Question 1.

k=5

$2^{k-1} = 2^4 = 16$

48/50

Data:

Treatment	Height	A	B	C	D	E	ABCDE	ABD
ac	7.96	+	−	+	−	−	−	+
acde	8.31	+	−	+	+	+	−	−
abcd	7.42	+	+	+	+	−	−	+
abce	7.74	+	+	+	−	+	−	−
ad	8.12	+	−	−	+	−	−	−
ae	8.33	+	−	−	−	+	−	+
ab	7.61	+	+	−	−	−	−	−
abde	7.85	+	+	−	+	+	−	+
c	7.14	−	−	+	−	−	+	−
cde	7.86	−	−	+	+	+	+	+
bcd	7.18	−	+	+	+	−	+	−
bce	7.47	−	+	+	−	+	+	+
d	7.24	−	−	−	+	−	+	+
e	7.79	−	−	−	−	+	+	−
b	6.65	−	+	−	−	−	+	+
bde	7.48	−	+	−	+	+	+	−

a) as above.

b) $A = -ABCDE$, so A and ABCDE are confounded.

c) $A \times I = -ABCDE$

$\Rightarrow I = -BCDE$

d). The resolution of a two-level fractional factorial design is equal to the number of letters in the shortest word in the defining relation.

Thus, resolution IV

e)

$$\begin{aligned}
A &\times I = A \times -BCDE = -ABCDE \\
B &\times I = B \times -BCDE = -CDE \\
C &\times I = C \times -BCDE = -BDE \\
D &\times I = D \times -BCDE = -BCE \\
AB &\times I = AB \times -BCDE = -ACDE \\
AC &\times I = AC \times -BCDE = -ABDE \\
AD &\times I = AD \times -BCDE = -ABCE \\
BC &\times I = BC \times -BCDE = -DE \\
BD &\times I = BD \times -BCDE = -CE \\
CD &\times I = CD \times -BCDE = -BE \\
ABC &\times I = ABC \times -BCDE = -ADE \\
ABD &\times I = ABD \times -BCDE = -ACE \\
ACD &\times I = ACD \times -BCDE = -ABE \\
BCD &\times I = BCD \times -BCDE = -E \\
ABCD &\times I = ABCD \times -BCDE = -AE
\end{aligned}$$

f.) $k=5, P=1,$

$$\left.\begin{aligned} \text{Effect}_i &= \frac{1}{2^{k-P-1}} \cdot \text{Contrast}_i \\ SS_{\text{Effect}_i} &= \frac{1}{2^{k-P}} \cdot \text{contrast}_i^2 \end{aligned}\right\}$$

(overall effects confounded with their aliases.)

effects with No confounding:

$A = \frac{1}{2}([A]+[A]')$

$A = \frac{1}{2^{5-1-1}} \cdot ((7.96+8.31+7.42+7.74+8.12+8.33+7.61+7.85)$

$-(7.14+7.86+7.18+7.47+7.24+7.79+6.65+7.48))$

$= \frac{1}{2^3} \times 4.53$

$= 0.56625$ ✓

$ABD = \frac{1}{2^{5-1-1}}((7.96+7.42+8.33+7.85+7.86+7.47+7.24+6.65)$

$-(8.31+7.74+8.12+7.61+7.14+7.18+7.79+7.48))$

$= \frac{1}{2^3} \times (-0.59)$

$= -0.07375$ ✓

$SS_A = \frac{1}{2^{5-1}} \times (4.53)^2 = 1.28256$ ✓

$SS_{ABD} = \frac{1}{2^{5-1}} \times (-0.59)^2 = 0.02176$ ✓

Not sure whether it is still required to create Q-Q plot in h.) using a different method than in g.) It's not. !! :)

g.) See as Attached File (PROC UNIVARIATE)

h.) points corresponding to estimates that deviate from the straight Line Indicate potential significance.

Thus, A, B, BCD are potentially Significant.

$A \times -BCDE = -ABCDE$

$B \times -BCDE = -CDE$

$BCD \times -BCDE = -E$

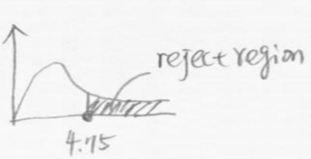
	DF	SS
Model	15	3.08785
Error	0	0
Total	15	3.08785

i)

	DF	SS	MS	F
A	1	1.28256	1.28256	46.102
B	1	0.70141	0.70141	25.212
E ~~BCD~~	1	0.77001	0.77001	27.678
Error	12	0.33387		
Total	15	3.08785		

- when performing analysis, only include one effect from each set of aliases in the ANOVA/regression model.
- choose "important" effects for the final model and pool the sums of squares of negligible effects into error sum of squares.

j) $F_{0.05,1,12} = 4.75$



$F_A = 46.102 > 4.75$, reject H_0, Significant. ✓

$F_B = 25.212 > 4.75$, reject H_0. Significant.

$F_{BCD} = 27.678 > 4.75$, reject H_0. Significant.

Thus, Effects of A, B, BCD E are significant.

$k=5,\ p=1$

k.) The complementary second experiment:

$I = BCDE$

$BC = BC \times I = BC \times BCDE = DE$

$= \frac{1}{2^3}\,\text{contrast}_{BC2}$

$= 0.17625$ (confounded with its alias)

$\text{Contrast}_{BC2} = 0.17625 \times 2^3$

$[BC] = BC + DE$

$= \frac{1}{2}\,\text{contrast}_{BC2}$

$= \frac{1}{2} \times 2^3 \times 0.17625$

$= 2^2 \times 0.17625$

$= 0.705$

The first experiment:

$I = -BCDE$

$BC = BC \times I = BC \times -BCDE = -DE$

$= \frac{1}{2^3}\,\text{contrast}_{BC1}$

$= 0.05375$ (confounded with its alias)

$\text{Contrast}_{BC1} = 2^3 \times 0.05375$

$[BC]' = BC - DE$

$= \frac{1}{2}\,\text{contrast}_{BC1}$

$= \frac{1}{2} \times 2^3 \times 0.05375$

$= 2^2 \times 0.05375$

$= 0.215$

$\frac{1}{2}([BC] + [BC]') = BC$

$= \frac{1}{2} \times (0.705 \cancel{+} 0.215)$ -1

$= 0.46$

l) $\frac{1}{2}([BC] - [BC]') = DE$

$= \frac{1}{2} \times (0.705 \cancel{-} 0.215)$ -1

$= 0.245$

m) See Attached File.

checked ANOVA Table in i).

	DF	SS	MS	F
A	1	1.28256	1.28256	46.102
B	1	0.70141	0.70141	25.212
BCD	1	0.77001	0.77001	27.678
Error	12	0.33387		
Total	15	3.08785		

checked.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	2.75396875	0.91798958	33.00	<.0001
Error	12	0.33382500	0.02781875		
Corrected Total	15	3.08779375			

Question 2. → Graded on completion. See solutions.

k=6, P=3, Factors: A, B, C, D, E, F

a) I = ABD = ACE = -BCF

I = ABD × ACE = BCDE

I = ABD × -BCF = -ACDF

I = ACE × -BCF = -ABEF

I = ABD × ACE × -BCF = -DEF

defining relations.

b) The resolution of a two-level fractional factorial design is equal to the number of letters in the shortest word in the defining relation.

Thus, Resolution III

c) 2^{6-3}_{III}

d)

B × ABD = AD

B × ACE = ABCE

B × -BCF = -CF

B × BCDE = CDE

B × -ACDF = -ABCDF

B × -ABEF = -AEF

B × -DEF = -BDEF

e)

CDF × ABD = ABCF

CDF × ACE = ADEF

CDF × -BCF = -BD

CDF × BCDE = BEF

CDF × -ACDF = -A

CDF × -ABEF = -ABCDE

CDF × -DEF = -CE

f.)

Treatment	observation	A	B	C	D	E	F	CDF
a	42.1	+	−	−	−	−	−	−
bc	26.7	−	+	+	−	−	−	+
de	32.7	−	−	−	+	+	−	+
abcde	36.9	+	+	+	+	+	−	−
abdf	42.5	+	+	−	+	−	+	−
cdf	39.6	−	−	+	+	−	+	+
bef	28.2	−	+	−	−	+	+	+
acef	39.8	+	−	+	−	+	+	−

$k=6$

$P=3$

$k-P-1=6-3-1=2$

$$\text{Effect}_i = \frac{1}{2^{k-P-1}} \text{ Contrast}_i$$ (confounded with their aliases)

$$B = \frac{1}{2^{6-3-1}} \times ((26.7+36.9+42.5+28.2)-(42.1+32.7+39.6+39.8))$$

$$= \frac{1}{2^2} \times (-19.9)$$

$$= -4.975$$

$$CDF = \frac{1}{2^{6-3-1}} \times ((26.7+32.7+39.6+28.2)-(42.1+36.9+42.5+39.8))$$

$$= \frac{1}{2^2} \times (-34.1)$$

$$= -8.525$$

g.) $$SS_{\text{Effect}_i} = \frac{1}{2^{k-P}} \cdot \text{Contrast}_i^2$$ (confounded with their aliases)

$$SS_B = \frac{1}{2^{6-3}} \times (-19.9)^2 = 49.501$$

$$SS_{CDF} = \frac{1}{2^{6-3}} \times (-34.1)^2 = 145.351$$

```
ods rtf file='hw5_output.rtf' startpage=NO;

data effects;
infile 'estimates.csv' firstobs=2 dsd;
input effect $ estimate;
run;

proc print data=effects;
run;

/* g)/h). use proc univariate to creat QQ plot */
/* not sure whether need to use some other software to create QQ plot, since
g) has already created */

proc univariate data=effects;
qqplot;
run;


proc sort data=effects;
by estimate;
run;

proc print data=effects;
run;


/* m). Effects of A B BCD are included. */

data obs;
infile 'springs.csv' firstobs=2 dsd;
input trt $ A B C D E y;
run;

proc print data=obs;
run;

data obs1;
set obs;
BCD=B*C*D;
run;

proc print data=obs1;
run;

proc glm data=obs1;
class A B C D E;
model y = A B BCD;
run;

ods rtf close;
```

Data: estimates.cvs

	A	B	C
1	effect	estimate	
2	A	0.56625	
3	B	-0.41875	
4	C	0.00125	
5	D	0.09625	
6	AB	-0.10625	
7	AC	-0.12125	
8	AD	-0.08125	
9	BC	0.05375	
10	BD	0.01875	
11	CD	0.01875	
12	ABC	-0.08375	
13	ABD	-0.07375	
14	ACD	-0.01875	
15	BCD	-0.43875	
16	ABCD	0.15875	
17			

Data: springs.cvs

	A	B	C	D	E	F	G	H
1	Treatment	A	B	C	D	E	y	
2	ac	1	-1	1	-1	-1	7.96	
3	acde	1	-1	1	1	1	8.31	
4	abcd	1	1	1	1	-1	7.42	
5	abce	1	1	1	-1	1	7.74	
6	ad	1	-1	-1	1	-1	8.12	
7	ae	1	-1	-1	-1	1	8.33	
8	ab	1	1	-1	-1	-1	7.61	
9	abde	1	1	-1	1	1	7.85	
10	c	-1	-1	1	-1	-1	7.14	
11	cde	-1	-1	1	1	1	7.86	
12	bcd	-1	1	1	1	-1	7.18	
13	bce	-1	1	1	-1	1	7.47	
14	d	-1	-1	-1	1	-1	7.24	
15	e	-1	-1	-1	-1	1	7.79	
16	b	-1	1	-1	-1	-1	6.65	
17	bde	-1	1	-1	1	1	7.48	
18								

The SAS System

Obs	effect	estimate
1	A	0.56625
2	B	-0.41875
3	C	0.00125
4	D	0.09625
5	AB	-0.10625
6	AC	-0.12125
7	AD	-0.08125
8	BC	0.05375
9	BD	0.01875
10	CD	0.01875
11	ABC	-0.08375
12	ABD	-0.07375
13	ACD	-0.01875
14	BCD	-0.43875
15	ABCD	0.15875

9).

The UNIVARIATE Procedure
Variable: estimate

Moments			
N	15	Sum Weights	15
Mean	-0.0285833	Sum Observations	-0.42875
Std Deviation	0.23294594	Variance	0.05426381
Skewness	0.58494888	Kurtosis	2.88611473
Uncorrected SS	0.77194844	Corrected SS	0.75969333
Coeff Variation	-814.97121	Std Error Mean	0.06014638

Basic Statistical Measures			
Location		Variability	
Mean	-0.02858	Std Deviation	0.23295
Median	-0.01875	Variance	0.05426
Mode	0.01875	Range	1.00500
		Interquartile Range	0.16000

The SAS System

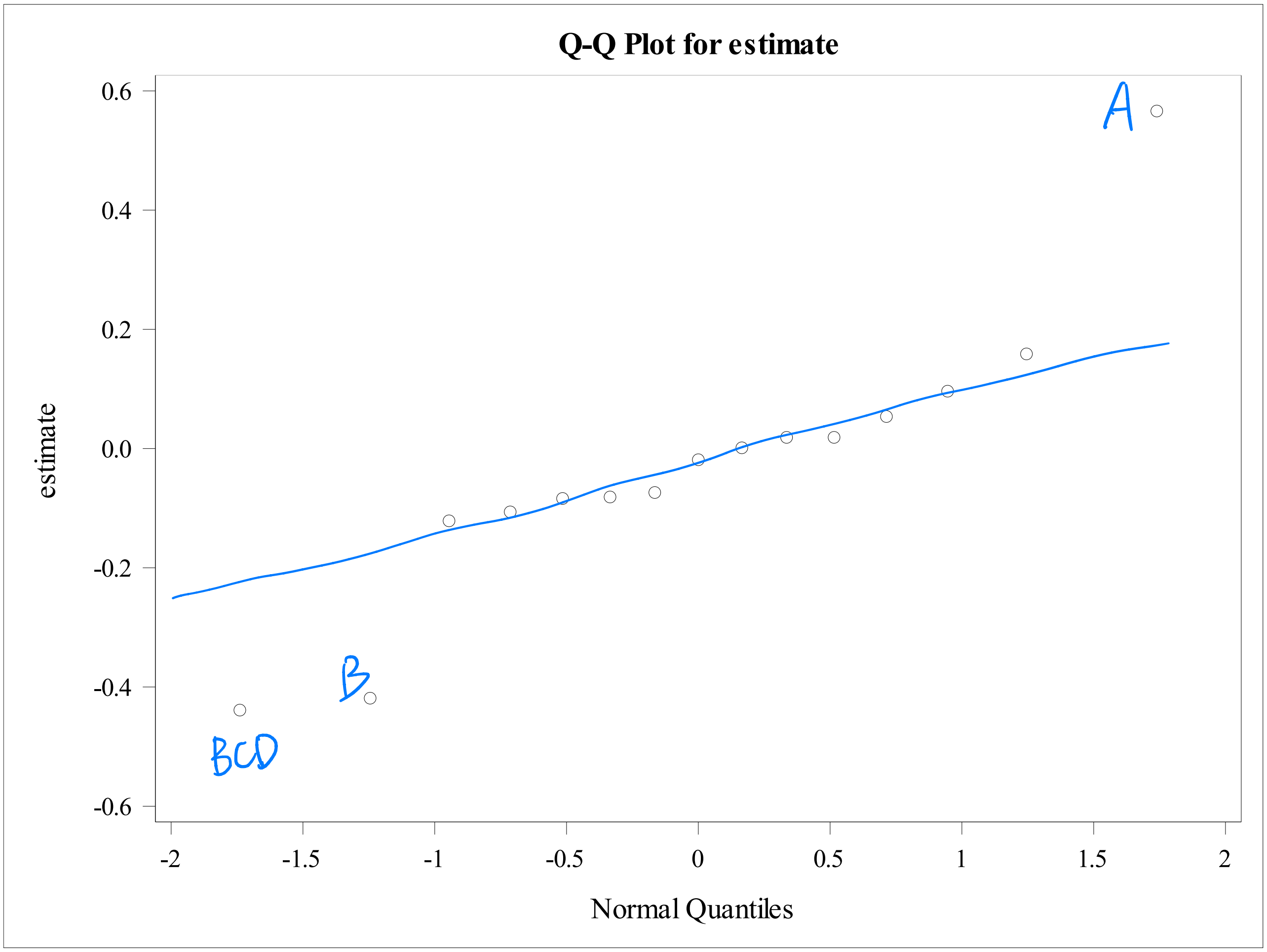
Tests for Location: Mu0=0				
Test	**Statistic**		**p Value**	
Student's t	**t**	-0.47523	**Pr > |t|**	0.6420
Sign	**M**	-0.5	**Pr >= |M|**	1.0000
Signed Rank	**S**	-12	**Pr >= |S|**	0.5242

Quantiles (Definition 5)	
Level	**Quantile**
100% Max	0.56625
99%	0.56625
95%	0.56625
90%	0.15875
75% Q3	0.05375
50% Median	-0.01875
25% Q1	-0.10625
10%	-0.41875
5%	-0.43875
1%	-0.43875
0% Min	-0.43875

Extreme Observations			
Lowest		**Highest**	
Value	**Obs**	**Value**	**Obs**
-0.43875	14	0.01875	10
-0.41875	2	0.05375	8
-0.12125	6	0.09625	4
-0.10625	5	0.15875	15
-0.08375	11	0.56625	1

The UNIVARIATE Procedure

The SAS System



h.) Choose the significant effects.

Obs	effect	estimate
1	BCD	-0.43875
2	B	-0.41875
3	AC	-0.12125
4	AB	-0.10625
5	ABC	-0.08375
6	AD	-0.08125
7	ABD	-0.07375
8	ACD	-0.01875
9	C	0.00125
10	BD	0.01875
11	CD	0.01875
12	BC	0.05375
13	D	0.09625
14	ABCD	0.15875
15	A	0.56625

The SAS System

Obs	trt	A	B	C	D	E	y
1	ac	1	-1	1	-1	-1	7.96
2	acde	1	-1	1	1	1	8.31
3	abcd	1	1	1	1	-1	7.42
4	abce	1	1	1	-1	1	7.74
5	ad	1	-1	-1	1	-1	8.12
6	ae	1	-1	-1	-1	1	8.33
7	ab	1	1	-1	-1	-1	7.61
8	abde	1	1	-1	1	1	7.85
9	c	-1	-1	1	-1	-1	7.14
10	cde	-1	-1	1	1	1	7.86
11	bcd	-1	1	1	1	-1	7.18
12	bce	-1	1	1	-1	1	7.47
13	d	-1	-1	-1	1	-1	7.24
14	e	-1	-1	-1	-1	1	7.79
15	b	-1	1	-1	-1	-1	6.65
16	bde	-1	1	-1	1	1	7.48

Obs	trt	A	B	C	D	E	y	BCD
1	ac	1	-1	1	-1	-1	7.96	1
2	acde	1	-1	1	1	1	8.31	-1
3	abcd	1	1	1	1	-1	7.42	1
4	abce	1	1	1	-1	1	7.74	-1
5	ad	1	-1	-1	1	-1	8.12	1
6	ae	1	-1	-1	-1	1	8.33	-1
7	ab	1	1	-1	-1	-1	7.61	1
8	abde	1	1	-1	1	1	7.85	-1
9	c	-1	-1	1	-1	-1	7.14	1
10	cde	-1	-1	1	1	1	7.86	-1
11	bcd	-1	1	1	1	-1	7.18	1
12	bce	-1	1	1	-1	1	7.47	-1
13	d	-1	-1	-1	1	-1	7.24	1
14	e	-1	-1	-1	-1	1	7.79	-1
15	b	-1	1	-1	-1	-1	6.65	1
16	bde	-1	1	-1	1	1	7.48	-1

The SAS System

The GLM Procedure

Class Level Information		
Class	**Levels**	**Values**
A	2	-1 1
B	2	-1 1
C	2	-1 1
D	2	-1 1
E	2	-1 1

Number of Observations Read	16
Number of Observations Used	16

The GLM Procedure

Dependent Variable: y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	2.75396875	0.91798958	33.00	<.0001
Error	12	0.33382500	0.02781875		
Corrected Total	15	3.08779375			

R-Square	Coeff Var	Root MSE	y Mean
0.891889	2.184718	0.166790	7.634375

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	1.28255625	1.28255625	46.10	<.0001
B	1	0.70140625	0.70140625	25.21	0.0003
BCD	1	0.77000625	0.77000625	27.68	0.0002

Source	DF	Type III SS	Mean Square	F Value	Pr > F
A	1	1.28255625	1.28255625	46.10	<.0001
B	1	0.70140625	0.70140625	25.21	0.0003
BCD	1	0.77000625	0.77000625	27.68	0.0002

The SAS System