Answer to Question 1: Modeling the Temporary Impact Function g_t(x)

Introduction

The Temporary Impact Function, $g_t(x)$, quantifies the slippage incurred when executing an order of size x at time t. Unlike permanent impact, temporary impact is a transient price deviation. The common simplification to a linear model, $g_t(x) \approx \beta_t * x$, is an oversimplification that fails to capture market complexities.

Financial markets are non-linear. Large orders consume deeper order book levels, leading to disproportionate slippage. Market impact from large trades can alter order book dynamics, and liquidity is not constant. Linear models also fail to account for sophisticated trader behaviors like order splitting. Therefore, a more realistic, non-linear approach is necessary.

Proposed Model: Power Law with Dynamic Factors

We propose a non-linear power law model for $g_t(x)$ that incorporates dynamic market conditions:

$g_t(x) = \alpha_t * x^k * (1 + \beta * Volatility_t) * Liquidity\_Factor_t$

Where:

- **$g_t(x)$:** Temporary impact (slippage) for order size x at time t.
- **x:** Order size (number of shares).
- **α_t (Volume Coefficient):** Sensitivity of slippage to order size at time t, reflecting general market conditions or asset liquidity.
- **k (Power Law Exponent):** A non-linear exponent (typically 0.5 to 1.0) capturing the non-linear relationship between order size and slippage. A value closer to 0.5 suggests slippage increases at a slower rate than order size.

- **β (Volatility Coefficient):** Determines how volatility affects slippage; higher volatility leads to higher expected slippage.
- **Volatility_t:** Market volatility at time t (e.g., standard deviation of price returns).
- **Liquidity_Factor_t:** Measure of order book liquidity depth at time t (e.g., inverse of spread, total volume in best bid/ask levels).

This model is superior because it: 1. **Captures Non-linearity:** The exponent `k` accurately reflects the non-linear relationship. 2. **Integrates Market Dynamics:** `Volatility_t` and `Liquidity_Factor_t` adapt to changing market conditions. 3. **Offers Interpretability:** Each parameter has a clear economic meaning. 4. **Provides Flexibility:** Can be extended with additional market factors.

Model Estimation Using Provided Ticker Data

To estimate the model parameters (α_t, k, β) for CRWV, FROG, and SOUN, we need historical data on order size, slippage, volatility, and liquidity. Slippage is calculated as the difference between the volume-weighted average execution price and the mid-price before the trade. Volatility can be derived from price returns, and liquidity from order book depth metrics.

Estimation Steps:

1. **Data Processing:** Clean and synchronize historical data. Calculate slippage, volatility, and liquidity factors from the provided order book data.
2. **Parameter Estimation:** Use non-linear regression techniques (e.g., `scipy.optimize.curve_fit`) to estimate α_t, k, and β. For instance, a logarithmic transformation can linearize the power law component: `log(g_t(x)) = log(α_t) + k * log(x)`, allowing for linear regression to estimate `k` and `log(α_t)`.
3. **Model Validation:** Evaluate performance using metrics like R-squared, MAE, and RMSE, and analyze residuals.

Even with limited data from three tickers, this methodology demonstrates a robust approach. Comparing parameters across tickers can reveal unique asset characteristics (e.g., higher `k` for an asset sensitive to order size, higher `α_t` for less liquid assets). Observing changes in `α_t`, `β`, and `Liquidity_Factor_t` over time provides insights into market dynamics.

Conclusion

Modeling temporary impact requires moving beyond simplistic linear models. The proposed power law model, incorporating dynamic market factors, offers a more realistic and adaptable framework. By carefully processing data and applying non-linear regression, we can build a model that captures market complexities and provides valuable insights for trading strategies, even with limited data.