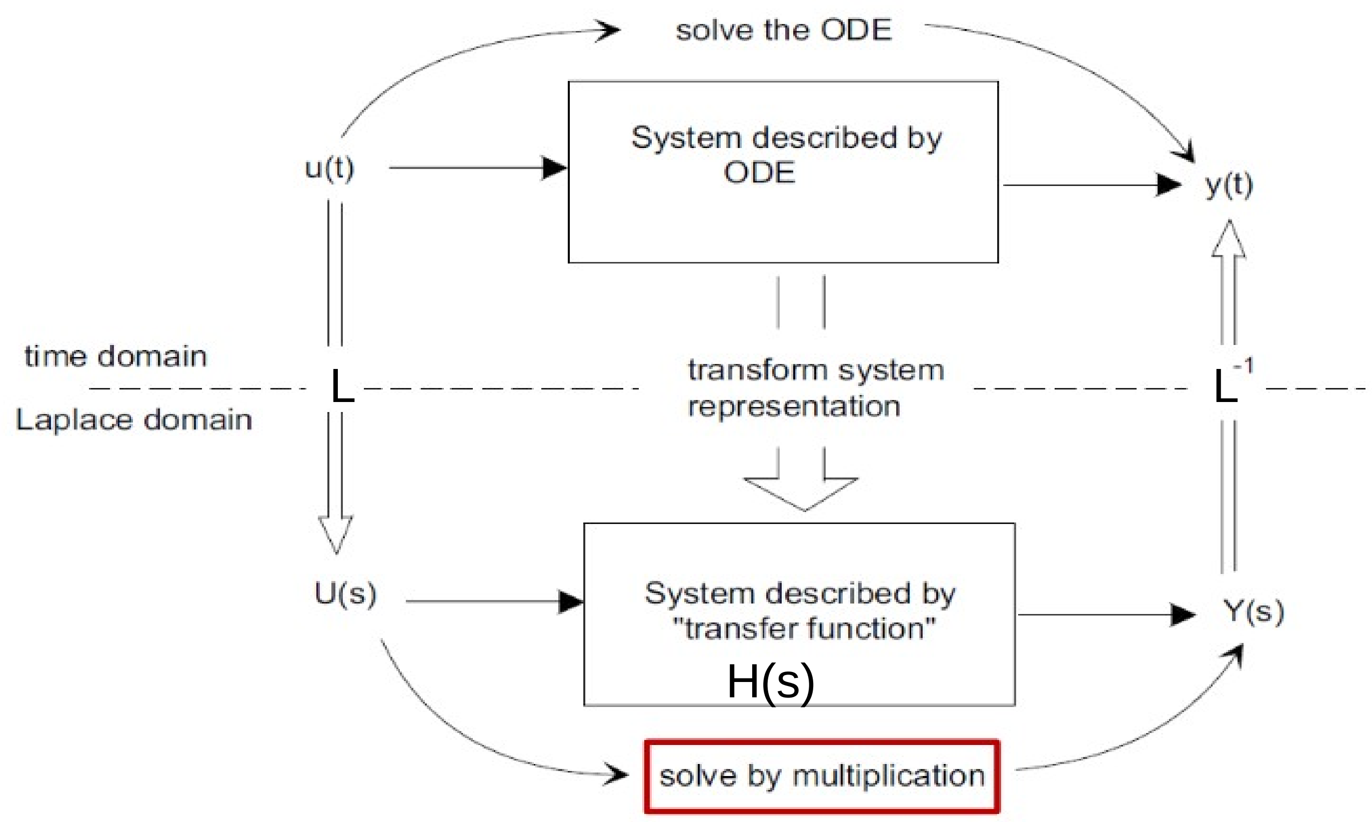
Analog IC Design

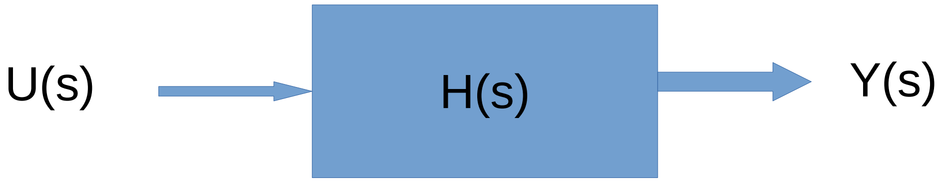
Lecture 2 circuits and systems review

Laplace Transform (LT)



Laplace Transform (LT)

- Laplace transform helps you to solve differential equations by multiplication and division



Y(S)=H(s)U(s)

$$H(s) = \frac{Y(s)}{U(s)}$$

Laplace Transform (LT)

Time domain	Laplace domain
e^{at}	$\frac{1}{s-a}$
$\int_0^t f(t)dt$	$\frac{1}{s}F(s)$
$\frac{df(t)}{dt}$	$sF(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$

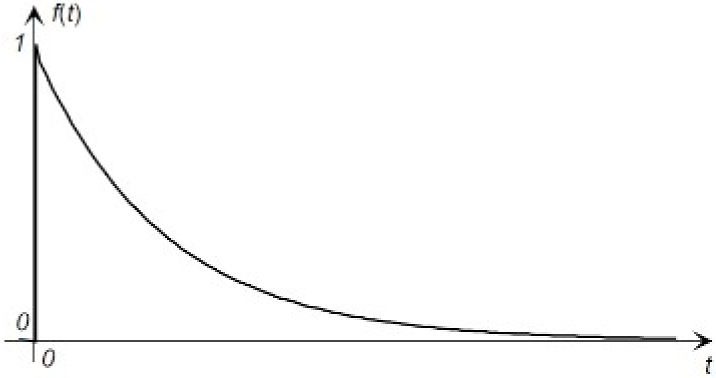
System input	System response (output) in Laplace domain
Unit impulse: $\delta(t)$	$H(s)$
Unit step: $u(t)$	$\frac{1}{s}H(s)$

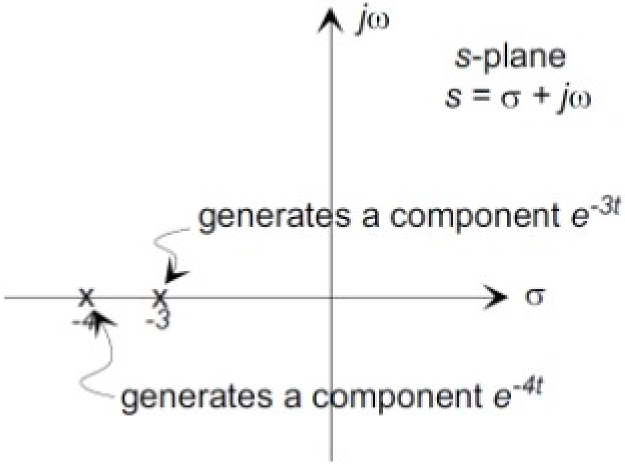
Poles and Zeros

- Transfer function $$H(s) = \frac{N(s)}{D(s)}$$
- Zeros:roots of the numerator N(s) = 0
- Poles: roots of the denominator (characteristic eq.) D(s) =0
- For physical systems, poles & zeros are real or complex conjugate

  Notes:
  - If the system input is impulse in the time domain then the output is the transfer function in laplace domain

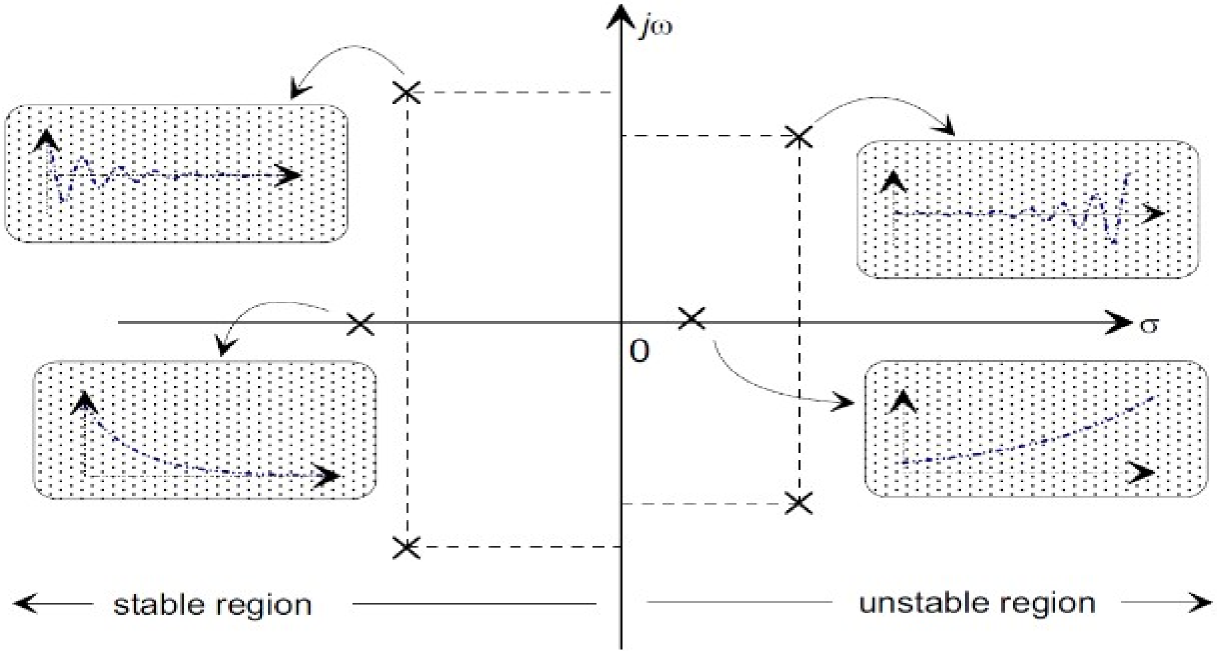
Real and Complex Poles





LHP and RHP Poles

- Poles in LHP: Decaying exponential (stable system)
- Poles in RHP: Growing exponential (unstable system)

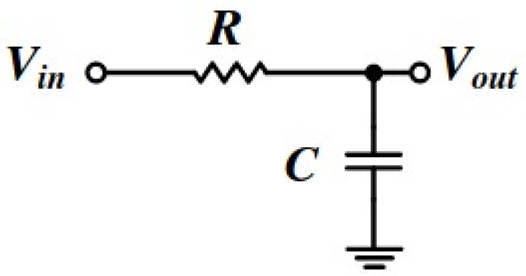


First-Order LPF

$$H(s) = \frac{v_{out}(s)}{v_{in}(s)} = \frac{1/sC}{R + 1/sC} = \frac{1}{1 + sRC} = \frac{1}{1 + s\tau}$$

$$H(j\omega) = \frac{v_{out}(j\omega)}{v_{in}(j\omega)} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + \frac{j\omega}{\omega_c}}$$

- $\tau = RC$: time constant
- $\omega_c = \frac{1}{\tau} = \frac{1}{RC}$: cutoff/corner frequency



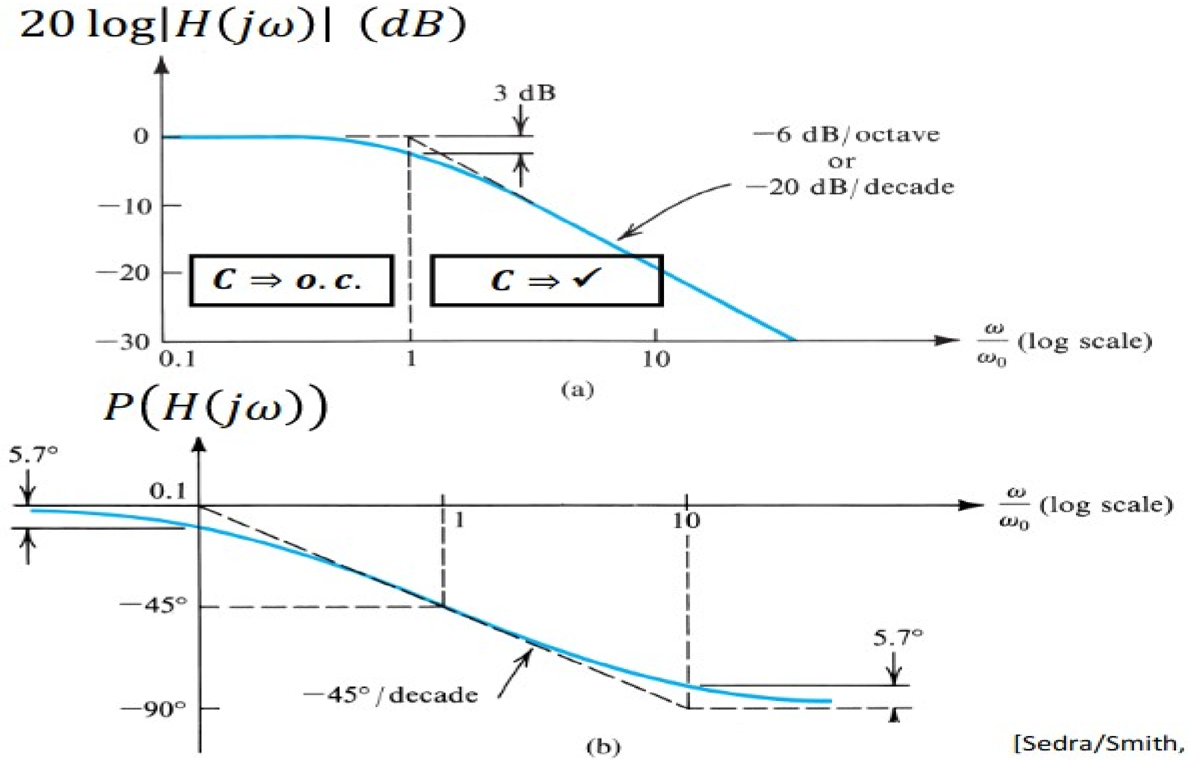
Poles: $s_p = -\frac{1}{\tau} = -\omega_c$

Zeros: ? @ w = infinity

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

$$P\big(H(j\omega)\big) = -\tan^{-1}\frac{\omega}{\omega_c}$$

First-Order LPF Bode Plot



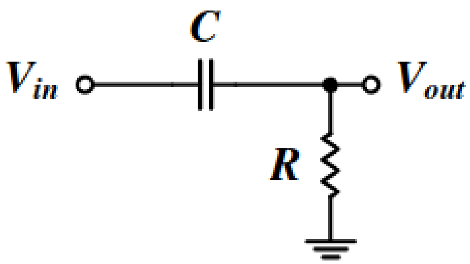
[Sedra/Smith, 2015]

First-Order HPF

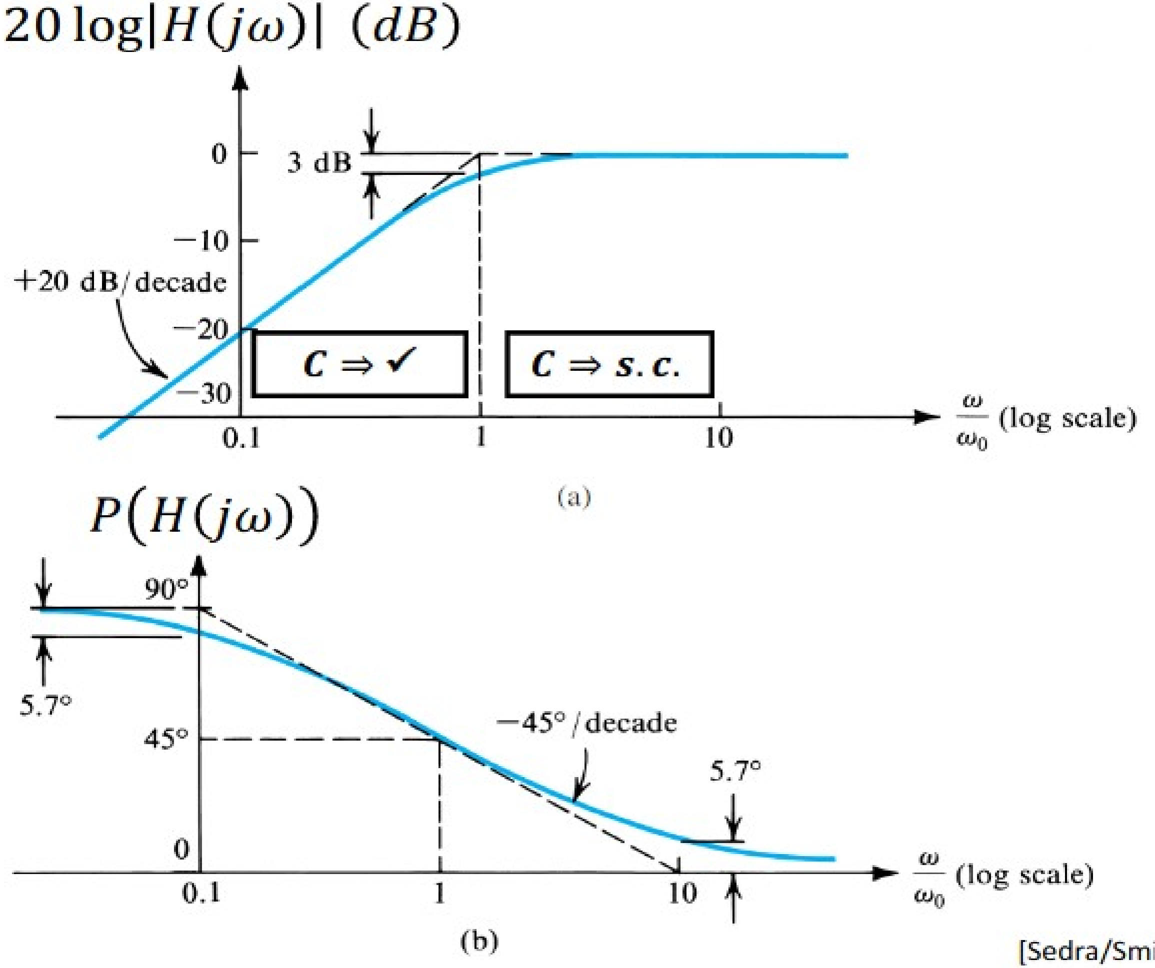
$$H(s) = \frac{v_{out}(s)}{v_{in}(s)} = \frac{R}{R + 1/sC} = \frac{sRC}{1 + sRC} = \frac{s\tau}{1 + s\tau}$$

$$H(j\omega) = \frac{v_{out}(j\omega)}{v_{in}(j\omega)} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{1 + j\omega RC} = \frac{\frac{j\omega}{\omega_c}}{1 + \frac{j\omega}{\omega_c}}$$

- Poles: $s_p = -\frac{1}{\tau} = -\omega_c$
- Zeros: $s_z = 0$
- $|H(j\omega)| = \frac{\frac{\omega}{\omega_c}}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$
- $P\big(H(j\omega)\big) = 90^o - \tan^{-1}\frac{\omega}{\omega_c}$



First-Order HPF Bode Plot



[Sedra/Smith, 2015]

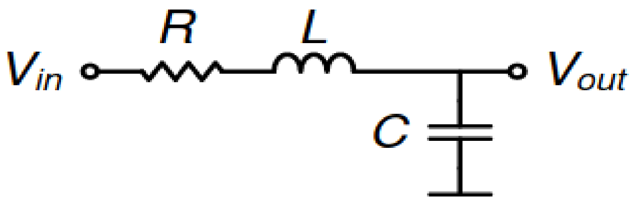
Second-Order System: LC LPF

$$H(s) = \frac{Z_C}{R + Z_L + Z_C} = \frac{1}{LCs^2 + RCs + 1}$$

$$\omega_o^2 = \frac{1}{LC} \quad and \quad Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC} = \frac{1}{2\zeta}$$

- Higher R means higher damping (ζ) and lower quality factor (Q)

$$H(s) = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{\omega_o Q} + 1} = \frac{\omega_o^2}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2} = \frac{\omega_o^2}{s^2 + (2\zeta\omega_o)s + \omega_o^2}$$

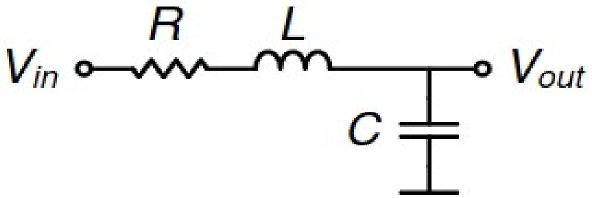


Second-Order Passive LC LPF

$$H(s) = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{\omega_o Q} + 1} = \frac{\omega_o^2}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2}$$

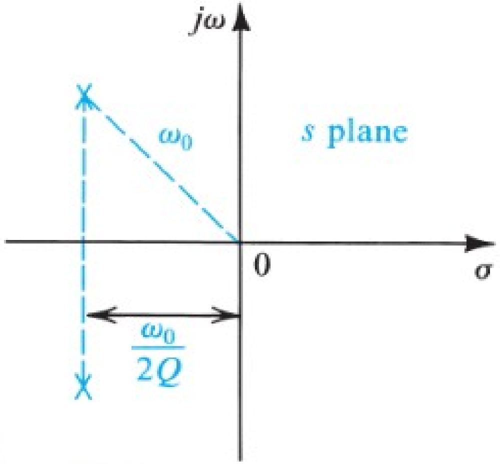
- The poles occur at $s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2 = 0$

$$s_{p1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{\omega_o}{2Q} \pm \frac{\omega_o}{2}\sqrt{\frac{1}{Q^2} - 4}$$



Second-Order System Poles

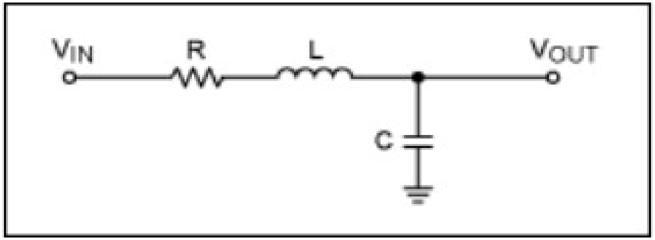
$$H(s) = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{\omega_o Q} + 1} = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{\omega_o / 2\zeta} + 1}$$



$= \frac{1}{2}$

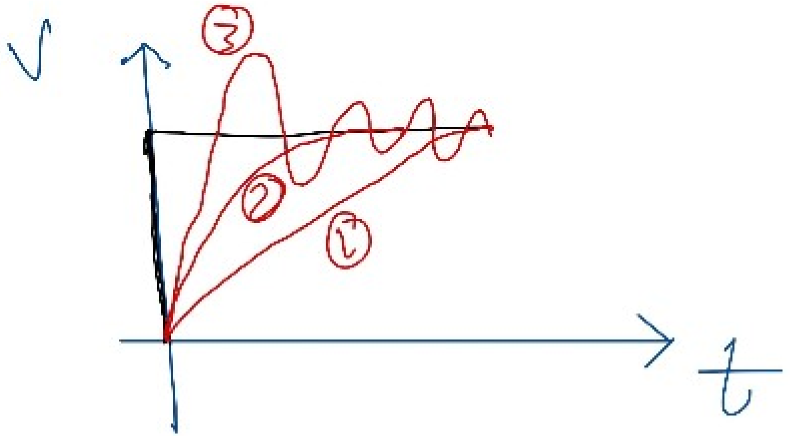
$$s_{p1,2} = -\frac{\omega_o}{2Q} \pm \frac{\omega_o}{2}\sqrt{\frac{1}{Q^2} - 4}$$

- If $Q < 0.5$ $(\zeta > 1)$: overdamped system, roots are real, negative, and distinct, like two first-order RC filters in cascade
- If $Q = 0.5$ $(\zeta = 1)$: critical damped system, roots are real, negative, and equal
- If $Q > 0.5$ $(\zeta < 1)$: underdamped system, roots are complex conjugate



Ringing and Peaking

1) $Q < 0.5$ ($\zeta > 1$) over damped system roots are real, negative, and distinct
2) $Q = 0.5$ ($\zeta = 1$):critical damped system, roots are real, negative, and equal
3) $Q > 0.5$ ($\zeta < 1$):under damped system,roots are complex conjugate



Problems:

- Increase settling time
- Overshoot and undershooting which is calculated by

$$\%\text{ overshoot} = 100\, e^{\frac{-\pi}{\sqrt{4Q^2-1}}}$$

Ringing and Peaking

- Ringing at time happens first at Q > 0.5 then peaking at frequency domain at Q > 0.707