

Multilayer Perceptron (MLP)

Neural Network Fundamentals

A Comprehensive Introduction

Marc Reyes

Lecturer

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Multilayer Perceptron (MLP)

The Building Blocks of Deep Learning

```
graph TD
  A[Input Layer] --> B[Hidden Layer]
  B --> C[Output Layer]
```

- Foundation of modern neural networks
- Versatile architecture for diverse problems
- Combines simplicity with powerful learning capabilities

From Neurons to Networks

```
flowchart TD
    D[Dendrites] --> S[Cell Body]
    S --> A[Axon]
    A --> O[Synapse]
```

- **Biological inspiration:** Mimics brain's neural structure
 - Neurons receive, process, and transmit information
- **Artificial neuron:** Weighted sum + activation function
 - Processes inputs through mathematical operations

From Neurons to Networks (cont.)

```
flowchart TD
    D[Dendrites] --> S[Cell Body]
    S --> A[Axon]
    A --> O[Synapse]
```

- **Network topology:** Input layer → Hidden layers → Output layer
 - Organized structure for information processing
- **Information flow:** Forward propagation for predictions
 - Data travels from input to output through the network

The Perceptron Journey

```
gantt
    dateFormat YYYY
    title Perceptron Journey
    section Milestones
    Rosenblatt's Perceptron      :a1, 1958, 1y
    Minsky & Papert (XOR issue)  :a2, 1969, 1y
    Backpropagation introduced   :a3, 1986, 1y
    Modern Neural Networks       :a4, 2025, 1y
```

- **1958:** Rosenblatt's single-layer perceptron
 - First implementation of a neural learning algorithm
- **1969:** Minsky & Papert expose limitations (XOR problem)
 - Demonstrated that single-layer networks couldn't solve nonlinear problems

The Perceptron Journey (cont.)

```
gantt
    dateFormat YYYY
    title Perceptron Journey
    section Milestones
    Rosenblatt's Perceptron      :a1, 1958, 1y
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    Modern Neural Networks       :a4, 2025, 1y
```

- **1986:** Rumelhart, Hinton & Williams introduce backpropagation
 - Breakthrough algorithm enabling training of multi-layer networks
- **Today:** Foundation for advanced architectures (CNNs, RNNs, Transformers)
 - Core concepts extended to specialized network designs

MLP Architecture

```
graph TD
    I[Input Layer] --> H[Hidden Layer]
    H --> O[Output Layer]
    I -.->|Weights| H
    H -.->|Weights| O
```

Key Components:

- **Input layer:** Raw data reception
 - Receives and standardizes input features
- **Hidden layers:** Feature extraction and transformation
 - Learns hierarchical representations of data
- **Output layer:** Final prediction/classification
 - Produces the network's answer to the given problem

MLP Architecture (cont.)

```
graph TD
    I[Input Layer] --> H[Hidden Layer]
    H --> O[Output Layer]
    I -.->|Weights| H
    H -.->|Weights| O
```

Key Components (continued):

- **Weights & biases:** Learnable parameters
 - Adjusted during training to minimize error
- **Activation functions:** Introduce non-linearity
 - Enable the network to learn complex patterns

Activation Functions

flowchart TD

AF[Activation Functions]

AF --> SIG[Sigmoid]

AF --> TANH[Tanh]

AF --> RELU[ReLU]

AF --> LRELU[Leaky ReLU]

Function	Formula	Characteristics
Sigmoid	$\sigma(x) = \frac{1}{1+e^{-x}}$	Output range $[0,1]$, vanishing gradient
Tanh	$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$	Output range $[-1,1]$, zero-centered

Activation Functions (cont.)

```
flowchart TD
    AF[Activation Functions]
    AF --> SIG[Sigmoid]
    AF --> TANH[Tanh]
    AF --> RELU[ReLU]
    AF --> LRELU[Leaky ReLU]
```

Function	Formula	Characteristics
ReLU	$f(x) = \max(0, x)$	Computationally efficient, sparse activation
Leaky ReLU	$f(x) = \max(0.01x, x)$	Prevents dying ReLU problem

Forward Propagation

```
graph LR
  IN[Input]
  WB[Weights & Biases]
  ACT[Activation]
  OUT[Output]
  IN --> WB
  WB --> ACT
  ACT --> OUT
```

For each layer (l):

$$Z^{[l]} = W^{[l]} \cdot A^{[l-1]} + b^{[l]}$$
$$A^{[l]} = g^{[l]}(Z^{[l]})$$

Where:

- ($W^{\{[l]\}}$) = weights matrix

Forward Propagation (cont.)

```
graph LR
  IN[Input]
  WB[Weights & Biases]
  ACT[Activation]
  OUT[Output]
  IN --> WB
  WB --> ACT
  ACT --> OUT
```

Where:

- $(g^{\{[l]\}})$ = activation function
- $(A^{\{[l]\}})$ = activation output

Backpropagation: Learning Process

```
flowchart TD
    FP[Forward Pass] --> EC[Error Calculation]
    EC --> BP[Backward Pass]
    BP --> UP[Update Parameters]
```

1. **Forward pass:** Compute predictions
 - Process inputs through the network
2. **Error calculation:** Compare with ground truth

Backpropagation: Learning Process (cont.)

flowchart TD

FP[Forward Pass]

EC[Error Calculation]

BP[Backward Pass]

UP[Update Parameters]

FP --> EC

EC --> BP

BP --> UP

3. Backward pass: Compute gradients

4. Parameter update: Adjust weights and biases

$$W^{[l]} = W^{[l]} - \alpha \frac{\partial J}{\partial W^{[l]}}$$

$$b^{[l]} = b^{[l]} - \alpha \frac{\partial J}{\partial b^{[l]}}$$

Loss Functions

Task	Loss Function	Formula
Regression	Mean Squared Error	$(\frac{1}{n}\sum_{i=1}^n(y_i - \hat{y}_i)^2)$
Binary Classification	Binary Cross-Entropy	$(-\frac{1}{n}\sum_{i=1}^n[y_i\log(\hat{y}_i) + (1-y_i)\log(1-\hat{y}_i)])$

Loss Functions (cont.)

Task	Loss Function	Formula
Multi-class Classification	Categorical Cross-Entropy	$(-\frac{1}{n} \sum_{i=1}^n \sum_{j=1}^m y_{ij} \log(\hat{y}_{ij}))$

- Loss guides the learning process
- Different tasks use specialized error measurements
- Optimization aims to minimize loss

Universal Approximation Theorem

```
graph LR
  IN[Input] --> HL[Hidden Layer]
  HL --> OUT[Output]
  HL --- NOTE[Approximates any continuous function]
```

"A feedforward network with a single hidden layer containing a finite number of neurons can approximate any continuous function, under mild assumptions on the activation function."

- The theoretical foundation for MLP capabilities

Universal Approximation Theorem (cont.)

```
graph LR
  IN[Input]
  HL[Hidden Layer]
  OUT[Output]
  IN --> HL
  HL --> OUT
  HL --- NOTE[Approximates any continuous function]
```

- More complex functions may require more neurons
- Practical implementations must balance capacity and training challenges

Visualizing Decision Boundaries

```
graph LR
  IN[Input]
  WB[Weights & Biases]
  ACT[Activation]
  OUT[Output]
  IN --> WB
  WB --> ACT
  ACT --> OUT
```

- **Linear boundaries:** Single-layer perceptrons
 - Separate data with straight lines
- **Non-linear boundaries:** MLPs with hidden layers
 - Can form complex separation surfaces

Visualizing Decision Boundaries (cont.)

```
graph LR
  IN[Input]
  WB[Weights & Biases]
  ACT[Activation]
  OUT[Output]
  IN --> WB
  WB --> ACT
  ACT --> OUT
```

- **Complexity increases** with deeper architectures
- Explore an interactive demo at perceptron.marcr.xyz

Quick Quiz: Test Your Knowledge!

Which of these problems can a single-layer perceptron solve?

- A) XOR problem
- B) Linear classification
- C) Image recognition
- D) All of the above

Use the poll feature to submit your answer!

Practical Implementation Challenges

Poll: What's your biggest challenge with neural networks?

- ☐ Understanding the math
- ☐ Choosing the right architecture
- ☐ Overfitting/underfitting
- ☐ Computational resources
- ☐ Interpreting results

Share your thoughts in the chat!

Practical Implementation

Thank You!

```
graph TD
  A[Input Layer] --> B[Hidden Layer]
  B --> C[Output Layer]
```

Contact Information

- Email: hi@marcr.xyz
- Website: marcr.xyz

Resources

- Interactive Demo: perceptron.marcr.xyz
- Slides: github.com/mabreyes/dlsu-lecture-slides