# HW 6

Q7 a)

$$z = F \frac{1/m}{s^2 + b/m\,s + k/m}$$

$$s^2 + b/m\,s + k/m$$

$$p_0 = \frac{-b/m \pm \sqrt{b^2/m^2 - 4k/m}}{2}$$

b)

$$z = \mathcal{L}^{-1}(A\,U_s)$$

$$z = A \cdot (Y(k_d s) + k_p E)$$

$$= A \cdot (-Y k_d s + k_p(Y_r - Y))$$

$Y = z$

$Y_r = z_r$

$$z = \frac{b_0}{s^2 + a_1 s + a_0}(k_p(Y_r - Y) - Y k_d s)$$

$$z(s^2 + a_1 s + a_0) = b_0 k_p Y_r - b_0 k_p Y - b_0 Y k_d s$$

$$z(s^2 + a_1 s + a_0 + b_0 k_p + b_0 k_d s) = b_0 k_p Y_r$$

$$z = z_r \frac{b_0 k_p}{s^2 + s(a_1 + b_0 k_d) + b_0 k_p + a_0}$$

$$z = z_r \frac{k_p/m}{s^2 + s(b/m + k_d/m) + k_p/m + k/m}$$

$$p_c = \frac{-(b/m + k_d/m) \pm \sqrt{(b/m + k_d/m)^2 - 4(k_p/m + k/m)}}{2}$$

$k_p = k_d - 7.5$

$k_p = 1.5 k_d - 1.5(a)$

$k_p = 1.5(k_p + 7.5) - 1.5(a)$

c.

$k_p = 4.5$

$k_d = 12$

$$w_n = \frac{2.2}{2} = 1.1 \qquad t_r = \frac{2.2}{w_n} =$$

D.8 $t_r = 2s$ $\qquad \zeta = \sqrt{\left(\frac{w_d}{w_n}\right)^2 + 1}$ $\qquad k =$

$\zeta = .7$ $\qquad b =$

a) $w_n = 1.1$ $\qquad p_{1,2} = -\zeta w_n \pm w_n\sqrt{\zeta^2 - 1}$

$$\frac{b_0}{s^2 + a_1 s + a_0} \qquad \boxed{p_{1,2} = -.77 \pm j.786}$$

$$\frac{K w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

$b_0 = 1/m = .2$

$a_0 = k/m = 3/5$

$a_1 = b/m = .1$

$$\boxed{\Delta(s) = s^2 + 1.54s + 1.21} = s^2 + \alpha_1 s + \alpha_0$$

$a_1 + b_0 k_d = 2\zeta w_n \qquad a_0 + b_0 k_p = w_n^2$

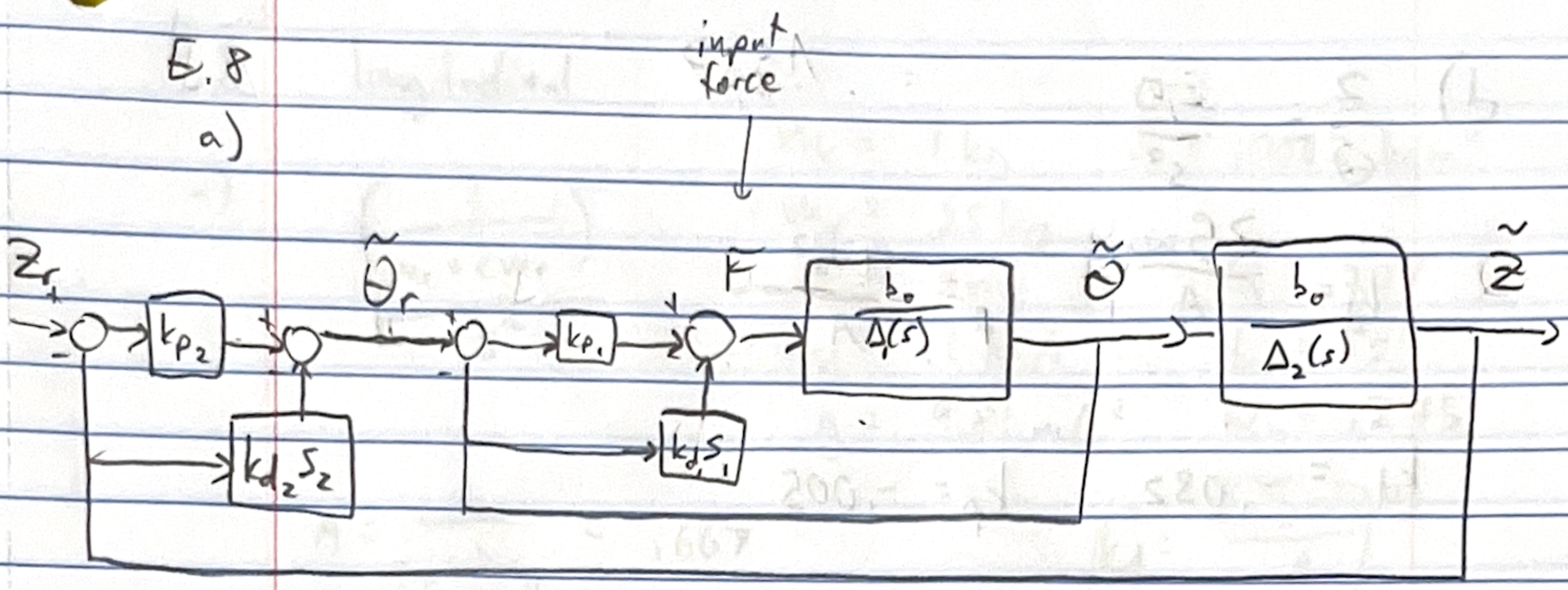
$$k_p = \frac{\alpha_0 - a_0}{b_0} \qquad k_d = \frac{\alpha_1 - a_1}{b_0}$$

$\boxed{k_p = 3.05}$ $\qquad \boxed{k_d = 7.2}$

b) $F_{max} = 6$

E.8

a)




b) $z_e = l/2$

$t_{r_\theta} = 1\,s$

$\zeta_\theta = .707$

$w_n = \frac{2.2}{1} : 2.2$

$m_1 = .35\,kg$

$m_2 = 2\,kg$

$l = .5m$

$g = 9.8\,m/s^2$

$z_e = l/2 = .25m$

$$\frac{\theta}{F} = \frac{\left(\frac{l}{\frac{m_2 l^2}{3} + m_1 z_c^2}\right)}{s^2}$$

$$\frac{l}{\frac{m_2 l^2}{3} + m_1 z_c^2} = A = 2.652$$

$$\Rightarrow \frac{k_p A}{s^2 + k_d A s + k_p A}$$

$$2\zeta w_n = k_d A$$

$$k_d = \frac{2\zeta w_n}{A} \quad k_p = \frac{w_n^2}{A}$$

$$\boxed{k_{d_\theta} = 1.173 \quad k_{p_\theta} = 1.83}$$

c) $$k_{dc_\theta} = \lim_{s \to 0} \frac{k_p A}{s^2 + k_d A s + k_p A}$$

$$= \frac{k_p A}{k_p A} = \boxed{1}$$

$$a_0 + b_0 k_p = w_n^2$$

$$a_1 + b_0 k_d = 2\zeta w_n$$

$\zeta = .707$

$t_r = 10 t_{r_\theta} = 10 \qquad \omega_n = \frac{3.2}{10}$

d) $\frac{z}{\theta} = \frac{-9}{s^2} \qquad A = -9$

$k_d = \frac{2\zeta\omega_n}{A} \qquad k_p = \frac{\omega_n^2}{A}$

$k_d = -.032 \qquad k_p = -.005$

e)

f)

F.8 longitudinal

$m_c = 1\,kg$ $\quad \bar{J}_c = .0042\,kg m^2$

$m_r = .25\,kg$

$d = .3m$ $\quad \zeta = .707$

$\mu = .1\,kg/s$ $\quad t_r = 8s$

$g = 9.81\,m/s^2$ $\quad w_n = .275$

a) $\left(\frac{1}{m_c + 2m_r}\right)\frac{}{s^2}$

$A = \frac{1}{m_c + 2m_r} = .667$

$k_{d_h} = \frac{2\zeta w_n}{A} = .583$

$\Delta(s) = s^2 + .389s + .076$

$p = \frac{-.389 \pm \sqrt{.389^2 - 4(.076)}}{2} = -.195 \pm j.195$

$k_{p_h} = \frac{w_n^2}{A} = .113$

b) inner loop

$\left(\frac{1}{J_c + 2m_r d^2}\right)\frac{}{s^2}$

$A = 20.325$

$t_{r_0} = .8s$

$w_n = 2.75s$

$k_{p_0} = \frac{w_n^2}{A} = .372$

$k_{d_0} = \frac{2\zeta w_n}{A} = .191$

$F_e = g(m_c + 2m_r)$
$= 14.7$

c) $K_{DC_0} = 1$

$w_{n_z} = .275$

d) $\frac{\left(\frac{-F_e}{m_c + 2m_r}\right)}{s\left(s + \frac{\mu}{m_c + 2m_r}\right)}\left(\frac{-F_e}{m_c + 2m_r}\right)^{b_0}$

$t_{r_z} = 10 t_{r_0} = 8s$

$\zeta = .707$

$b_0 = -9.8$

$a_0 = 0$

$a_1 = .067$

$\frac{}{s^2 + s\left(\frac{\mu}{m_c + 2m_r}\right)_{a_1}}$

$$k_p = \frac{-\omega_n^2 - a_0}{b_0} = -.0077$$

$$k_d = \frac{2\zeta\omega_n - a_1}{b_0} = -.0328$$