

STAT 662 - Homework . 2

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Xiangian Yao

1253177

problem A

problem C

problem B

output from SAS code

problem A.

Data:

	shower Head					
	1	2	3	4	5	6
80	75	74	67	62	60	
83	75	73	72	62	61	
83	79	76	74	67	64	
85	79	77	74	69	66	
Sum	331	308	300	287	260	251
\bar{y}_{ij}	82.75	77	75	71.75	65	62.75
$\hat{\tau}_i$	10.375	4.625	2.625	-0.625	-7.375	-9.625

1. factor level $a=6$

Sample size of each treatment group: $n=4$ ✓

Total sample size: $N=24$

2. factor effects model:

describe each observation as an overall population mean (μ) of Radar Release,
plus a treatment effect (τ_i) of showerHead,
plus a random error (ϵ_{ij}).

$$Y_{ij} = \mu + \tau_i + \epsilon_{ij}, \quad \left\{ \begin{array}{l} i=1, 2, 3, 4, 5, 6 \\ j=1, 2, 3, 4 \end{array} \right. \quad \checkmark$$

3. overall sample mean to estimate population mean:

$$\mu = \bar{y}_{..} = 72.375$$

estimator of i th treatment effect:

$$\hat{\tau}_i = \bar{y}_{i\cdot} - \bar{y}_{..}$$



1	2	3	4	5	6	\leftarrow treatment levels
10.375	4.625	2.625	-0.625	-7.375	-9.625	$\leftarrow \hat{\tau}_i$
82.75	77	75	71.75	65	62.75	$\leftarrow \hat{\mu}_i = \mu + \hat{\tau}_i$

4. $\epsilon_{ij} \stackrel{iid}{\sim} N(0, \sigma^2)$

The random errors are mutually independent and normally distributed with constant variance. The treatment level does not impact the variance of the random error term.

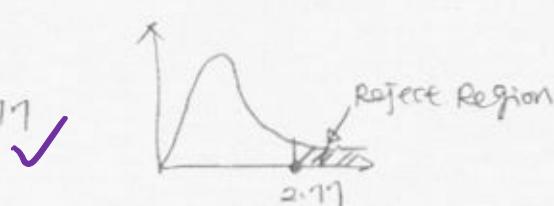
5.

a) $H_0: \tau_i = 0 \quad \forall i=1, 2, 3, 4, 5, 6$

$H_a: \tau_i \neq 0$ for at least one i

b) Critical Value: $F_{\alpha, a-1, N-a}$

$= F_{0.05, 5, 18} = 2.71$



c). $SS_{Treat} = 4 \times [10.375^2 + 4.625^2 + 2.625^2 + (-0.625)^2 + (-7.375)^2 + (-9.625)^2]$

$= 1133.375$

$MS_{Treat} = \frac{SS_{Treat}}{a-1} = \frac{1133.375}{5} = 226.675$

$$SS_E = (80-82.75)^2 + (83-82.75)^2 + (83-82.75)^2 + (85-82.75)^2 + \\ (75-77)^2 + (75-77)^2 + (79-77)^2 + (79-77)^2 + \\ (74-75)^2 + (73-75)^2 + (76-75)^2 + (77-75)^2 + \\ (69-71.75)^2 + (72-71.75)^2 + (74-71.75)^2 + (74-71.75)^2 + \\ (62-65)^2 + (62-65)^2 + (67-65)^2 + (69-65)^2 + \\ (60-62.75)^2 + (61-62.75)^2 + (64-62.75)^2 + (66-62.75)^2 = 132.25$$

$MS_E = \frac{SS_E}{N-a}$

$= \frac{132.25}{18} = 7.347$

$F_0 = \frac{MS_{Treat}}{MS_E} = \frac{226.675}{7.347} = 30.853$

Source	df	SS	MS	F	ANOVA Table
Treatment / Between	5	1133.375	226.675	30.853	
Error / Within	18	132.250	7.347		
Total	23				

d) Reject H_0 if $F_o > F_{\alpha, a-1, n-a} = F_{0.05, 5, 18} = 2.77$

$$F_o = 30.853 > F_{0.05, 5, 18}$$

Thus, Reject H_0 .

There's enough evidence to claim that the diameter of shower head has an effect on the mean value of Radon Release percentage at the significance level of 0.05.

6.

a) $P_i = \frac{\alpha}{6} C_i \bar{M}_i$

$$= \frac{1}{3} M_1 + \frac{1}{3} M_2 + \frac{1}{3} M_3 - \frac{1}{3} M_4 - \frac{1}{3} M_5 - \frac{1}{3} M_6 \quad \checkmark$$

b) Critical Value:

$$F_{\alpha, 1, N-a} = F_{0.05, 1, 18} = 4.41$$



c) $C_1 = \frac{1}{3} \times 82.75 + \frac{1}{3} \times 77 + \frac{1}{3} \times 75 - \frac{1}{3} \times 71.75 - \frac{1}{3} \times 65 - \frac{1}{3} \times 62.75$
 $= 11.75 \quad \checkmark$

d) Sum of squares for the contrast:-

$$SS_{C_1} = \frac{\left(\sum_{i=1}^a C_i \bar{M}_i \right)^2}{n \sum_{i=1}^a C_i^2} = \frac{11.75^2}{4 \times \left[\left(\frac{1}{3} \right)^2 \times 6 \right]} = 828.375 \quad \checkmark$$

e) $H_0: \sum_{i=1}^a C_i \bar{M}_i = 0$

$$H_a: \sum_{i=1}^a C_i \bar{M}_i \neq 0$$

$$\text{Test statistics: } F_o = \frac{SS_{C_1}/1}{MS_{\text{Z}}} = \frac{828.375}{7.347} = 112.750 \quad \checkmark$$

Reject H_0 if $F_o > F_{0.05, 1, 18}$.

$$F_o = 112.750 > 4.41,$$

Thus, Reject H_0 .

pp. mean There's enough evidence to claim that one third of mean values of random release percentage from shower head diameter of (1, 2, 3) is different from shower head diameter of (4, 5, 6).
one third of

$$f). H_0: \frac{1}{3}(M_1 + M_2 + M_3) = \frac{1}{3}(M_4 + M_5 + M_6)$$

$$\text{Population contrast: } l_1 = \frac{1}{3}M_1 + \frac{1}{3}M_2 + \frac{1}{3}M_3 - \frac{1}{3}M_4 - \frac{1}{3}M_5 - \frac{1}{3}M_6$$

$$\text{Sample contrast: } C_1 = 11.75$$

$$\begin{aligned}\text{Standard error: } S_{C_1} &= \sqrt{MS_E \sum_{i=1}^a \frac{c_{ri}^2}{n_i}} \\ &= \sqrt{7.347 \times \frac{1}{6} \times \left(\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right)} \\ &= \sqrt{7.347 \times \frac{1}{6} \times 6} \\ &= 1.107\end{aligned}$$

$$\begin{aligned}\text{Critical value: } S_{a,1} &= S_{C_1} \cdot \sqrt{(a-1) F_{\alpha, a-1, N-a}} \quad \checkmark \\ &= 1.107 \times \sqrt{5 \times F_{0.05, 5, 18}} \\ &= 1.107 \times \sqrt{5 \times 2.77} \\ &= 4.120\end{aligned}$$

Reject H_0 if $|C_1| \geq S_{a,1}$

$$|C_1| = 11.75 > 4.120$$

Thus, Reject H_0 . \checkmark

There's enough evidence to claim that one third of mean values of random release percentage from shower head diameter of (1, 2, 3) is different from one third of mean values of random release percentage from shower head diameter of (4, 5, 6).

9)

$$\text{i. } \Gamma_1 = \frac{1}{3}\mu_1 + \frac{1}{3}\mu_2 + \frac{1}{3}\mu_3 - \frac{1}{3}\mu_4 - \frac{1}{3}\mu_5 - \frac{1}{3}\mu_6$$

$$\Gamma_2 = \frac{1}{2}\mu_1 + \frac{1}{2}\mu_2 - \mu_3 - 0\cdot\mu_4 - 0\cdot\mu_5 - 0\cdot\mu_6 \quad \checkmark$$

$$\Gamma_3 = 0\cdot\mu_1 + 0\cdot\mu_2 + 0\cdot\mu_3 + 0\cdot\mu_4 + \mu_5 - \mu_6$$

ii.

$$\Gamma_4 = \mu_1 - \mu_2 + 0\cdot\mu_3 + 0\cdot\mu_4 + 0\cdot\mu_5 + 0\cdot\mu_6 \quad \checkmark$$

$$\Gamma_5 = 0\cdot\mu_1 + 0\cdot\mu_2 + 0\cdot\mu_3 - \frac{1}{2}\mu_4 + \frac{1}{2}\mu_5 + \frac{1}{2}\mu_6$$

i	C_{1i}	C_{2i}	C_{3i}	C_{4i}	C_{5i}
1	$\frac{1}{3}$	$\frac{1}{2}$	0	1	0
2	$\frac{1}{3}$	$\frac{1}{2}$	0	-1	0
3	$\frac{1}{3}$	-1	0	0	0
4	$-\frac{1}{3}$	0	0	0	-1
5	$-\frac{1}{3}$	0	1	0	$\frac{1}{2}$
6	$-\frac{1}{3}$	0	-1	0	$-\frac{1}{2}$

The two contrasts are orthogonal if $\sum_{i=1}^6 C_i d_i = 0$

$$\text{to verify: } SS_{C_1} = \frac{\frac{1}{3} \times 82.75 + \frac{1}{3} \times 77 + \frac{1}{3} \times 75 - \frac{1}{3} \times 71.75 - \frac{1}{3} \times 65 - \frac{1}{3} \times 62.75}{\frac{1}{4} \times (\frac{1}{3})^2 \times 6} \doteq 828.376$$

$$SS_{C_2} = \frac{\frac{1}{2} \times 82.75 + \frac{1}{2} \times 77 - 75 + 0}{\frac{1}{4} \times (\frac{1}{2} + \frac{1}{2} + 1)} \doteq 63.376$$

$$SS_{C_3} = \frac{1 \times 65 - 1 \times 62.75}{\frac{1}{4} \times 2} \doteq 10.124$$

$$SS_{C_4} = \frac{82.75 - 77}{\frac{1}{4} \times 2} \doteq 66.124$$

$$SS_{C_5} = \frac{-71.75 + \frac{1}{2} \times 65 + \frac{1}{2} \times 62.75}{\frac{1}{4} \times (1 + \frac{1}{2} + \frac{1}{2})} \doteq 166.004$$

$$SS_{C_1} + SS_{C_2} + SS_{C_3} + SS_{C_4} + SS_{C_5} \doteq 1134 \doteq \text{treatment sum of squares}$$

Thus, the five contrasts above are mutually orthogonal from ANOVA Table.

Graded on completion

7. multiple comparisons analysis

$$a) |\bar{y}_1 - \bar{y}_2| = |82.75 - 77| = |5.75|$$

$$|\bar{y}_1 - \bar{y}_3| = |82.75 - 75| = |7.75|$$

$$|\bar{y}_1 - \bar{y}_4| = |82.75 - 71.75| = |11|$$

$$|\bar{y}_1 - \bar{y}_5| = |82.75 - 65| = |17.75|$$

$$|\bar{y}_1 - \bar{y}_6| = |82.75 - 62.75| = |20|$$

$$|\bar{y}_2 - \bar{y}_3| = |77 - 75| = |2|$$

$$|\bar{y}_2 - \bar{y}_4| = |77 - 71.75| = |5.25|$$

$$|\bar{y}_2 - \bar{y}_5| = |77 - 65| = |12|$$

$$|\bar{y}_2 - \bar{y}_6| = |77 - 62.75| = |14.25|$$

$$|\bar{y}_3 - \bar{y}_4| = |75 - 71.75| = |3.25|$$

$$|\bar{y}_3 - \bar{y}_5| = |75 - 65| = |10|$$

$$|\bar{y}_3 - \bar{y}_6| = |75 - 62.75| = |12.25|$$

$$|\bar{y}_4 - \bar{y}_5| = |71.75 - 65| = |6.75|$$

$$|\bar{y}_4 - \bar{y}_6| = |71.75 - 62.75| = |9|$$

$$|\bar{y}_5 - \bar{y}_6| = |65 - 62.75| = |2.25|$$

b) Scheffé's method:

$$H_0: \mu_i = \mu_j \quad H_a: \mu_i \neq \mu_j$$

$$\text{population contrast: } \bar{\mu}_i - \bar{\mu}_j$$

$$\text{estimator: } C_{ij} = \bar{\mu}_i - \bar{\mu}_j$$

$$\text{critical value: } S_{x,\mu} = S_{C_{ij}} \cdot \sqrt{(a-1) \cdot F_{\alpha/2, a-1, n-a}}$$

$$= \sqrt{MSE \frac{\alpha}{n} \frac{\alpha^2}{n-1}} \cdot \sqrt{(a-1) \cdot F_{\alpha/2, a-1, n-a}}$$

$$= \sqrt{7.347 \times (\frac{1}{6} + \frac{1}{4})} \cdot \sqrt{5 \times F_{0.05, 5, 18}}$$

$$= \sqrt{7.347 \times (\frac{1}{6} + \frac{1}{4})} \cdot \sqrt{5 \times 2.77}$$

$$= 7.134$$

c) Reject H_0 if $|\bar{\mu}_i - \bar{\mu}_j| > 7.134$

By calculation in a), we can see that:

$$|\bar{y}_{1.} - \bar{y}_{3.}|$$

$$|\bar{y}_{1.} - \bar{y}_{4.}|$$

$$|\bar{y}_{1.} - \bar{y}_{5.}|$$

$$|\bar{y}_{1.} - \bar{y}_{6.}|$$

$$|\bar{y}_{2.} - \bar{y}_{5.}|$$

$$|\bar{y}_{2.} - \bar{y}_{6.}|$$

$$|\bar{y}_{3.} - \bar{y}_{5.}|$$

$$|\bar{y}_{3.} - \bar{y}_{6.}|$$

$$|\bar{y}_{4.} - \bar{y}_{6.}|$$

> 7.134 , thus, these pairs of shower heads have a significant difference in mean radon release percentage.

$$d) T_\alpha = q_{\alpha}(a, f) \cdot \sqrt{\frac{MS_e}{n}}$$

$$= 4.49 \times \sqrt{\frac{7.347}{4}}$$

$$= 6.085$$

$$\begin{aligned} a &= 6 \\ f &= N - a = 18 \\ \alpha &= 0.05 \\ n &= 4 \\ MS_e &= 7.347 \\ q_{\alpha}(a, f) \text{ by Table V} &= 4.49 \end{aligned}$$

e) Reject Ho if $|\bar{y}_{j_1} - \bar{y}_{j_2}| \geq T_\alpha$

$$|\bar{y}_{j_1} - \bar{y}_{j_2}|$$

$$|\bar{y}_{j_1} - \bar{y}_{j_4}|$$

$$|\bar{y}_{j_1} - \bar{y}_{j_5}|$$

$$|\bar{y}_{j_1} - \bar{y}_{j_6}|$$

$$|\bar{y}_{j_2} - \bar{y}_{j_5}|$$

$$|\bar{y}_{j_2} - \bar{y}_{j_6}|$$

$$|\bar{y}_{j_3} - \bar{y}_{j_5}|$$

$$|\bar{y}_{j_3} - \bar{y}_{j_6}|$$

$$|\bar{y}_{j_4} - \bar{y}_{j_5}|$$

$$|\bar{y}_{j_4} - \bar{y}_{j_6}|$$

> 6.085 , thus these pairs of shower heads have significant difference in mean radon release percentage.

$$f) \quad LSD = t_{\alpha/2, N-a} \cdot \sqrt{\frac{2MS_e}{n}}$$

$$= 2.101 \times \sqrt{\frac{2 \times 7.347}{4}}$$

$$= 4.027$$

$$\alpha = 0.05$$

$$N-a = 18$$

$$t_{0.025, 18} = 2.101$$

$$MS_e = 7.347$$

$$n = 4$$

Fisher's LSD

g) Reject H_0 if $|\bar{y}_i - \bar{y}_j| \geq LSD$

$$|\bar{y}_1 - \bar{y}_2|$$

$$|\bar{y}_1 - \bar{y}_3|$$

$$|\bar{y}_1 - \bar{y}_4|$$

$$|\bar{y}_1 - \bar{y}_5|$$

$$|\bar{y}_1 - \bar{y}_6|$$

$$|\bar{y}_2 - \bar{y}_4|$$

$$|\bar{y}_2 - \bar{y}_5|$$

$$|\bar{y}_2 - \bar{y}_6|$$

$$|\bar{y}_3 - \bar{y}_5|$$

$$|\bar{y}_3 - \bar{y}_6|$$

$$|\bar{y}_4 - \bar{y}_5|$$

$$|\bar{y}_4 - \bar{y}_6|$$

> 4.027 , thus these pairs of shower heads have significant difference in mean radon release percentage.

$$\text{h) } d_{\alpha}(a-1, f) \cdot \sqrt{\frac{2MS_{\bar{x}}}{n}}$$

$$= 2.76 \times \sqrt{\frac{2 \times 7.347}{4}}$$

$$= 5.291$$

$$\begin{aligned} \alpha &= 0.05 \\ a &= 6 \\ f &= N - a = 18 \\ n &= 4 \\ MS_{\bar{x}} &= 7.347 \\ d_{\alpha}(a-1, f) &= d_{0.05}(5, 18) \\ &= 2.76 \text{ by table VI} \end{aligned}$$

i) $i = 2, 3, 4, 5, 6$

control group $i = 1$

$$|\bar{y}_{2.} - \bar{y}_{1.}| = 5.75$$

$$|\bar{y}_{3.} - \bar{y}_{1.}| = 7.75$$

$$|\bar{y}_{4.} - \bar{y}_{1.}| = 11$$

$$|\bar{y}_{5.} - \bar{y}_{1.}| = 17.75$$

$$|\bar{y}_{6.} - \bar{y}_{1.}| = 20$$

> 5.291 , thus these pairs of shower heads have significant difference in mean radon release percentage.

j) Dunnett's Method: NOT Recommended, because this method is not controlling every pair.

Fisher's Method:

Tukey's Method:

Scheffe's Method:

→ Does not control FWER

Recommend. See Solutions

→ Less powerful than Tukey's

problem C

Derive: $E(MS_{TTE})$

$$\begin{aligned}
 E(MS_{TTE}) &= E\left(\frac{SS_{TTE}}{\alpha-1}\right) \\
 &= \frac{1}{\alpha-1} E\left[\sum_{i=1}^{\alpha} \sum_{j=1}^n (\bar{y}_{ij} - \bar{y}_{..})^2\right] \\
 &= \frac{1}{\alpha-1} E\left[\sum_{i=1}^{\alpha} \sum_{j=1}^n (\bar{y}_{i.}^2 - 2\bar{y}_{i.}\bar{y}_{..} + \bar{y}_{..}^2)\right] \\
 &= \frac{1}{\alpha-1} E\left[n \sum_{i=1}^{\alpha} \bar{y}_{i.}^2 - 2n \bar{y}_{..} \sum_{i=1}^{\alpha} \bar{y}_{i.} + n \bar{y}_{..}^2\right] \\
 &= \frac{1}{\alpha-1} E\left[n \sum_{i=1}^{\alpha} \bar{y}_{i.} - 2n \bar{y}_{..} \sum_{i=1}^{\alpha} \bar{y}_{i.} + \alpha n \bar{y}_{..}^2\right] \quad \text{①}
 \end{aligned}$$

The true factor effects model:

$$\begin{aligned}
 y_{ij} &= \gamma + \tau_i + \epsilon_{ij} \quad \left\{ \begin{array}{l} i=1, 2 \\ j=1, 2, 3, 4, 5 \end{array} \right. , \quad \tau_1 = \delta, \quad \tau_2 = -\delta, \quad \sum_{i=1}^{\alpha} \tau_i = 0, \\
 \alpha &= 2, \quad \mu_1 = \mu + \tau_1 = \gamma + \delta \\
 n &= 5 \quad \mu_2 = \mu + \tau_2 = \gamma - \delta \\
 \mu &= \gamma \quad \bar{y}_{..} = \frac{1}{\alpha} \sum_{i=1}^{\alpha} \bar{y}_{i.}
 \end{aligned}$$

by ① we have:

$$\begin{aligned}
 &= \frac{1}{2-1} \cdot E\left[5 \times (\bar{y}_{1.}^2 + \bar{y}_{2.}^2) - 2 \times 5 \times \frac{1}{2} \times (\bar{y}_{1.} + \bar{y}_{2.}) \cdot (\bar{y}_{1.} + \bar{y}_{2.}) + 2 \times 5 \times [\frac{1}{2} \times (\bar{y}_{1.} + \bar{y}_{2.})]^2\right] \\
 &= E\left[5 \times (\bar{y}_{1.}^2 + \bar{y}_{2.}^2) - 5 \times (\bar{y}_{1.} + \bar{y}_{2.})^2 + \frac{5}{2} (\bar{y}_{1.} + \bar{y}_{2.})^2\right] \\
 &= E\left[5 \times (\bar{y}_{1.}^2 + \bar{y}_{2.}^2) - \frac{5}{2} (\bar{y}_{1.} + \bar{y}_{2.})^2\right] \quad \checkmark \\
 &= E\left[\frac{5}{2} \bar{y}_{1.}^2 + \frac{5}{2} \bar{y}_{2.}^2 - 5 \bar{y}_{1.} \bar{y}_{2.}\right] \\
 &= \frac{5}{2} E(\bar{y}_{1.}^2) + \frac{5}{2} E(\bar{y}_{2.}^2) - 5 E(\bar{y}_{1.} \bar{y}_{2.}) \quad \text{②}
 \end{aligned}$$

Thus by ② we have: \leftarrow

$$= \frac{5}{2} \left[\frac{\delta^2}{5} + \mu_1^2 \right] + \frac{5}{2} \left[\frac{\delta^2}{5} + \mu_2^2 \right] - 5 \mu_1 \mu_2$$

③

$$\left\{
 \begin{aligned}
 \because E(x_i^2) &= \text{Var}(x_i) + [E(x_i)]^2 \\
 \therefore E(\bar{y}_{i.}^2) &= \text{Var}(\bar{y}_{i.}) + [E(\bar{y}_{i.})]^2 \\
 E(\bar{y}_{i.}^2) &= \text{Var}(\bar{y}_{i.}) + [E(\bar{y}_{i.})]^2 \\
 &= \frac{\delta^2}{n} + \mu_i^2 \\
 E(\bar{y}_{2.}^2) &= \frac{\delta^2}{n} + \mu_2^2 \\
 \because E(x_i x_j) &= E(x_i) E(x_j) \\
 \therefore E(\bar{y}_{1.} \bar{y}_{2.}) &= E(\bar{y}_{1.}) E(\bar{y}_{2.}) \\
 &= \mu_1 \mu_2
 \end{aligned}
 \right. \quad \text{④}$$

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$$M_1 = \eta + \delta,$$

$$M_2 = \eta - \delta,$$

Thus by ③ we have: ✓

$$= \delta^2 + \frac{5}{2}(\eta + \delta)^2 + \frac{5}{2}(\eta - \delta)^2 - 5(\eta + \delta)(\eta - \delta)$$

116²

we know the given equation for $Z(MS_{re}) = \delta^2 + \frac{n \sum_{i=1}^{\alpha} z_i^2}{\alpha - 1}$

$$= \delta^2 + \frac{5 \times (z_1^2 + z_2^2)}{1}$$

$$= \delta^2 + 5 \times 2 \delta^2$$

116²

Thus, we can verify that the deduction above is correct.

problem B

Question 1.

Code and output file see Attached output file (At the end of this file)

- one way ANOVA Table ✓
- contrast analysis ✓
- multiple comparisons ✓
- methods / plots ✓

✓

problem B

Q2 Model:

$$y_{ij} = \mu + \tau_i + \epsilon_{ij} \quad \begin{cases} i=1, 2, 3, \dots, a \\ j=1, 2, 3, \dots, n \end{cases}$$

$$\mu_i = \mu + \tau_i$$

Random Error Assumption

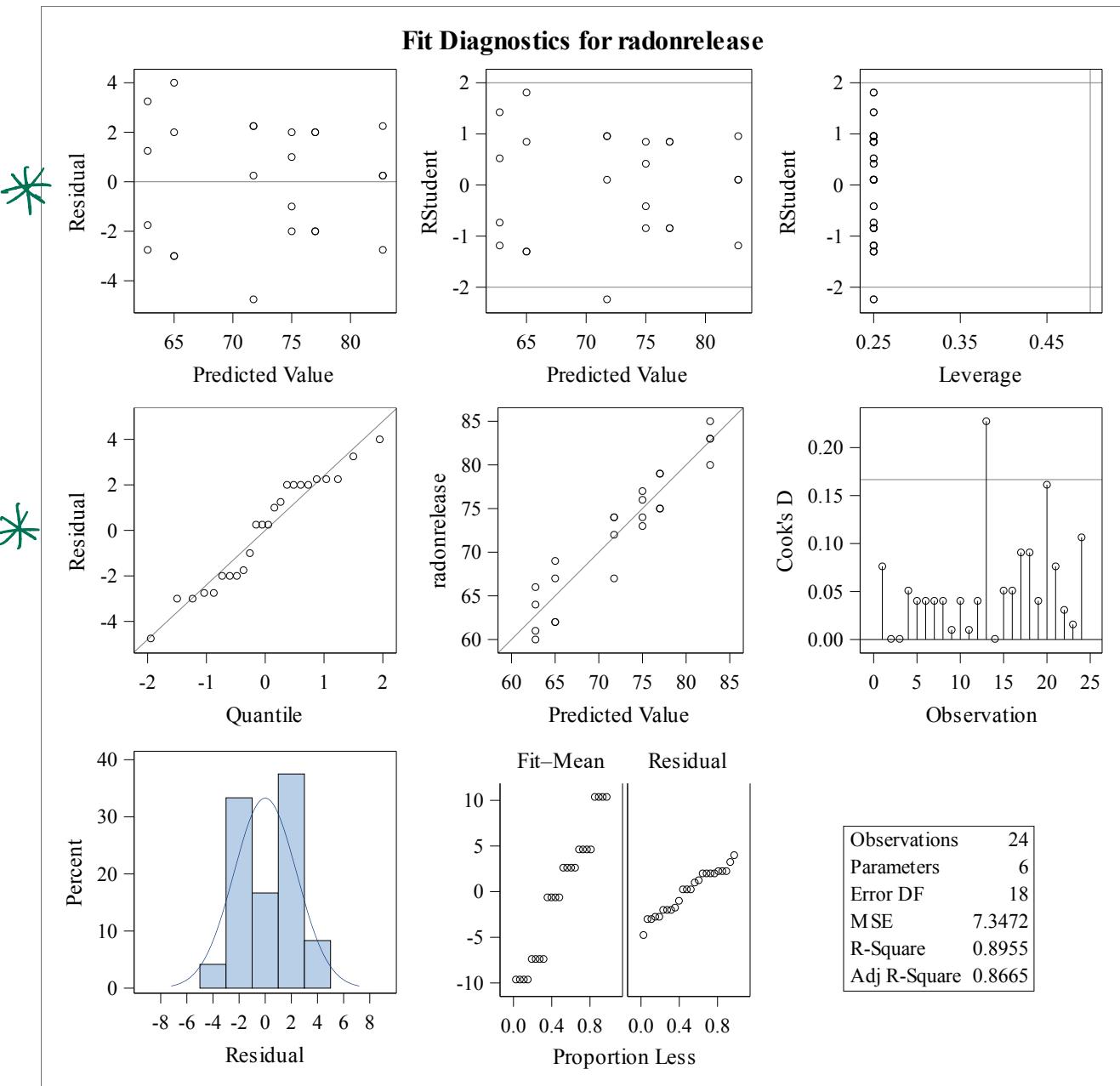
$$\epsilon_{ij} \stackrel{iid}{\sim} N(0, \sigma^2)$$

Random errors are mutually independent and normally distributed with constant variance.

By the Figures on next page,

Residual - predicted value Figure & Residual - Quantile Figure
Verify that assumptions on random errors are met.

* See solutions

The SAS System

problem B

Q3.

by SAS code running result on Next page, $N_{\text{total}} = 12$,

which means treatment Sample Size $n = \frac{12}{6} = 2$.

thus, treatment sample size $n=2$ is large enough to achieve 99% power
when controlling the Type I error rate at 5%.

✓

The SAS System

The GLMPOWER Procedure

Fixed Scenario Elements	
Dependent Variable	ymean
Source	treatment
Alpha	0.05
Error Standard Deviation	2.711
Nominal Power	0.99
Test Degrees of Freedom	5

$$MSE = \sigma^2 = 7.347$$

$$\sigma = \sqrt{7.347} = 2.711$$

$$N = 12$$

$$n = 12/6 = 2$$

Q3 in Problem B:

$n = 2$ is OK!

Computed N Total		
Error DF	Actual Power	N Total
6	0.995	12

The GLMPOWER Procedure

Fixed Scenario Elements	
Dependent Variable	ymean
Source	treatment
Alpha	0.05
Error Standard Deviation	3
Nominal Power	0.99
Test Degrees of Freedom	5

$$\text{if } \sigma = 3$$

$$N = 18$$

$$n = 18/6 = 3$$

Computed N Total		
Error DF	Actual Power	N Total
12	>.999	18

The GLMPOWER Procedure

Fixed Scenario Elements	
Dependent Variable	ymean
Source	treatment
Alpha	0.05
Test Degrees of Freedom	5

The SAS System

CODE:

```

ods rtf file='hw2_output.rtf' startpage=NO;

data shower;
infile 'shower.csv' firsttobs=2 dsd;
input showerhead radonrelease;
run;

proc print data=shower;
run;

/*Insert SAS code for Homework 2 here (i.e., before the "ods rtf close;" command*/
proc glm data=shower plots=all;
class showerhead;
model radonrelease = showerhead;
lsmeans showerhead/stderr pdiff cl;
means showerhead/cldiff scheffe tukey lsd dunnett('1') hovtest=bf;
contrast 'contrast 1' showerhead 0.333 0.333 0.333 -0.333 -0.333 -0.333;
contrast 'contrast 2' showerhead 0.5 0.5 -1 0 0 0;
contrast 'contrast 3' showerhead 0 0 0 0 1 -1;
estimate 'contrast 1' showerhead 0.333 0.333 0.333 -0.333 -0.333 -0.333;
estimate 'contrast 2' showerhead 0.5 0.5 -1 0 0 0;
estimate 'contrast 3' showerhead 0 0 0 0 1 -1;
output out=res p=yhat r=resid;

run;

proc print data=res;
run;

proc univariate data=res normal;
var resid;
probplot;
run;

proc plot data=res;
plot resid*yhat;
run;

data sscalcalc;
input treatment ymean;
datalines;
1 82.75
2 77
3 75
4 71.75
5 65
6 62.75
;
run;

proc glmpower data=sscalcalc;
class treatment;
model ymean=treatment;
power
stddev=2.711
ntotal= .
alpha = 0.05
power = 0.99;
run;

```

This is all you need

The SAS System

```
proc glmpower data=sscalc;
  class treatment;
  model ymean=treatment;
  power
    stddev=3
    ntotal= .
    alpha = 0.05
    power = 0.99;
run;
```

```
proc glmpower data=sscalc;
class treatment;
model ymean=treatment;
power
stddev = 10 5 2.711 1
ntotal = 10 20 24 30 40
alpha = 0.05
power = .;
plot x=n min=10 max=40;
run;
```

```
ods rtf close;
```

The SAS System

Obs	showerhead	radonrelease
1	1	80
2	1	83
3	1	83
4	1	85
5	2	75
6	2	75
7	2	79
8	2	79
9	3	74
10	3	73
11	3	76
12	3	77
13	4	67
14	4	72
15	4	74
16	4	74
17	5	62
18	5	62
19	5	67
20	5	69
21	6	60
22	6	61
23	6	64
24	6	66

The SAS System

The GLM Procedure

Class Level Information		
Class	Levels	Values
showerhead	6	1 2 3 4 5 6

Number of Observations Read	24
Number of Observations Used	24

The GLM Procedure

Dependent Variable: radonrelease

ANOVA
Table

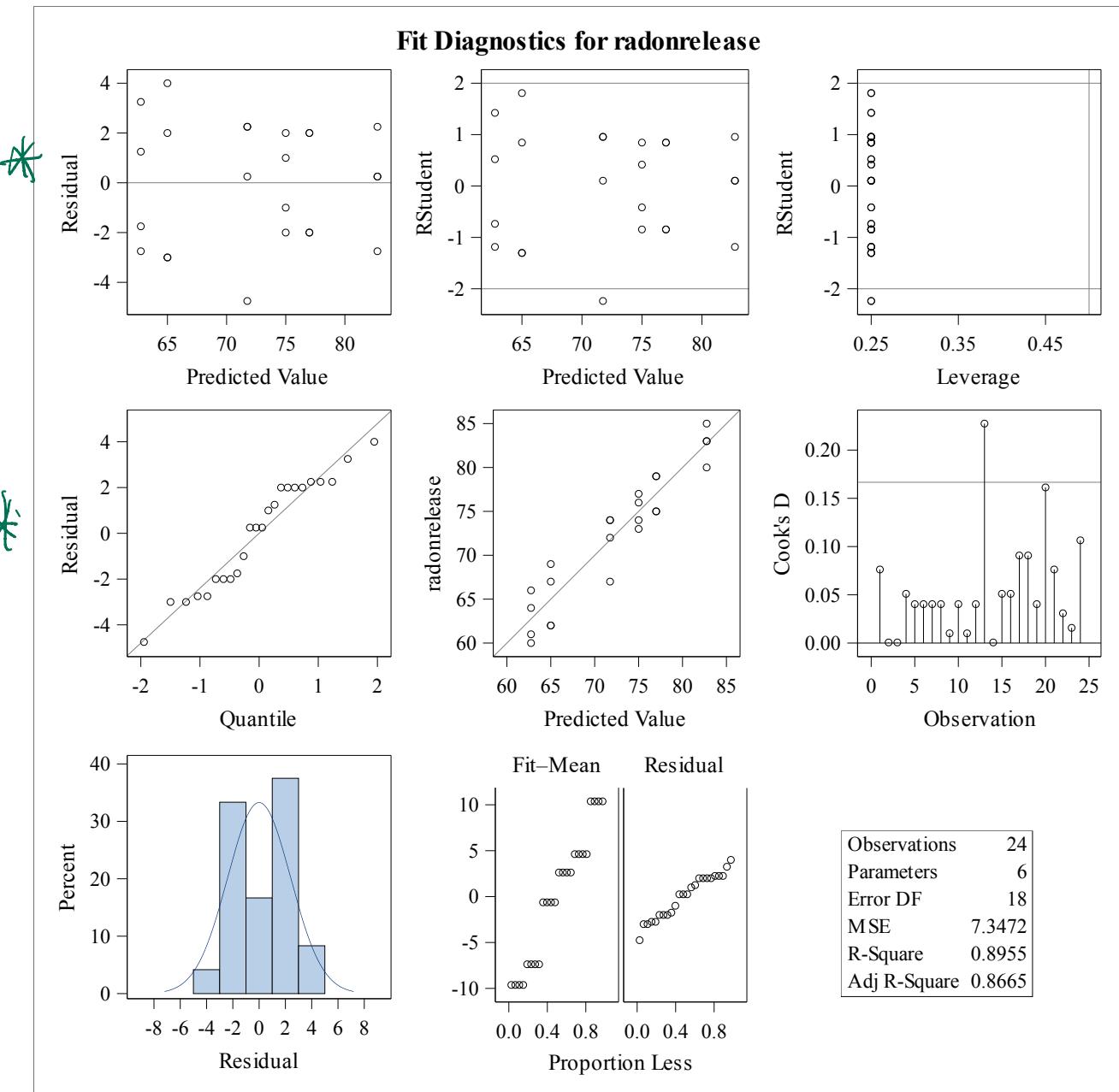
$$MSE = 7.347$$

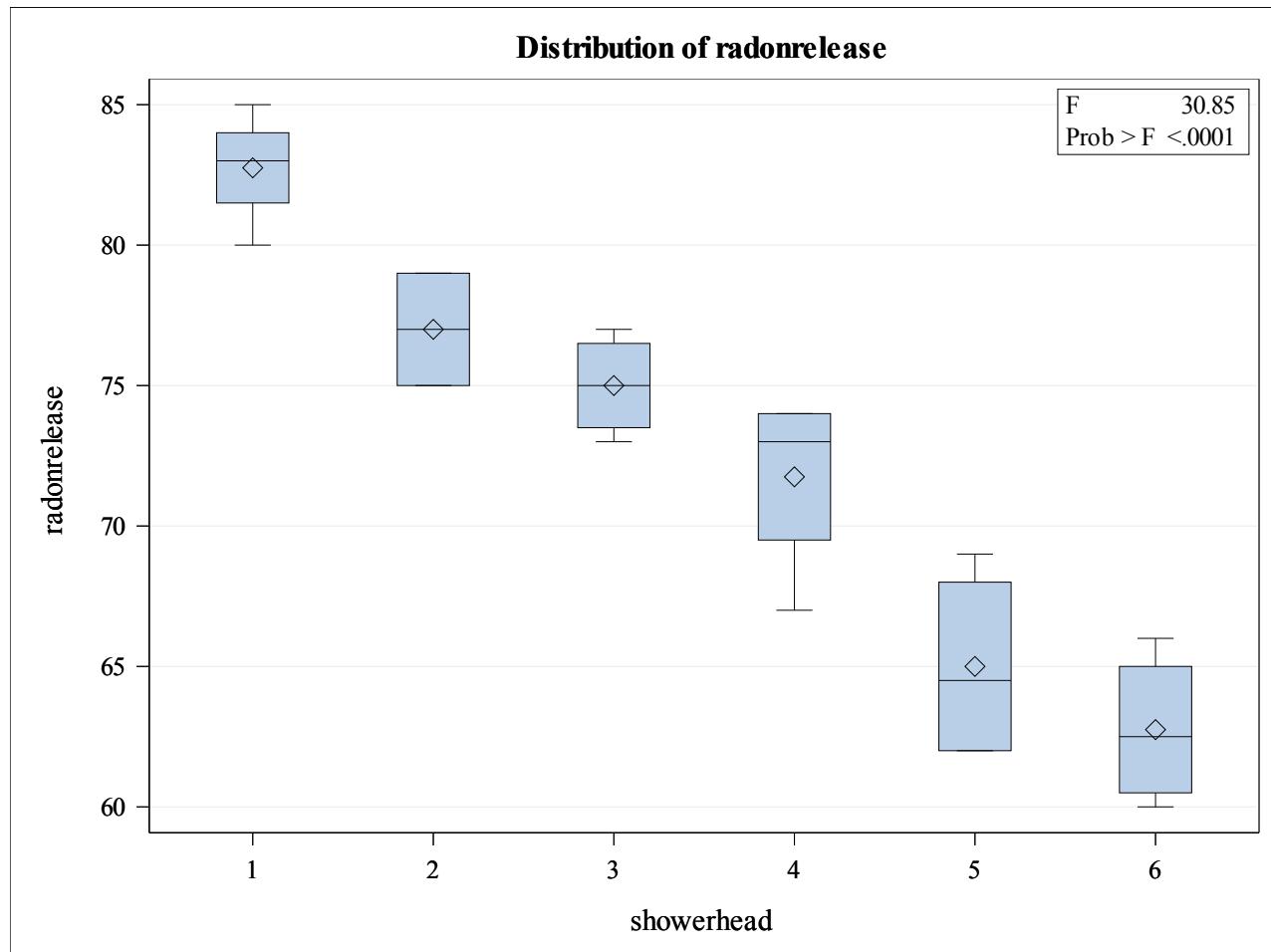
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	1133.375000	226.675000	30.85	<.0001
Error	18	132.250000	7.347222		
Corrected Total	23	1265.625000			

R-Square	Coeff Var	Root MSE	radonrelease Mean
0.895506	3.745183	2.710576	72.37500

Source	DF	Type I SS	Mean Square	F Value	Pr > F
showerhead	5	1133.375000	226.675000	30.85	<.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
showerhead	5	1133.375000	226.675000	30.85	<.0001

The SAS System

The SAS System*The GLM Procedure*
Least Squares Means

showerhead	radonrelease LSMEAN	Standard Error	Pr > t	LSMEAN Number
1	82.7500000	1.3552880	<.0001	1
2	77.0000000	1.3552880	<.0001	2
3	75.0000000	1.3552880	<.0001	3
4	71.7500000	1.3552880	<.0001	4
5	65.0000000	1.3552880	<.0001	5
6	62.7500000	1.3552880	<.0001	6

The SAS System

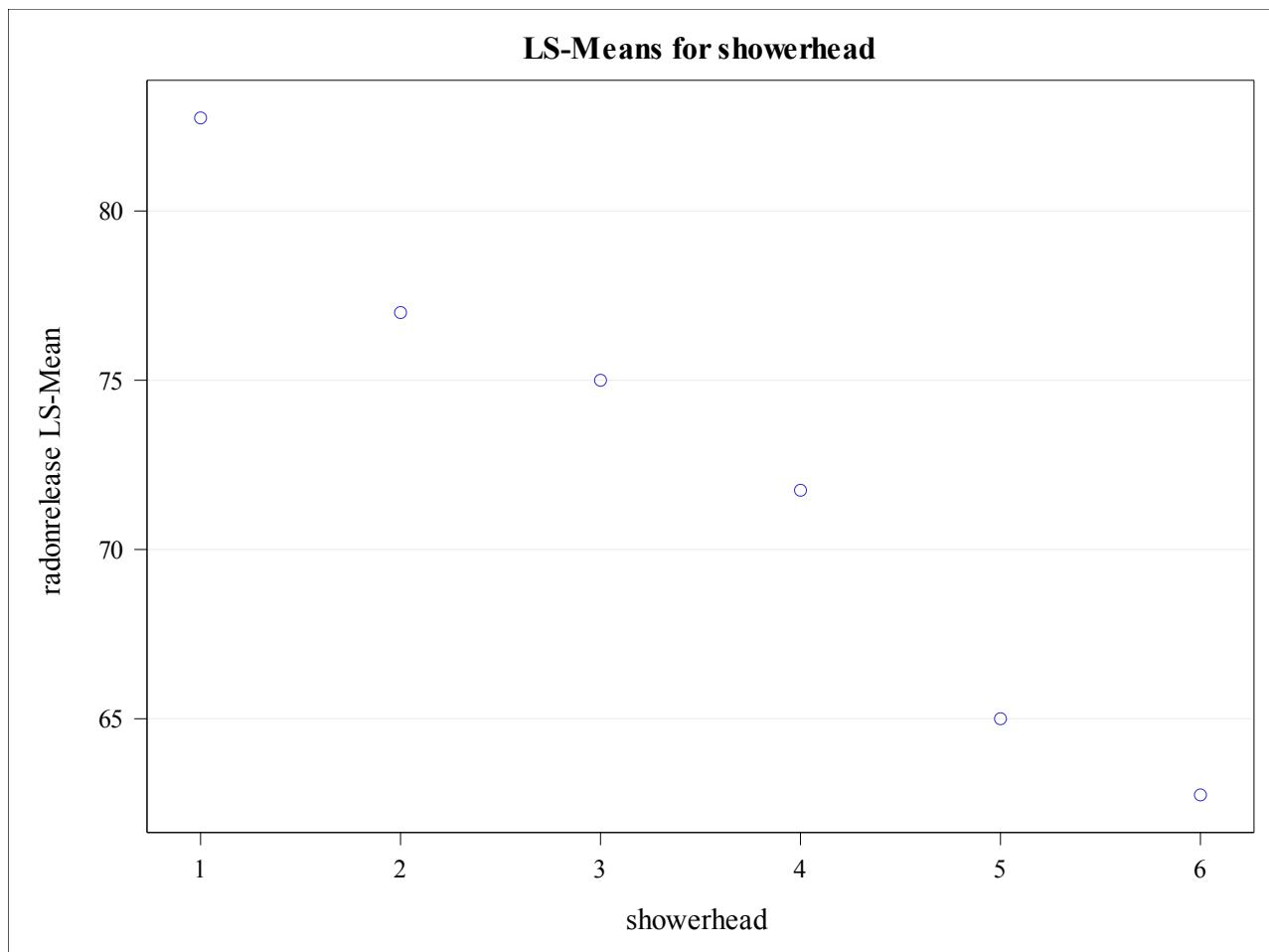
Least Squares Means for effect showerhead $\Pr > t \text{ for } H_0: \text{LSMean}(i) = \text{LSMean}(j)$						
Dependent Variable: radonrelease						
i/j	1	2	3	4	5	6
1		0.0077	0.0008	<.0001	<.0001	<.0001
2	0.0077		0.3105	0.0135	<.0001	<.0001
3	0.0008	0.3105		0.1072	<.0001	<.0001
4	<.0001	0.0135	0.1072		0.0024	0.0002
5	<.0001	<.0001	<.0001	0.0024		0.2557
6	<.0001	<.0001	<.0001	0.0002	0.2557	

showerhead	radonrelease LSMEAN	95% Confidence Limits	
1	82.750000	79.902646	85.597354
2	77.000000	74.152646	79.847354
3	75.000000	72.152646	77.847354
4	71.750000	68.902646	74.597354
5	65.000000	62.152646	67.847354
6	62.750000	59.902646	65.597354

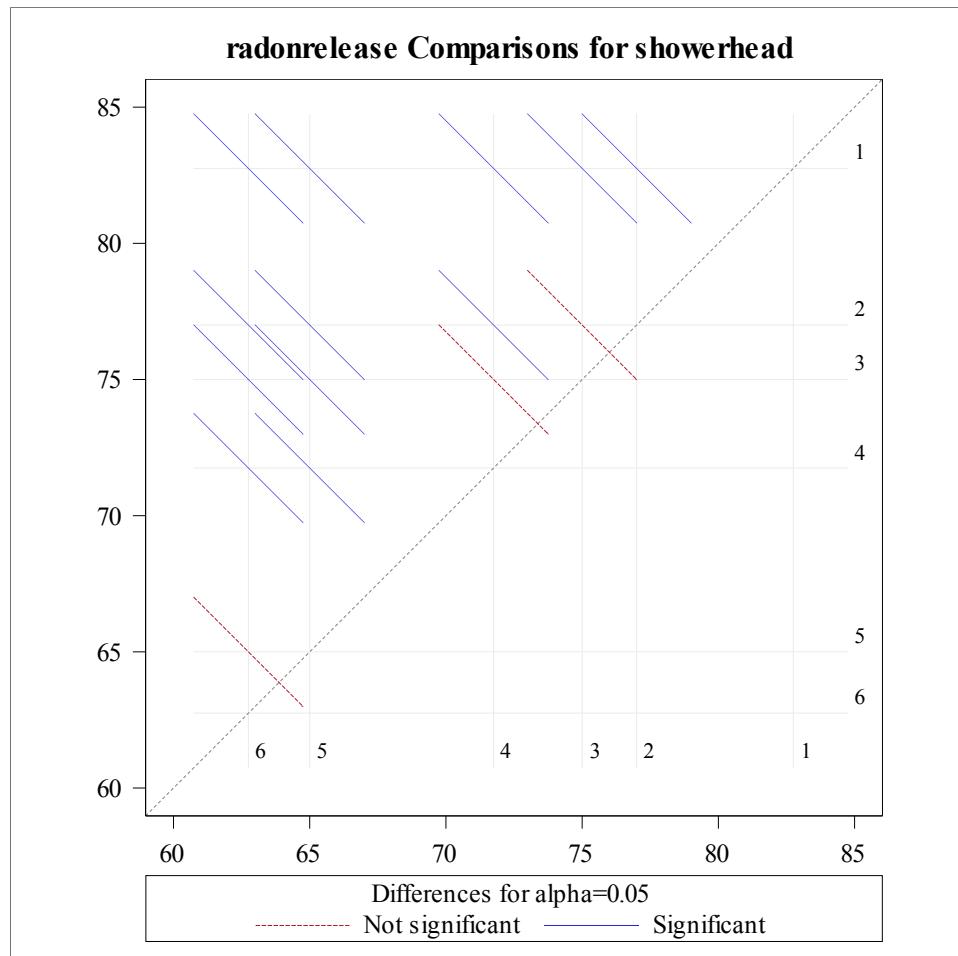
Least Squares Means for Effect showerhead				
i	j	Difference Between Means	95% Confidence Limits for LSMean(i)-LSMean(j)	
1	2	5.750000	1.723233	9.776767
1	3	7.750000	3.723233	11.776767
1	4	11.000000	6.973233	15.026767
1	5	17.750000	13.723233	21.776767
1	6	20.000000	15.973233	24.026767
2	3	2.000000	-2.026767	6.026767
2	4	5.250000	1.223233	9.276767
2	5	12.000000	7.973233	16.026767
2	6	14.250000	10.223233	18.276767
3	4	3.250000	-0.776767	7.276767
3	5	10.000000	5.973233	14.026767
3	6	12.250000	8.223233	16.276767

The SAS System

Least Squares Means for Effect showerhead				
i	j	Difference Between Means	95% Confidence Limits for LSMean(i)-LSMean(j)	
4	5	6.750000	2.723233	10.776767
4	6	9.000000	4.973233	13.026767
5	6	2.250000	-1.776767	6.276767



The SAS System



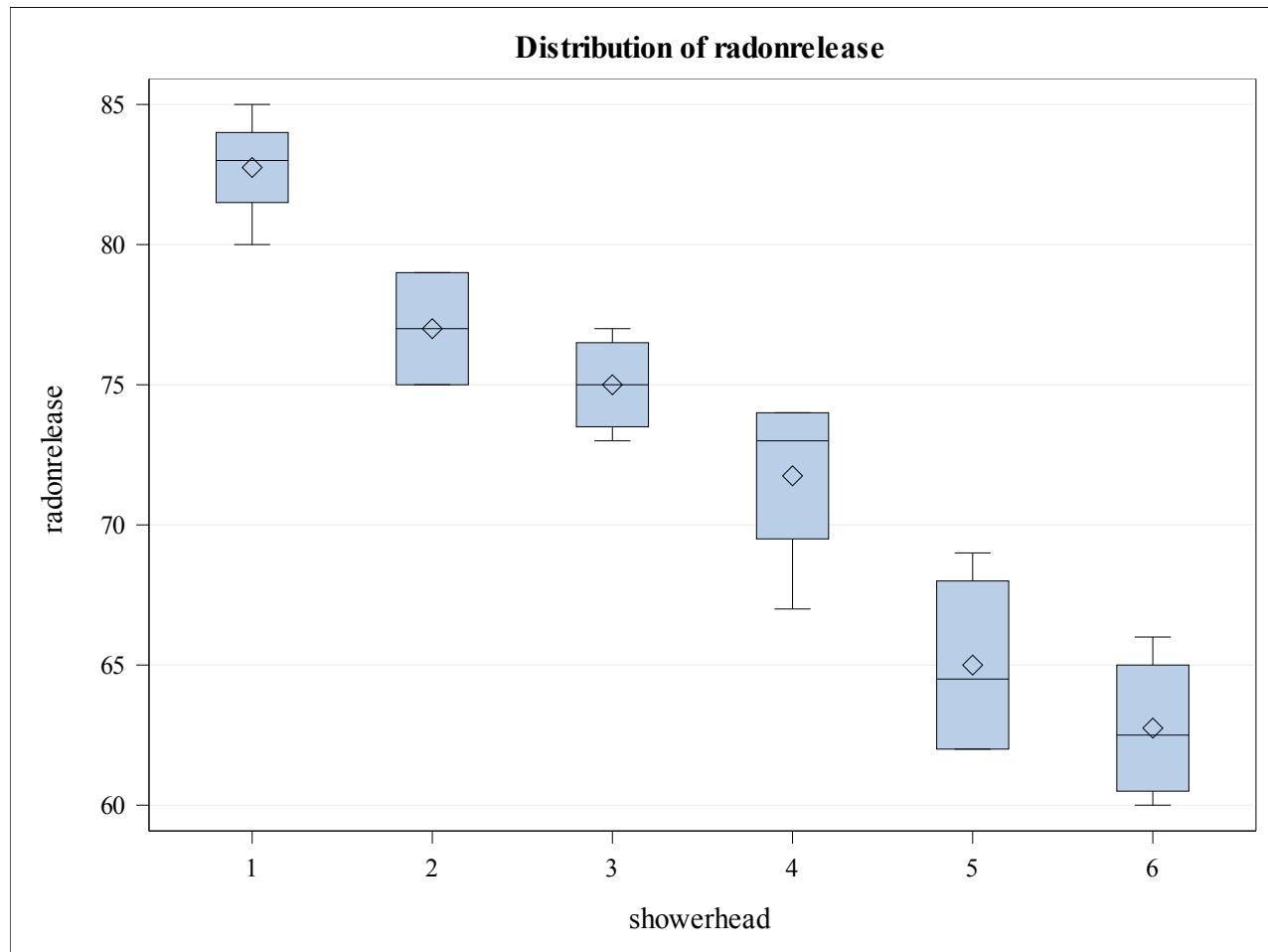
Note: To ensure overall protection level, only probabilities associated with pre-planned comparisons should be used.

The GLM Procedure

Brown and Forsythe's Test for Homogeneity of radonrelease Variance					
ANOVA of Absolute Deviations from Group Medians					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
showerhead	5	7.7083	1.5417	0.86	0.5261
Error	18	32.2500	1.7917		

The GLM Procedure

The SAS System



The GLM Procedure

t Tests (LSD) for radonrelease

Note: This test controls the Type I comparisonwise error rate, not the experimentwise error rate.

Alpha	0.05
Error Degrees of Freedom	18
Error Mean Square	7.347222
Critical Value of t	2.10092
Least Significant Difference	4.0268

The SAS System

Comparisons significant at the 0.05 level are indicated by ***.				
showerhead Comparison	Difference Between Means	95% Confidence Limits		
1 - 2	5.750	1.723	9.777	***
1 - 3	7.750	3.723	11.777	***
1 - 4	11.000	6.973	15.027	***
1 - 5	17.750	13.723	21.777	***
1 - 6	20.000	15.973	24.027	***
2 - 1	-5.750	-9.777	-1.723	***
2 - 3	2.000	-2.027	6.027	
2 - 4	5.250	1.223	9.277	***
2 - 5	12.000	7.973	16.027	***
2 - 6	14.250	10.223	18.277	***
3 - 1	-7.750	-11.777	-3.723	***
3 - 2	-2.000	-6.027	2.027	
3 - 4	3.250	-0.777	7.277	
3 - 5	10.000	5.973	14.027	***
3 - 6	12.250	8.223	16.277	***
4 - 1	-11.000	-15.027	-6.973	***
4 - 2	-5.250	-9.277	-1.223	***
4 - 3	-3.250	-7.277	0.777	
4 - 5	6.750	2.723	10.777	***
4 - 6	9.000	4.973	13.027	***
5 - 1	-17.750	-21.777	-13.723	***
5 - 2	-12.000	-16.027	-7.973	***
5 - 3	-10.000	-14.027	-5.973	***
5 - 4	-6.750	-10.777	-2.723	***
5 - 6	2.250	-1.777	6.277	
6 - 1	-20.000	-24.027	-15.973	***
6 - 2	-14.250	-18.277	-10.223	***
6 - 3	-12.250	-16.277	-8.223	***
6 - 4	-9.000	-13.027	-4.973	***
6 - 5	-2.250	-6.277	1.777	

The SAS System

The GLM Procedure

Tukey's Studentized Range (HSD) Test for radonrelease

Note: This test controls the Type I experimentwise error rate.

Alpha	0.05
Error Degrees of Freedom	18
Error Mean Square	7.347222
Critical Value of Studentized Range	4.49442
Minimum Significant Difference	6.0912

Comparisons significant at the 0.05 level are indicated by ***.				
showerhead Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
1 - 2	5.750	-0.341	11.841	
1 - 3	7.750	1.659	13.841	***
1 - 4	11.000	4.909	17.091	***
1 - 5	17.750	11.659	23.841	***
1 - 6	20.000	13.909	26.091	***
2 - 1	-5.750	-11.841	0.341	
2 - 3	2.000	-4.091	8.091	
2 - 4	5.250	-0.841	11.341	
2 - 5	12.000	5.909	18.091	***
2 - 6	14.250	8.159	20.341	***
3 - 1	-7.750	-13.841	-1.659	***
3 - 2	-2.000	-8.091	4.091	
3 - 4	3.250	-2.841	9.341	
3 - 5	10.000	3.909	16.091	***
3 - 6	12.250	6.159	18.341	***
4 - 1	-11.000	-17.091	-4.909	***
4 - 2	-5.250	-11.341	0.841	
4 - 3	-3.250	-9.341	2.841	
4 - 5	6.750	0.659	12.841	***
4 - 6	9.000	2.909	15.091	***

The SAS System

Comparisons significant at the 0.05 level are indicated by ***.				
showerhead Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
5 - 1	-17.750	-23.841	-11.659	***
5 - 2	-12.000	-18.091	-5.909	***
5 - 3	-10.000	-16.091	-3.909	***
5 - 4	-6.750	-12.841	-0.659	***
5 - 6	2.250	-3.841	8.341	
6 - 1	-20.000	-26.091	-13.909	***
6 - 2	-14.250	-20.341	-8.159	***
6 - 3	-12.250	-18.341	-6.159	***
6 - 4	-9.000	-15.091	-2.909	***
6 - 5	-2.250	-8.341	3.841	

The GLM Procedure

Scheffe's Test for radonrelease

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than Tukey's for all pairwise comparisons.

Alpha	0.05
Error Degrees of Freedom	18
Error Mean Square	7.347222
Critical Value of F	2.77285
Minimum Significant Difference	7.1367

Comparisons significant at the 0.05 level are indicated by ***.				
showerhead Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
1 - 2	5.750	-1.387	12.887	
1 - 3	7.750	0.613	14.887	***
1 - 4	11.000	3.863	18.137	***
1 - 5	17.750	10.613	24.887	***

The SAS System

Comparisons significant at the 0.05 level are indicated by ***.				
showerhead Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
1 - 6	20.000	12.863	27.137	***
2 - 1	-5.750	-12.887	1.387	
2 - 3	2.000	-5.137	9.137	
2 - 4	5.250	-1.887	12.387	
2 - 5	12.000	4.863	19.137	***
2 - 6	14.250	7.113	21.387	***
3 - 1	-7.750	-14.887	-0.613	***
3 - 2	-2.000	-9.137	5.137	
3 - 4	3.250	-3.887	10.387	
3 - 5	10.000	2.863	17.137	***
3 - 6	12.250	5.113	19.387	***
4 - 1	-11.000	-18.137	-3.863	***
4 - 2	-5.250	-12.387	1.887	
4 - 3	-3.250	-10.387	3.887	
4 - 5	6.750	-0.387	13.887	
4 - 6	9.000	1.863	16.137	***
5 - 1	-17.750	-24.887	-10.613	***
5 - 2	-12.000	-19.137	-4.863	***
5 - 3	-10.000	-17.137	-2.863	***
5 - 4	-6.750	-13.887	0.387	
5 - 6	2.250	-4.887	9.387	
6 - 1	-20.000	-27.137	-12.863	***
6 - 2	-14.250	-21.387	-7.113	***
6 - 3	-12.250	-19.387	-5.113	***
6 - 4	-9.000	-16.137	-1.863	***
6 - 5	-2.250	-9.387	4.887	

The SAS System

The GLM Procedure

Dunnett's t Tests for radonrelease

Note: This test controls the Type I experimentwise error for comparisons of all treatments against a control.

Alpha	0.05
Error Degrees of Freedom	18
Error Mean Square	7.347222
Critical Value of Dunnett's t	2.76150
Minimum Significant Difference	5.2929

Comparisons significant at the 0.05 level are indicated by ***.				
showerhead Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
2 - 1	-5.750	-11.043	-0.457	***
3 - 1	-7.750	-13.043	-2.457	***
4 - 1	-11.000	-16.293	-5.707	***
5 - 1	-17.750	-23.043	-12.457	***
6 - 1	-20.000	-25.293	-14.707	***

The GLM Procedure

Dependent Variable: radonrelease



Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
contrast 1	1	828.3750000	828.3750000	112.75	<.0001
contrast 2	1	63.3750000	63.3750000	8.63	0.0088
contrast 3	1	10.1250000	10.1250000	1.38	0.2557

Parameter	Estimate	Standard Error	t Value	Pr > t
contrast 1	11.7382500	1.10548143	10.62	<.0001
contrast 2	4.8750000	1.65988202	2.94	0.0088
contrast 3	2.2500000	1.91666667	1.17	0.2557

The SAS System

Obs	showerhead	radonrelease	yhat	resid
1	1	80	82.75	-2.75
2	1	83	82.75	0.25
3	1	83	82.75	0.25
4	1	85	82.75	2.25
5	2	75	77.00	-2.00
6	2	75	77.00	-2.00
7	2	79	77.00	2.00
8	2	79	77.00	2.00
9	3	74	75.00	-1.00
10	3	73	75.00	-2.00
11	3	76	75.00	1.00
12	3	77	75.00	2.00
13	4	67	71.75	-4.75
14	4	72	71.75	0.25
15	4	74	71.75	2.25
16	4	74	71.75	2.25
17	5	62	65.00	-3.00
18	5	62	65.00	-3.00
19	5	67	65.00	2.00
20	5	69	65.00	4.00
21	6	60	62.75	-2.75
22	6	61	62.75	-1.75
23	6	64	62.75	1.25
24	6	66	62.75	3.25

The UNIVARIATE Procedure

Variable:

resid

Moments			
N	24	Sum Weights	24
Mean	0	Sum Observations	0
Std Deviation	2.39791576	Variance	5.75
Skewness	-0.2257503	Kurtosis	-1.1196952
Uncorrected SS	132.25	Corrected SS	132.25
Coeff Variation	.	Std Error Mean	0.48947251

The SAS System

Basic Statistical Measures				
Location		Variability		
Mean	0.00000	Std Deviation	2.39792	
Median	0.25000	Variance	5.75000	
Mode	-2.00000	Range	8.75000	
		Interquartile Range	4.00000	

Note: The mode displayed is the smallest of 4 modes with a count of 3.

Tests for Location: Mu0=0				
Test	Statistic		p Value	
Student's t	t	0	Pr > t 	1.0000
Sign	M	2	Pr >= M 	0.5413
Signed Rank	S	5	Pr >= S 	0.8897

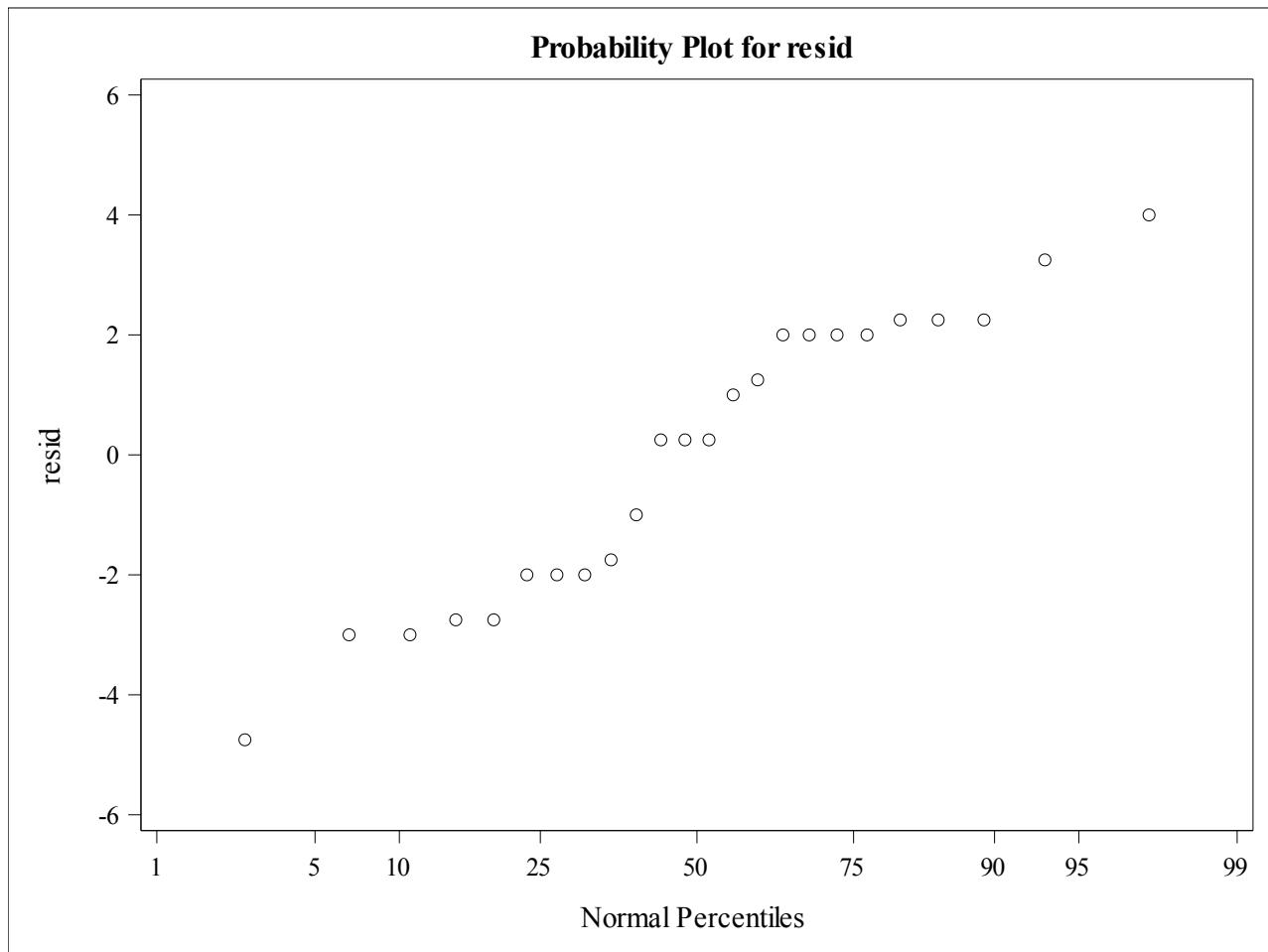
Tests for Normality				
Test	Statistic		p Value	
Shapiro-Wilk	W	0.934783	Pr < W	0.1247
Kolmogorov-Smirnov	D	0.172876	Pr > D	0.0630
Cramer-von Mises	W-Sq	0.127141	Pr > W-Sq	0.0457
Anderson-Darling	A-Sq	0.724826	Pr > A-Sq	0.0507

Quantiles (Definition 5)	
Level	Quantile
100% Max	4.00
99%	4.00
95%	3.25
90%	2.25
75% Q3	2.00
50% Median	0.25
25% Q1	-2.00
10%	-3.00
5%	-3.00

The SAS System

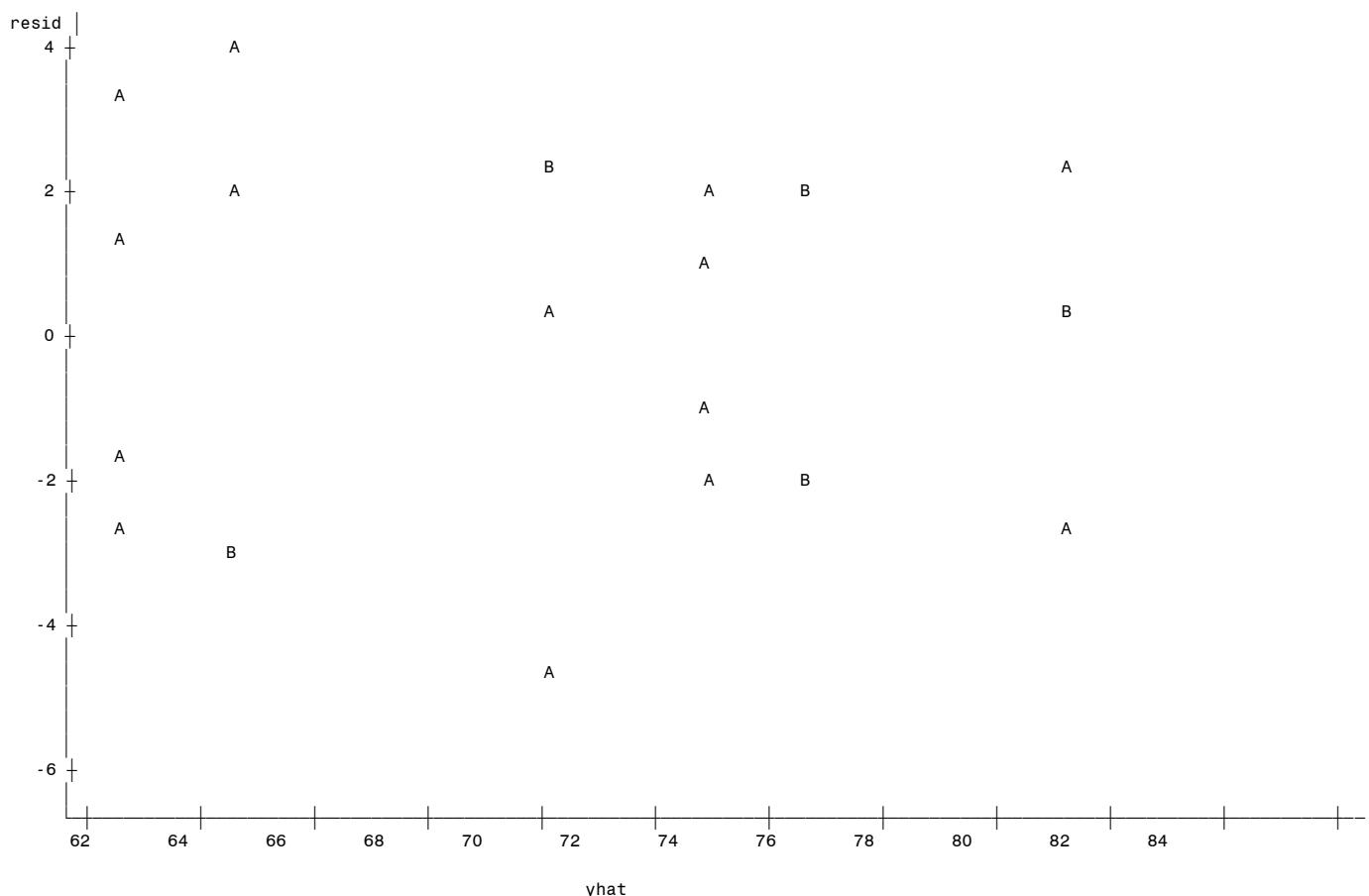
Quantiles (Definition 5)	
Level	Quantile
1%	-4.75
0% Min	-4.75

Extreme Observations			
Lowest		Highest	
Value	Obs	Value	Obs
-4.75	13	2.25	4
-3.00	18	2.25	15
-3.00	17	2.25	16
-2.75	21	3.25	24
-2.75	1	4.00	20

The UNIVARIATE Procedure

The SAS System

Plot of resid*yhat. Legend: A = 1 obs, B = 2 obs, etc.



The SAS System

The GLMPOWER Procedure

Fixed Scenario Elements	
Dependent Variable	ymean
Source	treatment
Alpha	0.05
Error Standard Deviation	2.711
Nominal Power	0.99
Test Degrees of Freedom	5

Computed N Total		
Error DF	Actual Power	N Total
6	0.995	12

The GLMPOWER Procedure

Fixed Scenario Elements	
Dependent Variable	ymean
Source	treatment
Alpha	0.05
Error Standard Deviation	3
Nominal Power	0.99
Test Degrees of Freedom	5

Computed N Total		
Error DF	Actual Power	N Total
12	>.999	18

The GLMPOWER Procedure

Fixed Scenario Elements	
Dependent Variable	ymean
Source	treatment
Alpha	0.05
Test Degrees of Freedom	5

The SAS System

Computed Power						
Index	Std Dev	Nominal N Total	Actual N Total	Error DF	Power	Error
1	10.00	10	6	0	.	Invalid input
2	10.00	20	18	12	0.403	
3	10.00	24	24	18	0.592	
4	10.00	30	30	24	0.740	
5	10.00	40	36	30	0.844	
6	5.00	10	6	0	.	Invalid input
7	5.00	20	18	12	0.966	
8	5.00	24	24	18	0.998	
9	5.00	30	30	24	>.999	
10	5.00	40	36	30	>.999	
11	2.71	10	6	0	.	Invalid input
12	2.71	20	18	12	>.999	
13	2.71	24	24	18	>.999	
14	2.71	30	30	24	>.999	
15	2.71	40	36	30	>.999	
16	1.00	10	6	0	.	Invalid input
17	1.00	20	18	12	>.999	
18	1.00	24	24	18	>.999	
19	1.00	30	30	24	>.999	
20	1.00	40	36	30	>.999	

The SAS System***The GLMPOWER Procedure***