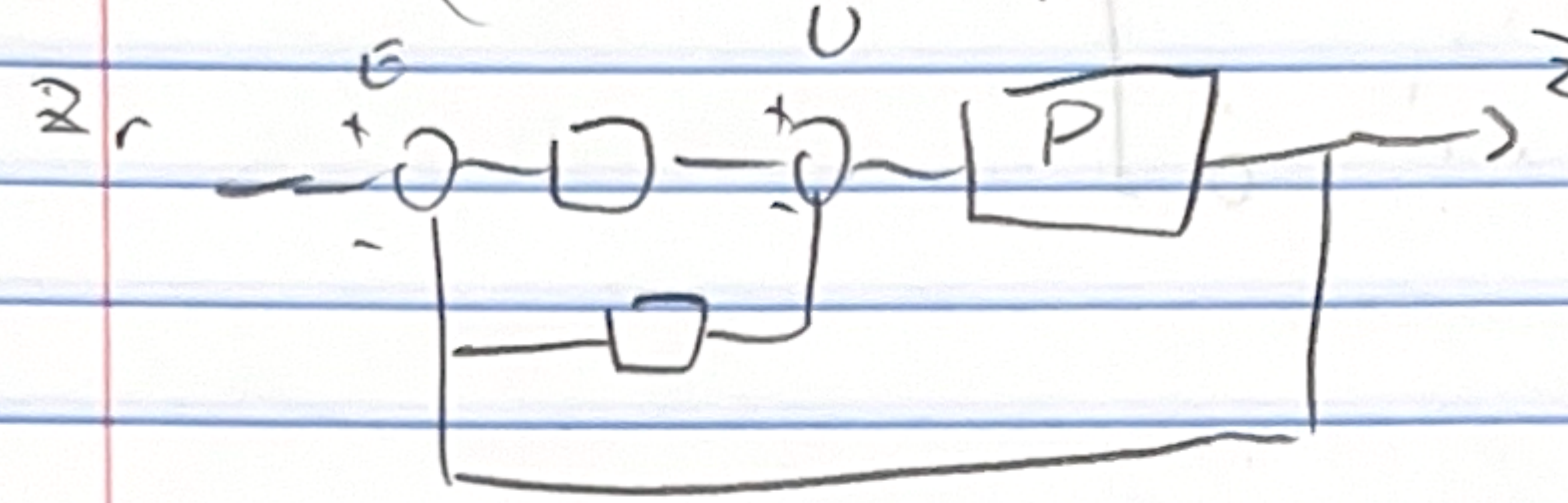
HW 7 D.9, E.9, $\overset{ac}{F.9}$

D.9

a) $Z = \left(\dfrac{1/m}{s^2 + b/m\, s + k/m}\right) = P$



$\boxed{\text{free integrator} = \text{pole @ } 0}$

$Z = PU$

$= P(k_p E - k_d s Z)$

$= P(k_p(Z_r - Z) - k_d s Z)$

$Z = P k_p s Z_r - P k_p Z - k_d s Z$

$Z(1 - P k_p - k_d s) = P k_p s Z_r$

$\dfrac{Z}{Z_r} = \dfrac{P k_p s}{1 - P k_p - k_d s}$

$\boxed{\text{no free int, type} = 0}$

step $\lim_{s \to 0} \dfrac{P k_p}{1 - P k_p - k_d s} \cdot \dfrac{1}{s}$

$\boxed{\begin{aligned} \text{step} &= \frac{1}{1 + M_p} \\ \text{ramp} &= \infty \\ \text{parabola} &= \infty \end{aligned}}$

$\dfrac{1}{1 + P(s)C(s)} = \dfrac{1}{1 + \left(\frac{1/m}{s^2 + b/m\, s + k/m}\right)(k_p + k_d s)}$

$\lim_{s \to 0} \dfrac{1}{1 + \left(\frac{1/m}{k/m}\right)(k_p)} = \boxed{\dfrac{1}{1 + k_p/k}}$

b) $E = \dfrac{P}{1 + PC} D$

$\lim_{s \to 0} \left(\dfrac{1/k}{1 + k_p/k}\right) \dfrac{A}{s^q} = \dfrac{A/k}{1 + k_p/k}$ w/ $q = 0$

$$\lim_{s\to 0} \frac{P}{1+PC}\left(\frac{A}{s^2}\right)$$

$$\frac{\frac{1/m}{s^2+b/m\,s+k/m}}{1+\left(\frac{1/m}{s^2+\frac{b}{m}s+k/m}\right)\left(\frac{k_d s^2+k_p s+k_i}{s}\right)}\left(\frac{A}{s^2}\right)$$

$$= \frac{s\,\frac{1/m}{s^2+b/m\,s+k/m}}{s+\frac{s/m}{s^2+b/m\,s+k/m}+k_d s^2+k_p s+k_i}\left(\frac{A}{s^2}\right)$$

$$\lim_{s\to 0} \frac{A/k}{k_i} \approx \boxed{\frac{A}{k_i}}$$

E.9

a) $$\frac{\left(\frac{l}{\frac{m_2 l^2}{2}+m_1 z_e^2}\right)}{s^2} = B$$

$$\frac{B}{s^2} = P \qquad \boxed{\text{type 2}}$$

step: 0
ramp: 0
parabola: $1/M_a$ $\qquad M_a = s^2 PC$

$$\frac{1}{s^2\left(\frac{B}{s^2}\right)(k_p+k_d s)} = \frac{1}{B(k_p+k_d s)} = \frac{1}{Bk_p+Bk_d s}$$

$$\lim_{s\to 0} = \frac{1}{Bk_p} \quad \text{steady state}$$

w/ $D(s)$

$$\frac{P}{1+PC} D(s)$$

$q = 2$

$$= \left(\frac{P}{1+PC}\right)\frac{A}{s^q}$$

$$= \frac{\left(\frac{B}{s^2}\right)}{1+\frac{B}{s^2}(k_p + k_d s)} \left(\frac{A}{s^q}\right)$$

$$= \frac{B \cdot A/s}{s^2 + B(k_p + k_d s)}$$

$$\lim_{s\to 0} \frac{B \cdot A/s}{0^2 + B(k_p + 0)} = \frac{A}{k_p}$$ type 0 w/ disturbance

b) outer loop

$$P = \frac{-g}{s^2}$$ $B = -g$

$$= \frac{B}{s^2}$$ $\boxed{\text{type } 2}$

step = 0
ramp = 0
parabola = $1/m_a$

steady state e = $\boxed{1/Bk_p}$

$\boxed{\text{now a type 3}}$

w/ integrator, $$\frac{1}{s^2 PC} = \frac{1}{s^2\left(\frac{B}{s^2}\right)\left(\frac{k_p s + k_d s^2 + k_i}{s}\right)}$$

$\boxed{\begin{matrix}\text{step} = 0 \\ \text{ramp} = 0 \\ \text{para} = 0\end{matrix}}$

sse = $\boxed{\frac{1}{Bk_i}}$

disturbance w/ PD control

$$\frac{P}{1+PC}\left(\frac{A}{s^q}\right) \quad \text{type} = 2 = q$$

$$\frac{\frac{B}{s^2}}{1+\frac{B}{s^2}(k_p+k_d s)}\left(\frac{A}{s^q}\right)$$

$$\frac{1}{\frac{s^2}{B}+(k_p+k_d s)}\left(\frac{A}{s^q}\right)$$

$$\frac{1}{k_p}\left(\frac{A}{s^q}\right) \quad q=0$$

type 0

$$= \frac{B}{s^2+B(k_p+k_d s)} = \frac{B}{s^2+B(k_p+k_d s)} \quad \lim_{s\to 0} \frac{0}{0+Bk_p} = 0$$

$$\lim_{s\to 0} = \frac{\frac{BA}{0}}{Bk_p} \quad \text{und} \qquad \text{type } 0$$

disturbance w/ PID

$$\frac{P}{1+PC}\left(\frac{A}{s^q}\right)$$

type $= q = 3$

$$\frac{1}{\frac{1}{P}+C}\left(\frac{A}{s^q}\right)$$

$$= \frac{1}{\frac{s^2}{B}+\left(\frac{k_d s^2+k_p s+k_i}{s}\right)}\left(\frac{A}{s^q}\right)$$

$$= \frac{s}{\frac{s^3}{B}+k_d s^2+k_p s+k_i}\left(\frac{A}{s^q}\right)$$

$$\lim_{s\to 0} \frac{s}{0+0+0+k_i}\left(\frac{A}{s^q}\right)$$

$$\boxed{\frac{1}{k_i} \text{ iff } q=1 \quad \text{type } 1}$$

F.9

a) longitude

$$\frac{\left(\frac{1}{m_c+2m_r}\right)}{s^2} = \frac{A}{s^2}$$ $\boxed{\text{type } 2}$

SSE	
step:	0
ramp:	0
para:	$1/M_a$

$$\frac{1}{s^2 PC} = \frac{1}{s^2(\cancel{B}/s^2)(k_p + k_d s)} = \frac{1}{\cancel{B}(k_p+k_d s)} \quad \lim_{s\to 0} = \boxed{\frac{1}{\cancel{B}k_p}}$$

w/ k_i

$$\frac{1}{s^2\left(\frac{\cancel{B}}{s^2}\right)\left(\frac{k_d s^2 + k_p s + k_i}{s}\right)} = s^2\frac{1}{\frac{\sim\sim}{s^3}}$$

this isn't going to give a valid answer, needs to eliminate s so we need a $\frac{1}{s^3 P(s) C(s)}$ SSE

$$\frac{1}{B\left(\frac{k_d s^2 + k_p s + k_i}{s}\right)} = \frac{s}{B(k_d s^2 + k_p s + k_i)}$$

this means $\boxed{\text{type } 3}$ $\boxed{\frac{1}{Bk_i}}$

PD dist

$$\frac{P}{1+PC}D \Rightarrow \frac{1}{\frac{1}{P}+C}D$$

$$\frac{1}{\frac{s^2}{B}+(k_p+k_d s)}D$$

$$\lim_{s\to 0} \frac{1}{\frac{0}{B}+(k_p+0)}\left(\frac{A}{s^q}\right) = \frac{1}{k_p}\left(\frac{A}{s^q}\right)$$

$q=0$

type 0

PID dist

$$\frac{1}{\frac{s^2}{B}+\left(\frac{k_d s^2+k_p s+k_i}{s}\right)}\left(\frac{A}{s^q}\right)$$

$$\frac{s}{\frac{s^3}{B}+k_d s^2+k_p s+k_i}\left(\frac{A}{s^q}\right)$$

$$\lim_{s\to 0} \frac{s}{0+0+0+k_i}\left(\frac{A}{s^q}\right) = \frac{sA}{k_i s^q}$$

$\boxed{q=1 \quad \text{type } 1}$

b) lateral

$$\frac{\left(\frac{1}{J_c+2m_r d^2}\right)}{s^2} = \frac{B}{s^2}$$

$\boxed{\text{type } 2}$

step = 0

ramp = 0

para = $1/M_a$

SSE = $1/k_p$ PD dist type? = $\frac{\frac{1}{P}}{\frac{1}{P}+C} D = \frac{1}{\frac{s^2}{B}+k_p+k_d s}\left(\frac{A}{s^2}\right)$

type 0

c) $\frac{B}{s(s+A)}$ $B = \left(\frac{-F_e}{m_c+2m_r}\right)$ $A = \left(\frac{\mu}{m_c+2m_r}\right)$

$\boxed{\text{type } 1}$

$$\boxed{\begin{array}{l} \text{step} = 0 \\ \text{ramp} = 1/M_v \\ \text{para} = \infty \end{array}}$$

$M_v = sPC$

$$\frac{1}{sPC}$$

$$= \frac{1}{s\left(\frac{B}{s(s+A)}\right)(k_p+k_d s)}$$

$$= \frac{1}{\left(\frac{B}{s+A}\right)(k_p+k_d s)}$$

$$\lim_{s\to 0} = \frac{1}{\left(\frac{B}{A}\right)(k_p)} = \boxed{\frac{A}{Bk_p} \text{ SSE}}$$

w/ k_i...

$$PC = \frac{B}{s(s+A)}\left(\frac{k_d s^2+k_p s+k_i}{s}\right)$$

type 2

step = 0

ramp = 0

para = $1/M_a$

$$\frac{1}{s^2\left(\frac{B}{s(s+A)}\right)\left(\frac{k_d s^2+k_p s+k_i}{s}\right)}$$

$$= \frac{1}{\left(\frac{B}{s+A}\right)(k_d s^2+k_p s+k_i)}$$

$\boxed{\frac{A}{Bk_i} \text{ SSE}_{k_i}}$

PD dist

$$\left(\frac{1}{\frac{1}{P}+C}\right)D$$

$$\left(\frac{1}{\frac{s(s+A)}{B}+k_p+k_d s}\right)\frac{A}{s^q}$$

$$\lim_{s\to 0}\ \frac{1}{k_p}\left(\frac{A}{s^q}\right)$$

$$\boxed{\frac{A}{k_p}}$$

$\boxed{q=0 \quad \text{type } 0}$

PID dist

$$\left(\frac{1}{\frac{s(s+A)}{B}+\left(\frac{k_d s^2+k_p s+k_i}{s}\right)}\right)\frac{A}{s^q}$$

$$\left(\frac{s}{\left(\frac{s^2(s+A)}{B}\right)+k_d s^2+k_p s+k_i}\right)\frac{A}{s^q}$$

$$\lim_{s_b\to 0}\ \frac{s}{0+0+0+k_i}\ \frac{A}{s^q}$$

$$=\ \frac{A}{k_i}$$

$\boxed{q=1 \quad \text{type } 1}$