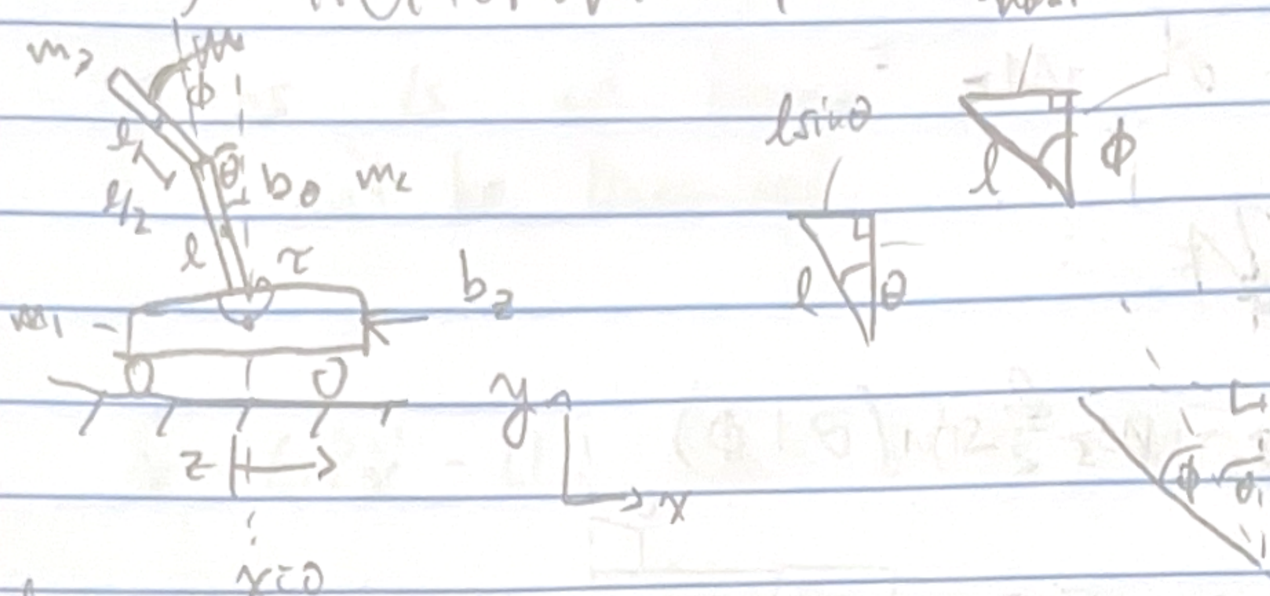
Midterm



1. ~~P~~

$$P_{3x} = z - l\sin\theta - l/2\sin(\phi+\theta)$$
$$P_{3y} = l\cos\theta + l/2\cos(\phi+\theta)$$

$$P_{2x} = z - l/2\sin\theta$$
$$P_{2y} = l/2\cos\theta$$

$$P_{1x} = z$$
$$P_{1y} = 0$$

3. $K_{r_1} = 0$

$K = \frac{1}{2}\dot{\theta}^T J \dot{\theta}$ $K_{r_3} = \frac{1}{2}\dot{\phi}^T J \dot{\phi}$ only ϕ because angle change is relative to m_3 & m_2

4. U_e?

$$m_3 = m_3 g(P_{3y}) = m_3 g(l\cos\theta + l/2\cos(\phi+\theta))$$
$$m_2 = m_2 g(P_{2y}) = m_2 g(l/2\cos\theta)$$
$$m_1 = m_1 g(P_{1y}) = 0$$

need cos, not sin

none

5. torque of m_3 =

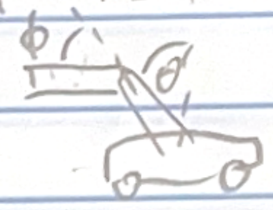
$$\tau_3 = m_3 \frac{\ell}{2} \sin(\theta + \phi)$$

if $\theta = 90$,

max τ_3 only if $\phi = 0$

or, if $\theta = 45$

max τ_3 iff $\phi = 45$,



none

6. b_q, given we have b_z & b_θ

$$-n \begin{bmatrix} b_z & 0 & 0 \\ 0 & b_\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{z} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}$$

$$-\begin{bmatrix} b_z \dot{z} \\ b_\theta \dot{\theta} \\ 0 \end{bmatrix}$$

7. cos is not linear,
can't be linearized

8.

$$\dddot{x} - 6.8\ddot{x} - 1.1\dot{x} + 3x = 4.2F_1 + 3.1F_2 - 5\phi$$

$$\phi + 7.1\phi = 1.6\ddot{x} - 2.9x - 9.9F_2$$

$$z = [x\ \dot{x}\ \ddot{x}\ \phi]^T \quad u = [F_1\ F_2]^T \quad y = [6.4\dot{x}\ \ \phi + F_1]^T$$

(2×4)(4×1) + (2×2)(2×1) → 2×1

$Cx + Du$

$$\begin{bmatrix} 0.64 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dddot{x} \\ \phi \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

b or d

9.

b or d again?

$x = [z, \dot{z}, \ddot{z}]^T = [i, z, \dot{z}]^T$

$u = \int V(t)$

10. $\dot{z} = z$

$\ddot{z} = \frac{k}{m}\left(\frac{1}{z}\right)^2 - g$

$V = L\dot{i} + Ri$

$\dot{i} = \frac{V - Ri}{L}$

$$\begin{bmatrix} \dot{i} \\ \dot{z} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} i \\ z \\ \dot{z} \end{bmatrix}$$

11. VAS

$U = (G_2 + G_3)E$

$U = (G_2 + G_3)(Y_r - G_4 Y)$

$U = (G_2 + G_3)(Y_r - G_4(G_1 U))$

$= G_2 Y_r + G_3 Y_r - (G_4 G_3 U - G_4 G_2 U) G_1$

$U(1 + G_4 G_3 + G_4 G_2) = Y(G_2 + G_3)$

$$\frac{U}{Y_r} = \frac{G_2 + G_3}{1 + G_4(G_3 + G_2)G_1}$$

13.

sys 1

$\dot{x} = -.4x + 1.2\dot{x} + 0$

$\ddot{x} = -.7(\dot{x}) + .2u$

$=$

$\dddot{x} = -.28x - .84\ddot{x} + .2u$

$s^3X = -.28X - .84s^2X + .2U$

$s^3X + .84s^2X + .28X = .2U$

$X(s^3 + .84s^2 + .28) = .2U$

$\frac{X}{U} = \frac{.2}{s^3 + .84s^2 + .28}$

$\frac{X}{\theta} = \frac{X}{U} \cdot \left(\frac{\theta}{U}\right)^{-1}$

sys 2

$\dot{\theta} = -.5\theta - .1\dot{\theta} + 0$

$\ddot{\theta} = .4\theta + 1.8\dot{\theta} + u$

$= .4\theta - .9\theta - .18\dot{\theta} + u$

$\dddot{\theta} = -.5\theta - .18\ddot{\theta} + u$

$s^3\theta = -.5\theta - .18s^2\theta + U$

$U = \theta s^3 + .18s^2\theta + .5\theta + \theta s^3$

$\frac{\theta}{U} = \frac{1}{s^3 + .18s^2 + .5}$

$\frac{X}{U} = \frac{X}{\theta} = \frac{.2}{s^3 + .84s^2 + .28}$

$\frac{\theta}{u} = \frac{1}{s^3 + .18s^2 + .5}$

nspire time!

?

14.

OL:

$$Y = PU$$
$$= P(k_p E - k_d s Y)$$
$$Y = P(k_p(Y_r - Y) - k_d s Y)$$
$$Y = Pk_p Y_r - (Pk_p Y) - Pk_d s Y$$
$$Y + Pk_p Y + Pk_d s Y = Pk_p Y_r$$
$$\frac{Y(s)}{Y_r} = \frac{Pk_p}{1 + Pk_p + Pk_d s}$$

$$P(s) = \frac{.87s - 3.3}{s^2 - 7.6s + 13.95}$$

$$s = \frac{7.6 \pm \sqrt{7.6^2 - 4(1)(13.95)}}{2}$$
$$= \frac{7.6 \pm 1.4}{2} = \boxed{4.5, 3.1}$$

15. $Y = MPU$

$= P(k_p E - k_d s Y)$

$= P(k_p(Y_r - Y) - k_d s Y)$

$= Pk_p Y_r - Pk_p Y - k_d s Y$

$Y + Pk_p Y + Pk_d s Y = Pk_p Y_r$

$Y(1 + Pk_p + k_d s) = Y_r(Pk_p)$

$$\frac{Y}{Y_r} = \frac{Pk_p}{1 + Pk_p + k_d s}$$

16. $t_r = 1.5$

$\zeta = .6$

$w_d = 2.2\sqrt{.6^2 - 1}$

$1.76?$

assuming $K = 1$

$$\frac{1.76}{s^2 + 2\zeta w_n s + w_n^2}$$

$1.2 w_n$ w_n^2

assuming $\frac{4}{5}$

$k_p =$

$k_d =$

$w_{max} = 2\pi f$

$= 2\pi(\frac{1}{5})$

$= 4\pi?$

w_n

17. $\frac{7.1}{s^2 + 8.5s + 16.24}$

$\alpha_0 = 8.5$

$\alpha_1 = 16.24$

$k_p = \frac{\alpha_0 - a_0}{b_0}$

$7.1 k_p$

$\alpha_1 = a_1 + b_0 k_d$

$b_0 = 7.1$

$k_d = \frac{\alpha_1 - a_1}{b_0}$

$\alpha_1 = 10.8$

$\alpha_0 = 27.47$

$\alpha_0 = b_0 k_p + a_0$

$k_p = \frac{\alpha_0 - a_0}{b_0}$

$7.1 = b_0 k_p$

18. $\frac{P k_p}{s^3 + 12 P k_p s^2 + k_d s}$

$$\frac{3.76}{(2.46s+1)(s^2+8.55s+16.24)+3.76}$$

$$\lim_{s \to 0} \frac{3.76}{(0+1)(0-0+16.24)}$$

$$\frac{3.76}{16.24+3.76} = \frac{3.76}{16.24+3.76}$$