Question 3

$$q(\bar{x}^{(t)} \mid x^{(t-1)}) = (1/2)^{1-\bar{x}^{(t)}} (1/2)^{\bar{x}^{(t)}}$$

$$q(0|1) = q(1|0) = 1/2$$

$$q(0|0) = q(1|1) = 1/2$$

$$p(x^{(t)}) = (1/3)^{1-x^{(t)}} (2/3)^{x^{(t)}} \longrightarrow p(0) = 1/3$$

$$p(1) = 2/3$$

$$A(\bar{x}^{(t)} \mid x^{(t-1)}) = \min\left(1, \frac{p(\bar{x}^{(t)})}{p(\bar{x}^{(t-1)})}\right)$$

$$\alpha(1 \to 0) = \min\left(1, \frac{p(0)}{p(1)}\right) = \min\left(1, \frac{1/3}{2/3}\right) = 1/2$$

$$\alpha(0 \to 1) = \min\left(1, p(1)/p(0)\right) = \min\left(1, 2/3 / 1/3\right) = 1$$

Compute the transition kernal

$$k\,(0 \to 0) = q(0|0) + q(1|0) \cdot (1 - \alpha(0 \to 1))$$

$$= 1/2 \cdot 0 + 1/2 = 1/2$$

$$k(0 \to 1) = q(1|0) \times \alpha(0 \to 1) = 1/2 \times 1 = 1/2$$

$$k(1 \to 0) = q(0|1) \times (1 - \alpha(1 \to 0)) = 1/2 \times 1/2 = 1/4$$

$$k(1 \to 1) = q(0|1) \cdot (1 - \alpha(1 \to 0)) + q(1,1)$$

$$= 3/4$$

$$k = \begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix}$$