Question 1 Random Effects Model for a Single Factor

Graded on completion. See solutions.

47.5/50

Factor: Location/Field

Response: Sunflower yield

Treatments: fields

a) Researchers are interested in a factor that has a large number of possible levels, Researchers randomly select a # of factor levels from this large population, In this case, a random effects model should be used, where the treatment effects are considered random variables.

b) $y_{ij} = \mu + \tau_i + \epsilon_{ij}, \quad \begin{cases} i = 1, 2, \ldots 10, & (a=10) \\ j = 1, 2, 3, 4 & (n=4) \end{cases}$ ← Index Range

y_{ij} Represents the jth observation of yield of sunflower in ith field.

μ Represents the overall population mean response of yield of sunflower.

τ_i Represents the effect of the ith treatment group on yield of sunflower.

ϵ_{ij} Represents the random error component of y_{ij}

$\tau_i \overset{iid}{\sim} N(0, \sigma_\tau^2)$

$\epsilon_{ij} \overset{iid}{\sim} N(0, \sigma^2)$

c) make inference on variance component σ_τ^2,

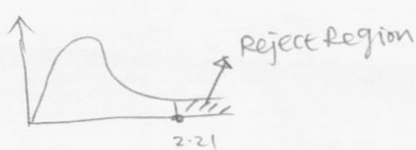
If $\sigma_\tau^2 = 0$ then all treatment effects are identical,

If $\sigma_\tau^2 > 0$ then variability exists among the treatment effects.

$H_0: \sigma_\tau^2 = 0 \qquad H_a: \sigma_\tau^2 > 0$

d) test statistic $F_0 = \dfrac{MS_{Tre}}{MS_E} = \dfrac{5.211}{3.406} = 1.530$

Critical Value $= F_{\alpha, a-1, N-a} = F_{0.05, 9, 30} = 2.21$



Reject H_0 if $F_0 > F_{0.05, 9, 30} = 2.21$.

$F_0 = 1.530 < F_{0.05, 9, 30} = 2.21$,

Thus, Do Not Reject H_0.

There's No sufficient evidence to claim that the variability exists among the treatment effects.

e) $a=10,$
$n=4$
$N=40$

$$\hat{\sigma}^2 = MS_E = 3.406$$

$$\hat{\sigma}_\tau^2 = \frac{MS_{Trt} - MS_E}{n} = \frac{5.211 - 3.406}{4} = 0.45125$$

f) 95% C.I.

$$\frac{(N-a)MS_E}{\chi^2_{\alpha/2,\, N-a}} \le \sigma^2 \le \frac{(N-a)MS_E}{\chi^2_{1-\alpha/2,\, N-a}}$$

$N-a=30$
$\alpha=0.05$
$\chi^2_{0.025,\,30} = 46.979$
$\chi^2_{0.975,\,30} = 16.791$

$$\frac{30 \times 3.406}{46.979} \le \sigma^2 \le \frac{30 \times 3.406}{16.791}$$

$$2.175 \le \sigma^2 \le 6.085$$

Question 2. Two-Factor Factorials with Random Effects

sunflower varieties : fixed

fields : random

a) $y_{ijk} = \mu + \underbrace{\alpha_i}_{\text{(fixed)}} + \underbrace{\gamma_j}_{\text{(Random)}} + (\alpha\gamma)_{ij} + \epsilon_{ijk}$

y_{ijk} Represents k th observation of sunflower yield at ith level of variety and j th level of field

μ: Represents overall population mean

α_i: Represents fixed effect corresponding to the ith level of variety.

γ_j: Represents random effect corresponding to the jth level of field.

$(\alpha\gamma)_{ij}$: Random interaction effect for the ith level of variety of sunflower and jth level of field.

α_i: $i = 1, 2, \cdots a$; $a = 3$; $\sum_{i=1}^{a} \alpha_i = 0$

K = ?

✓

γ_j: $j = 1, 2, \cdots b$; $b = 8$; $\gamma_j \overset{iid}{\sim} N(0, \sigma_\gamma^2)$

$(\alpha\gamma)_{ij} \overset{iid}{\sim} N(0, \sigma_{\alpha\gamma}^2)$

$\epsilon_{ijk} \overset{iid}{\sim} N(0, \sigma^2)$

γ_j, $(\alpha\gamma)_{ij}$, ϵ_{ijk} are all independent.

b) α_i: $H_0: \alpha_i = 0 \ \forall i = 1, 2, 3$ ~~8~~, H_a: at least one is nonzero ← sunflower variety

γ_j: $H_0: \sigma_\gamma^2 = 0$, $H_a: \sigma_\gamma^2 > 0$ ✓ ← fields

$(\alpha\gamma)_{ij}$: $H_0: \sigma_{\alpha\gamma}^2 = 0$, $H_a: \sigma_{\alpha\gamma}^2 > 0$ ← Interactions

Test statistics	Fixed	Random	Mixed-Restricted	Mixed-unrestricted
A α_i	$\frac{MS_A}{MS_E}$	$\frac{MS_A}{MS_{AB}}$	$\frac{MS_A}{MS_{AB}}$	$\frac{MS_A}{MS_{AB}}$
B γ_j	$\frac{MS_B}{MS_E}$	$\frac{MS_B}{MS_{AB}}$	$\frac{MS_B}{MS_E}$	$\frac{MS_B}{MS_{AB}}$
AB $(\alpha\gamma)_{ij}$	$\frac{MS_{AB}}{MS_E}$	$\frac{MS_{AB}}{MS_E}$	$\frac{MS_{AB}}{MS_E}$	$\frac{MS_{AB}}{MS_E}$

e) Reject Ho
Fail to Reject Ho
Fail to Reject Ho

d) α_i: Reject Ho
γ_j: Reject Ho
$(\alpha\gamma)_{ij}$: Fail to Reject Ho

c). α_i: Reject Ho
γ_j: Fail to Reject Ho
$(\alpha\gamma)_{ij}$: Fail to Reject Ho

{The differences of Decisions made in c). d). e). calculation Next Page} unrestricted

c) α_i :

$F_0 = \frac{MS_A}{MS_{AB}} = \frac{32.602}{2.937} = 11.100$ Critical Value = $F_{0.05, 2, 14} = 3.74$

Reject H_0 if $F_0 > F_{0.05, 2, 14} = 3.74$. ✓

$F_0 = 11.100 > 3.74$, Thus, Reject H_0. There's sufficient evidence to claim that the fixed factor of sunflower variety have different effects on sunflower yield.

γ_j :

$F_0 = \frac{MS_B}{MS_{AB}} = \frac{5.830}{2.937} = 1.985$ Critical Value = $F_{0.05, 7, 14} = 2.76$

Reject H_0 if $F_0 > F_{0.05, 7, 14} = 2.76$. ✓

$F_0 = 1.985 < 2.76$.

Thus, Do not Reject H_0. There's no sufficient evidence to claim that the variability exist among the treatment effects of fields.

$(\alpha\gamma)_{ij}$:

$F_0 = \frac{MS_{AB}}{MS_E} = \frac{2.937}{1.807} = 1.625$ Critical Value = $F_{0.05, 14, 24} = 2.145$

Reject H_0 if $F_0 > F_{0.05, 14, 24} = 2.145$. ✓

$F_0 = 1.625 < 2.145$

Thus, Do Not Reject H_0. There's no sufficient evidence to claim that the variability exist among the interaction treatment effects.

d) restricted

α_i $F_0 = \frac{MS_A}{MS_{AB}} = \frac{32.602}{2.937} = 11.100$

$F_{0.05, 2, 14} = 3.74$

$F_0 = 11.100 > 3.74$,

Reject H_0.

Sunflower variety has different effects.

γ_j $F_0 = \frac{MS_B}{MS_E} = \frac{5.830}{1.807} = 3.226$

$F_{0.05, 7, 24} = 2.42$

$F_0 = 3.226 > 2.42$.

Reject H_0.

Variability of fields effects exists.

$(\alpha\gamma)_{ij}$ $F_0 = \frac{MS_{AB}}{MS_E} = \frac{2.937}{1.807} = 1.625$

$F_{0.05, 14, 24} = 2.145$

$F_0 = 1.625 < 2.145$,

Do Not Reject H_0.

No sufficient evidence to claim variability of interaction treatments exists.

e) Random effects

$F_0 = \frac{MS_A}{MS_{AB}} = 11.100$

$F_{0.05, 2, 14} = 3.74$

$F_0 = 11.100 > 3.74$

Reject H_0.

Sunflower variety has different effects.

$F_0 = \frac{MS_B}{MS_{AB}} = 1.985$

$F_{0.05, 7, 14} = 2.76$

$F_0 = 1.985 < 2.76$

Do Not Reject H_0

No sufficient evidence to claim variability fields effects exists.

$F_0 = \frac{MS_{AB}}{MS_E} = 1.625$

$F_{0.05, 14, 24} = 2.145$

$F_0 = 1.625 < 2.145$

Do Not Reject H_0.

No sufficient evidence to claim variability of interaction treatments exists.

-1 * Clearly indicate which conclusions differ / do not differ.

f) unrestricted:

$a=3$
$n=2$
$b=8$

$\sigma^2 = MS_E = 1.807$

$\sigma_\gamma^2 = \frac{MS_B - MS_{AB}}{an} = \frac{5.830 - 2.937}{3\times 2} = 0.482$ ✓

$\sigma_{\alpha\gamma}^2 = \frac{MS_{AB} - MS_E}{n} = \frac{2.937 - 1.807}{2} = 0.565$

g) restricted

$\sigma^2 = MS_E = 1.807$

$\sigma_\gamma^2 = \frac{MS_B - MS_E}{an} = \frac{5.830 - 1.807}{3\times 2} = 0.6705$ ✓

$\sigma_{\alpha\gamma}^2 = \frac{MS_{AB} - MS_E}{n} = \frac{2.937 - 1.807}{2} = 0.565$

h) $\sigma_{\tau\beta}^2 = 0.5651$ ← Interaction effect. $SE(\sigma_{\tau\beta}^2) = 0.6133$, $\alpha = 0.05$

① Normal distribution:

$\sigma_{\tau\beta}^2 \pm Z_{\alpha/2} \cdot SE(\sigma_{\tau\beta}^2)$

$(0.5651 \pm 1.96 \times 0.6133) \Rightarrow (-0.637,\ 1.767)$ ✓

② χ^2 distribution

$Z_0 = \frac{\sigma_{\tau\beta}^2}{SE(\sigma_{\tau\beta}^2)} = \frac{0.5651}{0.6133} = 0.921$

$v = 2 \cdot Z_0^2 = 2\times(0.921)^2 = 1.698$

if use df = 1 ($v=1$)

$\chi^2_{1,\alpha/2} = 5.024$

$\chi^2_{1,1-\alpha/2} = 0.001$

$\frac{v\sigma_{\tau\beta}^2}{\chi^2_{v,\alpha/2}} \le \sigma_{\tau\beta}^2 \le \frac{v\sigma_{\tau\beta}^2}{\chi^2_{v,1-\alpha/2}}$

$\frac{1\times 0.5651}{5.024} \le \sigma_{\tau\beta}^2 \le \frac{1\times 0.5651}{0.001}$

$0.112 \le \sigma_{\tau\beta}^2 \le 565.1$

if df = 2 ($v=2$)

$\chi^2_{2,0.025} = 7.378$

$\chi^2_{2,0.975} = 0.051$

$\frac{v\sigma_{\tau\beta}^2}{\chi^2_{v,\alpha/2}} \le \sigma_{\tau\beta}^2 \le \frac{v\sigma_{\tau\beta}^2}{\chi^2_{v,1-\alpha/2}}$

$\frac{2\times 0.5651}{7.378} \le \sigma_{\tau\beta}^2 \le \frac{2\times 0.5651}{0.051}$ ✓

$0.153 \le \sigma_{\tau\beta}^2 \le 22.161$

Question 3.

Random A α_i

fixed → B β_j

Random C γ_k

Random D δ_l

Random:

- AB $(\alpha\beta)_{ij}$ $\sigma^2_{\alpha\beta}$ other terms contain AB: ABC, ABD, ABCD
  C. D are random,
  SO, ABC, ABD, ABCD are considered in MS(AB)
- AC $(\alpha\gamma)_{ik}$
- AD $(\alpha\delta)_{il}$
- BC $(\beta\gamma)_{jk}$
- BD $(\beta\delta)_{jl}$
- CD $(\gamma\delta)_{kl}$ $\sigma^2_{\gamma\delta}$ other terms contain CD: ACD, BCD, ABCD
  A is random, B is fixed,
  SO ACD is included in MS(CD)
- ABC $(\alpha\beta\gamma)_{ijk}$
- ABD $(\alpha\beta\delta)_{ijl}$
- ACD $(\alpha\gamma\delta)_{ikl}$
- BCD $(\beta\gamma\delta)_{jkl}$
- ABCD $(\alpha\beta\gamma\delta)_{ijkl}$

a) If an interaction contains at least one random effect, the entire interaction is considered random. ✓

* All interaction terms are random.

* A. C. D random

* B: fixed

b) $$MS(AB) = \sigma^2 + cdm\,\sigma^2_{AB} + \boxed{dm\,\sigma^2_{ABC} + cm\,\sigma^2_{ABD} + m\,\sigma^2_{ABCD}}$$ ← Additional terms

$$= \sigma^2 + cdm\,\sigma^2_{\alpha\beta} + dm\,\sigma^2_{\alpha\beta\gamma} + cm\,\sigma^2_{\alpha\beta\delta} + m\,\sigma^2_{\alpha\beta\gamma\delta}$$

* Use specific values of a, b, c, n -0.5

c) $$MS(CD) = \sigma^2 + abm\,\sigma^2_{CD} + \boxed{bm\,\sigma^2_{ACD}}$$ ← Additional term.

$$= \sigma^2 + abm\,\sigma^2_{\gamma\delta} + bm\,\sigma^2_{\alpha\gamma\delta}$$ $+ n\sigma^2_{\alpha\beta\gamma\delta}$ -1