Extracting Dataflow Communication from Object-Oriented Code

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Abstract

Object graphs help developers understand the runtime structure of an object-oriented system, in terms of objects and their runtime relations (points-to, call, or dataflow, depending on the intent of the diagram). Ideally, an object graph is sound and shows all possible objects and the relations between them. The object graph should also be hierarchical to scale and convey architectural abstraction. Achieving soundness requires a static analysis, but architectural hierarchy is not available in code written in general-purpose programming languages. To achieve hierarchy in a statically extracted object graph, we leverage ownership types in the code. We then abstractly interpret the annotated program and extract a global, sound, hierarchical object graph with dataflow communication edges that show the flow of objects due to field reads, field writes, and method invocations. We formalize the static analysis using a constraint-based specification and prove that the object graph is sound. We evaluate our analysis on an extended example and we show that the extracted edges are similar to those drawn by developers.

Keywords: hierarchical object graphs, dataflow communication, ownership domains

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1 Introduction

During software evolution, reverse-engineered diagrams of the code structure and of the runtime structure help developers to understand the system in order to modify it. Diagrams of the code structure are supported by many tools. Diagrams of the runtime structure, however, are more challenging and less mature.

One challenge with diagrams of the runtime structure is soundness, i.e., showing all possible objects and all possible relations between them. Achieving soundness requires static analysis since, by definition, dynamic analysis shows partial diagrams from a finite number of executions. Another challenge is to create a graph that scales and supports program understanding. A flat object graph, with its profusion of objects, does not meet this challenge. One solution is to use hierarchy, which provides both high-level and detailed understanding.

Architectural hierarchy is not observable in legacy object-oriented code, so we follow a previous approach [2] and use ownership types in the code, specifically, the Ownership Domains type system [4]. To support legacy code, we define annotations that implement the type system, using available language support for annotations. Developers use the annotations and specify, within the code, their design intent in the form of strict encapsulation, logical containment and architectural tiers. These annotations enable a static analysis to extract a sound, global, hierarchical Ownership Object Graph (OOG) [2]. An OOG provides architectural abstraction by ownership hierarchy and by types, where architecturally significant objects appear near the top of the hierarchy and data structures are further down.

In related work, we evaluated in a controlled experiment if OOGs, as diagrams of the runtime structure, help developers with program comprehension during coding activities, and thus complement widely-used class diagrams [6, 5]. We found that developers who used OOGs succeeded on code modification tasks, took less time, or explored less irrelevant code compared to developers who used only class diagrams or who just explored the code. In our previous experiment, developers wondered why the OOG did not show some relations between objects. The OOG showed only points-to edges due to field references that capture persistent relations between objects. In addition to points-to edges, developers need usage edges that capture more transient relations between

objects [11]. In this paper, we add to the OOG usage edges that make visually obvious the flow of objects in the program, and that we refer to as dataflow communication.

For instance, in object-oriented code that implements the Observer design pattern, understanding "what" gets notified during a change notification is crucial for understanding the system. "What" does not usually mean a class, "what" means a particular instance. Indeed, with many design patterns, developers need to understand the various instances in the system, and object graphs give insights into instances better than class diagrams. To understand what instances point to what other instances, points-to edges are useful. To understand not just "what" gets notified but also "what kind" of notification the subject of the notification sends to its observers, usage edges may be useful.

Contributions. In this paper, we propose a static analysis to extract a hierarchical object graph with usage edges showing dataflow communication. Our contributions are:

- We formalize the analysis using a constraint-based specification, showing the static and dynamic semantics;
- We prove the soundness of the extracted object graph;
- We evaluate our analysis on an extended example; we compare an OOG with dataflow edges to a diagram of the runtime structure with dataflow communication drawn by an expert, and to an OOG with points-to edges.

Outline. The rest of this report is organized as follows. In Section 2, we describe the challenges of designing a static analysis that extracts a runtime structure. In Section 3, we define dataflow communication. In Section 4, we formalize our analysis, and prove its soundness. We introduce a small example, and describe the analysis on a worked example in Section 5. We discuss related work in Section 6 and conclude.

2 Challenges of Static Analysis

We first discuss the challenges of extracting object graphs statically. At runtime, the structure of an object-oriented program can be represented as a Runtime Object Graph (ROG), where nodes represent objects, i.e., instances of classes, and edges represent relations between objects, such as

one object calling another object's methods. A sound static analysis extracts an object graph that approximates all possible ROGs, for any program execution. We refer to the extracted object graph as an OGraph that has nodes that are OObjects and edges that are OEdges. An OObject is a canonical object that represents multiple runtime objects. Similarly, an OEdge is a canonical edge that represents runtime dataflow communication between the corresponding runtime objects. An OGraph has the following requirements:

Object soundness. The OGraph must show a unique representative for each runtime object. While one OObject can represent multiple runtime objects, the same runtime object cannot map to two separate OObjects. Since we are using OGraphs to gain high-level understanding, it would be misleading to have one runtime entity appear as two boxes on an architectural diagram.

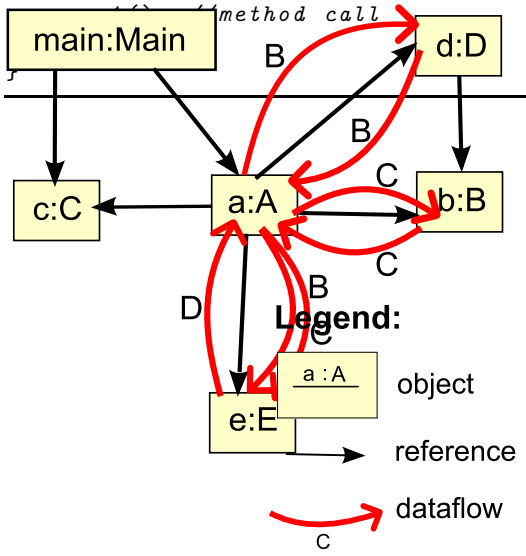
Aliasing. In particular, the static analysis must soundly handle possible aliasing in the program by enforcing the unique representatives invariant. For two variables in the program that may alias and refer to the same runtime object, the analysis must create a single OObject.

Edge soundness. If there is a runtime dataflow communication between two runtime objects, the OGraph must show an OEdge between the representative of these objects.

Summarization. An ROG can have an unbounded number of runtime objects. For example, in the presence of recursive types, the ROG might have an unbounded depth. The OGraph must be a finite representation of all ROGs and must have a finite depth. The static analysis must stop creating new nodes in the OGraph at some level, and instead use already created nodes. A common heuristic is for the analysis to stop when it gets to a node of the same type as a node it previously created.

Hierarchy. A global OGraph must convey architectural abstraction by object hierarchy and support both high-level and detailed understanding of the runtime structure. It must show architecturally significant OObjects near the top of the hierarchy and OObjects representing data structures further down.

```
1  class Main{
2    A a;   C c;
3    void main(){
4      main = new Main();
5      main.run();
6    }
7    void run(){
8      a = new A(c);
9      //no dataflow communication
10     //method call
11
12 }
```




```
1  class A{
2    B b; C c; D d; E e;
3    A(C c){
4     //no dataflow communication
5     //field initialization
6      this.c = c;
7    }
8    void m1(){
9    //method call (a -B-> d)
10     d.setB(b);
11   }
12   B m2(){
13   //method call (d -B-> a)
14     return d.getB();
15   }
16   C m3(){
17   //field read (b -C-> a)
18     return b.c;
19   }
20   void m4(){
21   //field write (a -C-> b)
22     b.c = c;
23   }
24   D m5(){
25   //method call(a -B-> e, a -C-> e, e -D-> a)
26     return e.me(b,c);
27   }
28 }
```

Figure 1: Example of export and import dataflow communication.

Precision. The analysis must not merge objects excessively. For example, an OGraph that represents all the runtime objects with one node is sound but very imprecise. Ideally, the OGraph must have no more OEdges than soundness requires. Like any sound static analysis, however, the OGraph may have false positives and may show OObjects or OEdges that do not correspond to a runtime object or runtime relation, due to infeasible paths in the program.

3 Dataflow Communication

Dataflow communication: *Let* `a` *and* `b` *be two objects. A dataflow communication exists from* `a` *to* `b` *if* `a` *reads or writes to* `b`*'s fields or calls* `b`*'s methods.*

In object-oriented code, a dataflow communication between two references `a` and `b` corresponds to field writes, field reads, or method invocations. Since the communication can be bidirectional,

we introduce two additional definitions.

Import dataflow communication: *An import dataflow communication exists from the source* `b` *of type* `B` *to the destination* `a` *of type* `A` *if* `a` *receives data from* `b`. That is, there is a method `ma` of *A* such that `ma` refers to `b.f` or uses the result returned by a method `mb` of `B`.

Export dataflow communication: *An export dataflow communication exists from the source* `a` *of type* `A` *to the destination* `b` *of type* `B` *if one of* `b`*'s field* `f` *may be modified when one of* `a`*'s methods is invoked.* That is, there is a method `ma` of `a` such that `ma` contains the statement `b.f = c` or `b.mb(c)`, where `c` is in the scope of `ma`, i.e., a field of `A`, an argument of `ma`, an object instantiated by `ma`, or an object returned by another method invoked by `ma`.

To understand the above definitions, consider the example in Figure 1, which has the code for the classes `Main` and `A`, and the corresponding flat object graph. For brevity consider that all the variables are correctly initialized, we do not include the code for the classes `B..E`. In the object graph, the nodes corresponds to objects, and there are two types of edges. Straight arrows means that an object refer another object, while curved arrows correspond to dataflow communications between objects. The curve arrows are labeled with the type of the data communicated between objects.

Dataflow communication exists due to the statements of the methods of `A`, `m1()` to `m5()`. Import dataflow communication exist from `b` to `a` and from `d` to `a` due to the field read and method invocation expressions of `m3()` and `m2()`, respectively. Also, export dataflow communication exist from `a` to `b` and from `a` to `d` due to the field write and method invocation expressions of `m4()` and `m1()`, respectively. Due to only one method invocation expression in `m5()`, two export dataflow communication exist from `a` to `e`, and due to the same expression, an import dataflow communication exists from `e` to `a`.

On the other hand, in the last statement of `run()`, the invocation of `m1()` does not correspond to any import or export dataflow communication, since the method has no arguments, and it returns `void`. Also, there is no dataflow communication from `main` to `a` even though the constructor of `A` has an argument. Dataflow communication definitions ignore object allocation because we consider creation and usage of objects as separate relations, and we distinguish between field initialization

$$\frac{\Gamma, \Sigma, \theta \vdash e : T_0 \qquad fields(T_0) = \overline{T}\ \overline{f} \qquad \Gamma, \Sigma, \theta \vdash e' : T \qquad T <: T_i}{\Gamma, \Sigma, \theta \vdash e.f_i = e' : T}[\text{T-WRITE}]$$

$$\frac{S[\ell] = C\text{<}\overline{p}\text{>}(\overline{v}) \qquad fields(C\text{<}\overline{p}\text{>}) = \overline{T}\ \overline{f} \qquad S' = S[\ell \mapsto C\text{<}\overline{p}\text{>}([v/v_i]\overline{v})]}{\ell.f_i = v; S \rightsquigarrow v; S'}[\text{R-WRITE}]$$

$$\frac{\theta \vdash e_0; S \to e_0'; S'}{\theta \vdash e_0.f_i = e_1; S \to e_0'.f_i = e_1; S'}[\text{RC-WRITE-RCV}]$$

$$\frac{\theta \vdash e_1; S \to e_1'; S'}{\theta \vdash v.f_i = e_1; S \to v.f_i = e_1'; S'}[\text{RC-WRITE-ARG}]$$

Figure 2: Field write semantics.

in a constructor and field write. That is why, there is no dataflow communication between `main` and `a` and between `a` and `c`.

4 Formalization

4.1 Abstract Syntax

We formally describe our static analysis using Featherweight Domain Java (FDJ), which models a core of the Java language with ownership domain annotations [4]. To keep the language simple and easier to reason about, FDJ uses Featherweight Java (FJ), and it ignores Java language constructs such as interfaces and static fields and methods.

We adopt the FDJ abstract syntax (Fig. 3) but with the following changes. We exclude cast expressions and domain links, which are part of FDJ, but not crucial to our discussion. We also include a field write expression $e.f = e'$, which can lead to dataflow communication. (Fig. 2)

In FDJ, C ranges over class names; T ranges over types; f ranges over field names; v ranges over values; d ranges over domain names; e ranges over expressions; x ranges over variable names; n ranges over values and variable names; S ranges over stores; ℓ and θ ranges over locations in a store; a store S maps locations ℓ to their contents; the set of variables includes the distinguished

$$
\begin{array}{rcl}
CT & ::= & \overline{cdef} \\
cdef & ::= & \texttt{class}\ C\texttt{<}\overline{\alpha},\overline{\beta}\texttt{>}\ \texttt{extends}\ C'\texttt{<}\overline{\alpha}\texttt{>} \\
 & & \{\ \overline{dom};\ \ \overline{T}\ \overline{f};\ \ C(\overline{T'}\ \overline{f'},\overline{T}\ \overline{f})\{\texttt{super}(f');\texttt{this}.\overline{f}=\overline{f};\}\ \ \overline{md}\ \} \\
dom & ::= & [\texttt{public}]\ \texttt{domain}\ d; \\
md & ::= & T_R\ m(\overline{T}\ \overline{x})\ \ T_{this}\ \{\texttt{return}\ e_R;\} \\
e & ::= & x\ \mid \texttt{new}\ C\texttt{<}\overline{p}\texttt{>}(\overline{e})\ \mid e.f\ \mid e.f=e'\ \mid e.m(\overline{e})\ \mid \ell\ \mid \ell \triangleright e \\
n & ::= & x\ \mid\ v \\
p & ::= & \alpha\ \mid\ n.d\ \mid\ \texttt{SHARED} \\
T & ::= & C\texttt{<}\overline{p}\texttt{>} \\
v,\ell,\theta & \in & \textit{locations} \\
S & ::= & \ell\ \rightarrow\ C\texttt{<}\overline{\ell'.d}\texttt{>}(\overline{v}) \\
\Sigma & ::= & \ell\ \rightarrow\ T \\
\Gamma & ::= & x\ \rightarrow\ T
\end{array}
$$

Figure 3: Simplified FDJ abstract syntax [4].

variable `this` of type T_{this} used to refer to the receiver of a method; the result of the computation is a location ℓ, which is sometimes referred to as a value v; θ represents the value of `this`; $S[\ell]$ denotes the store entry of ℓ; $S[\ell, i]$ denotes the value of i^{th} field of $S[\ell]$; $S[\ell \mapsto C\texttt{<}\overline{\ell'.d}\texttt{>}(\overline{v})]$ denotes adding an entry for location ℓ to S; α and β range over formal domain parameters; m ranges over method names; p ranges over formal domain parameters, actual domains, or the special domain `SHARED`; the expression form $\ell \triangleright e$ represents a method body e executing with a receiver ℓ; an overbar denotes a sequence; the fixed class table CT maps classes to their definitions; a program is a tuple (CT, e) of a class table and an expression; Γ is the typing context; and Σ is the store typing.

4.2 Data Type Declarations

Our analysis produces a hierarchical object graph (OGraph), which has nodes representing objects and domains, and edges representing dataflow communication (Fig. 4). The OGraph is a triplet $G = \langle DO, DD, DE \rangle$, where DO is a set of OObjects, and DD maps a pair $(O, C :: d)$ to an ODomain D, i.e., DD maintains a mapping from a local domain or a domain parameter d of an OObject O to an actual domain D. Each E in DE is a directed edge from a source O_{src} to a destination O_{dst}, and the label C is the class of the object being communicated. Multiple edges with different labels might exists between two OObjects. To keep the OGraph representation simpler, DE does not distinguish between import and export edges.

Our analysis distinguishes between different instances of the same class C that are in different

$G \in \mathsf{OGraph}$	$::= \langle\ \mathbf{Objects} = DO,\ \mathbf{Domains} = DD,\ \mathbf{Edges} = DE\ \rangle$	
$D \in \mathsf{ODomain}$	$::= \langle\ \mathbf{Id} = D_{id},\ \mathbf{Domain} = C{::}d\ \rangle$	
$O \in \mathsf{OObject}$	$::= \langle\ \mathbf{Id} = O_{id},\ \mathbf{Type} = C\!<\!\overline{D}\!>\ \rangle$	
$E \in \mathsf{OEdge}$	$::= \langle\ \mathbf{From} = O_{src},\ \mathbf{To} = O_{dst}, \mathbf{Class} = C\ \rangle$	
DD	$::= \emptyset \mid DD \cup \{\ (O, C{::}d) \mapsto D\ \}$	Dataflow Domain
DO	$::= \emptyset \mid DO \cup \{\ O\ \}$	Dataflow Object
DE	$::= \emptyset \mid DE \cup \{\ E\ \}$	Dataflow Edge
Υ	$::= \emptyset \mid \Upsilon \cup \{\ C\!<\!\overline{D}\!>\ \}$	Visited objects
H	$::= \emptyset \mid H \cup \{\ \ell \mapsto O\ \}$	Object map
K	$::= \emptyset \mid K \cup \{\ \ell.d \mapsto D\ \}$	Domain map
L_I	$::= \emptyset \mid L_I \cup \{\ (\ell_{src}, \ell_{dst}) \mapsto \{E\}\}$	Edge map import
L_E	$::= \emptyset \mid L_E \cup \{\ (\ell_{src}, \ell_{dst}) \mapsto \{E\}\}$	Edge map export

Figure 4: Data type declarations for the OGraph.

domains, even if created at the same `new` expression. In addition, the analysis treats an instance of class C with actual parameters $\overline{p}$ differently from another instance that has actual parameters $\overline{p'}$. Hence, the data type of an OObject uses $C\!<\!\overline{D}\!>$ instead of just a type and an owning ODomain. We follow the FDJ convention and consider an OObject's owning ODomain as the first element D_1 of $\overline{D}$. As a result of the aliasing precision provided by ownership domains, our analysis avoids merging objects excessively. It only merges two objects of the same class if all their domains are the same. The context Υ records the combination of class and domain parameters $C\!<\!\overline{D}\!>$ analyzed in the call stack to avoid non-termination of the analysis due to recursive calls.

To invoke the analysis, a developer picks a root class, which is instantiated into a root object. The root class can take only one domain parameter to represent the owning domain. Typically, the root object is in the global ODomain $D_{\texttt{SHARED}}$, the root of the OGraph.

Although a domain d is declared by class C, each instance of C gets its own runtime domain $\ell.d$. For example, if there are two distinct object locations ℓ and ℓ' of class C, then the analysis distinguishes between $\ell.d$ and $\ell'.d$. Since an ODomain represents a runtime domain $\ell_i.d_i$, one domain declaration d in the code can create multiple ODomains D_i in the OGraph. We qualify a domain d by the class that declares it, as $C{::}d$ alongside with a unique D_{id}. Since no class declares the `SHARED` domain, we qualify it as ::`SHARED`.

Instrumentation. The maps H, K, L_I, and L_E are part of the instrumented dynamic semantics (Fig. 4). H maps a location ℓ to the corresponding OObject, and K maps a runtime domain $\ell.d$ to an ODomain. The multi-valued maps L_I and L_E map a pair of locations (ℓ_{src}, ℓ_{dst}) to a set of OEdges $\{E\}$. We use two maps for edges because a pair ($H[\ell_1]$, $H[\ell_2]$) can be associated with an import edge from $H[\ell_1]$ to $H[\ell_2]$, or with an export edge from $H[\ell_1]$ to $H[\ell_2]$.

Notation. For a map M, a key k, and a value v, we use $M[k]$ to denote the lookup of k, and $M' = M[k \mapsto v]$ for adding an entry for k to M. For a multi-valued map M, we use the notation $M' = M[k \mapsto_\cup \{v\}]$ for adding an entry for k to M. If the map already has an entry for k, the resulting value is the union of the existing value set and $\{v\}$.

Static Semantics. We formalize our static analysis using a constraint-based specification, as a set of inference rules, then prove that the OGraph is sound, i.e., it has all the required OObjects, ODomains, and OEdges.

In FDJ, a program is a tuple (CT, e) that consists of a class table CT, which maps classes to their definitions, and an expression e. Our analysis starts with a root expression e_{root}, that explicitly instantiates the root class C_{root}. The analysis result is the least solution $G = \langle DO, DD, DE \rangle$ of the following constraint system:

$$\emptyset, \emptyset, DO, DD, DE \vdash (CT, e_{root})$$

The analysis creates the OObject O_{root} and its owning ODomain $D_{\texttt{SHARED}}$:

$$D_{\texttt{SHARED}} = \ \langle\ D_s,\ \texttt{::SHARED}\ \rangle$$
$$O_{root} = \ \langle\ O_{root},\ C_{root}\texttt{<}D_{\texttt{SHARED}}\texttt{>}\ \rangle$$

The analysis then abstractly interprets e_{root} in the context of O_{root}:

$$\emptyset, \emptyset, DO, DD, DE \vdash_{O_{root}} e_{root}$$

$$CT(C) = \texttt{class}\ C\texttt{<}\overline{\alpha}, \overline{\beta}\texttt{>}\ \texttt{extends}\ C'\texttt{<}\overline{\alpha}\texttt{>}\ \{\ \overline{T}\ \overline{f};\ \overline{dom};\ \ldots;\ \overline{md};\ \}$$

$$CT(\texttt{Object}) = \texttt{class Object<}\alpha_o\texttt{>}\ \ \{\ \ \}$$

$$\frac{\begin{array}{c}\forall i \in 1..|\overline{p}| \qquad D_i = DD[(O, p_i)] \qquad params(C) = \overline{\alpha} \\ O_C = \langle\ O_{id},\ C\texttt{<}\overline{D}\texttt{>}\ \rangle \qquad \{O_C\} \subseteq DO \qquad \alpha_i \in \overline{\alpha} \\ \{(O_C, \alpha_i) \mapsto D_i\} \subseteq DD \\ DO, DD, DE \vdash_O ddomains(C, O_C) \\ \forall m \in \overline{md}\ mbody(m, C\texttt{<}\overline{p}\texttt{>}) = (\overline{x} : \overline{T},\ e_R) \\ C\texttt{<}\overline{D}\texttt{>} \notin \Upsilon \implies \{\overline{x} : \overline{T},\ \texttt{this} : C\texttt{<}\overline{p}\texttt{>}\}, \Upsilon \cup \{C\texttt{<}\overline{D}\texttt{>}\}, DO, DD, DE \vdash_{O_C} e_R \\ \Gamma, \Upsilon, DO, DD, DE \vdash_O \overline{e}\end{array}}{\Gamma, \Upsilon, DO, DD, DE \vdash_O \texttt{new}\ C\texttt{<}\overline{p}\texttt{>}(\overline{e})}\ [\textsc{Df-New}]$$

$$\frac{\forall(\texttt{domain}\ d_j) \in \overline{dom} \qquad D_j = \langle D_{id_j},\ C{::}d_j\rangle \qquad \{(O_C, C{::}d_j) \mapsto D_j\} \subseteq DD \qquad DO, DD, DE \vdash_O ddomains(C', O_C)}{DO, DD, DE \vdash_O ddomains(C, O_C)}\ [\textsc{Aux-Dom}]$$

$$\frac{}{DO, DD, DE \vdash_O ddomains(\texttt{Object}, O_C)}\ [\textsc{Aux-Obj1}]$$

Figure 5: Static semantics.

The judgement form for expressions is as follows:

$$\Gamma, \Upsilon, DO, DD, DE \vdash_{O,H} e$$

The O subscript on the turnstile captures the context-sensitivity, and represents the context object that the analysis uses to abstractly interpret e. For readability, we add the second subscript H only to the rules that use it. $CT(C)$ and $CT(\texttt{Object})$ represent a lookup of a class C and the class `Object` in the class table, and is an implicit clause in all the static rules. (We list these clauses once at the top of Fig. 5 to avoid repetition.)

In Df-New, the analysis interprets a `new` object allocation in the context of O. The analysis first ensures that DO contains an OObject O_C for the newly allocated object. Then, Df-New ensures that DD has a representative ODomain D_i for each domain parameter p_i passed to the constructor of the class C. Based on the binding of each formal domain parameter α_i to actual p_i, DD maps each α_i to a corresponding D_i in the context of O_C ($(O_C, \alpha_i) \mapsto D_i$) (Fig. 5).

Then, Df-New uses the auxiliary judgement Aux-Dom to ensure that DD has an ODomain

corresponding to each domain that C locally declares $((O_C, C{::}d_j) \mapsto D_j)$. Aux-Dom recursively includes inherited domains from base classes as well. Aux-Obj1, the base case of the recursion, deals with the class `Object`, for which Aux-Obj1 does nothing, because `Object` has no fields, domains, or methods in FDJ.

Df-New then obtains each expression e_R in each method m of C, and recursively processes e_R in the context of the new OObject O_C. To avoid infinite recursion, before Df-New analyzes e_R, it checks if the combination of the class C and actual domains $\overline{D}$ have been previously analyzed by looking for this combination in Υ. If this combination does not exist, Df-New extends Υ with the current combination. As a side note, Υ tracks previously analyzed OObjects only at the call stack level. It does not do so globally across the program because similar combinations of the same class and domain parameters can occur in different contexts, and must be analyzed separately. Finally, Df-New analyzes each argument of the constructor. Since our analysis distinguishes between a field initialization in a constructor and a field write, Df-New does not require dataflow edges in DE.

Df-Lookup defines the auxiliary judgement *lookup* that returns the set of the OObjects O_k in DO such that the class of O_k is C' or one of its subclasses. It also ensures that each domain D_i of O' corresponds to D'_i, a domain associated with O in DD (Fig. 6). The second condition increases the precision of our analysis, because *lookup* returns only a subset of all the objects of class C' or its subclasses in DO. From this subset, our analysis picks the source or destination OObjects, and finds the class representing the label of an OEdge, as follows.

The auxiliary judgements Aux-Import and Aux-Export ensure import and export edges between the context OObject O and the OObjects O_i, where O_i is the result of *lookup* (T_{src}), and *lookup* (T_{dst}), respectively. The direction of the edge is from O_i to the context O for Aux-Import, and from the context O to O_i for Aux-Export. To identify edge's labels, Aux-Export calls *lookup* in the context of O, while Aux-Import calls the second *lookup* in the context of O_i. As a result, there could be multiple edges with different labels between the same two OObjects, depending on the size of the set that *lookup* returns.

Df-Read and Df-Write abstractly interpret field read and write expressions, respectively. In

$$\frac{e_0 : T \qquad T = C\langle\overline{p}\rangle \qquad (T_k\ f_k) \in fields(C\langle\overline{p}\rangle) \qquad T_k = C'_k\langle\overline{p'}\rangle \qquad DO, DD, DE \vdash_O import(T, T_k) \qquad \Gamma, \Upsilon, DO, DD, DE \vdash_O e_0}{\Gamma, \Upsilon, DO, DD, DE \vdash_O e_0.f_k} [\textsc{Df-Read}]$$

$$\frac{e_0 : T \qquad T = C\langle\overline{p}\rangle \qquad (T_k\ f_k) \in fields(C\langle\overline{p}\rangle) \qquad T_k = C'_k\langle\overline{p'}\rangle \qquad e_1 : T' \qquad T' = C_1\langle\overline{p''}\rangle \qquad T' <: T_k \qquad DO, DD, DE \vdash_O export(T, T') \qquad \Gamma, \Upsilon, DO, DD, DE \vdash_O e_0 \qquad \Gamma, \Upsilon, DO, DD, DE \vdash_O e_1}{\Gamma, \Upsilon, DO, DD, DE \vdash_O e_0.f_k = e_1} [\textsc{Df-Write}]$$

$$\frac{O_k = \langle\ O_{id}, C\langle\overline{D}\rangle\ \rangle \in DO \qquad T' = C'\langle\overline{p'}\rangle \qquad C <: C' \qquad \forall i \in 1..|\overline{p'}| \qquad D'_i = DD[(O, p'_i)] \qquad D'_i = D_i}{DO, DD, DE \vdash_O lookup\ (T') = \{O_k\}_{k \in 1..sz}} [\textsc{Df-Lookup}]$$

$$\frac{DO, DD, DE \vdash_O lookup\ (T_{src}) = \{O_i\}_{i \in 1..sz} \qquad DO, DD, DE \vdash_{O_i} lookup\ (T_{label}) = \{O_j\}_{j \in 1..sz'} \qquad \forall i \in 1..sz\ \forall j \in 1..sz'\ O_j = \langle O_{id}, C_j\langle\overline{D}\rangle\rangle \in DO\ \{\langle O_i, O, C_j\rangle\} \subseteq DE}{DO, DD, DE \vdash_O import\ (T_{src}, T_{label})} [\textsc{Aux-Import}]$$

$$\frac{DO, DD, DE \vdash_O lookup\ (T_{dst}) = \{O_i\}_{i \in 1..sz} \qquad DO, DD, DE \vdash_O lookup\ (T_{label}) = \{O_j\}_{j \in 1..sz'} \qquad \forall i \in 1..sz\ \forall j \in 1..sz'\ O_j = \langle O_{id}, C_j\langle\overline{D}\rangle\rangle \in DO\ \{\langle O, O_i, C_j\rangle\} \subseteq DE}{DO, DD, DE \vdash_O export\ (T_{dst}, T_{label})} [\textsc{Aux-Export}]$$

$$\frac{e_0 : T \qquad T = C\langle\overline{p}\rangle \qquad mtype(m, C\langle\overline{p}\rangle) = \overline{T} \to T_R \qquad DO, DD, DE \vdash_O import(T, T_R) \qquad \forall k \in 1..|\overline{e}|\ e_k : T'_k \qquad T'_k <: T_k \qquad T_k \in \overline{T} \qquad DO, DD, DE \vdash_O export(T, T'_k) \qquad \Gamma, \Upsilon, DO, DD, DE \vdash_O e_0 \qquad \Gamma, \Upsilon, DO, DD, DE \vdash_O \overline{e}}{\Gamma, \Upsilon, DO, DD, DE \vdash_O e_0.m(\overline{e})} [\textsc{Df-Invk}]$$

Figure 6: Static semantics (continued).

turn, they use Aux-Import and Aux-Export. Both auxiliary judgements take the type T of e_0 as the first argument, and pass it to *lookup* to search for source and destination OObjects. To search for the label, Df-Read uses the type T_k of the field f_k, while Df-Write uses the type T' of the right-hand side expression e_1. The labels are the classes of these types or one of their subclasses.

Df-Invk abstractly interprets method invocation expressions. First, it ensures the existence of an import edge from the receiver of the method to the context OObject O. The label of the import edge is the class of the return type, or one of its subclasses. Next, for each argument e_k, Df-Invk

$$\frac{}{\Gamma, \Upsilon, DO, DD, DE \vdash_O x}[\text{Df-Var}] \qquad \frac{}{\Gamma, \Upsilon, DO, DD, DE \vdash_O \ell}[\text{Df-Loc}]$$

$$\frac{O_C = H[\ell] \qquad \Gamma, \Upsilon, DO, DD, DE \vdash_{O_C} e}{\Gamma, \Upsilon, DO, DD, DE \vdash_{O,H} \ell \triangleright e}[\text{Df-Context}]$$

$$\frac{\begin{array}{c}\forall \ell \in dom(S), \Sigma[\ell] = C\!<\!\overline{p}\!> \qquad H[\ell] = O = \langle O_{id}, C\!<\!\overline{D}\!>\rangle \in DO \\ \forall m.\ mbody(m, C\!<\!\overline{p}\!>) = (\overline{x} : \overline{T},\ e_R) \qquad \{\overline{x} : \overline{T},\ \texttt{this} : C\!<\!\overline{p}\!>\}, \emptyset, DO, DD, DE \vdash_O e_R\end{array}}{DO, DD, DE \vdash_{CT,H} \Sigma}[\text{Df-Sigma}]$$

Figure 7: Static semantics (continued).

ensures the existence of an export edge from O to the receiver of the method. The label of each export edge is the class of the argument or one of its subclasses. The rule ensures export edges only for a method invocation with at least one argument. Finally, the rule evaluates recursively the expressions e_0 and $\overline{e}$.

Df-Var, and Df-Loc, and the rest of the rules complete our formalization and make the induction go through (Fig. 7). Df-Context analyzes expressions of the form $\ell \triangleright e$. The context for analyzing e changes from O to O_C, where O_C is the result of looking up the receiver ℓ in H. Finally, the induction requires an augmented store typing rule, Df-Sigma, to ensures that the method bodies have been analyzed for all the locations ℓ in the store, and that every ℓ has a corresponding OObject in DO. To denote all the objects in the store, we use the CT subscript instead of O.

Dynamic Semantics. To complete the formalization, we instrumented the dynamic semantics (Fig. 8). The instrumentation extends the dynamic semantics of FDJ [4] (the common parts are highlighted), but is safe since discarding it produces exactly the FDJ dynamic semantics. The instrumented evaluation rule is of the following form:

$$\theta \vdash e; S; H; K; L_I; L_E \rightsquigarrow_G e'; S'; H'; K'; L'_I; L'_E$$

where $G = \langle DO, DD, DE\rangle$ is the statically computed object graph, and $\rightsquigarrow_G$ means that the expression e evaluates to e' in the context of θ, the value of `this`. The dynamic semantics keep G unchanged, but change the store S and the maps H, K, L_I, and L_E.

$$\frac{\boxed{\ell \notin dom(S) \qquad S' = S[\ell \mapsto C\!<\!\overline{p}\!>\!(\overline{v})]} \quad G = \langle DO, DD, DE\rangle \quad \overline{p} = \overline{\ell'.d} \quad \forall i \in 1..|\overline{\ell'.d}|\ D_i = K[\ell'_i.d_i] \quad O_C = \langle O_{id}, C\!<\!\overline{D}\!>\rangle \quad O_C \in DO \quad H' = H[\ell \mapsto O_C] \quad \forall(\texttt{domain}\ d_j) \in domains(C\!<\!\overline{p}\!>) \quad D_j = DD[(O_C, C{::}d_j)] \quad K' = K[\ell.d_j \mapsto D_j]}{\theta \vdash \boxed{\texttt{new}\ C\!<\!\overline{p}\!>\!(\overline{v}); S}; H; K; L_I; L_E \rightsquigarrow_G \boxed{\ell; S'}; H'; K'; L_I; L_E}[\text{IR-New}]$$

$$\frac{\boxed{S[\ell] = C\!<\!\overline{p}\!>\!(\overline{v}) \qquad fields(C\!<\!\overline{p}\!>) = \overline{T}\ \overline{f}} \quad O = H[\theta] \quad O_\ell = H[\ell] \quad T_i = C_i\!<\!\overline{p'}\!> \quad T_i \in \overline{T} \quad E = \langle O_\ell, O, C_v\rangle \in DE \quad C_v <: C_i \quad L'_I = L_I[(\ell, \theta) \mapsto_\cup \{E\}]}{\theta \vdash \boxed{\ell.f_i; S}; H; K; L_I; L_E \rightsquigarrow_G \boxed{v_i; S}; H; K; L'_I; L_E}[\text{IR-Read}]$$

$$\frac{\boxed{S[\ell] = C\!<\!\overline{p}\!>\!(\overline{v}) \qquad fields(C\!<\!\overline{p}\!>) = \overline{T}\ \overline{f}} \quad \boxed{S' = S[\ell \mapsto C\!<\!\overline{p}\!>\!([v/v_i]\overline{v})]} \quad O = H[\theta] \quad O_\ell = H[\ell] \quad T_i = C_i\!<\!\overline{p'}\!> \quad T_i \in \overline{T} \quad E = \langle O, O_\ell, C_v\rangle \in DE \quad C_v <: C_i \quad L'_E = L_E[(\theta, \ell) \mapsto_\cup \{E\}]}{\theta \vdash \boxed{\ell.f_i = v; S}; H; K; L_I; L_E \rightsquigarrow_G \boxed{v; S'}; H; K; L_I; L'_E}[\text{IR-Write}]$$

$$\frac{\boxed{S[\ell] = C\!<\!\overline{p}\!>\!(\overline{v}) \qquad mbody(m, C\!<\!\overline{p}\!>) = (\overline{x}, e_R)} \quad O = H[\theta] \quad O_\ell = H[\ell] \quad mtype(m, C\!<\!\overline{p}\!>) = \overline{T} \to T_R \quad T_R = C_R\!<\!\overline{p'}\!> \quad E' = \langle O_\ell, O, C'_R\rangle \in DE \quad C'_R <: C_R \quad L'_I = L_I[(\ell, \theta) \mapsto_\cup \{E'\}] \quad \forall k \in 1..|\overline{T}|\ T_k = C_k\!<\!\overline{p''}\!> \quad E_k = \langle O, O_\ell, C'_k\rangle \in DE \quad C'_k <: C_k \quad L'_E = L_E[(\theta, \ell) \mapsto_\cup \{E_k\}]}{\theta \vdash \boxed{\ell.m(\overline{v}); S}; H; K; L_I; L_E \rightsquigarrow_G \boxed{\ell \triangleright [\overline{v}/\overline{x}, \ell/\texttt{this}]e_R; S}; H; K; L'_I; L'_E}[\text{IR-Invk}]$$

$$\frac{}{\theta \vdash \boxed{\ell \triangleright v; S}; H; K; L_I; L_E \rightsquigarrow_G \boxed{v;\ S}; H; K; L_I; L_E}[\text{IR-Context}]$$

Figure 8: Instrumented dynamic semantics (core rules).

IR-New adds a new location ℓ to the store S, where ℓ maps to an object of type C with the specified ownership domain parameters, and the fields set to the values $\overline{v}$ passed to the constructor. The rule extends H by mapping ℓ and the OObject O_C from DO. The rule requires that each actual domains p_i passed during instantiation corresponds to an actual domain D_i of O_C. Next, the rule extends K such that for all the domains $C{::}d_j$, the pair (O_C, $C{::}d_j$) has a corresponding D_j in DD.

IR-Read and IR-Write ensure that an OEdge E exists between the context OObject O and

the receiver O_ℓ. They use θ and ℓ to lookup these OObjects in H. They also ensure that the edge label C_v is a subclass of the field class C_i. Finally, the rules extend the maps L_I and L_E, respectively, by adding E to the set of edges associated with (ℓ, θ) in L_I, and (θ, ℓ) in L_E.

IR-INVK ensures that an import OEdge E' exists from the receiver O_ℓ to the context O, having as the edge's label a subclass of the return class C_R. IR-INVK also ensures that an export OEdge E_k exist from O to O_ℓ for every parameter, having as edge label a subclass of the method's parameter class C_k. The rule uses θ and ℓ to lookup O and O_ℓ in H . It extends both L_I and L_E by adding E' to the set of import edges between the locations ℓ and θ in L_I, and by adding each E_k to the set of export edges between the locations θ and ℓ in L_E.

When the method expression reduces to a value v, IR-CONTEXT propagates v outside of its method context. This rule does not affect the execution of the program. Finally, the dynamic semantics include standard congruence rules.

The congruence rules are similar to those in FDJ [4] (Fig. 9). In addition, there are two congruence rules for field-write: IRC-WRITE-RCV and IRC-WRITE-ARG. IRC-WRITE-RCV states that the receiver expression e_0 reduces to e'_0, while IRC-WRITE-ARG states that the right-hand side expression e_1 reduces to e'_1.

$$\frac{\theta \vdash e_i; S; H; K; L_I; L_E \rightsquigarrow_G e_i'; S'; H'; K'; L_I'; L_E'}{\theta \vdash \texttt{new}\ C\!<\!\overline{p}\!>\!(v_{1..i-1}, e_i, e_{i+1..n}); S; H; K; L_I; L_E \rightsquigarrow_G \texttt{new}\ C\!<\!\overline{p}\!>\!(v_{1..i-1}, e_i', e_{i+1..n}); S'; H'; K'; L_I'; L_E'}[\text{IRC-New}]$$

$$\frac{\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'}{\theta \vdash e_0.f_i; S; H; K; L_I; L_E \rightsquigarrow_G e_0'.f_i; S'; H'; K'; L_I'; L_E'}[\text{IRC-Read}]$$

$$\frac{\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'}{\theta \vdash e_0.f_i = e_1; S; H; K; L_I; L_E \rightsquigarrow_G e_0'.f_i = e_1; S'; H'; K'; L_I'; L_E'}[\text{IRC-Write-Rcv}]$$

$$\frac{\theta \vdash e_1; S; H; K; L_I; L_E \rightsquigarrow_G e_1'; S'; H'; K'; L_I'; L_E'}{\theta \vdash v.f_i = e_1; S; H; K; L_I; L_E \rightsquigarrow_G v.f_i = e_1'; S'; H'; K'; L_I'; L_E'}[\text{IRC-Write-Arg}]$$

$$\frac{\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'}{\theta \vdash e_0.m(\overline{e}); S; H; K; L_I; L_E \rightsquigarrow_G e_0'.m(\overline{e}); S'; H'; K'; L_I'; L_E'}[\text{IRC-RecvInvk}]$$

$$\frac{\theta \vdash e_i; S; H; K; L_I; L_E \rightsquigarrow_G e_i'; S'; H'; K'; L_I'; L_E'}{\theta \vdash v.m(v_{1..i-1}, e_i, e_{i+1..n}); S; H; K; L_I; L_E \rightsquigarrow_G v.m(v_{1..i-1}, e_i', e_{i+1..n}); S'; H'; K'; L_I'; L_E'}[\text{IRC-ArgInvk}]$$

$$\frac{\theta \vdash e; S; H; K; L_I; L_E \rightsquigarrow_G e'; S'; H'; K'; L_I'; L_E'}{\theta \vdash \ell \triangleright e; S; H; K; L_I; L_E \rightsquigarrow_G \ell \triangleright e'; S'; H'; K'; L_I'; L_E'}[\text{IRC-Context}]$$

Figure 9: Instrumented dynamic semantics (congruence rules).

```
1 Main<SHARED> main = new Main<SHARED>();
2 class Main<OWNER> {
3   domain OWNED;
4   QuadTree<OWNED> aQT = new QuadTree<OWNED>();
5 }
6 class QuadTree<M> {
7    domain OWNED;
8    QuadTree<OWNED> nwQT = new QuadTree<OWNED>
9 }
```

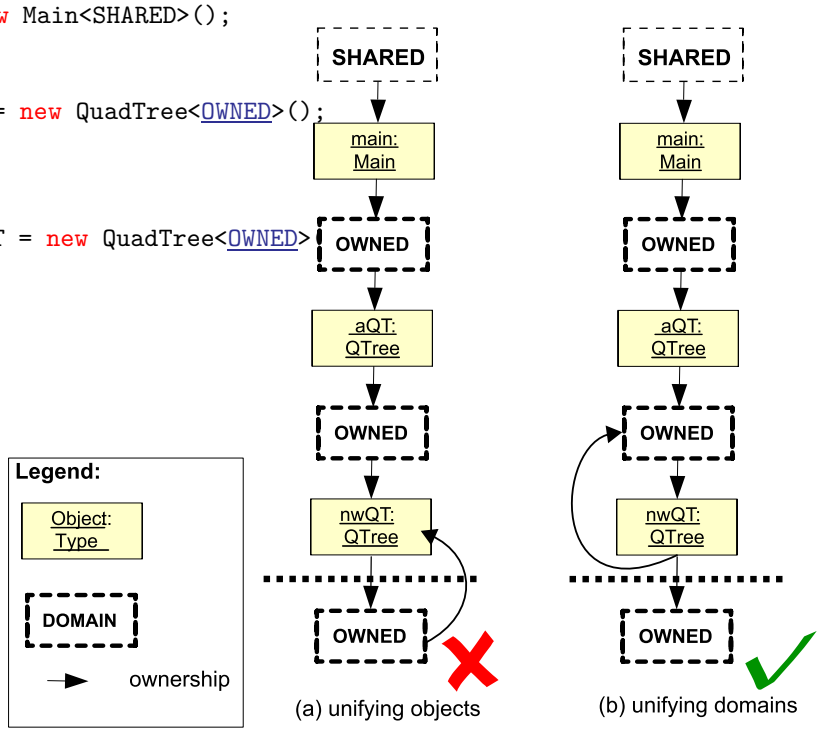


Figure 10: Handling recursive types, revised from [1, Figure 2.22].

4.3 Recursive Types.

The analysis must handle recursive types which can lead an unbound number of nodes in the OGraph. As an example, consider a class `QuadTree`, which declares a field `nwQT` of type `QuadTree` in its `OWNED` private domain (Fig. 10). To get a finite OGraph and ensure the analysis terminates, the analysis could stop expanding an OGraph after a certain depth. However, truncating the recursion at an arbitrary depth may fail to show when a child object beyond the visible depth communicates to external objects. Instead, the analysis creates a cycle in the OGraph when it reaches a similar context. There are two possible choices: to unify objects or to unify domains.

The analysis creates objects until it detects that it is creating objects similar to the one it created before. In this case, the analysis uses an existing similar object. One can imagine multiple notion of similarity; it can be any equivalence relation as long as the number of dissimilar objects is finite. We adopt the following similarity relation between two objects a and b: a and b are of the

```
1  Main<SHARED> main = new Main<SHARED>();
2  OObject(main, null, Main)
3  analyze(main, [Main::OWNER ↦ TORAD ])
4  this ↦ main
5  class Main<OWNER> {
6    domain OWNED;
7    ODomain(main.OWNED, Main::OWNED)
8    OObject(main.OWNED.aQT, main.OWNED, QuadTree)
9    QuadTree<OWNED> aQT = new QuadTree<OWNED>();
10     analyze(main.OWNED.aQT, [QuadTree::M ↦ Main::OWNED])
11 }
12 this ↦ main.OWNED.aQT
13 [QuadTree::M ↦ Main::OWNED]
14 class QuadTree<M> {
15   domain OWNED;
16   ODomain(main.OWNED.aQT.OWNED, QuadTree::OWNED)
17   OObject(main.OWNED.aQT.OWNED.nwQT, main.OWNED.aQT.OWNED, QuadTree)
18   QuadTree<M> nwQT = new QuadTree<M>();
19     analyze(main.OWNED.aQT.OWNED.nwQT, [QuadTree::M ↦ QuadTree::OWNED])
20 }
21 this ↦ main.OWNED.aQT.OWNED.nwQT
22 [QuadTree::M ↦ QuadTree::OWNED]
23 class QuadTree<M> {
24   domain OWNED;
25   ODomain(main.OWNED.aQT.OWNED, QuadTree::OWNED)
26   OObject(main.OWNED.aQT.OWNED.nwQT, <main.OWNED.aQT.OWNED, QuadTree)
27   QuadTree<OWNED> nwQT = new QuadTree<OWNED>();
28     analyze(main.OWNED.aQT.OWNED.nwQT, [QuadTree::M ↦ QuadTree::OWNED])
29 }
```

Figure 11: Worked example with recursive types, revised from [1, Figure 2.24].

same type, including actual domain parameters ($C<\overline{D}>$). Unifying objects is problematic, because for two objects to be similar, it is necessary to detect they have the same owning ODomain. But, if the ODomain has a unique owning OObject, the problem is circular. Moreover, in order to add edges, we lookup objects in a given domain by their type.

Since recognizing domains is important, we adopt the solution of unifying domains. It is simpler to recognize that two ODomains have the same underlying domain declaration $C::d$, than to recognize similar objects. The analysis creates a cycle in the OGraph when the same ODomain appears as the child of two OObjects. This justifies an ODomain not having a unique owning OObject (Fig. 4).

4.4 Soundness

An OGraph is a *sound* approximation of a ROG, represented by a well-typed store S, if the OGraph relates to the ROG as follows:

Object soundness. There is a map H that maps each object ℓ in S to exactly one representative OObject in the OGraph. Similarly, there is a map K such that each runtime domain $\ell.d$ has exactly one representative ODomain in the OGraph.

Edge soundness. If there is a dataflow communication from an object ℓ_1 to ℓ_2 in a ROG, with their representatives OObjects O_1 and O_2 in the OGraph, then there are two maps L_I and L_E that map the pair (ℓ_1, ℓ_2) to a set of OEdges in the OGraph that represent the dataflow communication between O_1 and O_2.

To relate the dynamic and the static semantics of the analysis, we define an approximation relation (DF-APPROX) between a runtime state (S,H,K,L_I,L_E) and an analysis result (DO, DD, DE). It ensures that the runtime objects, runtime domains and runtime edges are consistent with their representatives in the statically extracted OGraph (Fig 12).

DF-APPROX states that given a well-typed store S of a program and an OGraph $\langle DO, DD, DE\rangle$ of the same program, there are maps H, K, L_I, and L_E, such that H maps each runtime object ℓ in the store to a unique OObject O_C from DO, K maps each runtime domain $\ell.d_i$ in the store to a unique ODomain D_i, and L_I and L_E map each pair of runtime objects (ℓ_{src}, ℓ) and (ℓ, ℓ_{dst}) to OEdges from DE. DF-APPROX ensures the consistency of these mappings with the ownership

Approximation Relation (Df-Approx).

$$
\begin{array}{l}
\forall\ \Sigma \vdash S,\quad (S, H, K, L_I, L_E) \sim (DO, DD, DE) \\
\iff \\
\forall \ell \in dom(S), \Sigma[\ell] = C\langle\overline{\ell'.d}\rangle \\
\implies \\
H[\ell] = O_C = \langle O_{id}, C\langle\overline{D}\rangle\rangle \in DO \\
\quad and\ \forall \ell'_j.d_j \in \overline{\ell'.d}\ K[\ell'_j.d_j] = D_j = \langle D_{id_j}, d_j\rangle \in rng(DD) \\
\quad and\ \forall d_i \in domains(C\langle\overline{\ell'.d}\rangle)\ K[\ell.d_i] = D_i = \langle D_{id_i}, d_i\rangle\ \{(O_C, C::d_i) \mapsto D_i\} \in DD \\
\quad and\ \forall \ell_{src} \in dom(H),\ fields(\Sigma[\ell_{src}]) = \overline{T_{src}}\ \overline{f} \\
\qquad \forall m.\ mtype(m, \Sigma[\ell_{src}]) = \overline{T} \to T_R \\
\qquad\quad \forall T_k \in \{\overline{T_{src}}\} \cup \{T_R\} \quad T_k = C_k\langle\overline{p}\rangle \\
\qquad\qquad E'_k \in L_I[(\ell_{src}, \ell)]\ E'_k = \langle H[\ell_{src}], H[\ell], C'_k\rangle \in DE \quad C'_k <: C_k \\
\quad and\ \forall \ell_{dst} \in dom(H),\ fields(\Sigma[\ell_{dst}]) = \overline{T_{dst}}\ \overline{f} \\
\qquad \forall m.\ mtype(m, \Sigma[\ell_{dst}]) = \overline{T} \to T_R \\
\qquad\quad \forall T_k \in \{\overline{T_{dst}}\} \cup \{\overline{T}\} \quad T_k = C_k\langle\overline{p}\rangle \\
\qquad\qquad E_k \in L_E[(\ell, \ell_{dst})]\ E_k = \langle H[\ell], H[\ell_{dst}], C'_k\rangle \in DE \quad C'_k <: C_k
\end{array}
$$

Figure 12: Approximation Relation.

relation, and with the dataflow communication.

The last two conditions relate runtime dataflow communication back to field reads, field writes, and method invocations that produce the corresponding import and export edges in DE. L_I maps a runtime dataflow communication from a runtime object ℓ_{src} to another runtime object ℓ back to an import OEdge E'_k from DE. By our definition of import dataflow communication, E'_k exists in DE due to a field read or a method invocation expression that has ℓ_{src} as its receiver. The condition also ensures that the edge's label is a subclass of C_k, the class of a field of ℓ_{src}'s class, or the return class of a method of ℓ_{src}'s class.

Similarly, L_E maps a runtime dataflow communication from a runtime object ℓ to another runtime object ℓ_{dst} back to an export OEdge E_k from DE. By our definition of export dataflow communication, E_k exists in DE due to a field write or a method invocation expression that has ℓ_{dst} as its receiver. The condition also ensures that the edge's label is a subclass of C_k, the class of a field of ℓ_{dst}'s class, or the class of a parameter on a method of ℓ_{dst}'s class.

Theorem: Dataflow Object Graph Soundness.

$$
\begin{aligned}
&\textit{If } G = \langle DO, DD, DE \rangle \\
&\quad DO, DD, DE \vdash (CT, e_{root}) \\
&\quad \forall e,\ \theta_0 \vdash e; \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rightsquigarrow_G^* e; S; H; K; L_I; L_E \\
&\quad \Sigma \vdash S \\
&\textit{then } DO, DD, DE \vdash_{CT,H} \Sigma \\
&\quad (S, H, K, L_I, L_E) \sim (DO, DD, DE)
\end{aligned}
$$

where $\rightsquigarrow_G^*$ relation is the reflexive and transitive closure of $\rightsquigarrow_G$ relation, and θ_0 is the location of the first object instantiated by e_{root}. To prove the Object Graph Soundness theorem, we prove the Dataflow Preservation and Dataflow Progress theorems, which extend the standard FDJ Preservation and Progress. The common parts are highlighted (Fig. 13).

Theorem: Dataflow Preservation (Subject reduction).

$$
\begin{array}{l}
\textit{If } \boxed{\emptyset, \Sigma, \theta \vdash e : T} \\
\quad \boxed{\Sigma \vdash S} \\
\quad DO, DD, DE \vdash_{CT,H} \Sigma \\
\quad \emptyset, \emptyset, DO, DD, DE \vdash_{O} e \\
\quad (S, H, K, L_I, L_E) \sim (DO, DD, DE) \\
\quad \theta \vdash \boxed{\text{e; S}}; H; K; L_I; L_E \rightsquigarrow_G \boxed{\text{e'; S'}}; H'; K'; L'_I; L'_E \\
\textit{then } \boxed{\textit{there exists } \Sigma' \supseteq \Sigma \textit{ and } T' <: T \textit{ such that}} \\
\quad \boxed{\emptyset, \Sigma', \theta \vdash e' : T' \textit{ and } \Sigma' \vdash S'} \\
\quad (S', H', K', L'_I, L'_E) \sim (DO, DD, DE) \\
\quad \emptyset, \emptyset, DO, DD, DE \vdash_{O} e' \\
\quad \textit{and } DO, DD, DE \vdash_{CT,H} \Sigma'
\end{array}
$$

Theorem: Dataflow Progress.

$$
\begin{array}{l}
\textit{If } \boxed{\emptyset, \Sigma, \theta \vdash e : T} \\
\quad \boxed{\Sigma \vdash S} \\
\quad DO, DD, DE \vdash_{CT,H} \Sigma \\
\quad \emptyset, \emptyset, DO, DD, DE \vdash_{O} e \\
\quad (S, H, K, L_I, L_E) \sim (DO, DD, DE) \\
\textit{then either } \boxed{e \textit{ is a value}} \\
\quad \textit{or else } \theta \vdash \boxed{e; S}; H; K; L_I; L_E \rightsquigarrow_G \boxed{e'; S'}; H'; K'; L'_I; L'_E
\end{array}
$$

Figure 13: Dataflow Progress and Data Preservation theorems.

4.5 Theorem: Dataflow Preservation (Subject reduction)

If

$\boxed{\emptyset, \Sigma, \theta \vdash e : T}$

$\boxed{\Sigma \vdash S}$

$DO, DD, DE \vdash_{CT,H} \Sigma$

$\emptyset, \emptyset, DO, DD, DE \vdash_O e$

$(S, H, K, L_I, L_E) \sim (DO, DD, DE)$

$\theta \vdash \boxed{e; S}; H; K; L_I; L_E \rightsquigarrow_G \boxed{e'; S'}; H'; K'; L'_I; L'_E$

then

$\boxed{\textit{there exists } \Sigma' \supseteq \Sigma \textit{ and } T' <: T \textit{ such that } \emptyset, \Sigma', \theta \vdash e' : T' \textit{ and } \Sigma' \vdash S'}$

$(S', H', K', L'_I, L'_E) \sim (DO, DD, DE)$

$\emptyset, \emptyset, DO, DD, DE \vdash_O e'$

and $DO, DD, DE \vdash_{CT,H} \Sigma'$

The Dataflow Preservation theorem extends the FDJ Type Preservation theorem (the common parts are highlighted). Those parts are proved by induction over the derivation of the FDJ evaluation relation : $e; S \rightsquigarrow e'; S'$.

Proof: We prove preservation by induction on the instrumented evaluation relation

$$\theta \vdash e; S; H; K; L_I; L_E \rightsquigarrow_G e'; S'; H'; K'; L'_I; L'_E$$

The most interesting cases are IR-NEW, IR-READ (page 27), IR-WRITE (page 28), and IR-INVK (page 29).

Case Ir-New: e = `new` $C<\overline{\ell'.d}>(\overline{v})$, and $e' = \ell$.

To Show:

(1) $(S', H', K', L'_I, L'_E) \sim (DO, DD, DE)$

(2) $\emptyset, \emptyset, DO, DD, DE \vdash_O e'$

(3) $DO, DD, DE \vdash_{CT,H'} \Sigma'$

$\theta \vdash e; S; H; K; L_I; L_E \rightsquigarrow_G e'; S'; H'; K'; L'_I; L'_E$ — By assumption

$(S, H, K, L_I, L_E) \sim (DO, DD, DE)$ — By assumption

$\forall \ell \in dom(S),\ \Sigma(\ell) = C\langle\overline{\ell'.d}\rangle$ — Since $\Sigma \vdash S$

$H[\theta] = O = \langle O_{id}, C\langle\overline{D}\rangle\rangle \in DO$ — By Df-Approx

$\forall \theta'_j.d_j \in \overline{\theta'.d}\ K[\theta'_j.d_j] = D_j = \langle D_{id_j}, d_j\rangle \in rng(DD)$ — By Df-Approx

$\forall d_i \in domains(C\langle\overline{\theta'.d}\rangle)\ K[\theta.d_i] = D_i = \langle D_{id_i}, d_i\rangle$

$\{(O, d_i) \mapsto D_i\} \in DD$ — By Df-Approx

$\forall \ell_{src} \in dom(H),\ fields(\Sigma[\ell_{src}]) = \overline{T_{src}}\ \overline{f}$,

$\forall m.\ mtype(m, \Sigma[\ell_{src}]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T_{src}}\} \cup \{T_R\} \quad T_k = C_k\langle\overline{p}\rangle$

$E'_k \in L_I[(\ell_{src}, \theta)] = \langle H[\ell_{src}], H[\theta], C'_k\rangle \in DE \quad C'_k <: C_k$ — By Df-Approx

$\forall \ell_{dst} \in dom(H),\ fields(\Sigma[\ell_{dst}]) = \overline{T_{dst}}\ \overline{f}$,

$\forall m.\ mtype(m, \Sigma[\ell_{dst}]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T_{dst}}\} \cup \{\overline{T}\} \quad T_k = C_k\langle\overline{p}\rangle$

$E_k \in L_E[(\theta, \ell_{dst})] = \langle H[\theta], H[\ell_{dst}], C'_k\rangle \in DE \quad C'_k <: C_k$ — By Df-Approx

$O_C = \langle O_{id}, C\langle\overline{D}\rangle\rangle \in DO$ — By sub-derivation of Ir-New

$S' = S[\ell \mapsto C\langle\overline{p}\rangle(\overline{v})]$ — By sub-derivation of Ir-New

$H' = H[\ell \mapsto O_C]$ — By sub-derivation of Ir-New

$\overline{p} = \overline{\ell'.d}\ \forall i \in 1..|\overline{\ell'.d}|\ D_i = K[\ell'_i.d_i]$ — By sub-derivation of Ir-New

$\forall(\texttt{domain}\ d_j) \in domains(C\langle\overline{p}\rangle) \quad D_j = DD[(O_C, d_j)]$

$K' = K[\ell.d_j \mapsto D_j]$ — By sub-derivation of Ir-New

$L'_I = L_I \quad L'_E = L_E$ — By sub-derivation of Ir-New

$\exists \Sigma' \supseteq \Sigma \ \ and\ T' <: T\ s.t.\ \emptyset, \Sigma', \theta \vdash e' : T'\ and\ \Sigma' \vdash S'$ — By FDJ Type Preservation

$\Sigma'[\ell] = C_\ell\langle\overline{\ell'.d}\rangle$ — By $\Sigma' \vdash S'$

$(S', H', K', L'_I, L'_E) \sim (DO, DD, DE)$ — By Df-Approx

This proves (1).

$\emptyset, \emptyset, DO, DD, DE \vdash_O e'$ — By Df-Loc, since $e' = \ell$

This proves (2).

$DO, DD, DE \vdash_{CT,H} \Sigma$ — By assumption

$\forall \ell \in dom(S), \Sigma[\ell] = C_\ell\langle\overline{p}\rangle$ — By sub-derivation of Df-Sigma

$H[\ell] = O_\ell = \langle O_{id}, C_\ell\langle\overline{D_\ell}\rangle\rangle \in DO$ — By sub-derivation of Df-Sigma

$\forall m.\ mbody(m, C_\ell\langle\overline{p}\rangle) = (\overline{x} : \overline{T},\ e_R)$ — By sub-derivation of Df-Sigma

$\{\overline{x} : \overline{T},\ \texttt{this} : C_\ell\langle\overline{p}\rangle\}, \emptyset, DO, DD, DE \vdash_{O_\ell} e_R$ — By sub-derivation of Df-Sigma

$O_C = \langle O_{id}, C\langle\overline{D}\rangle\rangle \in DO$ — By sub-derivation of Ir-New

$S' = S[\ell \mapsto C\langle\overline{p}\rangle(\overline{v})]$ — By sub-derivation of Ir-New

$H' = H[\ell \mapsto O_C]$ — By sub-derivation of Ir-New

$\emptyset, \emptyset, DO, DD, DE \vdash_O e$ — By assumption with e, Υ below

$e = \texttt{new}\ C\langle\overline{\ell'.d}\rangle(\overline{v})$, and $\Upsilon = \emptyset$

$\forall m.\ mbody(m, C\langle\overline{p}\rangle) = (\overline{x} : \overline{T},\ e_R)$ By sub-derivation of Df-New

$C\langle\overline{D}\rangle \notin \Upsilon \Longrightarrow$

$\{\overline{x} : \overline{T}, \texttt{this} : C\langle\overline{p}\rangle\}, \Upsilon \cup \{C\langle\overline{D}\rangle\}, DO, DD, DE \vdash_{O_C} e_R$ By sub-derivation of Df-New

$\{\overline{x} : \overline{T}, \texttt{this} : C\langle\overline{p}\rangle\}, \emptyset, DO, DD, DE \vdash_{O_C} e_R$ By Df-Strengthening Lemma

$\forall \ell \in dom(S'), \Sigma'[\ell] = C_\ell\langle\overline{p}\rangle$

$H'[\ell] = O_\ell = \langle O_{id}, C_\ell\langle\overline{D_\ell}\rangle\rangle \in DO$

$\forall m.\ mbody(m, C_\ell\langle\overline{p}\rangle) = (\overline{x} : \overline{T},\ e_R)$

$\{\overline{x} : \overline{T},\ \texttt{this} : C_\ell\langle\overline{p}\rangle\}, \emptyset, DO, DD, DE \vdash_{O_\ell} e_R$ By above

$DO, DD, DE \vdash_{CT,H'} \Sigma'$ By Df-Sigma with above H' and Σ'

This proves (3).

Case Ir-Read: e = $\ell.f_i$, and $e' = v_i$.

To Show:

(1) $(S', H', K', L'_I, L'_E) \sim (DO, DD, DE)$

(2) $\emptyset, \emptyset, DO, DD, DE \vdash_O e'$

(3) $DO, DD, DE \vdash_{CT,H'} \Sigma'$

$\theta \vdash e; S; H; K; L_I; L_E \rightsquigarrow_G e'; S'; H'; K'; L'_I; L'_E$ By assumption

$(S, H, K, L_I, L_E) \sim (DO, DD, DE)$ By assumption

$\forall \ell \in dom(S),\ \Sigma(\ell) = C\langle\overline{\ell'.d}\rangle$ Since $\Sigma \vdash S$

$H[\theta] = O = \langle O_{id}, C\langle\overline{D}\rangle\rangle \in DO$ By Df-Approx

$\forall \theta'_j.d_j \in \overline{\theta'.d}\ K[\theta'_j.d_j] = D_j = \langle D_{id_j}, d_j\rangle \in rng(DD)$ By Df-Approx

$\forall d_i \in domains(C\langle\overline{\theta'.d}\rangle)\ K[\theta.d_i] = D_i = \langle D_{id_i}, d_i\rangle$

$\{(O, d_i) \mapsto D_i\} \in DD$ By Df-Approx

$\forall \ell_{src} \in dom(H),\ fields(\Sigma[\ell_{src}]) = \overline{T_{src}}\ \overline{f},$

$\forall m.\ mtype(m, \Sigma[\ell_{src}]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T_{src}}\} \cup \{T_R\} \quad T_k = C_k\langle\overline{p}\rangle$

$E'_k \in L_I[(\ell_{src}, \theta)] = \langle H[\ell_{src}], H[\theta], C'_k\rangle \in DE \quad C'_k <: C_k$ By Df-Approx

$\forall \ell_{dst} \in dom(H),\ fields(\Sigma[\ell_{dst}]) = \overline{T_{dst}}\ \overline{f},$

$\forall m.\ mtype(m, \Sigma[\ell_{dst}]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T_{dst}}\} \cup \{\overline{T}\} \quad T_k = C_k\langle\overline{p}\rangle$

$E_k \in L_E[(\theta, \ell_{dst})] = \langle H[\theta], H[\ell_{dst}], C'_k\rangle \in DE\ C'_k <: C_k$ By Df-Approx

$S' = S, H' = H, K' = K, L'_E = L_E$ By sub-derivation of Ir-Read

$S[\ell] = C_\ell\langle\overline{p}\rangle(\overline{v}) \quad fields(C_\ell\langle\overline{p}\rangle) = \overline{T'}\ \overline{f}$ By sub-derivation of Ir-Read

$O = H[\theta] \quad O_\ell = H[\ell] \quad T_i' = C_i{<}\overline{p'}{>}$ By sub-derivation of Ir-Read

$E' = \langle O_\ell, O, C_v\rangle \in DE \quad C_v <: C_i$ By sub-derivation of Ir-Read

$L_I' = L_I[(\ell,\theta) \mapsto_\cup \{E'\}]$ By sub-derivation of Ir-Read

$\forall \ell_{src} \in dom(H'),\ fields(\Sigma'[\ell_{src}]) = \overline{T_{src}}\ \overline{f},$

$\forall m.\ mtype(m, \Sigma'[\ell_{src}]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T_{src}}\} \cup \{T_R\} \quad T_k = C_k{<}\overline{p}{>}$

$E_k' \in L_I'[(\ell_{src},\theta)] = \langle H'[\ell_{src}], H'[\theta], C_k'\rangle \in DE\ C_k' <: C_k$ By above, since $\Sigma' = \Sigma$

$(S', H', K', L_I', L_E') \sim (DO, DD, DE)$ By Df-Approx

This proves (1).

$\emptyset, \emptyset, DO, DD, DE \vdash_O e'$ By Df-Loc, since $e' = v_i$

This proves (2).

$DO, DD, DE \vdash_{CT,H} \Sigma$ By assumption

$S' = S, H' = H$ By sub-derivation of Ir-Read

$DO, DD, DE \vdash_{CT,H'} \Sigma'$ By Df-Sigma with the above H' and $\Sigma' = \Sigma$

This proves (3).

Case Ir-Write: e $= \ell.f_i = v$, and $e' = v$

To Show:

(1) $(S', H', K', L_I', L_E') \sim (DO, DD, DE)$

(2) $\emptyset, \emptyset, DO, DD, DE \vdash_O e'$

(3) $DO, DD, DE \vdash_{CT,H'} \Sigma'$

$\theta \vdash e; S; H; K; L_I; L_E \rightsquigarrow_G e'; S'; H'; K'; L_I'; L_E'$ By assumption

$(S, H, K, L_I, L_E) \sim (DO, DD, DE)$ By assumption

$\forall \ell \in dom(S),\ \Sigma(\ell) = C{<}\overline{\ell'.d}{>}$ Since $\Sigma \vdash S$

$H[\theta] = O = \langle O_{id}, C{<}\overline{D}{>}\rangle \in DO$ By Df-Approx

$\forall \theta_j'.d_j \in \overline{\theta'.d}\ K[\theta_j'.d_j] = D_j = \langle D_{id_j}, d_j\rangle \in rng(DD)$ By Df-Approx

$\forall d_i \in domains(C{<}\overline{\theta'.d}{>})\ K[\theta.d_i] = D_i = \langle D_{id_i}, d_i\rangle$

$\{(O, d_i) \mapsto D_i\} \in DD$ By Df-Approx

$\forall \ell_{src} \in dom(H),\ fields(\Sigma[\ell_{src}]) = \overline{T_{src}}\ \overline{f},$

$\forall m.\ mtype(m, \Sigma[\ell_{src}]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T_{src}}\} \cup \{T_R\} \quad T_k = C_k{<}\overline{p}{>}$

$E_k' \in L_I[(\ell_{src},\theta)] = \langle H[\ell_{src}], H[\theta], C_k'\rangle \in DE \quad C_k' <: C_k$ By Df-Approx

$\forall \ell_{dst} \in dom(H),\ fields(\Sigma[\ell_{dst}]) = \overline{T_{dst}}\ \overline{f},$
$\quad \forall m.\ mtype(m, \Sigma[\ell_{dst}]) = \overline{T} \to T_R$
$\quad\quad \forall T_k \in \{\overline{T_{dst}}\} \cup \{\overline{T}\} \quad T_k = C_k\!<\!\overline{p}\!>$
$\quad\quad\quad E_k \in L_E[(\theta, \ell_{dst})] = \langle H[\theta], H[\ell_{dst}], C'_k\rangle \in DE\ C'_k <: C_k$ — By Df-Approx

$H' = H, K' = K, L'_I = L_I$ — By sub-derivation of Ir-Write

$S[\ell] = C_\ell\!<\!\overline{p}\!>\!(\overline{v}) \quad fields(C_\ell\!<\!\overline{p}\!>) = \overline{T'}\ \overline{f}$ — By sub-derivation of Ir-Write

$S' = S[\ell \mapsto C_\ell\!<\!\overline{p}\!>\!([v/v_i]\overline{v})]$ — By sub-derivation of Ir-Write

$O = H[\theta] \quad O_\ell = H[\ell] \quad T'_i = C_i\!<\!\overline{p'}\!>$ — By sub-derivation of Ir-Write

$E = \langle O, O_\ell, C_v\rangle \in DE \quad C_v <: C_i$ — By sub-derivation of Ir-Write

$L'_E = L_E[(\theta, \ell) \mapsto_\cup \{E\}]$ — By sub-derivation of Ir-Write

$\exists \Sigma' \supseteq \Sigma \ \ and\ T' <: T\ s.t.\ \emptyset, \Sigma', \theta \vdash e' : T'\ and\ \Sigma' \vdash S'$ — By FDJ Type Preservation

$\Sigma'[\ell] = C\!<\!\overline{\ell'.d}\!>$ — $\Sigma' \vdash S'$

$\forall \ell_{dst} \in dom(H'),\ fields(\Sigma'[\ell_{dst}]) = \overline{T_{dst}}\ \overline{f},$
$\quad \forall m.\ mtype(m, \Sigma'[\ell_{dst}]) = \overline{T} \to T_R$
$\quad\quad \forall T_k \in \{\overline{T_{dst}}\} \cup \{\overline{T}\} \quad T_k = C_k\!<\!\overline{p}\!>$
$\quad\quad\quad E_k \in L'_E[(\theta, \ell_{dst})] = \langle H'[\theta], H'[\ell_{dst}], C'_k\rangle \in DE\ C'_k <: C_k$ — By above

$(S', H', K', L'_I, L'_E) \sim (DO, DD, DE)$ — By Df-Approx

This proves (1).

$\emptyset, \emptyset, DO, DD, DE \vdash_O e'$ — By Df-Loc, since $e' = v_i$

This proves (2).

$DO, DD, DE \vdash_{CT,H} \Sigma$ — By assumption

$\forall \ell \in dom(S), \Sigma[\ell] = C_\ell\!<\!\overline{p}\!>$
$\quad H[\ell] = O_\ell = \langle O_{id}, C_\ell\!<\!\overline{D_\ell}\!>\rangle \in DO$
$\quad \forall m.\ mbody(m, C_\ell\!<\!\overline{p}\!>) = (\overline{x} : \overline{T},\ e_R)$
$\quad\quad \{\overline{x} : \overline{T},\ \texttt{this} : C_\ell\!<\!\overline{p}\!>\}, \emptyset, DO, DD, DE \vdash_{O_\ell} e_R$ — By sub-derivation of Df-Sigma

$H' = H$ — By sub-derivation of Ir-Write

$S[\ell] = C\!<\!\overline{p}\!>\!(\overline{v}) \quad fields(C\!<\!\overline{p}\!>) = \overline{T}\ \overline{f}$ — By sub-derivation of Ir-Write

$S' = S[\ell \mapsto C\!<\!\overline{p}\!>\!([v/v_i]\overline{v})]$ — By sub-derivation of Ir-Write

$DO, DD, DE \vdash_{CT,H'} \Sigma'$ — By Df-Sigma with the above H' and $\Sigma' = \Sigma$

This proves (3).

Case Ir-Invk: e $= \ell.m(\overline{v})$, and $e' = \ell \triangleright [\overline{v}/\overline{x}, \ell/\texttt{this}]e_R$

To Show:

(1) $(S', H', K', L'_I, L'_E) \sim (DO, DD, DE)$

(2) $\emptyset, \emptyset, DO, DD, DE \vdash_O e'$

(3) $DO, DD, DE \vdash_{CT,H'} \Sigma'$

$\theta \vdash e; S; H; K; L_I; L_E \rightsquigarrow_G e'; S'; H'; K'; L'_I; L'_E$	By assumption
$(S, H, K, L_I, L_E) \sim (DO, DD, DE)$	By assumption
$\forall \ell \in dom(S),\ \Sigma(\ell) = C\langle\overline{\ell'.d}\rangle$	Since $\Sigma \vdash S$
$H[\theta] = O = \langle O_{id}, C\langle\overline{D}\rangle\rangle \in DO$	By Df-Approx
$\forall \theta'_j.d_j \in \overline{\theta'.d}\ K[\theta'_j.d_j] = D_j = \langle D_{id_j}, d_j\rangle \in rng(DD)$	By Df-Approx
$\forall d_i \in domains(C\langle\overline{\theta'.d}\rangle)\ K[\theta.d_i] = D_i = \langle D_{id_i}, d_i\rangle$	
$\{(O, d_i) \mapsto D_i\} \in DD$	By Df-Approx
$\forall \ell_{src} \in dom(H),\ fields(\Sigma[\ell_{src}]) = \overline{T_{src}}\ \overline{f},$	
$\forall m.\ mtype(m, \Sigma[\ell_{src}]) = \overline{T} \to T_R$	
$\forall T_k \in \{\overline{T_{src}}\} \cup \{T_R\} \quad T_k = C_k\langle\overline{p}\rangle$	
$E'_k \in L_I[(\ell_{src}, \theta)] = \langle H[\ell_{src}], H[\theta], C'_k\rangle \in DE \quad C'_k <: C_k$	By Df-Approx
$\forall \ell_{dst} \in dom(H),\ fields(\Sigma[\ell_{dst}]) = \overline{T_{dst}}\ \overline{f},$	
$\forall m.\ mtype(m, \Sigma[\ell_{dst}]) = \overline{T} \to T_R$	
$\forall T_k \in \{\overline{T_{dst}}\} \cup \{\overline{T}\} \quad T_k = C_k\langle\overline{p}\rangle$	
$E_k \in L_E[(\theta, \ell_{dst})] = \langle H[\theta], H[\ell_{dst}], C'_k\rangle \in DE\ C'_k <: C_k$	By Df-Approx
$S' = S \quad H' = H \quad K' = K$	By sub-derivation of Ir-Invk
$S[\ell] = C_\ell\langle\overline{p}\rangle(\overline{v}) \quad mbody(m, C_\ell\langle\overline{p}\rangle) = (\overline{x}, e_R)$	By sub-derivation of Ir-Invk
$H[\theta] = O \quad H[\ell] = O_\ell$	By sub-derivation of Ir-Invk
$mtype(m, C_\ell\langle\overline{p}\rangle) = \overline{T} \to T_R \quad T_R = C_R\langle\overline{p'}\rangle$	By sub-derivation of Ir-Invk
$E' = \langle O_\ell, O, C'_R\rangle \in DE \quad C'_R <: C_R$	By sub-derivation of Ir-Invk
$L'_I = L_I[(\ell, \theta) \mapsto_\cup \{E'\}]$	By sub-derivation of Ir-Invk
$\forall i \in 1..\lvert\overline{T}\rvert\ T_i = C_i\langle\overline{p''}\rangle \quad E_i = \langle O, O_\ell, C'_i\rangle \in DE \quad C'_i <: C_i$	By sub-derivation of Ir-Invk
$L'_E = L_E[(\theta, \ell) \mapsto_\cup \{E_i\}]$	By sub-derivation of Ir-Invk
$\exists \Sigma' \supseteq \Sigma \ \ and\ T' <: T\ s.t.\ \emptyset, \Sigma', \theta \vdash e' : T'\ and\ \Sigma' \vdash S'$	By FDJ Type Preservation
$\Sigma'[\ell] = C_\ell\langle\overline{\ell'.d}\rangle$	$\Sigma' \vdash S'$

$\forall \ell_{src} \in dom(H'),\ fields(\Sigma'[\ell_{src}]) = \overline{T_{src}}\ \overline{f},$

$\quad \forall m.\ mtype(m, \Sigma'[\ell_{src}]) = \overline{T} \to T_R$

$\quad\quad \forall T_k \in \{\overline{T_{src}}\} \cup \{T_R\} \quad T_k = C_k\!<\overline{p}>$

$\quad\quad\quad E'_k \in L'_I[(\ell_{src}, \theta)] = \langle H'[\ell_{src}], H'[\theta], C'_k\rangle \in DE \quad C'_k <: C_k$ By above

$\forall \ell_{dst} \in dom(H'),\ fields(\Sigma'[\ell_{dst}]) = \overline{T_{dst}}\ \overline{f},$

$\quad \forall m.\ mtype(m, \Sigma'[\ell_{dst}]) = \overline{T} \to T_R$

$\quad\quad \forall T_k \in \{\overline{T_{dst}}\} \cup \{\overline{T}\} \quad T_k = C_k\!<\overline{p}>$

$\quad\quad\quad E_k \in L'_E[(\theta, \ell_{dst})] = \langle H'[\theta], H'[\ell_{dst}], C'_k\rangle \in DE \quad C'_k <: C_k$ By above

$(S', H', K', L'_I, L'_E) \sim (DO, DD, DE)$ By Df-Approx

This proves (1).

$\emptyset, \emptyset, DO, DD, DE \vdash_O e$ By assumption

$e = \ell.m(\overline{v}) \quad e_0 = \ell \quad \overline{e} = \overline{v}$ By assumption

$e' = \ell \triangleright [\overline{v}/\overline{x}, \ell/\texttt{this}]e_R$ By assumption

$\emptyset, \Sigma, \theta \vdash e : T$ By assumption

$\exists \Sigma' \supseteq \Sigma \ \ and\ T' <: T\ s.t.\ \emptyset, \Sigma', \theta \vdash e' : T'\ and\ \Sigma' \vdash S'$ By FDJ Type Preservation

$\quad e_0 : T_0 \quad T_0 = C_\ell\!<\overline{p}>$ By sub-derivation of Df-Invk

$mtype(m, C_\ell\!<\overline{p}>) = \overline{T} \to T_R$ By sub-derivation of Df-Invk

$\emptyset, \emptyset, DO, DD, DE \vdash_O e_0$ By sub-derivation of Df-Invk

$\emptyset, \emptyset, DO, DD, DE \vdash_O \overline{e}$ By sub-derivation of Df-Invk

$\{\overline{x} : \overline{T}, \texttt{this} : C_\ell\!<\alpha, \overline{\beta}>\}, \Sigma, \theta \vdash e_R : T_R \quad T_R <: T$ By FDJ $Meth$OK:

$S[\ell] = C_\ell\!<d, \overline{d'}>(\overline{v})$ By sub-derivation of Ir-Invk

$mbody(m, C_\ell\!<d, \overline{d'}>) = (\overline{x}, e_R)$ By sub-derivation of Ir-Invk

$\Sigma[\ell] = C_\ell\!<d, \overline{d'}> = T_0$ Since $e_0 = \ell$, by T-Store

$e_0 : C_\ell\!<d, \overline{d'}>$ Since $e_0 = \ell$, by T-Store

$mtype(m, C_\ell\!<d, \overline{d'}>) = \overline{T} \to T_R$ Since $e_0 = \ell$, by T-Store

$\overline{v} : \overline{T_a}$ By inversion

$\overline{T_a} <: [\overline{v}/\overline{x}, \ell/\texttt{this}]\overline{T}$ For some $\overline{T_a}$ and $\overline{T}$

there are some $D\!<\overline{d}>$ and T'_R so that: By Method Lemma

$T'_R <: T_R$ and $C_\ell\!<d, \overline{d'}> <: D\!<\overline{d}>$ By Method Lemma

so that $\{\overline{x} : \overline{T}, \texttt{this} : D\!<\overline{d}>\}, \Sigma, \theta \vdash e_R : T'_R$ By Method Lemma

there exists $T_S, T_S <: T'_R$ s. t. $[\overline{v}/\overline{x}, \ell/\texttt{this}]e_R : T_S$ Since term substitution preserves typing

$T_S <: T'_R$ and $T'_R <: T_R$ By above

$T_S <: T_R$ By transitivity of $<:$

Take $T = T' = T_R$ in FDJ Preservation

$\{\overline{x} : \overline{T}, \texttt{this} : C_\ell\!<\!d, \overline{d'}\!>\}, \emptyset, DO, DD, DE \vdash_{O_C} e_R$ By Df-Sigma

$O_C = H[\ell]$ By Df-Sigma

$\emptyset, \emptyset, DO, DD, DE \vdash_O \ell$ By Df-Loc

$\emptyset, \emptyset, DO, DD, DE \vdash_{O_C} [\overline{v}/\overline{x}, \ell/\texttt{this}]e_R$ By Df-Substitution Lemma

$\emptyset, \emptyset, DO, DD, DE \vdash_O \ell \triangleright [\overline{v}/\overline{x}, \ell/\texttt{this}]e_R$ By Df-Context

This proves (2).

$DO, DD, DE \vdash_{CT,H} \Sigma$ By assumption

$S' = S, H' = H$ By sub-derivation of Ir-Invk

$DO, DD, DE \vdash_{CT,H'} \Sigma'$ By Df-Sigma with the above H' and $\Sigma' = \Sigma$

This proves (3).

Case Ir-Context: e $= \ell \triangleright v$, and $e' = v$

To Show:

(1) $(S', H', K', L'_I, L'_E) \sim (DO, DD, DE)$

(2) $\emptyset, \emptyset, DO, DD, DE \vdash_O e'$

(3) $DO, DD, DE \vdash_{CT,H'} \Sigma'$

$(S, H, K, L_I, L_E) \sim (DO, DD, DE)$ By assumption

$S' = S, H' = H, K' = K, L'_I = L_I, L'_E = L_E$ By sub-derivation of Ir-Context

This proves (1).

$\emptyset, \emptyset, DO, DD, DE \vdash_O e'$ By Df-Loc, since $e' = v$

This proves (2).

$DO, DD, DE \vdash_{CT,H} \Sigma$ By assumption

$S' = S, H' = H$ By sub-derivation of Ir-Context

$DO, DD, DE \vdash_{CT,H'} \Sigma'$ Take $\Sigma' = \Sigma$

This proves (3).

Case Irc-New: e $= \texttt{new}\ C\!<\!\overline{p}\!>\!(v_{1..i-1}, e_i, e_{i+1..n})$, and $e' = \texttt{new}\ C\!<\!\overline{p}\!>\!(v_{1..i-1}, e'_i, e_{i+1..n})$.

To Show:

(1) $(S', H', K', L'_I, L'_E) \sim (DO, DD, DE)$

(2) $\emptyset, \emptyset, DO, DD, DE \vdash_O e'$

(3) $DO, DD, DE \vdash_{CT,H'} \Sigma'$

$\theta \vdash e_i; S; H; K; L_I; L_E \rightsquigarrow_G e_i'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-New
$(S', H', K', L_I', L_E') \sim (DO, DD, DE)$ By induction hypothesis
This proves (1).

$\theta \vdash e_i; S; H; K; L_I; L_E \rightsquigarrow_G e_i'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-New
$\emptyset, \emptyset, DO, DD, DE \vdash_O e_i'$ By induction hypothesis
$\emptyset, \emptyset, DO, DD, DE \vdash_O \texttt{new}\ C\texttt{<}\overline{p}\texttt{>}(v_{1..i-1}, e_i', e_{i+1..n})$ By Df-New
This proves (2).

$\theta \vdash e_i; S; H; K; L_I; L_E \rightsquigarrow_G e_i'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-New
$DO, DD, DE \vdash_{CT,H'} \Sigma'$ By induction hypothesis, take $\Sigma' = \Sigma$
This proves (3).

Case Irc-Read: e $= e_0.f_k$, and $e' = e_0'.f_k$.
To Show:
(1) $(S', H', K', L_I', L_E') \sim (DO, DD, DE)$
(2) $\emptyset, \emptyset, DO, DD, DE \vdash_O e'$
(3) $DO, DD, DE \vdash_{CT,H'} \Sigma'$

$\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-Read
$(S', H', K', L_I', L_E') \sim (DO, DD, DE)$ By induction hypothesis
This proves (1).

$\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-Read
$\emptyset, \emptyset, DO, DD, DE \vdash_O e_0'$ By induction hypothesis
$\emptyset, \emptyset, DO, DD, DE \vdash_O e_0'.f_k$ By Df-Read
This proves (2).

$\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-Read
$DO, DD, DE \vdash_{CT,H'} \Sigma'$ By induction hypothesis, take $\Sigma' = \Sigma$
This proves (3).

Case Irc-Write-Rcv: e $= (e_0.f_k = e_1)$, and $e' = (e_0'.f_k = e_1)$.
To Show:
(1) $(S', H', K', L_I', L_E') \sim (DO, DD, DE)$
(2) $\emptyset, \emptyset, DO, DD, DE \vdash_O e'$
(3) $DO, DD, DE \vdash_{CT,H'} \Sigma'$

$\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-Write-Rcv

$(S', H', K', L_I', L_E') \sim (DO, DD, DE)$ By induction hypothesis

This proves (1).

$\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-Write-Rcv

$\emptyset, \emptyset, DO, DD, DE \vdash_O e_0'$ By induction hypothesis

$\emptyset, \emptyset, DO, DD, DE \vdash_O e_1$ By Df-Write

$\emptyset, \emptyset, DO, DD, DE \vdash_O e_0'.f_k = e_1$ By Df-Write

This proves (2).

$\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-Write-Rcv

$DO, DD, DE \vdash_{CT,H'} \Sigma'$ By induction hypothesis, take $\Sigma' = \Sigma$

This proves (3).

Case Irc-Write-Arg: e $= (v.f_k = e_1)$, and $e' = (v.f_k = e_1')$.

To Show:

(1) $(S', H', K', L_I', L_E') \sim (DO, DD, DE)$

(2) $\emptyset, \emptyset, DO, DD, DE \vdash_O e'$

(3) $DO, DD, DE \vdash_{CT,H'} \Sigma'$

$\theta \vdash e_1; S; H; K; L_I; L_E \rightsquigarrow_G e_1'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-Write-Arg

$(S', H', K', L_I', L_E') \sim (DO, DD, DE)$ By induction hypothesis

This proves (1).

$\theta \vdash e_1; S; H; K; L_I; L_E \rightsquigarrow_G e_1'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-Write-Arg

$\emptyset, \emptyset, DO, DD, DE \vdash_O e_1'$ By induction hypothesis

$\emptyset, \emptyset, DO, DD, DE \vdash_O e_0'.f_k = e_1'$ By Df-Write

This proves (2).

$\theta \vdash e_1; S; H; K; L_I; L_E \rightsquigarrow_G e_1'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-Write-Arg

$DO, DD, DE \vdash_{CT,H'} \Sigma'$ By induction hypothesis, take $\Sigma' = \Sigma$

This proves (3).

Case Irc-Recvinvk: e $= e_0.m(\overline{e})$, and $e' = e_0'.m(\overline{e})$.

To Show:

(1) $(S', H', K', L_I', L_E') \sim (DO, DD, DE)$

(2) $\emptyset, \emptyset, DO, DD, DE \vdash_O e'$

(3) $DO, DD, DE \vdash_{CT,H'} \Sigma'$

$\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-RecvInvk
$(S', H', K', L_I', L_E') \sim (DO, DD, DE)$ By induction hypothesis
This proves (1).

$\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-RecvInvk
$\emptyset, \emptyset, DO, DD, DE \vdash_O e_0'$ By induction hypothesis
$\emptyset, \emptyset, DO, DD, DE \vdash_O \overline{e}$ By Df-Invk
$\emptyset, \emptyset, DO, DD, DE \vdash_O e_0'.m(\overline{e})$ By Df-Invk
This proves (2).

$\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-RecvInvk
$DO, DD, DE \vdash_{CT,H'} \Sigma'$ By induction hypothesis, take $\Sigma' = \Sigma$
This proves (3).

Case Irc-Arcinvk: e $= v.m(v_{1..i-1}, e_i, e_{i+1..n})$,and $e' = v.m(v_{1..i-1}, e_i', e_{i+1..n})$.
To Show:
(1) $(S', H', K', L_I', L_E') \sim (DO, DD, DE)$
(2) $\emptyset, \emptyset, DO, DD, DE \vdash_O e'$
(3) $DO, DD, DE \vdash_{CT,H'} \Sigma'$

$\theta \vdash e_i; S; H; K; L_I; L_E \rightsquigarrow_G e_i'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-ArgInvk
$(S', H', K', L_I', L_E') \sim (DO, DD, DE)$ By induction hypothesis
This proves (1).

$\theta \vdash e_i; S; H; K; L_I; L_E \rightsquigarrow_G e_i'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-ArgInvk
$\emptyset, \emptyset, DO, DD, DE \vdash_O e_i'$ By induction hypothesis
$\emptyset, \emptyset, DO, DD, DE \vdash_O v.m(v_{1..i-1}, e_i', e_{i+1..n})$ By Df-Invk
This proves (2).

$\theta \vdash e_i; S; H; K; L_I; L_E \rightsquigarrow_G e_i'; S'; H'; K'; L_I'; L_E'$ By sub-derivation of Irc-ArgInvk
$DO, DD, DE \vdash_{CT,H'} \Sigma'$ By induction hypothesis, take $\Sigma' = \Sigma$
This proves (3).

Case Irc-Context: e $= \ell \triangleright e_0$, and $e' = \ell \triangleright e_0'$.
To Show:
(1) $(S', H', K', L_I', L_E') \sim (DO, DD, DE)$

(2) $\emptyset, \emptyset, DO, DD, DE \vdash_O e'$
(3) $DO, DD, DE \vdash_{CT,H'} \Sigma'$

$\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'$	By sub-derivation of IRC-CONTEXT
$(S', H', K', L_I', L_E') \sim (DO, DD, DE)$	By induction hypothesis
This proves (1).	

$\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'$	By sub-derivation of IRC-CONTEXT
$O_\ell = H[\ell]$	By induction hypothesis
$\emptyset, \emptyset, DO, DD, DE \vdash_{O_\ell} e_0'$	By induction hypothesis
$\emptyset, \emptyset, DO, DD, DE \vdash_O \ell \triangleright e_0'$	By DF-CONTEXT
This proves (2).	

$\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e_0'; S'; H'; K'; L_I'; L_E'$	By sub-derivation of IRC-CONTEXT
$DO, DD, DE \vdash_{CT,H'} \Sigma'$	By induction hypothesis, take $\Sigma' = \Sigma$
This proves (3).	

■

4.6 Theorem: Dataflow Progress

$$
\begin{array}{l}
If \\
\quad \boxed{\emptyset, \Sigma, \theta \vdash e : T} \\
\quad \boxed{\Sigma \vdash S} \\
\quad DO, DD, DE \vdash_{CT,H} \Sigma \\
\quad \emptyset, \emptyset, DO, DD, DE \vdash_O e \\
\quad (S, H, K, L_I, L_E) \sim (DO, DD, DE) \\
then \\
\quad either \ \boxed{e \ is \ a \ value} \\
\quad or \ else \ \theta \vdash \boxed{e; S}; H; K; L_I; L_E \rightsquigarrow_G \boxed{e'; S'}; H'; K'; L'_I; L'_E
\end{array}
$$

Proof: We prove progress by derivation of $\emptyset, \emptyset, DO, DD, DE \vdash_O e$, with a case analysis on the last typing rule used. The most interesting cases are Df-New, Df-Read (page 38), Df-Write (page 40), and Df-Invk (page 42).

Case DF-NEW : e = $new\ C\overline{<p>}(\overline{e})$.

Subcase $\overline{e} = \overline{v}$ that is $e = new\ C\overline{<p>}(\overline{v})$. Take $e' = \ell$, then Ir-New can apply.

To show:
(1) $\forall i \in |\overline{\ell'.d}| \quad D_i = K[\ell'_i.d_i]$
(2) $O_C = \langle O_{id}, C\overline{<D>}\rangle \quad O_C \in DO$
(3) $\forall d_j \in domains(C\overline{<\ell'.d>}) \quad D_j = DD[(O_C, d_j)]$

$(S, H, K, L_I, L_E) \sim (DO, DD, DE)$	By assumption
$\forall \ell \in dom(S), \Sigma[\ell] = C\overline{<\ell'.d>}$	$\Sigma \vdash S$
$H[\ell] = O_C = \langle O_{id}, C\overline{<D>}\rangle \in DO$	By Df-Approx
$\forall \ell'_j.d_j \in \overline{\ell'.d} \quad K[\ell'_j.d_j] = D_j = \langle D_{id_j}, d_j\rangle \in rng(DD)$	By Df-Approx
$\forall d_i \in domains(C\overline{<\ell'.d>})\ K[\ell.d_i] = D_i = \langle D_{id_i}, d_i\rangle$	
$\{(O_C, d_i) \mapsto D_{\ell i}\} \in DD$	By Df-Approx

This proves (1).

$CT(C) = \texttt{class}\ C\texttt{<}\overline{\alpha}, \overline{\beta}\texttt{>}\ \texttt{extends}\ C'\texttt{<}\overline{\alpha}\texttt{>}\ \ldots\ \{\ \overline{T}\ \overline{f};\ \overline{dom};\ \ldots;\ \overline{md};\ \}$

$\emptyset, \emptyset, DO, DD, DE \vdash_O e$	By assumption
$\forall i \in 1..\lvert\overline{p}\rvert \quad D_i = DD[(O, p_i)]$	By sub-derivation of Df-New
$params(C) = \overline{\alpha}$	By sub-derivation of Df-New
$O_C = \langle\ O_{id},\ C\texttt{<}\overline{D}\texttt{>}\ \rangle \quad \{O_C\} \subseteq DO$	By sub-derivation of Df-New
This proves (2).	
$\{(O_C, \alpha_i) \mapsto D_i\} \subseteq DD$	By sub-derivation of Df-New
$DO, DD, DE \vdash_O ddomains(C, O_C)$	By sub-derivation of Df-New
This proves (3).	By Df-Domains Lemma

Subcase $e = new\ C\texttt{<}\overline{p}\texttt{>}(v_{1..i-1}, e_i, e_{i+1..n})$. Then Irc-New can apply.

$\Gamma, \Upsilon, DO, DD, DE \vdash_O e_i$	By sub-derivation of Df-New
$\theta \vdash e_i; S; H; K; L_I; L_E \rightsquigarrow_G S'; H'; K'; L'_I; L'_E$	By induction hypothesis
$\theta \vdash \texttt{new}\ C\texttt{<}\overline{p}\texttt{>}(v_{1..i-1}, e_i, e_{i+1..n}) S; H; K; L_I; L_E \rightsquigarrow_G$ $\texttt{new}\ C\texttt{<}\overline{p}\texttt{>}(v_{1..i-1}, e'_i, e_{i+1..n}); S'; H'; K'; L'_I; L'_E$	By Irc-New
This proves (2).	
$\{(O_C, \alpha_i) \mapsto D_i\} \subseteq DD$	By sub-derivation of Df-New
$DO, DD, DE \vdash_O ddomains(C, O_C)$	By sub-derivation of Df-New
Take $e' = \texttt{new}\ C\texttt{<}\overline{p}\texttt{>}(v_{1..i-1}, e'_i, e_{i+1..n})$	By Df-Domains Lemma

Case DF-VAR : $e = x$.
Not applicable since variable is not a closed term.

Case DF-LOC : $e = \ell$.
e is a value.

Case DF-READ : e = $e_0.f_i$. There are two subcases to consider depending on whether the receiver e_0 is a value.

Subcase $e_0 = \ell$. Then $e = \ell.f_i$
To show:
(1) $O = H[\theta]$
(2) $O_\ell = H[\ell]$
(3) $E = \langle O_\ell, O, C_v \rangle \in DE \quad C_v <: C_i$

$DO, DD, DE \vdash_{CT,H} \Sigma$ By assumption

$\forall \ell' \in dom(S), \Sigma[\ell'] = C'<\overline{p}>$ By sub-derivation of Df-Sigma

$H[\ell'] = O' = \langle O_{id}, C'<\overline{D'}>\rangle \in DO$ By sub-derivation of Df-Sigma

$H[\theta] = O = \langle O_{\theta id}, C<\overline{D}>\rangle \in DO$ Since $\theta \in dom(S)$

$H[\ell] = O_\ell = \langle O_{\ell id}, C_\ell<\overline{D_\ell}>\rangle \in DO$ Since $\ell \in dom(S)$

this proves (1), and (2).

$(S, H, K, L_I, L_E) \sim (DO, DD, DE)$ By assumption

$\forall \ell \in dom(S),\ \Sigma(\ell) = C<\overline{\ell'.d}>$ Since $\Sigma \vdash S$

$H[\theta] = O = \langle O_{id}, C<\overline{D}>\rangle \in DO$ By Df-Approx

$\forall \theta'_j.d_j \in \overline{\theta'.d}\ K[\theta'_j.d_j] = D_j = \langle D_{id_j}, d_j\rangle \in rng(DD)$ By Df-Approx

$\forall d_i \in domains(C<\overline{\theta'.d}>)\ K[\theta.d_i] = D_i = \langle D_{id_i}, d_i\rangle$

$\{(O, d_i) \mapsto D_i\} \in DD$ By Df-Approx

$\forall \ell_{src} \in dom(H),\ fields(\Sigma[\ell_{src}]) = \overline{T_{src}}\ \overline{f},$

$\forall m.\ mtype(m, \Sigma[\ell_{src}]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T_{src}}\} \cup \{T_R\} \quad T_k = C_k<\overline{p}>$

$E'_k \in L_I[(\ell_{src}, \theta)] = \langle H[\ell_{src}], H[\theta], C'_k\rangle \in DE \quad C'_k <: C_k$ By Df-Approx

$\forall \ell_{dst} \in dom(H),\ fields(\Sigma[\ell_{dst}]) = \overline{T_{dst}}\ \overline{f},$

$\forall m.\ mtype(m, \Sigma[\ell_{dst}]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T_{dst}}\} \cup \{\overline{T}\} \quad T_k = C_k<\overline{p}>$

$E_k \in L_E[(\theta, \ell_{dst})] = \langle H[\theta], H[\ell_{dst}], C'_k\rangle \in DE\ C'_k <: C_k$ By Df-Approx

$\emptyset, \emptyset, DO, DD, DE \vdash_O \ell.f_i$ By assumption

$fields(\Sigma[\ell]) = \overline{T'}\ \overline{f}$ By FDJ T-Store

Since $e_0 = \ell \in dom(H)$

$\ell : \Sigma[\ell] = C_\ell<\overline{p}>\ (T'_i\ f_i) \in fields(C_\ell<\overline{p}>)\ T'_i = C_i<\overline{p'}>$ By sub-derivation of Df-Read

$DO, DD, DE \vdash_O import(\Sigma[\ell], T'_i)$ By sub-derivation of Df-Read

$\emptyset, \emptyset, DO, DD, DE \vdash_O \ell$ By sub-derivation of Df-Read

Take $\ell_{src} = \ell$.

$\ell : \Sigma[\ell] = C_\ell\!<\overline{p}>\ (T_i'\ f_i) \in fields(C_\ell\!<\overline{p}>)\ T_i' = C_i\!<\overline{p'}>$ By above sub-derivation

$\forall m.\ mtype(m, \Sigma[\ell]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T'}\} \cup \{T_R\} \quad T_k = C_k\!<\overline{p''}>$

$\langle H[\ell], H[\theta], C_k' \rangle \in DE \quad C_k' <: C_k$ By above DF-APPROX

Take $T_k = T_i' \in \overline{T'}, C_i = C_k$, and $C_v = C_k'$, this proves (3).

Subcase $e_0 = e_0'.f_i$. That is, e_0 is not a value

From IRC-READ:

$\theta \vdash e_0'; S; H; K; L_I; L_E \rightsquigarrow_G e_0''; S'; H'; K'; L_I'; L_E'$ By induction hypothesis

$\theta \vdash e_0'.f_i; S; H; K; L_I; L_E \rightsquigarrow_G e_0''.f_i; S'; H'; K'; L_I'; L_E'$ By IRC-READ

Take $e' = e_0''.f_i$.

Case DF-WRITE : e = $(e_0.f_i = e_1)$. There are three subcases to consider depending on whether the receiver e_0, and e_1 are values.

Subcase $e_0 = \ell$, and $e_1 = v$. Then $e = (\ell.f_i = v)$

To show:

(1) $O = H[\theta]$

(2) $O_\ell = H[\ell]$

(3) $E = \langle O, O_\ell, C_v \rangle \in DE \quad C_v <: C_i$

$DO, DD, DE \vdash_{CT,H} \Sigma$ By assumption

$\forall \ell' \in dom(S), \Sigma[\ell'] = C'<\overline{p}>$ By sub-derivation of Df-Sigma

$H[\ell'] = O' = \langle O_{id}, C'<\overline{D'}>\rangle \in DO$ By sub-derivation of Df-Sigma

$H[\theta] = O = \langle O_{\theta id}, C<\overline{D}>\rangle \in DO$ Since $\theta \in dom(S)$

$H[\ell] = O_\ell = \langle O_{\ell id}, C_\ell<\overline{D_\ell}>\rangle \in DO$ Since $\ell \in dom(S)$

this proves (1), and (2).

$(S, H, K, L_I, L_E) \sim (DO, DD, DE)$ By assumption

$\forall \ell \in dom(S),\ \Sigma(\ell) = C<\overline{\ell'.d}>$ Since $\Sigma \vdash S$

$H[\theta] = O = \langle O_{id}, C<\overline{D}>\rangle \in DO$ By Df-Approx

$\forall \theta'_j.d_j \in \overline{\theta'.d}\ K[\theta'_j.d_j] = D_j = \langle D_{id_j}, d_j\rangle \in rng(DD)$ By Df-Approx

$\forall d_i \in domains(C<\overline{\theta'.d}>)\ K[\theta.d_i] = D_i = \langle D_{id_i}, d_i\rangle$

$\{(O, d_i) \mapsto D_i\} \in DD$ By Df-Approx

$\forall \ell_{src} \in dom(H),\ fields(\Sigma[\ell_{src}]) = \overline{T_{src}}\ \overline{f},$

$\forall m.\ mtype(m, \Sigma[\ell_{src}]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T_{src}}\} \cup \{T_R\} \quad T_k = C_k<\overline{p}>$

$E'_k \in L_I[(\ell_{src}, \theta)] = \langle H[\ell_{src}], H[\theta], C'_k\rangle \in DE \quad C'_k <: C_k$ By Df-Approx

$\forall \ell_{dst} \in dom(H),\ fields(\Sigma[\ell_{dst}]) = \overline{T_{dst}}\ \overline{f},$

$\forall m.\ mtype(m, \Sigma[\ell_{dst}]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T_{dst}}\} \cup \{\overline{T}\} \quad T_k = C_k<\overline{p}>$

$E_k \in L_E[(\theta, \ell_{dst})] = \langle H[\theta], H[\ell_{dst}], C'_k\rangle \in DE\ C'_k <: C_k$ By Df-Approx

$\emptyset, \emptyset, DO, DD, DE \vdash_O \ell.f_i = v$ By assumption:

Since $e_0 = \ell \in dom(H) \quad e_1 = v$:

$\ell : \Sigma[\ell] = C_\ell<\overline{p}>\ (T'_i\ f_i) \in fields(C_\ell<\overline{p}>) = \overline{T'}\ \overline{f}\ T'_i = C_i<\overline{p'}>$ By sub-derivation of Df-Write

$v : \Sigma[v] = C_v<\overline{p''}> \quad \Sigma[v] <: T'_i$ By sub-derivation of Df-Write

$DO, DD, DE \vdash_O export(\Sigma[\ell], \Sigma[v])$ By sub-derivation of Df-Write

$\emptyset, \emptyset, DO, DD, DE \vdash_O \ell$ By sub-derivation of Df-Write

$\emptyset, \emptyset, DO, DD, DE \vdash_O v$ By sub-derivation of Df-Write

Take $\ell_{dst} = \ell$.

$\ell : \Sigma[\ell] = C_\ell<\overline{p}>\ (T'_i\ f_i) \in fields(C_\ell<\overline{p}>) = \overline{T'}\ \overline{f}\ T'_i = C_i<\overline{p'}>$ By the sub-derivation above

$\forall m.\ mtype(m, \Sigma[\ell]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T'}\} \cup \{\overline{T}\} \quad T_k = C_k<\overline{p'''}>$

$\langle H[\theta], H[\ell], C'_k\rangle \in DE \quad C'_k <: C_k$ By above Df-Approx

Take $T_k = T'_i \in \overline{T'}, C_i = C_k$, and $C_v = C'_k$, this proves (3).

Subcase $e_0 = e_0'$. Then $e = (e_0'.f_i = e_1)$
From IRC-Write-Rcv:

$\theta \vdash e_0'; S; H; K; L_I; L_E \rightsquigarrow_G e_0''; S'; H'; K'; L_I'; L_E'$ By induction hypothesis
$\theta \vdash e_0'.f_i = e_1; S; H; K; L_I; L_E \rightsquigarrow_G e_0''.f_i = e_1; S'; H'; K'; L_I'; L_E'$ By IRC-Write-Rcv
Take $e' = (e_0''.f_i = e_1)$.

Subcase $e_0 = v$, and $e_1 = e_1'$. Then $e = (v.f_i = e_1')$
From IRC-Write-Arg:

$\Gamma, \Upsilon, DO, DD, DE \vdash_O e_1$ By sub-derivation of Df-Write
$\theta \vdash e_1; S; H; K; L_I; L_E \rightsquigarrow_G e_1'; S'; H'; K'; L_I'; L_E'$ By induction hypothesis
$\theta \vdash v.f_i = e_1; S; H; K; L_I; L_E \rightsquigarrow_G v.f_i = e_1'; S'; H'; K'; L_I'; L_E'$ By IRC-Write-Arg
Take $e' = (v.f_i = e_1')$.

Case DF-INVK : e = $e_0.m(\overline{e})$. There are three subcases to consider, depending on whether the receiver e_0, or the arguments $\overline{e}$ are values.

Subcase $e_0 = \ell$, and $\overline{e} = \overline{v}$ that is $e = \ell.m(\overline{v})$
To show:
(1) $O = H[\theta]$
(2) $O_\ell = H[\ell]$
(3) $mtype(m, C_\ell{<}\overline{p}{>}) = \overline{T} \to T_R$ $T_R = C_R{<}\overline{p'}{>}$
$E' = \langle O_\ell, O, C_R' \rangle \in DE$ $C_R' <: C_R$
(4) $\forall i \in 1..|\overline{T}|$ $T_i = C_i{<}\overline{p''}{>}$ $E_i = \langle O, O_\ell, C_i' \rangle \in DE$ $C_i' <: C_i$

$DO, DD, DE \vdash_{CT,H} \Sigma$ By assumption

$\forall \ell' \in dom(S), \Sigma[\ell'] = C'<\overline{p}>$ By sub-derivation of DF-SIGMA

$H[\ell'] = O' = \langle O_{id}, C'<\overline{D'}>\rangle \in DO$ By sub-derivation of DF-SIGMA

$H[\theta] = O = \langle O_{\theta id}, C<\overline{D}>\rangle \in DO$ Since $\theta \in dom(S)$

$H[\ell] = O_\ell = \langle O_{\ell id}, C_\ell<\overline{D_\ell}>\rangle \in DO$ Since $\ell \in dom(S)$

this proves (1), and (2).

$(S, H, K, L_I, L_E) \sim (DO, DD, DE)$ By assumption

$\forall \ell \in dom(S),\ \Sigma(\ell) = C<\overline{\ell'.d}>$ Since $\Sigma \vdash S$

$H[\theta] = O = \langle O_{id}, C<\overline{D}>\rangle \in DO$ By DF-APPROX

$\forall \theta'_j.d_j \in \overline{\theta'.d}\ K[\theta'_j.d_j] = D_j = \langle D_{id_j}, d_j\rangle \in rng(DD)$ By DF-APPROX

$\forall d_i \in domains(C<\overline{\theta'.d}>)\ K[\theta.d_i] = D_i = \langle D_{id_i}, d_i\rangle$

$\{(O, d_i) \mapsto D_i\} \in DD$ By DF-APPROX

$\forall \ell_{src} \in dom(H),\ fields(\Sigma[\ell_{src}]) = \overline{T_{src}}\ \overline{f},$

$\forall m.\ mtype(m, \Sigma[\ell_{src}]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T_{src}}\} \cup \{T_R\} \quad T_k = C_k<\overline{p}>$

$E'_k \in L_I[(\ell_{src}, \theta)] = \langle H[\ell_{src}], H[\theta], C'_k\rangle \in DE \quad C'_k <: C_k$ By DF-APPROX

$\forall \ell_{dst} \in dom(H),\ fields(\Sigma[\ell_{dst}]) = \overline{T_{dst}}\ \overline{f},$

$\forall m.\ mtype(m, \Sigma[\ell_{dst}]) = \overline{T} \to T_R$

$\forall T_k \in \{\overline{T_{dst}}\} \cup \{\overline{T}\} \quad T_k = C_k<\overline{p}>$

$E_k \in L_E[(\theta, \ell_{dst})] = \langle H[\theta], H[\ell_{dst}], C'_k\rangle \in DE\ C'_k <: C_k$ By DF-APPROX

$\emptyset, \emptyset, DO, DD, DE \vdash_O \ell.m(\overline{v})$ By assumption

$\ell : \Sigma[\ell] = C_\ell<\overline{\ell'.d}>$ By sub-derivation of DF-INVK

$mtype(m, C_\ell<\overline{\ell'.d}>) = \overline{T} \to T_R \quad T_R = C_R<\overline{p'}>$ By sub-derivation of DF-INVK

$DO, DD, DE \vdash_O import(\Sigma[\ell], T_R)$ By sub-derivation of DF-INVK

$\forall i \in 1..|\overline{v}|\ v_i : \Sigma[v_i]\ \Sigma[v_i] <: T_i\ DO, DD, DE \vdash_O export(\Sigma[\ell], \Sigma[v_i])$ By sub-derivation of DF-INVK

$\emptyset, \emptyset, DO, DD, DE \vdash_O \ell$ By sub-derivation of DF-INVK

$\emptyset, \emptyset, DO, DD, DE \vdash_O \overline{v}$ By sub-derivation of DF-INVK

Take $\ell_{src} = \ell$.

$mtype(m, \Sigma[\ell]) = \overline{T} \to T_R \quad T_R = C_R<\overline{p'}>$ By above sub-derivation

$fields(\Sigma[\ell]) = \overline{T'}\ \overline{f}$ By FDJ T-Store

$\forall T_k \in \{\overline{T'}\} \cup \{T_R\} \quad T_k = C_k<\overline{p'''}>$

$\langle H[\ell], H[\theta], C'_k\rangle \in DE \quad C'_k <: C_k$ By above DF-APPROX

Take $T_k = T_R, C_R = C_k$ and $C'_R = C'_k$, this proves (3).

Take $\ell_{dst} = \ell$.

$mtype(m, \Sigma[\ell]) = \overline{T} \to T_R \quad T_i = C_i\!<\!\overline{p''}\!>$ By above sub-derivation

$fields(\Sigma[\ell]) = \overline{T'}\ \overline{f}$ By FDJ T-Store

$\forall T_k \in \{\overline{T'}\} \cup \{\overline{T}\} \quad T_k = C_k\!<\!\overline{p'''}\!>$

$\langle H[\theta], H[\ell], C'_k\rangle \in DE \quad C'_k <: C_k$ By above Df-Approx

Take $\forall i \in 1..\overline{T}.\ T_k = T_i \in \overline{T}, C_i = C_k$ and $C'_i = C'_k$. This proves (4).

Subcase $e_0 = e'_0$ that is $e = e'_0.m(\overline{e})$.

From IRC-RecvInvk

$\theta \vdash e'_0; S; H; K; L_I; L_E \rightsquigarrow_G e''_0; S'; H'; K'; L'_I; L'_E$ By induction hypothesis

$\theta \vdash e'_0.m(\overline{e}); S; H; K; L_I; L_E \rightsquigarrow_G e''_0.m(\overline{e}); S'; H'; K'; L'_I; L'_E$ By IRC-RecvInvk

Take $e' = e''_0.m(\overline{e})$.

Subcase $e_0 = v$ that is $e = v.m(v_{1..i-1}, e_i, e_{i+1..n})$.

From IRC-ArgInvk:

$\Gamma, \Upsilon, DO, DD, DE \vdash_O e_i$ By sub-derivation of Df-Invk

$\theta \vdash e_i; S; H; K; L_I; L_E \rightsquigarrow_G e'_i; S'; H'; K'; L'_I; L'_E$ By induction hypothesis

$\theta \vdash v.m(v_{1..i-1}, e_i, e_{i+1..n}); S; H; K; L_I; L_E \rightsquigarrow_G$

$v.m(v_{1..i-1}, e'_i, e_{i+1..n}); S'; H'; K'; L'_I; L'_E$ By IRC-ArgInvk

Take $e' = v.m(v_{1..i-1}, e'_i, e_{i+1..n})$.

Case DF-CONTEXT : e $= \ell \triangleright e_0$. there are two subcases to consider, depending on whether e_0 is a value

Subcase e_0 **is a value** that is $e = \ell \triangleright v$.

From Ir-Context:

Then Ir-Context can apply. Take $e' = v$.

Subcase e_0 **is a value** that is $e = \ell \triangleright e'_0$.

From Irc-Context:

$\theta \vdash e_0; S; H; K; L_I; L_E \rightsquigarrow_G e'_0; S'; H'; K'; L'_I; L'_E$ By induction hypothesis

$\theta \vdash \ell \triangleright e_0; S; H; K; L_I; L_E \rightsquigarrow_G \ell \triangleright e'_0; S'; H'; K'; L'_I; L'_E$ By Irc-Context

Take $e' = \ell \triangleright e'_0$.

■

$$\frac{}{\theta \vdash e; S; H; K; L_I; L_E \rightsquigarrow^*_G e; S; H; K; L_I; L_E}[\text{Df-Reflex}]$$

$$\frac{\theta \vdash e; S; H; K; L_I; L_E \rightsquigarrow^*_G e''; S''; H''; K''; L''_I; L''_E \quad \theta \vdash e''; S''; H''; K''; L''_I; L''_E \rightsquigarrow_G e'; S'; H'; K'; L'_I; L'_E}{\theta \vdash e; S; H; K; L_I; L_E \rightsquigarrow^*_G e'; S'; H'; K'; L'_I; L'_E}[\text{Df-Trans}]$$

Figure 14: Reflexive, transitive closure of the instrumented evaluation relation

4.7 Theorem: Object Graph Soundness

$$\begin{array}{l} \textit{If} \\ \quad G = \langle DO, DD, DE \rangle \\ \quad DO, DD, DE \vdash (CT, e_{root}) \\ \quad \forall e,\ \theta_0 \vdash e; \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rightsquigarrow^*_G e; S; H; K; L_I; L_E \\ \quad \Sigma \vdash S \\ \textit{then} \\ \quad DO, DD, DE \vdash_{CT,H} \Sigma \\ \quad (S, H, K, L_I, L_E) \sim (DO, DD, DE) \end{array}$$

where $\rightsquigarrow^*_G$ relation is the reflexive and transitive closure of $\rightsquigarrow_G$ relation (Fig. 14). θ_0 is the location of the first object instantiated by e_{root}.

To prove the Object Graph Soundness theorem, we need to show:

(1) $DO, DD, DE \vdash_{CT,H} \Sigma$

(2) $(S, H, K, L_I, L_E) \sim (DO, DD, DE)$

Proof: The proof is by induction on the $\rightsquigarrow^*_G$ relation. There are two cases to consider: [1]

Case Df-Reflex :

Since $S = \emptyset$:

$(S, H, K, L_I, L_E) \sim G$

Immediately, from Df-Sigma store constraint with $S = \emptyset$:

$DO, DD, DE \vdash_{CT,H} \Sigma$

Case Df-Trans :

By assumption:

$\theta_0 \vdash e; \emptyset; \emptyset; \emptyset; \emptyset; \emptyset \rightsquigarrow^*_G e; S; H; K; L_I; L_E$

Since $S = \emptyset$:

$(\emptyset, \emptyset, \emptyset, \emptyset, \emptyset) \sim G$

By inversion of Df-Trans:

[1] The soundness proof follows similar steps to the one of points-to analysis [1].

$\theta_0 \vdash e; \emptyset; \emptyset; \emptyset; \emptyset; \emptyset \rightsquigarrow^*_G e'; S'; H'; K'; L'_I; L'_E$
By induction hypothesis:
$(S'; H'; K'; L'_I; L'_E) \sim G$
By inversion of Df-Trans:
$\theta_0 \vdash e'; S'; H'; K'; L'_I; L'_E \rightsquigarrow_G e; S; H; K; L_I; L_E$
By preservation:
$(S; H; K; L_I; L_E) \sim G$

By assumption:
$\theta_0 \vdash e; \emptyset; \emptyset; \emptyset; \emptyset; \emptyset \rightsquigarrow^*_G e; S; H; K; L_I; L_E$
Since $S = \emptyset$:
$(\emptyset, \emptyset, \emptyset, \emptyset, \emptyset) \sim G$
By inversion of Df-Trans:
$\theta_0 \vdash e; \emptyset; \emptyset; \emptyset; \emptyset; \emptyset \rightsquigarrow^*_G e'; S'; H'; K'; L'_I; L'_E$
By induction hypothesis:
$DO, DD, DE \vdash_{CT,H'} \Sigma'$
By inversion of Df-Trans:
$\theta_0 \vdash e'; S'; H'; K'; L'_I; L'_E \rightsquigarrow_G e; S; H; K; L_I; L_E$
By preservation:
$DO, DD, DE \vdash_{CT,H} \Sigma$

■

4.7.1 Lemmas

To prove the Progress and Preservation theorems, we use the following lemmas. We intended to use the first four lemmas, (i.e. the import and export lemmas) in the Progress theorem proof. However, we complete the Progress proof without their use. We keep them for backward compatibility with the previous version of this report.

Df-Substitution Lemma.
If
$\Gamma \cup \{\overline{x} : \overline{T_f}\}, \Sigma, \theta \vdash e : T$
$\Gamma \cup \{\overline{x} : \overline{T_f}\}, \Upsilon, DO, DD, DE \vdash_O e$
$\Gamma, \Sigma, \theta \vdash \overline{v} : \overline{T_a}$ where $\overline{T_a} <: [\overline{v}/\overline{x}]\overline{T_f}$
then
$\Gamma, \Sigma, \theta \vdash [\overline{v}/\overline{x}]e : T'$ *for some* $T' <: [\overline{v}/\overline{x}]T$
$\Gamma, \Upsilon, DO, DD, DE \vdash_O [\overline{v}/\overline{x}]e$
Proof: By induction on the $\Gamma, \Upsilon, DO, DD, DE \vdash_O e$ relation. ■

Df-Weakening Lemma.
If
$\Gamma, \Upsilon, DO, DD, DE \vdash_O e$
then
$\Gamma, \Upsilon \cup \{C<\overline{D}>\}, DO, DD, DE, \vdash_O e$

Proof: By induction on the $\Gamma, \Upsilon, DO, DD, DE \vdash_O e$ relation. ■

Df-Strengthening Lemma.

If

$\Gamma, \emptyset, DO, DD, DE \vdash_O \texttt{new}\ C{<}\overline{p}{>}(v)$

$\forall i \in 1..|\overline{p}| \quad D_i = DD[(O, p_i)]$

$\Gamma, \Upsilon \cup \{C{<}\overline{D}{>}\}, DO, DD, DE, \vdash_{O'} e'$

then

$\Gamma, \Upsilon, DO, DD, DE, \vdash_O e$

Proof: By induction on the $\Gamma, \Upsilon, DO, DD, DE \vdash_O e$ relation. ■

Df-Domains Lemma.

If

$\emptyset, \Sigma, \theta \vdash e : T$

$\Sigma \vdash S$

$DO, DD, DE \vdash_{CT,H} \Sigma$

$\emptyset, \emptyset, DO, DD, DE \vdash_O \ new\ C{<}\overline{p}{>}(\overline{v})$

$(S, H, K, L_I, L_E) \sim (DO, DD, DE)$

$DO, DD, DE \vdash_O ddomains(C, O_C)$

$\forall i \in 1..|\overline{p}| \quad D_i = DD[(O, p_i)]$

$O_C = \langle O_{id}, C{<}\overline{D}{>}\rangle \quad \{O_C\} \subseteq DO$

then

$\forall d_j \in domains(C{<}\overline{p}{>}) \quad D_j = DD[(O_C, d_j)]$

Proof: By induction on the $DO, DD, DE \vdash_O ddomains(C, O_C)$ relation. ■

Differences with earlier versions of this work. Our formalization refines the one in Rawshdeh's thesis [17]. Our analysis does not consider creational edges, it focuses on usage edges only. We completed the formalization of the static and dynamic semantics, and proved progress, preservation, and soundness. We defined the approximation relation, and used the additional maps L_I and L_E to track import and export edges.

Differences with OOG with points-to edges. Our formalization is similar to the one for the points-to analysis [1, Section 3.2 and 3.3]. The two analyses create the same object-domain hierarchy, but the points-to analysis ignores field reads, field writes, and method invocations. The analysis in this paper shows additional edges that are missing from an OOG with points-to edges. The key differences in the formalization deal with generating the dataflow edges and the soundness proof.

5 Evaluation

5.1 Running Example

As a running example, we use a small system that follows a Document-View architecture and implements the Observer design pattern. We refer to this example as Listeners, and we selected it because empirical data shows that developers often struggle while understanding listeners in object-oriented code [12].

In Listeners, a `BarChart` and a `PieChart` render a `Model`. `BarChart` and `PieChart` implements different charts views. If the user changes the model, the charts are updated. Similarly, if the user edits a chart, the model is updated.

The code consists of several classes and uses various base classes, as is common in object-oriented code. The entry point of the application is the `Main` class, which instantiates `Model`, `PieChart`, and `BarChart`. `Model` extends the `Listener` abstract class, and contains the information displayed in the charts. `BarChart` and `PieChart` extend the `BaseChart` abstract class, which subsequently extends `Listener`. `BaseChart` and `Model` are both subject and observer: an object of type `Model` (i.e., the subject) may register objects of type `BaseChart` (i.e., the observers), and vice versa. Each of these classes has a field of type `List` that represents a collection of objects of type `Listener`. `Model` and viewers exchange messages of type `MsgMtoV` and `MsgVtoM`, which extend `Msg`.

To express the Document-View architecture, the class `Main` defines two domains `DOC` and `VIEW` using ownership domain annotations. Next, the `Model` object is placed in `DOC`, and`BarChart` `PieChart` in `VIEW`. Next, the class `Listener` has a declaration of a public domain `DATA` for messages. This domain is inherited by `BarChart`, `PieChart`, and `Model`. `Model` and `BaseChart` declares the private domain `OWNED` for collections of `Listener` objects registered for notification (Fig. 15). `BarChart` and `PieChart` inherits the `OWNED` domain from `BaseChart`. As a public domain, `DATA` gives access to messages, while the collections in the private domains are strictly encapsulated.

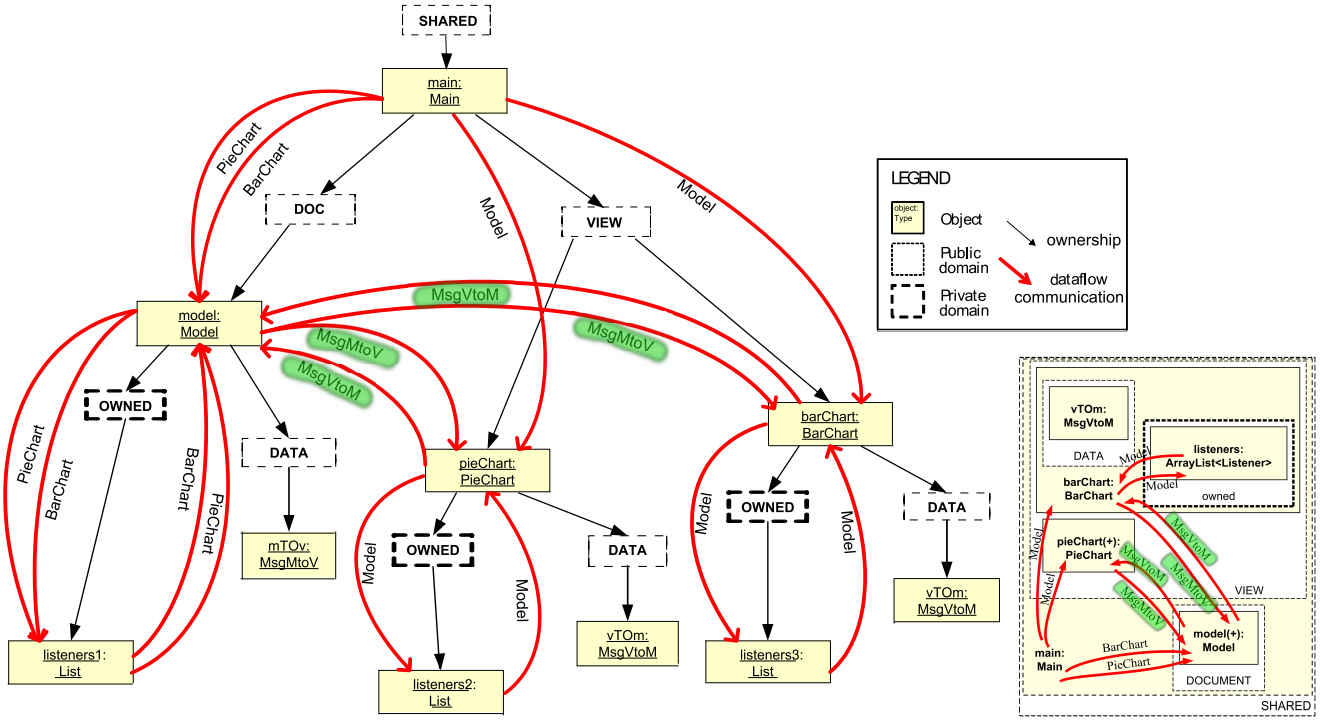
Our static analysis extracts a hierarchical object graph that distinguishes between different instances of `Listener`, and depicts the dataflow communication between objects. The object graph conveys architectural abstraction by organizing objects hierarchically with the architecturally

```
1  class Main<OWNER> {
2    public domain DOC, VIEW;
3    BarChart<VIEW, DOC> barChart = new BarChart();
4    PieChart<VIEW, DOC> pieChart = new PieChart();
5    Model<DOC, VIEW> pieChart = new PieChart();
6    void run(){
7     model.addListener(barChart); //(main --BarChart--> model)
8     model.notifyObservers(); //no dataflow
9     ...
10   }
11 }
12 class BarChart<OWNER, M> extends BaseChart<OWNER, M> { ... }
13 class PieChart<OWNER, M> extends BaseChart<OWNER, M> { ... }
14 class BaseChart<OWNER, M> extends Listener<OWNER> {
15   domain OWNED;
16   List<OWNED, Listener<M>> listeners = new List();
17   ...
18 }
19 class Model<OWNER, V> extends Listener<OWNER> {
20   domain OWNED;
21   List<OWNED, Listener<V>> listeners = new List();
22   public void addListener(Listener<V> l) {
23    listeners.add(l); //(model --BarChart--> listeners1)
24   }
25   public void notifyObservers() {
26    MsgMtoV<DATA> mTOv = new MsgMtoV();
27    Listener<V> l = listeners.value; //(listeners1 --BarChart--> model)
28    l.update(mTOv); //(model --MsgMtoV--> barChart, model --MsgMtoV--> pieChart)
29   }
30 }
31 class List<OWNER, T<ELTS>> {  //generic type T
32   T<ELTS> value; // ELTS is a domain parameter for list elements
33 }
34 abstract class Listener<OWNER> {
35   public domain DATA;
36   public abstract void update(Msg<DATA> msg);
37 }
```

Figure 15: Listeners code fragments. The code is available in Appendix A.

significant objects such as `BarChart`, `PieChart` and `Model` at a higher level of the hierarchy, and low-level objects such as collections and messages at a lower level (Fig. 16a).

Discussion. The resulting object graph makes visually obvious the dataflow communication occurring in the program. For example, the object graph shows two dataflow edges labeled `MsgMtoV` from `model` to `barChart` and `pieChart`, and two edges labeled `MsgVtoM`, from `barChart` and `pieChart` to `model`. Indeed, such communication is common in a Document-View architecture, followed by this example. An object graph with points-to edges does not show this communication since `model` does not have a field of type `BarChart` or `PieChart`. Although the objects representing messages appear



(a) OOG. **(b)** Collapsed OOG.

Figure 16: Dataflow communication for Listeners. Interesting edges are highlighted.

in the object graph, namely `mTOv:MsgMtoV` and `vTOm:MsgVtoM`, they have no incoming or outgoing dataflow edges. The graph still shows the classes of the messages as edge labels, which makes visually obvious the transient relations between `model`, `barChart`, and `pieChart` (the highlighted edges in Fig. 16).

Furthermore, the object graph shows three distinct objects of type `List`, which contain objects of the abstract type `Listener`. The dataflow edges to the `List` objects result from the analysis of the method invocations `listeners.add(l)` inside the methods `addListener(Listener l)`, in `BaseChart` and `Model`, respectively. From just reading these statements, it is not clear "what kind" of objects are added to each collection. But the object graph makes visually obvious (as edge labels) that in `BaseChart`, the reference `l` represents `Model` objects, while in `Model`, `l` represents `BarChart` or `PieChart` objects. These labels are more precise than the base class `Listener`, the declared type of `l` in the code.

Collapsed OOG. Having a hierarchical representation allows expanding or collapsing the substructure of an object to control the level of visual detail. For example, only the substructure of

`barChart` is visible, while the substructures of `pieChart` and `model` are collapsed (Fig. 16b). A (+) symbol indicate than an object has a collapsed substructure. While collapsing, the visualization also lifts the parent-child dataflow communication edges which makes the graph less cluttered showing only the interesting edges. The nested box visualization is similar to the one we used for OOG with points-to edges [1, Section 3.4].

5.2 Notation.

We use the following notation: `obj.DOM` refers to either a public or a private domain `DOM` inside object `obj`. We effectively treat a domain as a field of an object, e.g., `main.DOC`; `obj1.DOM.obj2` refers to the object `obj2` inside the domain `DOM` of `obj1`, e.g., `main.DOC.model`; $C{::}d$ refers to a domain d qualified by the class C that declares it. The first domain parameter corresponds to the owning domain, and we call it `OWNER`. We use capital letters for domain names to distinguish them from other program identifiers.

The analysis calls the *analyze* method for every expression e, with the bindings $p_i \mapsto D_i$, and the context O:

$$analyze(e, [..., p_i \mapsto D_i, ...], O)$$

When it encounters inheritance, *analyze* recursively analyzes the base class of the current class (in that case, we do not show the parameter e).

A boxed statement represents the analysis step performed while analyzing the preceding statement. When a boxed statement precedes a class declaration, it includes the mapping of formal domain parameters to actual domains, and the context OObject O. Consecutive boxed statements correspond to consecutive analysis steps.

For example, when the analysis encounters a method invocation expression, it calls *lookup* multiple times to find the type of the receiver expression, the type of method arguments, and the return type. Next, it creates dataflow edges, and continues in the body of m. For brevity, we include only the most interesting *lookup* calls.

```
1  Main<SHARED> main = new Main();
2  OObject(main, Main<SHARED>) (O0)
3  analyze ( new Main<OWNER>(), [Main::OWNER ↦ SHARED], O ↦ main )
4
5  [Main::OWNER↦ ::SHARED], O↦ main
6  class Main<OWNER> {
7    public domain DOC, VIEW;
8    ODomain(main.DOC, Main::DOC) (D1)
9    ODomain(main.VIEW, Main::VIEW) (D2)
10
11   BarChart<VIEW, DOC> barChart = new BarChart();
12   OObject(main.VIEW.barChart, BarChart<main.VIEW, main.DOC>) (O1)
13   analyze(new BarChart<OWNER, M>(), [BarChart::OWNER ↦ main.VIEW, BarChart::M ↦ main.DOC], O ↦
     main.VIEW.barChart)
14   // continue to Fig. 18
15
16   PieChart<VIEW, DOC> pieChart = new PieChart();
17   OObject(main.VIEW.pieChart, PieChart<main.VIEW, main.DOC>) (O2)
18   analyze(new PieChart<OWNER, M>(), [PieChart::OWNER ↦ main.VIEW, PieChart::M ↦ main.DOC], O ↦
     main.VIEW.pieChart)
19   // The analysis is similar to barChart, omitted for brevity
20
21   Model<DOC, VIEW> model = new Model();
22   OObject(main.DOC.model, Model<main.DOC, main.VIEW>) (O3)
23   analyze(new Model<OWNER, V>(), [Model::OWNER ↦ main.DOC, Model::V ↦ main.VIEW],O ↦
     main.DOC.model)
24    // continue to Fig. 19
25     ...
```

Figure 17: Abstractly interpreting the program, starting with the root class `Main`.

5.3 Worked Example.

Our analysis starts with the developer selecting the root type, in this case, the `Main` class (Fig. 17). The analysis creates the OObject (O0) for the `main` object allocation. Then, it analyzes `Main` in the context of `main`. Before analyzing `Main`, the analysis maps all formal domain parameters, if any, to their corresponding ODomains in the OGraph. In this case, the analysis maps `MAIN::OWNER` to the global domain `::SHARED`. The analysis also tracks the context OObject O.

The analysis continues inside `Main` and finds that the first statement is a domain declaration. In response, it creates two ODomains: `DOC` and `VIEW`. Next, for the object allocation statement of the `barChart` object, the analysis creates an OObject (O1), and proceeds to analyze the class `BarChart`,

```
1  [BarChart::OWNER ↦ main.VIEW, BarChart::M ↦ main.DOC], O ↦ main.VIEW.barChart
2  class BarChart<OWNER, M> extends BaseChart<OWNER, M> {
3    analyze([BaseChart::OWNER ↦ main.VIEW, BaseChart::M ↦ main.DOC], O ↦ main.VIEW.barChart)
4
5    public void update(Msg<DATA> msg) {...}
6  }
7
8  [BaseChart::OWNER ↦ main.VIEW, BaseChart::M ↦ main.DOC], O ↦ main.VIEW.barChart
9  class BaseChart<OWNER, M> extends Listener<OWNER> {
10   domain OWNED;
11   ODomain(main.VIEW.barChart.OWNED, BaseChart::OWNED) (D3)
12
13   public domain DATA;
14   ODomain(main.VIEW.barChart.DATA, BaseChart::DATA) (D4)
15
16   List<OWNED, Listener<M>> listeners = new List();
17   OObject(main.VIEW.barChart.OWNED.listeners, List<main.VIEW.barChart.OWNED,
     Listener<main.DOC>) (O4)
18   analyze(new List<OWNER, ELTS>(), [List::OWNER ↦ main.VIEW.barChart.OWNED, List::ELTS
     ↦ main.DOC], O ↦ main.VIEW.barChart.OWNED.listeners)
19   ...
20 }
21
22 [List::OWNER ↦ main.VIEW.barChart.OWNED, List::ELTS ↦ main.DOC], O
   ↦main.VIEW.barChart.OWNED.listeners
23 T = Listener //generic type
24 class List<OWNER, T<ELTS>> {
25   T<ELTS> value; // ELTS is a domain parameter for list elements
26   ...
27 }
```

Figure 18: Abstractly interpreting the program (continued): `BarChart`, `BaseChart` and `List`.

mapping O to `main.VIEW.barChart`. It also maps the domain parameter `BarChart::OWNER` to `main.VIEW`, and `BarChart::M` to `main.DOC`.

Next, the analysis covers `BarChart` and its base class `BaseChart` in the context of the OObject `barChart` (Fig. 18). The analysis proceeds into the base class `BaseChart` mapping `BaseChart::OWNER` to `main.VIEW`, and `BaseChart::M` to `main.DOC` while the context O remains unchanged. Inside `BaseChart`, the analysis encounters two domain declarations: `domain OWNED` and `public domain DATA`. As a result, it creates a private and a public ODomains for `barChart`. Next statement is an instantiation of the class `List`, the analysis creates the OObject `main.VIEW.barChart.OWNED.listeners` (O4) inside the `barChart.OWNED` domain. Since `List` has

```
1  [ Model::OWNER ↦ main.DOC, Model::V ↦ main.VIEW], O ↦ main.DOC.model
2  class Model<OWNER, V> extends Listener<OWNER> {
3    domain OWNED;
4    ODomain(main.DOC.model.OWNED, Model::OWNED) (D9)
5
6    public domain DATA;
7    ODomain(main.DOC.model.DATA, Model::DATA) (D10)
8
9    List<OWNED, Listener<V>> listeners = new List();
10   OObject(main.DOC.model.OWNED.listeners, List<main.DOC.model.OWNED,
     Listener<main.VIEW>>) (O6)
11   analyze(new List<OWNER, ELTS>(), [List::ELTS ↦ main.VIEW, List::OWNER ↦
     main.DOC.model.OWNED], O ↦ main.DOC.model.OWNED.listeners)
12   ...
13 } [List::OWNER ↦ main.DOC.model.OWNED, List::ELTS ↦ main.VIEW], O
     ↦main.DOC.model.OWNED.listeners
14 T = Listener //generic type
15 class List<OWNER, T<ELTS>> {
16
17   T<ELTS> value; // ELTS is a domain parameter for list elements
18   ...
19 }
```

Figure 19: Abstractly interpreting the program (continued): `Model` and `List`.

no domain declarations or `new` statements, the analysis backtracks to `BaseChart`, and then further on to `Main`.

The analysis of class `PieChart`, its base class `BaseChart`, and its `List` is similar to `BarChart`, so we omitted it for brevity.

Back in `Main` (Fig. 17), the analysis creates the OObject `main.DOC.model` (O3) corresponding to the instantiation of the class `Model` inside the ODomain `DOC`. Then it proceeds to analyze `Model` in the context of `main.DOC.model` (Fig. 19). The analysis maps the domain parameter `Model::OWNER` to `main.DOC`, and `Model::V` to `main.VIEW`. Similar to `BaseChart` the analysis creates a private and a public ODomain `OWNED` and `DATA`, and an OObject `main.DOC.model.OWNED.listeners`. Note that the analysis distinguishes between the `listeners` object owned by `model`, and the one owned by `barChart` although they are both instances of `List`.

Next, the analysis encounters the method invocation `main.run()` (Fig. 20). In this case, O correspond to `main`. Inside `run()`, the analysis encounters `model.addListener(barChart)`, and it

```
1  main.run();
2  analyze(main.run(), [Main::OWNER ↦ SHARED], O ↦ main)
3
4  public class Main<OWNER> {
5    ...
6    public void run() {
7
8      model.addListener(barChart);
9      analyze(model.addListener(barChart), [Model::OWNER ↦ main.DOC, Model::V ↦
       main.VIEW], O ↦ main.DOC.model
10     OObject(main.DOC.model, Model<main.DOC, main.VIEW>) ∈ lookup(Model<main.DOC,
       main.VIEW>)
11     OEdge(main, main.DOC.model, BarChart) (E1)
12     // continue to Fig. 21
13
14     model.addListener(pieChart);
15     // The analysis is similar to model.addListener(barChart)
16     // Omitted for brevity
17     OEdge(main, main.DOC.model, PieChart) (E4)
18
19     barChart.addListener(model);
20     analyze(barChart.addListener(model), [BaseChart::OWNER ↦ main.VIEW, BaseChart::M ↦
       main.DOC], O ↦ main.VIEW.barChart)
21     OObject(main.VIEW.barchart, BarChart<main.VIEW, main.DOC>) ∈
       lookup(BarChart<main.VIEW, main.DOC>)
22     OEdge(main, main.VIEW.barChart, Model) (E7)
23     // continue to Fig. 22
24
25     pieChart.addListener(model);
26     // The analysis is similar to barChart.addListener(model)
27     // Omitted for brevity
28     OEdge(main, main.VIEW.pieChart, Model) (E9)
29
30     model.notifyObservers();
31     analyze(model.notifyObservers(), [Model::OWNER ↦ main.DOC, Model::V ↦ main.VIEW], O
       ↦ main.DOC.model)
32     // continue to Fig. 23
33
34     barChart.notifyObservers();
35     analyze(barChart.notifyObservers(), [BaseChart::OWNER ↦ main.VIEW, BaseChart::M ↦
       main.DOC], O ↦ main.VIEW.barChart)
36     // continue to Fig. 24
37
38     pieChart.notifyObservers();
39     // The analysis is similar to barChart.notifyObservers()
40     // Omitted for brevity
41   }
42 }
```

Figure 20: Abstractly interpreting the program, class `Main`.

```
1  [Model::OWNER ↦ main.DOC, Model::V ↦ main.VIEW], O ↦ main.DOC.model
2  class Model<OWNER,V> extends Listener<OWNER> {
3    ...
4    l:BarChart<main.VIEW, main.DOC>
5    public void addListener(Listener<V> l) {
6      listeners.add(l);
7      analyze(listeners.add(l),[List::OWNER ↦ main.DOC.model.OWNED, List::ELTS ↦
       main.VIEW], O ↦ main.DOC.model.OWNED.listeners)
8      OObject(main.DOC.model.OWNED.listeners, List<main.DOC.model.OWNED,
       Listener<main.VIEW>>) ∈ lookup(List<main.DOC.model.OWNED, Listener<main.VIEW>>)
9      OEdge(main.DOC.model, main.DOC.model.OWNED.listeners, BarChart) (E2)
10   }
11 } [List::OWNER ↦ main.DOC.model.OWNED, List::ELTS ↦ main.VIEW], O
   ↦main.DOC.model.OWNED.listeners
12  T = Listener //generic type
13  class List<OWNER, T<ELTS>> {
14     T<ELTS> value; // ELTS is a domain parameter for list elements
15     public void add(T<ELTS> value) {...}
16     ...
17 }
```

Figure 21: Abstractly interpreting the program (continued): `Model` addListener method.

```
1  [BarChart::OWNER ↦ main.VIEW, BarChart::M ↦ main.DOC], O ↦ main.VIEW.barChart
2  class BarChart<OWNER, M> extends BaseChart<OWNER, M> {
3    analyze([BaseChart::OWNER ↦ main.VIEW, BaseChart::M ↦ main.DOC], O ↦
     main.VIEW.barChart)
4
5    public void update(Msg<DATA> msg) {...}
6  }
7
8  [BaseChart::OWNER ↦ main.VIEW, BaseChart::M ↦ main.DOC], O ↦ main.VIEW.barChart
9  class BaseChart<OWNER, M> extends Listener<OWNER> {
10   ...
11   l :  Model<main.DOC, main.VIEW>
12   public void addListener(Listener<M> l) {
13     listeners.value = l; // field write - export dataflow communication
14     OObject(main.VIEW.barChart.OWNED.listeners, List<main.VIEW.barChart.OWNED,
       Listener<main.DOC>>) ∈ lookup(List<main.VIEW.barChart.OWNED, Listener<main.DOC>>)
15     OEdge(main.VIEW.barChart, main.VIEW.barChart.OWNED.listeners, Model) (E8)
16   }
17 }
```

Figure 22: Abstractly interpreting the program (continued): `BaseChart` addListener method.

```
1  [Model::OWNER ↦ main.DOC, Model::V ↦ main.VIEW], O ↦ main.DOC.model
2  class Model<OWNER,V> extends Listener<OWNER> {
3    ...
4    public void notifyObservers() {
5      MsgMtoV<DATA> mTOv = new MsgMtoV();
6      OObject(main.DOC.model.DATA.mTOv, MsgMtoV<main.DOC.model.DATA>) (O7)
7      analyze(new MsgMtoV<OWNER>(), [MsgMtoV::OWNER ↦ main.DOC.model.DATA], O ↦
       main.DOC.model.DATA.mTOv)
8
9      Listener<V> l = listeners.value; //field read - import dataflow communication
10     OObject(main.DOC.model.OWNED.listeners, List<main.DOC.model.OWNED,
       Listener<main.VIEW>>) ∈ lookup(List<main.DOC.model.OWNED, Listener<main.VIEW>>)
11     OObject(main.VIEW.barChart, BarChart<main.VIEW, main.DOC>) ∈
       lookup(Listener<main.VIEW>)
12     OObject(main.VIEW.pieChart, PieChart<main.VIEW, main.DOC>) ∈
       lookup(Listener<main.VIEW>)
13     OEdge(main.DOC.model.OWNED.listeners, main.DOC.model, BarChart) (E11)
14     OEdge(main.DOC.model.OWNED.listeners, main.DOC.model, PieChart) (E12)
15
16     l.update(mTOv);
17     analyze(l.update(vTOm),[BarChart::OWNER ↦ main.VIEW, BarChart::M ↦ main.DOC], O
       ↦ main.VIEW.barChart)
18     OObject(main.VIEW.barChart, BarChart<main.VIEW, main.DOC>) ∈
       lookup(Listener<main.VIEW>)
19     OEdge(main.DOC.model, main.VIEW.barChart, MsgMtoV) (E13)
20
21     analyze(l.update(vTOm),[PieChart::OWNER ↦ main.VIEW, PieChart::M ↦ main.DOC], O
       ↦ main.VIEW.pieChart)
22     OObject(main.VIEW.pieChart, PieChart<main.VIEW, main.DOC>) ∈
       lookup(Listener<main.VIEW>)
23     OEdge(main.DOC.model, main.VIEW.pieChart, MsgMtoV) (E14)
24     }
25   }
26 }
```

Figure 23: Abstractly interpreting the program (continued): `Model` notifyObservers method.

changes the context to `main.DOC.model`. This method invocation introduces an export edge (E1) from `main` to `model` because `main` exports an object of type `BarChart` to `model` as the argument of `addListener(Listener)`. For edge label, the analysis calls *lookup* and finds one OObject of type `BarChart<main.VIEW, main.DOC>`, `main.VIEW.barChart`.

Inside `addListener(Listener)` of `Model`, the analysis encounters `listeners.add(l)`. A first *lookup* call returns the OObject `listeners` of type `List<main.DOC.model.OWNED,`

```
1  [BarChart::OWNER ↦ main.VIEW, BarChart::M ↦ main.DOC], O ↦ main.VIEW.barChart
2  class BarChart<OWNER, M> extends BaseChart<OWNER, M> {
3    analyze([BaseChart::OWNER ↦ main.VIEW, BaseChart::M ↦ main.DOC], O ↦
     main.VIEW.barChart)
4
5    public void update(Msg<DATA> msg) {...}
6  } [BaseChart::OWNER ↦ main.VIEW, BaseChart::M ↦ main.DOC], O ↦ main.VIEW.barChart
7  class BaseChart<OWNER, M> extends Listener<OWNER> {
8    ...
9    public void notifyObservers() {
10     MsgVtoM<DATA> vTOm = new MsgVtoM();
11     OObject(main.VIEW.barChart.DATA.vTOm, MsgVtoM<main.VIEW.barChart.DATA>) (O8)
12     analyze(new MsgVtoM<OWNER>(), [MsgVtoM::OWNER ↦ main.VIEW.barChart.DATA], O ↦
       main.VIEW.barChart.DATA.vTOm)
13
14     Listener<M> l = listeners.getFirst(); //generates import dataflow communication
15     analyze(listeners.getFirst(), [List::OWNER ↦ main.VIEW.barChartl.OWNED, List::ELTS
       ↦ main.DOC], O ↦ main.VIEW.barChart.OWNED.listeners)
16     OObject(main.VIEW.barChart.OWNED.listeners, List<main.VIEW.barChart.OWNED,
       Listener<main.DOC>>) ∈ lookup(List<main.VIEW.barChart.OWNED, Listener<main.DOC>>)
17     OObject(main.DOC.model, Model<main.DOC, main.VIEW>) ∈ lookup(Listener<main.DOC>)
18     OEdge(main.VIEW.barChart.OWNED.listeners, main.VIEW.barChart, Model) (E15)
19
20     l.update(vTOm);
21     analyze(l.update(vTOm),[Model::OWNER ↦ main.DOC, Model::V ↦ main.VIEW], O ↦
       main.DOC.model)
22     OObject(main.DOC.model, Model<main.DOC, main.VIEW>) ∈ lookup(Listener<main.DOC>)
23     OEdge(main.VIEW.barChart, main.DOC.model, MsgVtoM) (E16)
24   }
25 }
26
27 [List::OWNER ↦ main.VIEW.barChart.OWNED, List::ELTS ↦ main.DOC], O
   ↦main.VIEW.barChart.OWNED.listeners
28  T = Listener //generic type
29  class List<OWNER, T<ELTS>> {
30     T<ELTS> value; // ELTS is a domain parameter for list elements
31     public T<ELTS> getFirst() { return value; }
32 }
```

Figure 24: Abstractly interpreting the program (continued): `BaseChart` notifyObservers method.

`Listener<main.VIEW>>`, i.e., the object of type `List` of the ODomain `model.OWNED`, which is a collection of elements of type `Listener`, and each element of the collection is of the ODomain `main.VIEW` (Fig. 21). A second *lookup* returns the OObject `barChart` and, the analysis adds an OEdge (E2) between `model` and `listeners` labeled using `BarChart`. At this point the analysis

backtracks to `Main` (Fig. 20).

Similar to the previous analyzed statement, the analysis of the method invocation `model.addListener(pieChart)` creates two edges labeled `PieChart`: from `main` to `model` (E4), from `model` to `listeners` (E5).

For `barChart.addListener(model)` and `pieChart.addListener(model)`, the analysis creates two OEdges labeled with `Model`: from `main` to `barChart` (E7), and from `main` to `pieChart` (E9). In both cases, the analysis encounters statement `listeners.value = l` in the `addListener` method of the class `BaseChart`. In response to this field write statement, the analysis calls *lookup* with the parameter `List<main.VIEW.barChart.OWNED, Listener<main.DOC>>`, and `List<main.VIEW.pieChart.OWNED, Listener<main.DOC>>`, respectively. The resulting OObjects are the two `listeners` owned by `barChart` and `pieChart`, respectively, and in each case, the analysis creates an OEdge labeled with `Model` (i.e., the actual type of `l`): from `barChart` to `listeners` (E8), and from `pieChart` to its owned `listeners` (E10) (Fig. 22).

Back in `run()`, the analysis processes three method invocations `notifyObservers()` with different receivers (Fig. 20). Since the method has no parameters, the analysis does not include additional `OEdge` from `main` to the receivers. The analysis continues in the `notifyObservers()` methods of `Model`, `BarChart`, and `PieChart`.

While analyzing `notifyObservers()` of `Model` in the context of `model` (O), the analysis encounter the first statement, a class instantiation, and it creates a new OObject (O7) `mTOv` in the public domain `DATA` of `model` (Fig. 23).

The second statement contains the field read expression `listener.value`. In response, the analysis calls *lookup* twice. First *lookup* searches for object of type `List` in `model.OWNED`, while the second *lookup* searches for objects of type `Listener<main.VIEW>`. For the first call *lookup* returns `listeners`, while for the later call, *lookup* returns two OObjects: barChart and pieChart. As a result, the analysis creates two OEdges from `listeners` to `model`, one labeled with `BarChart` (E11), and the other labeled with `PieChart` (E12). Note that if the second *lookup* would not have been performed, the label would be `Listener`, which can be interpreted as any of the five classes extending `Listener`: `Model`, `BarChart`, `PieChart` or `BaseChart`. Therefore, by calling *lookup*,

the analysis produces more accurate labels. Another observation is that the field read expression introduces an import edge, and O is the destination of the edge, while in the previous cases O was the source.

The third and last statement contains the method invocation `l.update(mTOv)`. The analysis calls again *lookup* searching for OObjects of type `Listener<main.VIEW>`. The result are the two OObjects `barChart` and `pieChart`. The analysis includes two OEdges labeled as MsgMtoV: from `model` to `barChart` (E13), and from `model` to `pieChart` (E14).

Since the `update` methods are empty, the analysis returns to `Main` and proceeds to `notifyObservers()` with `barChart` as receiver (O) (Fig. 24). The method is implemented in the superclass `BaseChart`, and the analysis performs the similar steps as previously discussed with two major differences. First, the second statement is the method invocation `listeners.getFirst()` instead of field read. The `getFirst()` method returns an alias to the `value` field. After a first *lookup* identifies `listeners` as the receiver of `getFirst()`, the analysis calls a second *lookup* searching for OObjects of type `Listener<main.DOC>`, which corresponds to the returned type of `getFirst()`. The result of the second *lookup* is `model`, and its class constitutes the edge label. As for field read, O is the destination of the OEdge from `listeners` to `barChart` (E15). Second, the analysis looks up the OObjects of a subtype of the local variable `l` in the method invocation `l.update(vTOm)`, and it finds only one OObject in the `main.DOC` domain (i.e., `model`). Therefore, it creates the OEdge from `barChart` to `model` labeled with `MsgVtoM` (E16).

The analysis concludes with the method invocation `pieChart.notifyObservers()`, and its corresponding implementation from `BaseChart`. The analysis performs the same steps previously discussed in the context of `pieChart` (O).

5.4 Graphical notation.

In the visualization of the OOG, we graphically distinguish between objects and domains by using a rectangle-shape to represent an object and a dashed rectangle-shape to represent a domain. We further distinguish between public and private domains using a thin dashed border for a public domain, and a bold dashed border for a private domain. In all cases, we label each rectangle with

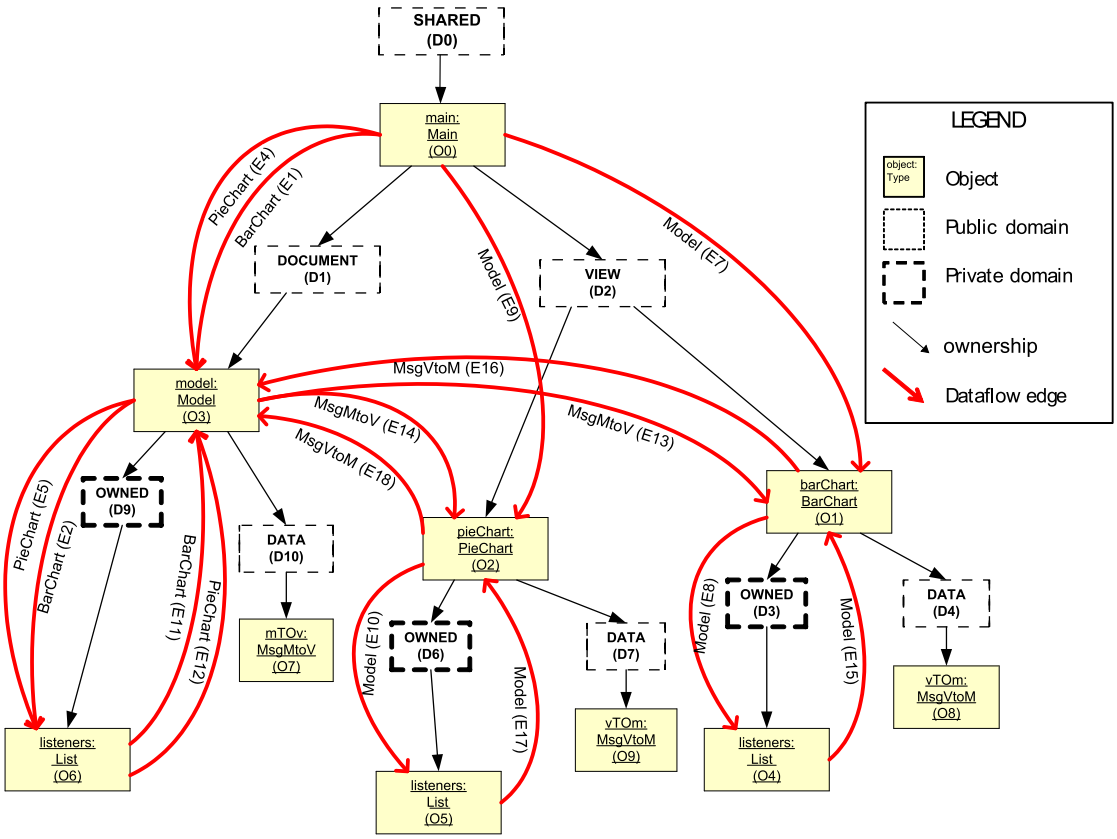


Figure 25: OOG extracted for Listeners

the name of the object or domain that corresponds to it. We represent the dataflow edges with a red solid arrow, and the ownership edges from domains to objects and vice-versa with a black arrow. We display as a root the `SHARED` domain, which constitutes the root node of the graph. The only object `SHARED` we display is the first object instantiated when the application starts, in our case `main`. On the dataflow edges, we also display the class of the object passed through the dataflow (Fig. 25). Developer can also opt for the hierarchical view (Fig. 26), hide the low level objects, and visualize only the dataflow communication between the high level objects (Fig. 27).

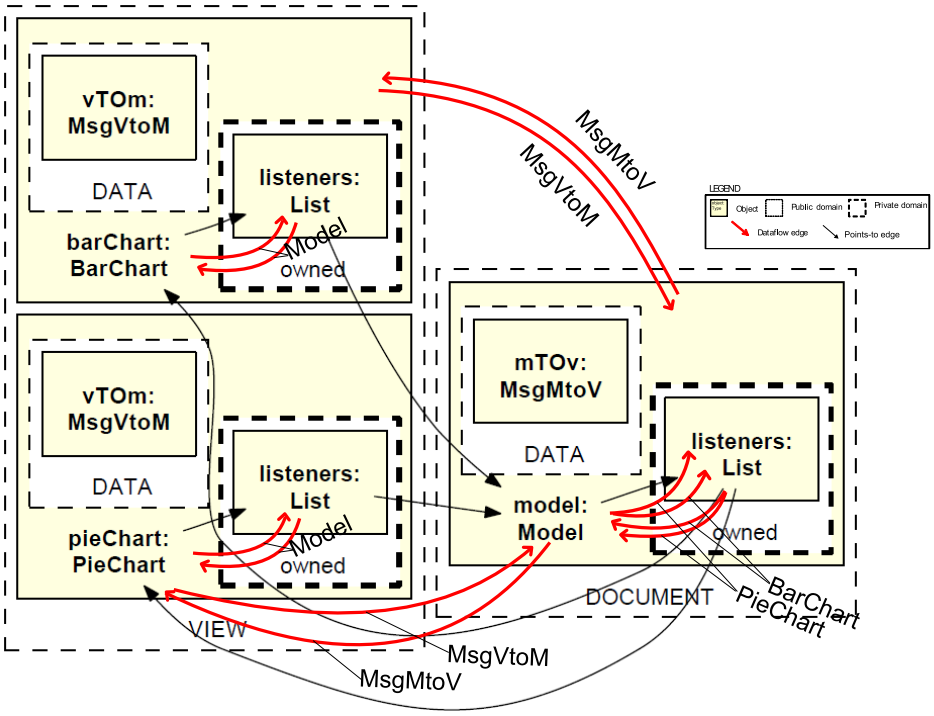


Figure 26: Display Graph for Listeners, with both points-to and dataflow edges.

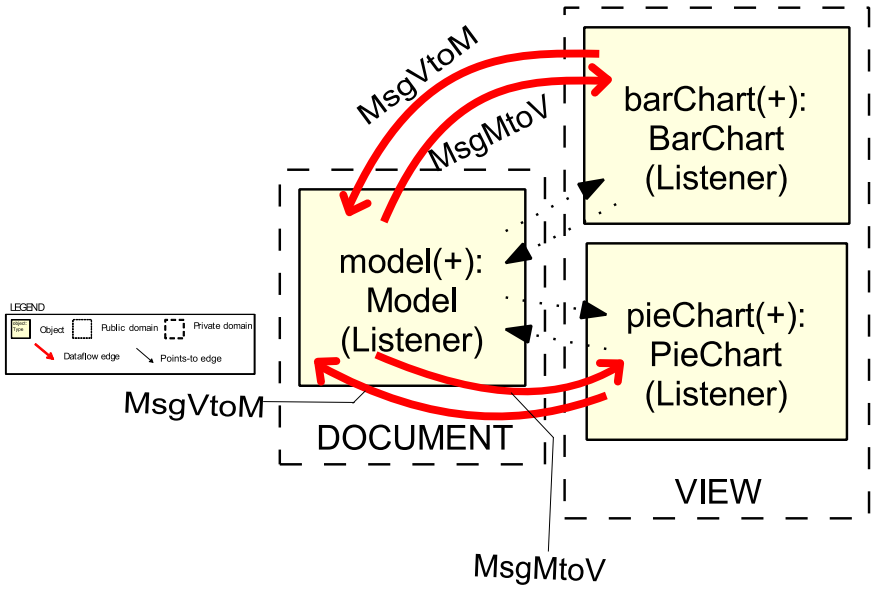


Figure 27: Display Graph for Listeners, after collapsing the sub-structures of top-level objects.

6 Related Work

Our work is focused on extracting one type of information which is usage relationship, i.e., an object is using a method or a field of another object. We discuss how the related analyses address the above challenges.

Dataflow Communication. Andersen's static analysis extracted dataflow and points-to information from programs written in C [7], and was extended to object oriented code [14, 20] including Java [18]. These analyses determined the memory locations that may be modified by the execution of a statement. A dataflow edge means that *an object a owns a reference to an object c, and passes it to an object b*, or *an object a owns a reference to an object b, from which it receives a reference to an object c which only b knew before* [18]. However, the results of these analyses are flat graphs [18, 9] and the analyses did not attempt to be sound. In contrast, our analysis extracts hierarchical object graphs, with a relatively small number of objects at the top levels [1, Section 4.6.2].

Sensitivity. An analysis can be flow-, and context- sensitive. A flow-sensitive analysis considers the order in which methods are called. A context-sensitive analysis analyzes the methods for each context under which a method is invoked. Object sensitive analyses for points-to and dataflow edges addressed the aliasing and precision challenges [20, 14]. However, the analysis might not scale for a large number of references. Such an analysis worked well for on-demand based approaches which refined the references analyzed [19]. Seeking for a tradeoff between soundness and precision, our analysis considers ownership domains as contexts and distinguishes objects of the same type but in different domains. That is, our analysis is *domain sensitive*, and object- and flow-insensitive.

Dynamic analyses. Object graphs were extracted by analyzing heap snapshots [15, 16], and execution traces [13]. Lienhard analyzed execution traces and extracted an Object Flow Graph (OFG) in which edges represent objects, and nodes represent code structures: classes, and groups of classes [13]. OFG analysis addressed aliasing challenge, and linked objects to field read, field write, and method invocation expressions in the code, the same expressions used by our analysis. The difference is that one class corresponds to one OFG node; therefore, an OFG is unable to show

communication between different instances of the same class and does not meet the soundness challenges. One advantage of dynamic analysis is that it can organize objects in an owner-as-dominator hierarchy which is limited in representing design idioms. To get a high-level picture of object graph, Mitchell et al. used extensively graph summarization and graph manipulation [16, 8]. Ownership domains, through public domains, also support logical containment and can express arbitrary design intent without restricting accessibility.

Annotation-based static analyses. Lam and Rinard [10] proposed a type system and a static analysis where by developer-specified annotations guide the static abstraction of an object model by merging objects based on tokens. Their approach supports a fixed set of statically declared global tokens, and their analysis shows a graph indicating which objects appear in which tokens. Since there is a statically fixed number of tokens, all of which are at the top level, an extracted object model is a top-level architecture that does not support hierarchical decomposition, thus limiting the scalability of the object model. In addition to their object model, Lam and Rinard extract models for "subsystem access", "call/return interaction", and "heap interaction", which is similar to the dataflow information our analysis extracts. From the challenges, they addressed aliasing, summarization in the presence of recursive types, and precision supported by tokens. Our approach extends Lam and Rinard's both to handle hierarchical object graphs and to support object-oriented language constructs such as inheritance.

7 Conclusion

We proposed a static analysis to extract a hierarchical object graph with dataflow communication edges, that show transient relations between objects. We formalized the analysis following ownership domains and Featherweight Domain Java, and proved the soundness of the resulting graph. We evaluated our analysis on an extended example and showed that the dataflow edges extracted by our analysis are similar to the ones drawn by developers who are reasoning about dataflow communication, and different from points-to edges.

Just as we found that developers benefit from hierarchical object graphs with points-to edges [5], we plan to evaluate if global object graphs that highlight dataflow communication help developers

with program comprehension. We also plan to extend the current evaluation to use OOG with dataflow communication edges to find security vulnerabilities in applications without DFDs [3].

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APPENDIX

A Source Code of Listeners Example

```
1
2  class Main<OWNER> {
3
4    public domain DOC, VIEW;
5    BarChart<VIEW, DOC> barChart = new BarChart();
6    PieChart<VIEW, DOC> pieChart = new PieChart();
7    Model<DOC, VIEW> model = new Model();
8
9    public void run() {
10     model.addListener(barChart);
11     model.addListener(pieChart);
12     barChart.addListener(model);
13     pieChart.addListener(model);
14
15     model.notifyObservers();
16     barChart.notifyObservers();
17     pieChart.notifyObservers();
18   }
19
20   public static void main(String[]<SHARED[SHARED]> args){
21     Main<SHARED> main = new Main();
22     main.run();
23   }
24 }
25
26 class BaseChart<OWNER, M> extends Listener<OWNER> {
27   domain OWNED;
28   List<OWNED, Listener<M>> listeners = new List();
29
30   public void addListener(Listener<M> l) {
31     listeners.value = l;
32   }
33
34   public void notifyObservers() {
35     MsgVtoM<DATA> vTOm = new MsgVtoM();
36     Listener<M> l = listeners.getFirst();
37     l.update(vTOm);
38   }
39 }
40
41 class BarChart<OWNER, M> extends BaseChart<OWNER, M> {
42   public void update(Msg<DATA> msg) {...}
43 }
44
45 class PieChart<OWNER, M> extends BaseChart<OWNER, M> {
46   public void update(Msg<DATA> msg) {...}
47 }
48
49
50
```

```
51 //generic type T
52 class List<OWNER, T<ELTS>> {
53   T<ELTS> value; // ELTS is a domain parameter for list elements
54   public T<ELTS> getFirst() {  return value;  }
55   ...
56 }
57
58 class Model<OWNER, V> extends Listener<OWNER> {
59   domain OWNED;
60   List<OWNED, Listener<V>> listeners = new List();
61
62   public void addListener(Listener<V> l) {
63     listeners.add(l);
64   }
65
66   public void notifyObservers() {
67     MsgMtoV<DATA> mTOv = new MsgMtoV();
68     Listener<V> l = listeners.value;
69     l.update(mTOv);
70     }
71   }
72
73 }
74
75 abstract class Listener<OWNER> {
76   public domain DATA;
77   public abstract void update(Msg<DATA> msg);
78 }
```