from the streamflow data.

Methods

The FFT package in R is used on the 74 years of daily streamflow data from a USGS gauge station on Chattahoochee River, GA.

Process approach is;

- Analyze the behavior of FFT on streamflow time series.
- Extract any underlying seasonal signal.

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-i2\pi kn/N}$$

'discreet fourier transform function'

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{i2\pi kn/N}$$

'inverse discreet fourier transform function'

Results & Discussion

Hydrological Insights

- *Dominant low frequencies;* FFT output shows a clear dominance of low-frequency components, representing long-term trends and seasonal fluctuations inherent in the hydrological data. (Fig: 1)
- *High correlation in signal reconstruction;* Reconstructed signal using an inverse FFT demonstrated a perfect correlation of '1' compared to the original time series. (Fig: 2)

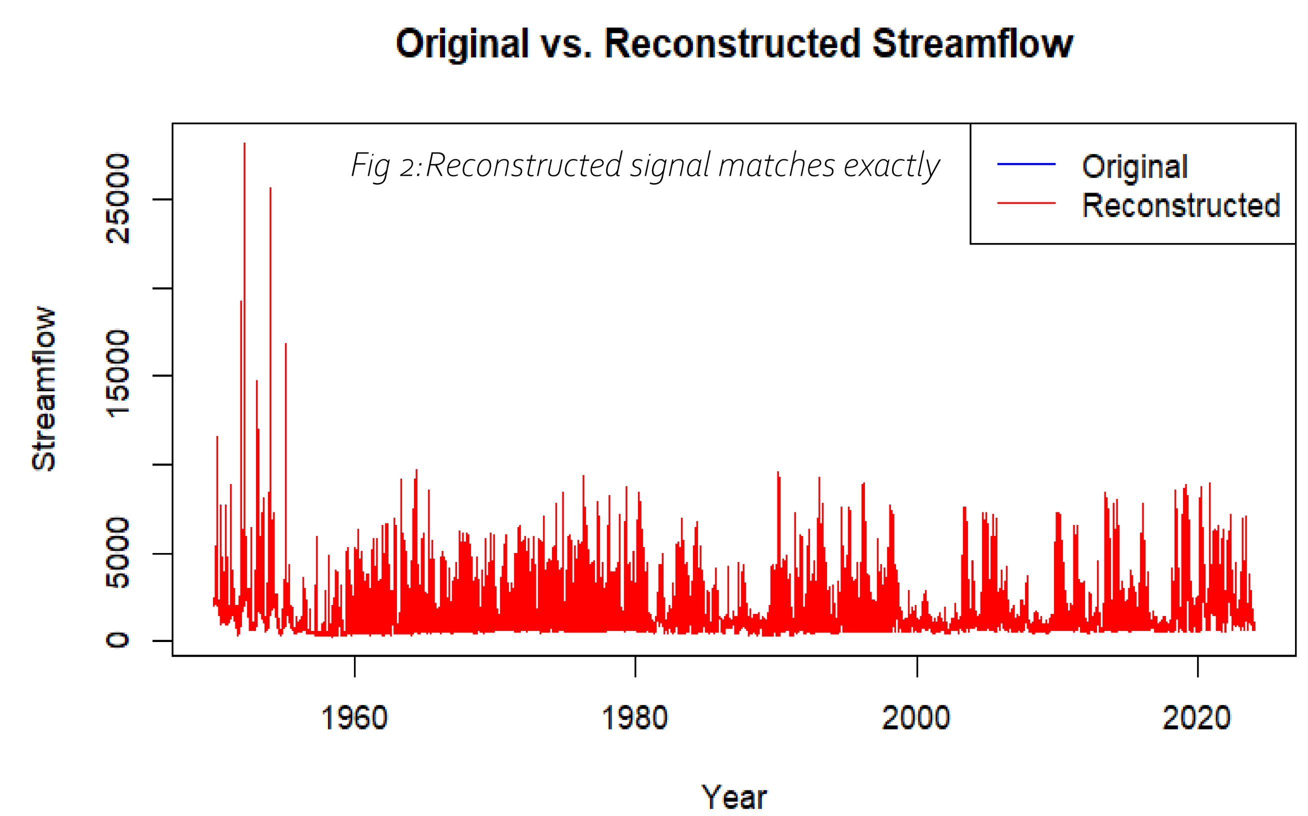

Fig 2:Reconstructed signal matches exactly

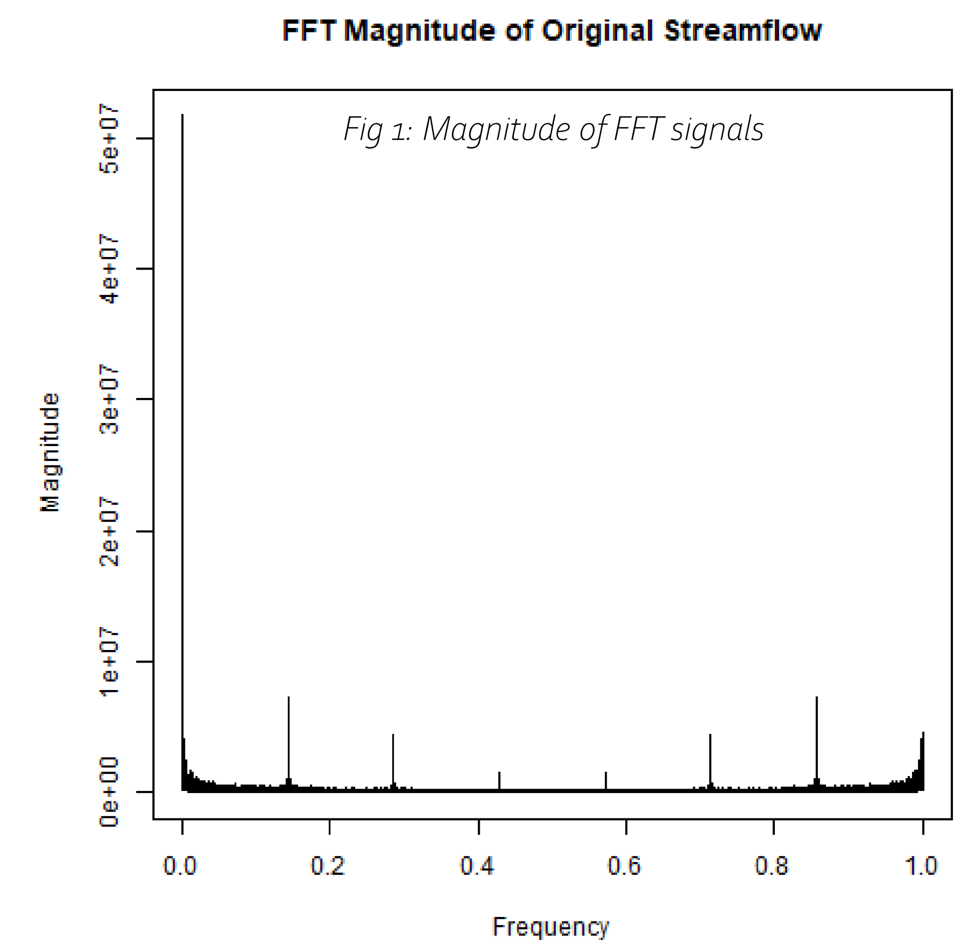

Fig 1: Magnitude of FFT signals

Extraction of Seasonal Trend:

Optimized denoising & Low flow season; Analysis of the effects of variable window size concluded 15–day window to be optimum (Fig: 3), also reconstructed low flow signal (Fig: 4).

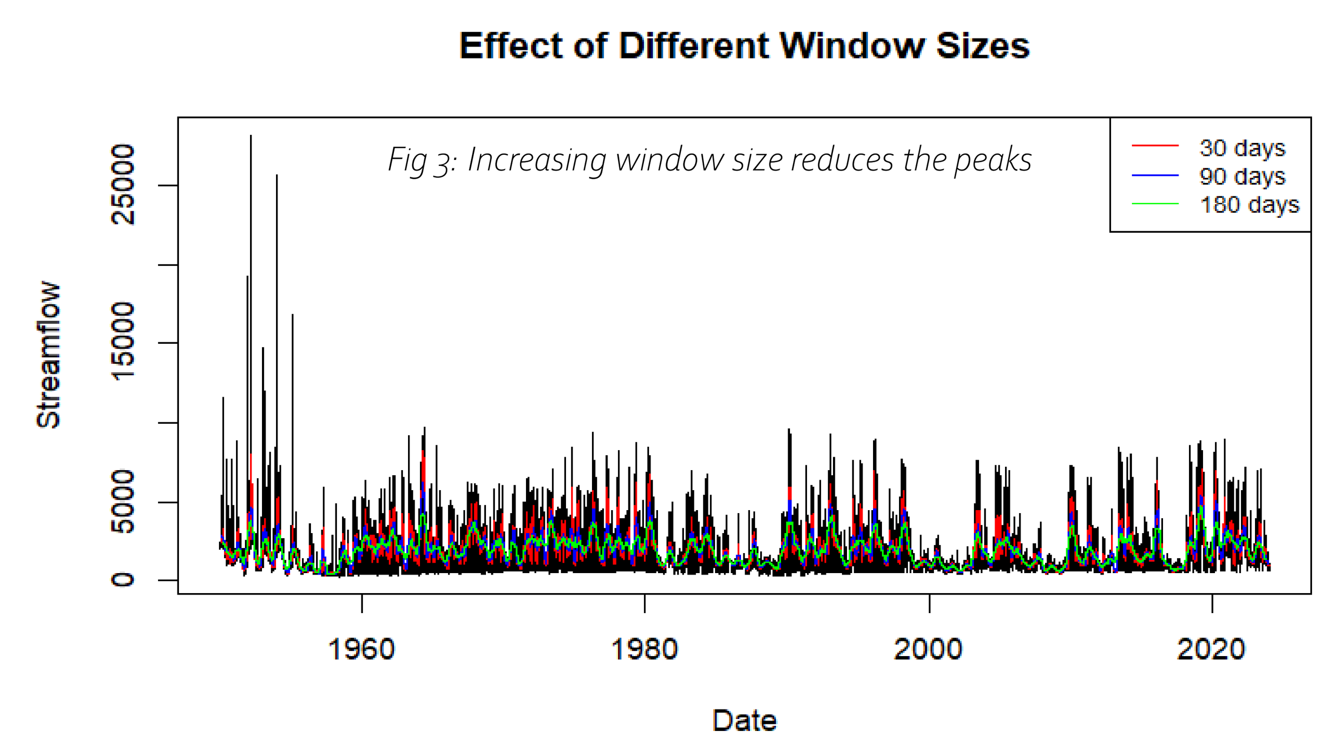

Fig 3: Increasing window size reduces the peaks

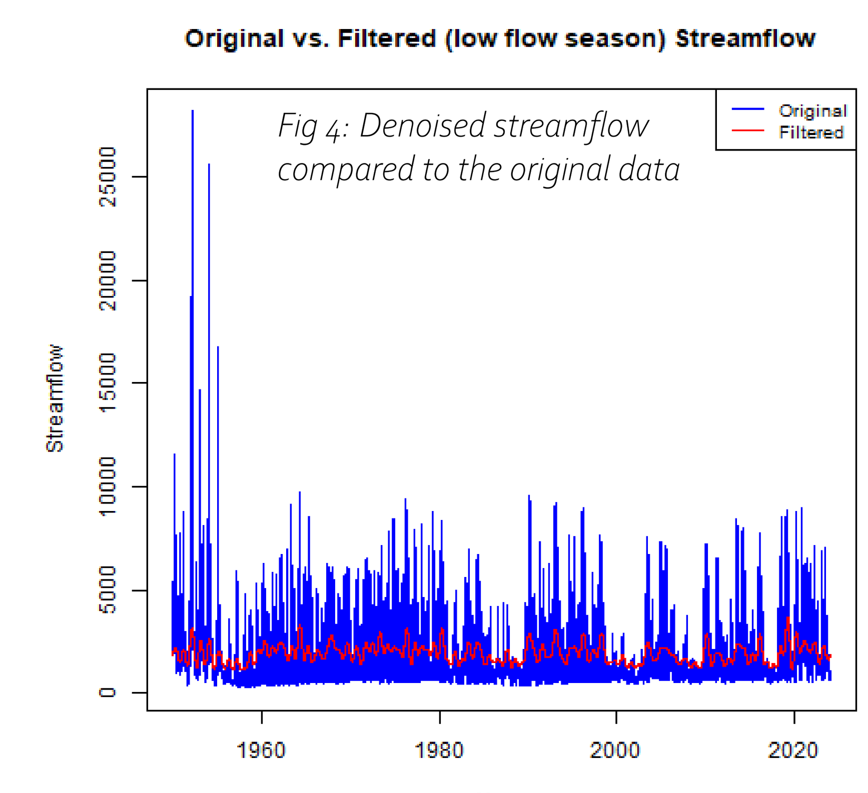

Fig 4: Denoised streamflow compared to the original data

Autocorrelation Function Analysis;

- The ACF of the filtered data exhibited a slower decline in autocorrelation values at higher lags, meaning the filtering process retained the core, slower-changing dynamics of the streamflow. (Fig: 4 and 5.)

Fig 5: Persistence in the low flows against time.

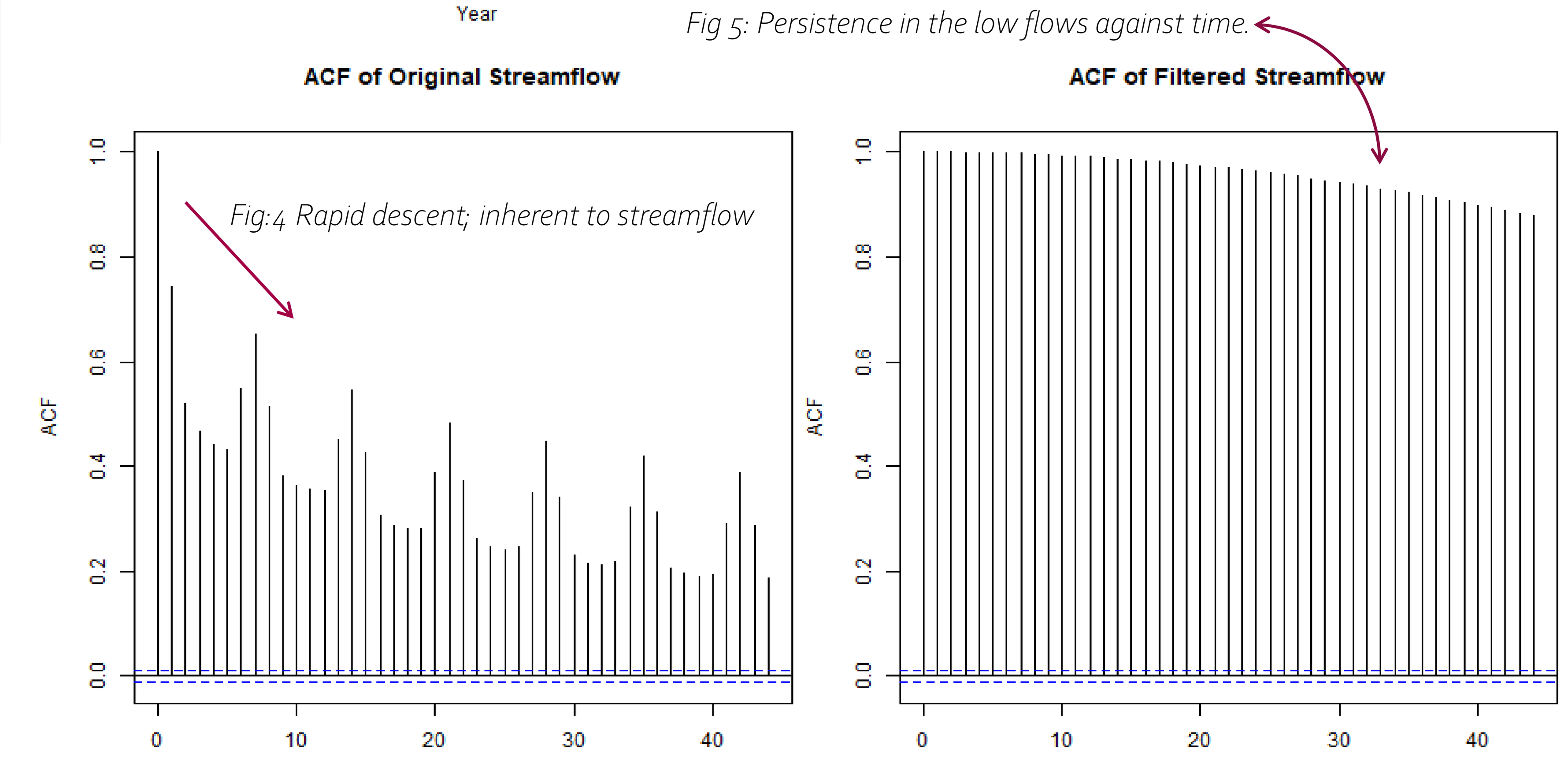

Fig:4 Rapid descent; inherent to streamflow

Conclusions

- FFT effectively identifies and isolates the major frequencies that characterize the hydrological data.
- Major seasonal and annual trends within the dataset were preserved during the transformations from time to frequency domain and back.

Implications

FFT can be useful in forecasting streamflow and water availability, enhancing the accuracy of flood forecasting models, and drought management strategies by isolating periods of interest

References

- Bracewell, R. N. (2000). The Fourier Transform & Its Applications. McGraw-Hill, 3rd edition.
- Korner, T. W. (1988). Fourier Analysis. Cambridge University Press.
- Sneddon, I.N. (1951). Fourier Transforms. McGraw-Hill.

Learn more about the lab;

Access the project source code here;