IL NUOVO CIMENTO VOL. 5 A, N. 4 21 Ottobre 1971

Electromagnetic Effects in the Decay of Polarized Neutrons.

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(ricevuto il 12 Marzo 1971)

Summary. — A finite, model-independent function $h(E, E_0, m)$ is found which, together with the Sirlin function $g(E, E_0, m)$, gives the energy dependence of the electron spectrum in the decay of polarized neutrons. Applied to asymmetry data, these functions determine a value for the ratio of the effective weak-coupling constants. This value is compared with an existing determination of the same quantity from decay data.

1. – Introduction.

The lowest-order contribution to the neutron decay matrix element can be written in the form

$$M_0 = \bar{u}_p \gamma_\nu (G_v - G_a \gamma_5) u_n M_l^\nu ,$$

where u_p, u_n are proton and neutron spinors and M_l^ν contains the leptonic factors. G_v and G_a (which we denote, generically, by G) are the weak charges, renormalized only by the strong interactions. SIRLIN (1) has shown that the next largest contribution, the order $O(\alpha G)$ matrix element M_1, can be written as

$$M_1 = M^u + M^k ,$$

where the « unknown » part M^u has the form

$$M^u = \bar{u}_p \gamma_\nu \left(G_v \frac{\alpha c}{2\pi} - G_a \frac{\alpha d}{2\pi} \gamma_5 \right) u_n M_l^\nu .$$

(1) A. SIRLIN: *Phys. Rev.*, **164**, 1767 (1967).

For the definition of c and d we refer to Sirlin's paper; note that c and d include the conventional renormalizations (which are real, momentum independent and for most models infinite), along with other terms, so that *a priori* they may be c-number functions of the external momenta and the coupling constants. However, SIRLIN argues that c and d are approximately momentum independent, due to the small energy release. The usefulness of this decomposition stems from this fact and the fact that the « known » part M^k involves the strong interactions only via the known on-mass-shell vertices. Indeed it follows that any model of the strong interactions whose vertices on the mass shell are correct will give M^u correctly along with model-dependent values for c and d.

If now we restrict ourselves to the (experimentally important) class of observables which to order $O(\alpha G)$ depend only on $\operatorname{Re} c$ and $\operatorname{Re} d$ then for these observables G_v and G_a are unobservable bare values and only

$$G'_v = G_v\left(1+\frac{\alpha}{2\pi}\operatorname{Re} c\right), \qquad G'_a = G_a\left(1+\frac{\alpha}{2\pi}\operatorname{Re} d\right) \tag{1}$$

are measurable. For these observables electromagnetic effects can be included by adding M^k to M_0 and changing the coupling constants to their primed values. This is not the whole story, of course, since eqs. (1) are not genuine renormalizations, and, even if they were, the bare values are still of interest in the light of symmetries such as universality. For this reason it is still of interest to use a model for one's calculations rather than Sirlin's model-independent formulae, so that one has estimates of c and d.

Within the restricted framework of (1) the neutron lifetime and $0^+\to 0^+$ β-decays of nuclei have been used to determine G'_v and G'_a (see BLIN-STOYLE and FREEMAN [2]), note that their notation is $C = 2\operatorname{Re} c$ and $D = 2\operatorname{Re} d$). The electron asymmetry coefficient in the decay of polarized neutrons is another observable to which (1) is applicable, and in this paper we use the recent measurement [3] of it to determine a second value of G'_v/G'_a.

2. – The calculations.

The order $O(G^2)$ expression for the differential decay rate of polarized neutrons is

$$\frac{\mathrm{d}\Gamma^0}{\mathrm{d}^3p} = \frac{2(E-E_0)^2}{(2\pi)^4}\left[(G_v^2+3G_a^2)+2G_v(G_v-G_a)\beta\cos\theta\right], \tag{2}$$

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[2] R. J. BLIN-STOYLE and J. M. FREEMAN: *Nucl. Phys.*, **150** A, 369 (1970).
[3] C. J. CHRISTENSEN, V. E. KROHN and C. R. RINGO: *Phys. Lett.*, **28** B, 411 (1969).

where E and β are the energy and velocity of the electron, E_0 is the neutron-proton mass difference, and θ is the angle between the electron momentum p and the neutron spin. Following the above discussion the order $O(\alpha G^2)$ expression can be written in the form

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}^3p}=\frac{2(E-E_0)^2}{(2\pi)^4}\left[(G_v'^2+3G_a'^2)\left\{1+\frac{\alpha}{2\pi}g(E,E_0,m)\right\}+\right.$$
$$\left.+2G_v'(G_v'-G_a')\beta\cos\theta\left\{1+\frac{\alpha}{2\pi}h(E,E_0,m)\right\}\right]. \tag{3}$$

Here m is the electron mass. G_v' and G_a' include the effects of M^u, while the functions g and h arise from M^k. (In principle we could have a more complicated θ-depencence, but this turns out not to be so. We include in (3) the rate to final states which include one photon, and assume that their only significant effect is to cancel the infra-red divergence.) Note that (3) can be rewritten as

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}^3p}=\frac{2(E-E_0)^2}{(2\pi)^4}\left(1+\frac{\alpha}{2\pi}R\right)\left[(G_v^2+3G_a^2)\left\{1+\frac{\alpha}{2\pi}g(E,E_0,m)\right\}+\right.$$
$$\left.+2G_v(G_v-G_a)\beta\cos\theta\left\{1+\frac{\alpha}{2\pi}\big(h(E,E_0,m)+T\big)\right\}\right], \tag{4}$$

where the overall «renormalization» R and the relative «renormalization» of the $\cos\theta$ part T are determined by (1) and (3). If we define $\lambda=G_a/G_v$, $C=2\,\mathrm{Re}\,c$ and $D=2\,\mathrm{Re}\,d$, then

$$R=\frac{C+3\lambda^2D}{1+3\lambda^2},\qquad T=\frac{2C-\lambda(C+D)}{2(1-\lambda)}-R, \tag{5}$$

so that (for $\lambda=1.26$)

$$\begin{cases} C=R-0.5T\,,\\ D=R+0.11T\,. \end{cases} \tag{6}$$

KINOSHITA and SIRLIN [4] computed R using perturbation theory with point nucleons, and found

$$R=9/4+6\log(\Lambda/m_p) \tag{7}$$

(see also footnote [23] of ref. [1]), where m_p is the proton mass and Λ is a cut-off which, hopefully, represents form factor effects. This value of R has been used for C in the calculation of pure vector decay. By eq. (6) this pro-

(4) T. KINOSHITA and A. SIRLIN: *Phys. Rev.*, **113**, 1652 (1959).

cedure contains the implicit assumption that $T = 0$. We have computed T using the bare nucleon model and found

$$T = -9/4 - 3\log(\Lambda/m_p)\,, \tag{8}$$

so that this procedure does not use the model consistently. In performing this calculation we use the same assumptions as KINOSHITA and SIRLIN [4] except that we refrained from making the approximation $\lambda = 1$ until ambiguities of the form $(1-\lambda)/(1-\lambda)$ had been removed. Equation (8) permits a determination in this model (via (6) and (7)) of the effective renormalizations C, D of G_v, G_a (whereas before we had only the effective renormalization R of $G_v^2 + 3G_a^2$). C is an experimental quantity in the sense that the Cabibbo angle and the muon lifetime determine G_v. Comparing this with the value of G_v' obtained from pure vector decays, BLIN-STOYLE and FREEMAN [2] found, for ^{26}Al and ^{14}O, respectively,

$$\begin{cases} (\alpha/2\pi)\,C = 2.3 \pm 0.3\,\%\,, \\ (\alpha/2\pi)\,C = 1.7 \pm 0.4\,\%\,, \end{cases} \tag{9}$$

where in both cases corrections whose (generally small) estimates tend to increase C have been neglected. Unambiguous comparison with the prediction of the bare-nucleon model is not possible because of the cut-off Λ, however, fitting Λ to these values of C we get $\Lambda = 9m_p$ or $5m_p$ and $D = 14$ or 10, for ^{26}Al and ^{14}O, respectively.

We also computed the functions g and h (which by Sirlin's argument are model independent) using the same model, and found g to be as given by SIRLIN (see eq. (20b) of ref. [1]) and

$$h(E, E_0, m) = 3\log(m_p/m) - \frac{3}{4} + 4(\beta^{-1}\,\mathrm{tgh}^{-1}\beta - 1)\left(\log\frac{2(E_0-E)}{m} - \frac{3}{2} + (E_0-E)/3E\beta^2 + (E_0-E)^2/24E^2\beta^2\right) + 4\beta^{-1}L\left(\frac{2\beta}{1+\beta}\right) + 4\beta^{-1}\,\mathrm{tgh}^{-1}\beta(1-\mathrm{tgh}^{-1}\beta)\,, \tag{10}$$

where $L(x) = \int_0^x (\log(1-t)/t)\,\mathrm{d}t$ and the Coulomb term in h (as in g) has been omitted. In the next Section we determine $\lambda' \equiv G_a'/G_v'$ using this function and electron asymmetry data.

3. – Asymmetry coefficients.

Of the asymmetry coefficients that have been measured [3], the most sensitive to λ' is the A-coefficient. A is determined in terms of the experimental quantities Γ^+, Γ^- by the expression of JACKSON *et al.* ([5]), so that

$$A = (J/I)\frac{\Gamma^+ - \Gamma^-}{\Gamma^+ + \Gamma^-}, \tag{11}$$

where

$$I = \int_0^{\sqrt{E_0^2-m^2}} p^2(E_0 - E)^2\beta\,\mathrm{d}p = 0.04\ (\mathrm{MeV})^5,$$

$$J = \int_0^{\sqrt{E_0^2-m^2}} p^2(E_0 - E)^2\,\mathrm{d}p = 0.06\ (\mathrm{MeV})^5$$

and

$$\Gamma^\pm = \pm\, 2\pi\int_0^{\pm 1}\mathrm{d}\cos\theta\int_0^{\sqrt{E_0^2-m^2}} p^2\,\mathrm{d}p\,\frac{\mathrm{d}\Gamma}{\mathrm{d}^3 p}. \tag{12}$$

Thus, the order $O(\alpha G^2)$ expression for A is

$$A = \frac{2\lambda'(1-\lambda')}{1+3\lambda'^2}\left[1 + (\alpha/2\pi)\left\{I^{-1}\int_0^{\sqrt{E_0^2-m^2}} p^2(E_0 - E)^2\beta H\,\mathrm{d}p - J^{-1}\int_0^{\sqrt{E_0^2-m^2}} p^2(E_0 - E)^2 G\,\mathrm{d}p\right\}\right], \tag{13}$$

where H and G include the Coulomb term:

$$H = h + 2\pi^2\beta^{-1}, \qquad G = g + 2\pi^2\beta^{-1}. \tag{14}$$

Numerical evaluation of the integrals yields

$$A = \frac{2\lambda'(1-\lambda')}{1+3\lambda'^2}\left[1 + \frac{\alpha}{2\pi}(-1.0 \pm 0.1)\right]. \tag{15}$$

The error quoted here is an upper limit on the estimated computer error.

We see that this is a very small radiative correction ($\delta\lambda'/\lambda' \sim 0.01\,\%$) which is due to the similarity of g and h. The model-independent result is then that

$$\lambda' = 1.26 \pm 0.02,$$

([5]) JACKSON *et al.*: *Phys. Rev.*, **106**, 517 (1957).

from $A = -0.115 \pm 0.008$ (CHRISTENSEN *et al.* [3]), to be compared with

$$\lambda' = 1.226 \pm 0.011 ,$$

$$\lambda' = 1.232 \pm 0.012 ,$$

from ^{26}Al and ^{14}O, respectively, together with the current neutron lifetime [6].

4. – Conclusion.

A straightforward shrinking of the error in A would reveal a clear discrepancy between the two values of λ'. On the other hand, a decrease in A (and another increase in the neutron rate?) may bring these into harmony. The experiments now in progress on the A-coefficient and the neutron lifetime should clear up the situation.

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The author thanks Prof. R. J. BLIN-STOYLE for suggesting the calculation, Dr. D. BAILIN for helpful discussions, and the S.R.C. for finance.

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(6) C. J. CHRISTENSEN, A. NIELSEN, A. BAHNSEN, W. K. BROWN and B. M. RUSTAD: *Phys. Lett.*, **26** B, 11 (1967).

● RIASSUNTO (*)

Si trova una funzione $h(E, E_0, m)$ finita ed indipendente da ogni modello che, assieme alla funzione di Sirlin $g(E, E_0, m)$, dà la dipendenza dello spettro elettronico dall'energia nel decadimento dei neutroni polarizzati. Applicate ai dati dell'asimmetria, queste funzioni determinano un valore del rapporto delle costanti effettive dell'accoppiamento debole. Si confronta questo valore con una determinazione esistente di questa quantità dai dati di decadimento.

(*) *Traduzione a cura della Redazione.*

Электромагнитные эффекты при распаде поляризованных нейтронов.

Резюме (*). — Определяется конечная, не зависящая от модели функция $h(E, E_0, m)$, которая, вместе с функцией Сирлина $g(E, E_0, m)$, дает энергетическую зависимость электронного спектра в распаде поляризованных нейтронов. Когда эти функции применяются к данным по асимметрии, то они определяют величину для отношения постоянных эффективных слабых связей. Это значение сравнивается с существующим определением той же величины отношения из данных по распаду.

(*) *Переведено редакцией.*