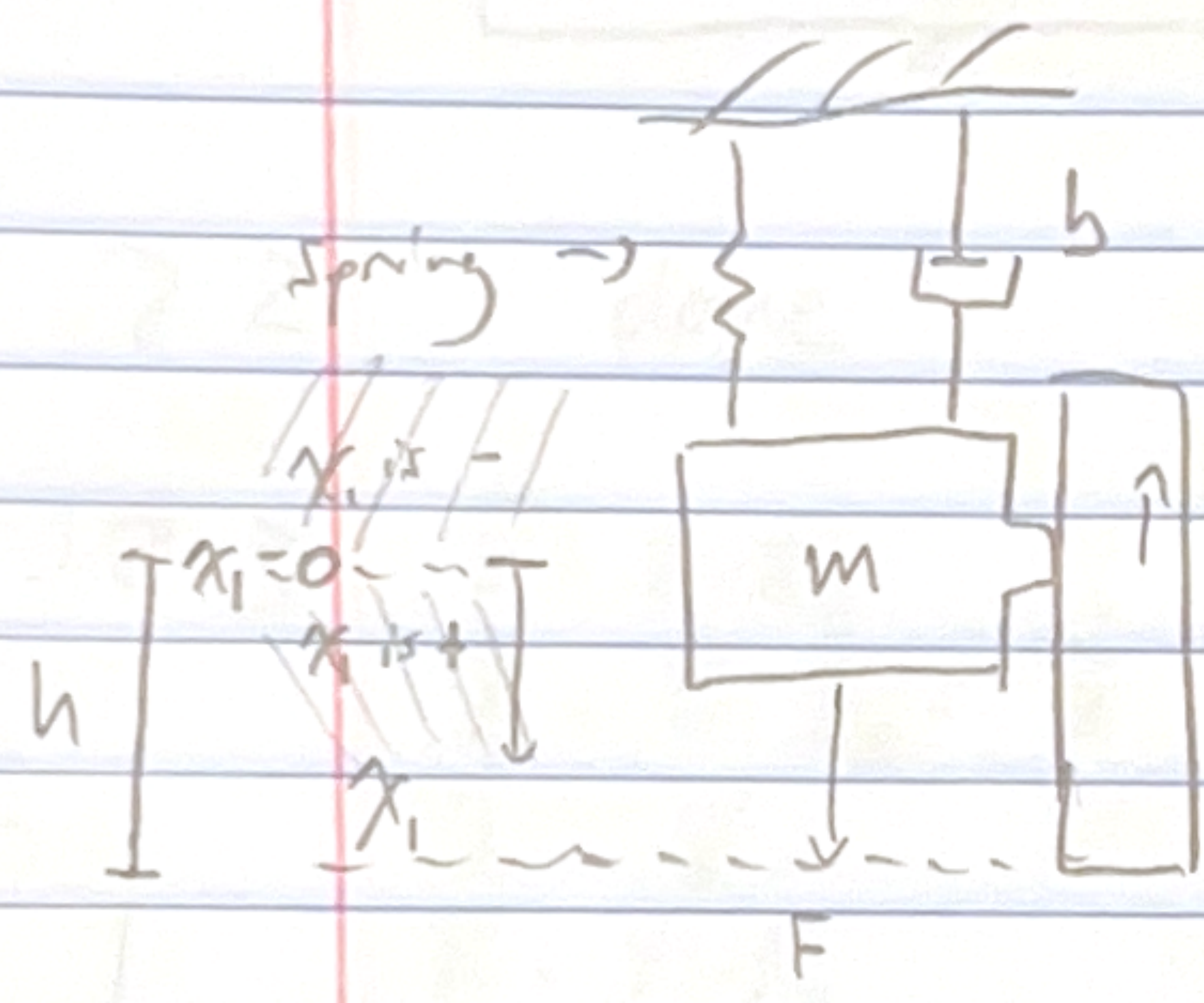
Final

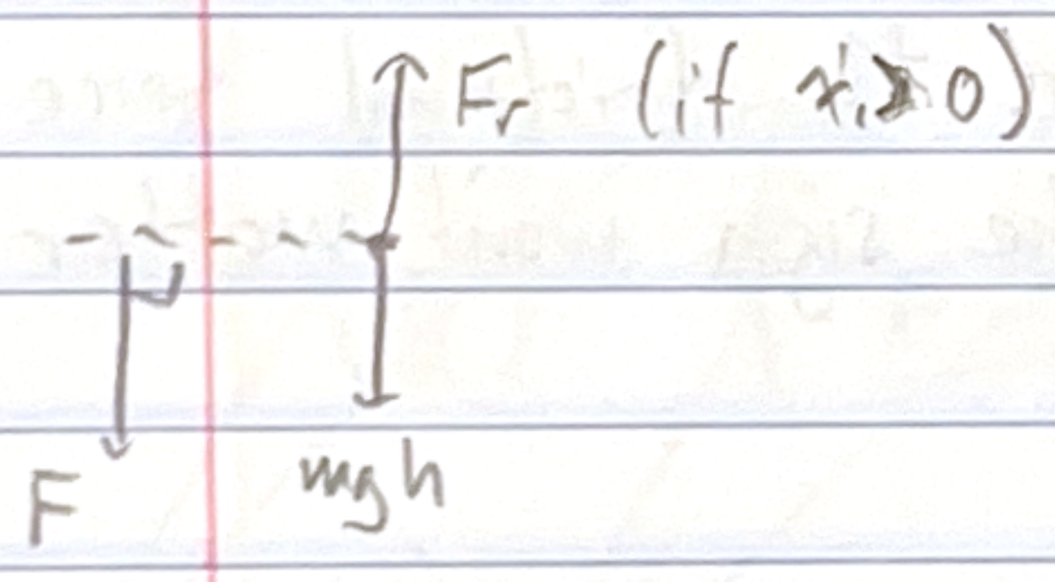


$$\text{sign}(\dot{x}_1) = \begin{cases} 1 & \dot{x}_1 > 0 \\ 0 & \dot{x}_1 = 0 \\ -1 & \dot{x}_1 < 0 \end{cases}$$

$F_r = c \cdot \text{sign}(\dot{x}_1)$

$h(x) = h - x_1$ $\qquad q = x_1$

$U_k = \frac{1}{2} k_1 x_1^2 + \frac{1}{4} k_2 x_1^4 \qquad U_g = mgh$



$q = x_1$
$\dot{q} = \dot{x}_1$

1.1 $K_e = \frac{1}{2} m v^T v$

$\boxed{K_e = \frac{1}{2} m \dot{x}^2}$

1.2 $\boxed{\begin{array}{l} U_g = mg(h - x_1) \\ U_s = \frac{1}{2} k_1 x_1^2 + \frac{1}{4} k_2 x_1^4 \end{array}}$

~~as x_1 is larger~~

when $h = x_1$
we are on the "ground" so $U_g = 0$

1.3 $L = K - P$
code!

$\boxed{L = m\ddot{x}_1 - k_2 x_1^3 - k_1 x_1 - mg}$

1.4 gen forces 1 gen coords so 1 gen force

$\tau - b\dot{q}$

$\tau = [F] \qquad q = [\dot{x}]$

$b = [b + c \cdot \text{sign}(\dot{x})]$

go to next page

code...

gen forces / dynamics

$$\boxed{m\ddot{x}_1 - k_2 x_1^3 - k_1 x_1 - gm = F - (b + c \cdot \text{sign}(\dot{x}_1))\dot{x}_1}$$

1.5 EoM

code...

$$\begin{bmatrix} x_1 \\ \dot{x}_1 \end{bmatrix} = \left(\begin{array}{c} \dot{x}_1 \\ \begin{cases} \dfrac{F + gm + k_1 x_1 + k_2 x_1^3}{m} & \text{when } \dot{x}_1 = 0 \\ \dfrac{F + gm + k_1 x_1 + k_2 x_1^3 - b\dot{x}_1}{m} - \dfrac{c\dot{x}_1^2}{m|\dot{x}_1|} & \text{otherwise} \end{cases} \end{array} \right)$$

this makes sense because the frictional force will always oppose motion, so the sign won't matter

Part 2

2.1 $F_e = \frac{-\sqrt{2}gm}{2}$ per code

2.2 done

2.3 $\tilde{z} = z - z_e$

$\dot{\tilde{z}} = \dot{z} - \dot{z}_e = \dot{\tilde{z}}$

$\ddot{\tilde{z}} = \ddot{\tilde{z}}$

$\tilde{F} = F - F_e$

$$F_e = \frac{-\sqrt{2}gm}{2} + k_1 z + k_2 z^3 + m(0) = F - b(0)$$

$$F = k_1 z + k_2 z^3 - \frac{\sqrt{2}}{2}gm + \tilde{F}$$

$$\cancel{\begin{pmatrix} \dot{\tilde{z}} \\ \ddot{\tilde{z}} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{F - \dot{\tilde{z}}b - \tilde{z}_e k_1 - \dot{z}b - k_1 z_e - k_2(\tilde{z}_e + z_e)^3 + m(\sqrt{2}g - \dot{z}_e)}{m} \end{pmatrix}}$$

$$\cancel{\begin{pmatrix} \dot{\tilde{z}} \\ \ddot{\tilde{z}} \end{pmatrix} = (}$$

$$\boxed{\tilde{F} - b\dot{\tilde{z}} = \ddot{\tilde{z}}m - \frac{\sqrt{2}gm}{2} + k_1(z_e + \tilde{z}) + k_2(z_e + \tilde{z})^3}$$

$[\tilde{z}, \dot{\tilde{z}}]$

2.4 jacobian

$$\begin{bmatrix} \dot{\tilde{z}} \\ \frac{-\dot{\tilde{z}}b + \tilde{F} - \tilde{z}^3 k_2 - k_1\tilde{z} + \frac{\sqrt{2}gm}{2}}{m} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ & \end{bmatrix}$$

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$z_e = 0$

$$\boxed{H = \frac{2}{s^2 + 0.2s + 0.1}}$$

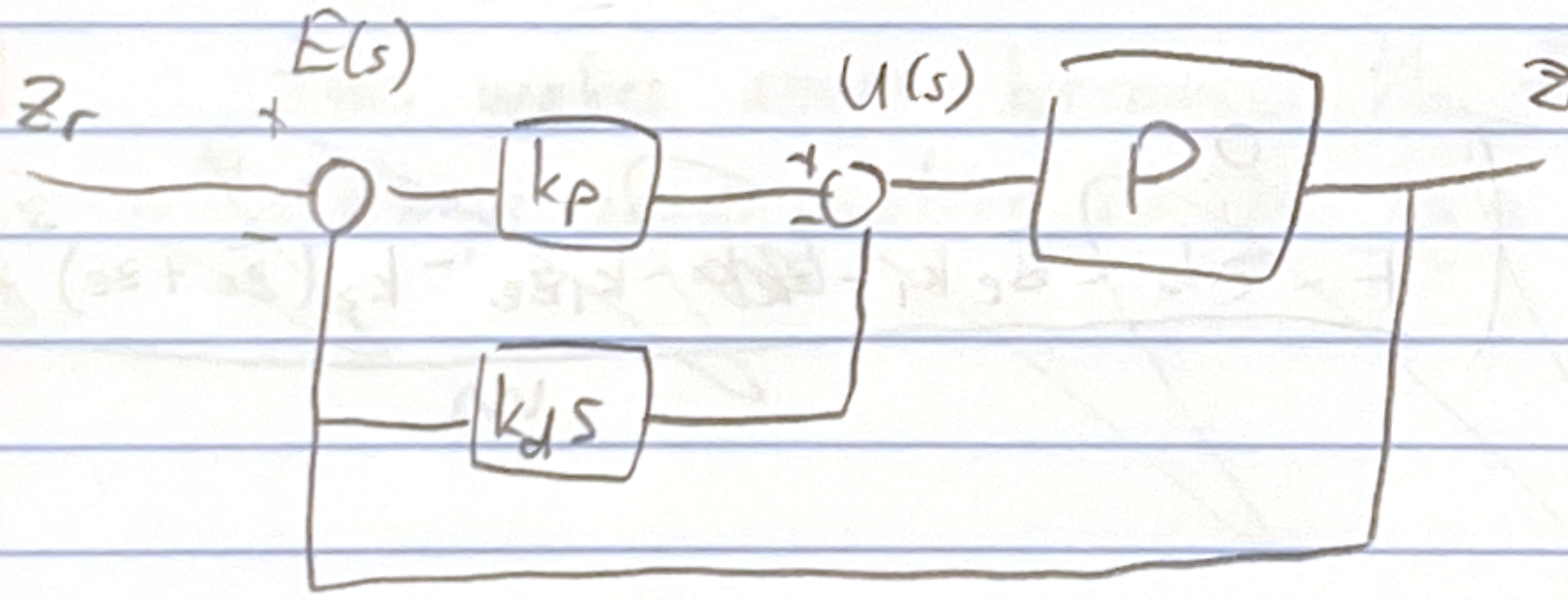
2.5

$\dot{x} = Ax + B\tilde{u}$

$y = Cx + D\tilde{u}$

$$\boxed{\begin{gathered}\begin{pmatrix}\dot{\tilde{z}} \\ \ddot{\tilde{z}}\end{pmatrix} = \begin{pmatrix}0 & 1 \\ -.1 & -.2\end{pmatrix}\begin{pmatrix}\tilde{z} \\ \dot{\tilde{z}}\end{pmatrix} + \begin{pmatrix}0 \\ 2\end{pmatrix}\tilde{u} \\ y = \begin{pmatrix}1 \\ 0\end{pmatrix}\begin{pmatrix}\tilde{z} \\ \dot{\tilde{z}}\end{pmatrix} + 0 \cdot \tilde{u}\end{gathered}}$$

3.1



3.2

$Z = PU$

$= P(k_p E - k_d s z)$

$= P(k_p(z_r - z) - k_d s z)$

$Z = P k_p z_r - P k_p z - k_d s z$

$z(1 + P k_p + k_d s) = P k_p z_r$

$$\boxed{\frac{z}{z_r} = \frac{P k_p}{1 + P k_p + k_d s}}$$

$F_{max} = 5N$ 1m step

3.3 $k_p = \pm \frac{u_{max}}{e_{max}} =$

$= \pm \frac{5}{1} = \boxed{\pm 5\ N/m} = k_p$

3.4 $\Delta_{cl}^{d} = s^2 + 2\zeta\omega_n s + \omega_n^2$

$$\frac{2}{s^2 + (2k_d + 0.2)s + 2k_p + 0.1}$$

$$\frac{1/m}{s^2 + s\frac{(b+k_d)}{m} + \frac{k_1 + k_p}{m}}$$

$$\boxed{\begin{aligned}\omega_n &= 3.178 \\ k_d &= 2.147\end{aligned}}$$

$$\frac{2}{s^2 + 4.49s + 10.1}$$

Part 4

4.1 poles $= (-2 \pm 2j)$

$(s-2+2j)(s-2-2j)$

$s^2 - 4s + 8$

$s^2 + .2s + .1 + 2K_1 + 2K_2 s$

$= s^2 + s(.2 + 2K_2) + .1 + 2K_1$

$$\boxed{K = (13.95, 4.4) \quad k_i = -20}$$