جامعة بيرزيت

BIRZEIT UNIVERSITY

Department of Electrical & Computer Engineering
ENEE3309 - Communication Systems

Course Assignment

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Section: 2

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1 Questions

1.1 Question 1

First, we need to write the expression in terms of singularity functions.

$$g(t) = \begin{cases} 1 & 0 \leq t \leq 0.05 \\ -20t + 2 & 0.05 < t \leq 0.1 \end{cases}$$

then we can get the coefficients by calculating the following integrals

$$a_0 = \frac{1}{T_0} \int_0^{T_0} g(t)dt \tag{1}$$

$$a_n = \frac{2}{T_0} \int_0^{T_0} g(t)cos(nw_0t)dt \tag{2}$$

$$b_n = \frac{2}{T_0} \int_0^{T_0} g(t)sin(nw_0t)dt \tag{3}$$

```
import prettytable as pt
import sympy
from sympy.plotting import plot
import numpy as np
an = {}
bn = {}
t = sympy.Symbol('t')
for n in range(4):
    w = 2*np.pi*n/0.1
    a = (2/0.1) * (sympy.integrate(sympy.cos(w*t), (t, 0, 0.1/2)) +
sympy.integrate((-20*t + 2)*sympy.cos(w*t), (t, 0.1/2, 0.1)))
    b = (2/0.1) * (sympy.integrate(sympy.sin(w*t), (t, 0, 0.1/2)) +
sympy.integrate((-20*t + 2)*sympy.sin(w*t), (t, 0.1/2, 0.1)))
    an[n] = a
    bn[n] = b
an[0] = an[0]/2
```

Snippet 1.1: Coefficients Evaluation

n	An	Bn
0	0.75000	0.00000
1	-0.20264	0.31831
2	0.00000	0.15915
3	-0.02252	0.10610

1.2 Question 2

in this question, we are asked to plot the approximated signal along with the actual signal on the same graph, we've plotted it using python and using Matlab.

1.2.1 Using Python

```
def approximate(t, r=4):
    return an[0]+ sum([an[n]*sympy.cos(2*np.pi*n*t/0.1) +
            bn[n]*sympy.sin(2*np.pi*n*t/0.1) for n in range(1, r)])

t = sympy.Symbol('t')
function = sympy.Piecewise((1, t <= 0.05), (-20*t + 2, True))
p = plot(function, approximate(t), (t, 0, 0.1), show=False)
p[0].line_color = 'r'
p[0].label = 'g(t)'
p[1].line_color = 'b'
p[1].label = 'g(t) approximated'
p.title = 'g(t) and g(t) approximated'
p.legend = True
p.xlabel = 't'
p.ylabel = 'g(t)'
p.show()
```

Snippet 1.2: Plotting using python

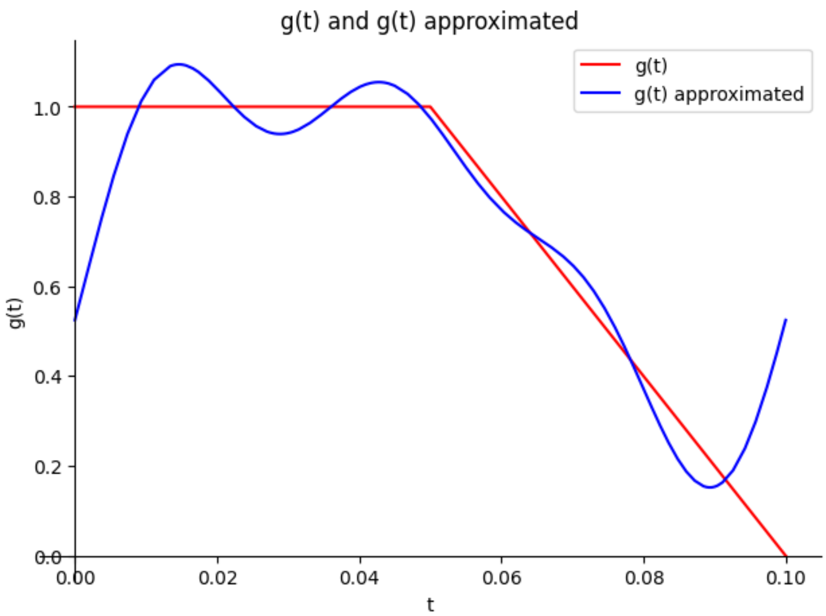

Figure 1.1: Python Plot

1.2.2 Using Matlab

```
t = (0:0.00000001:0.1)';
a0 = 0.75;
a = [-0.20264, 0, -0.02252];
b = [0.31831, 0.15915, 0.10610];
T0 = 0.1;
g = heaviside(t) - 20*(t-T0/2).*heaviside(t-T0/2) + 20*(t -
↪  T0).*heaviside(t-T0);
approx = a0;
for n = 1:3
    approx = approx + a(n)*cos(2*pi*n*t/T0) + b(n)*sin(2*pi*n*t/T0);
end
plot(t, approx, t, g)
xlabel('t')
ylabel('g(t)')
legend('ga(t)','g(t)')
axis([0 0.1 0 2])
```

Snippet 1.3: Plotting using Matlab

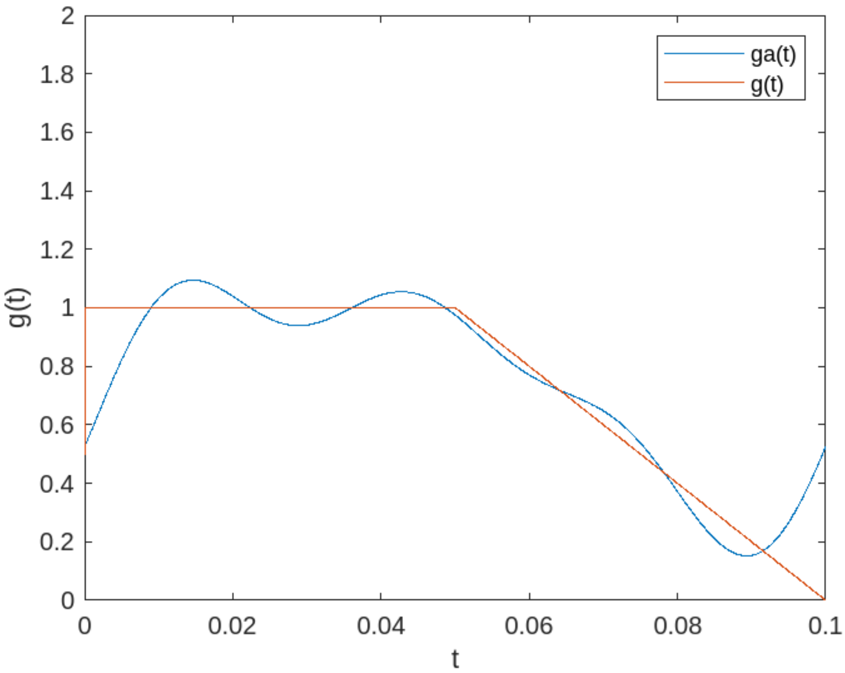

Figure 1.2: Matlab Plot

1.3 Question 3

In this question, we are going to evaluate the mean square error for the approximated signal with respect to the actual signal for K = 1, 2, and 3, according to the following equation.

$$MSE = \frac{1}{T_0} \int_0^{T_0} (g(t) - g_a(t))^2 dt \tag{4}$$

```
def mse(function, approximate):
    return 10 * sympy.integrate((function - approximate)**2, (t, 0,
    ↪  0.1))

table = pt.PrettyTable()
table.field_names = ["K", "MSE"]
table.set_style(pt.DOUBLE_BORDER)
table.max_width = 31
table.min_width = 31
table.align = "c"
for k in range(1, 4):
    table.add_row([k, mse(function, approximate(t, k))])
print(table)
```

Snippet 1.4: Evaluating MSE

K	MSE
1	0.104166666666667
2	0.0329741103361289
3	0.0203089623808363

1.4 Question 4

in this question, we are asked to multiply the approximated signal by a carrier, apply a band-pass filter to generate a single side-band signal s(t) and to evaluate its spectrum.

$$c(t) = 10cos(2\pi(200)t) \tag{5}$$
$$\tag{6}$$

$g_a(t)$ is considered the message signal, its highest frequency f_m = 30Hz. Hence, the band pass filter frequency with bandwidth of 30Hz and the cut-off frequency edges on f_c and $f_c + f_m$ for upper side band or f_c and $f_c - f_m$ for lower side so if we take the upper side the filter will range from [200, 230] Hz.

$$\begin{aligned} s(t) &= 10cos(2\pi(200t)) * [a_0 + \sum_{n=1}^{3} a_n cos(nw_0 t) + b_n sin(nw_0 t)] \\ &= 10a_0 cos(2\pi(200t)) + 5a_1[cos(2\pi(210)t) + cos(2\pi(190)t)] \\ &+ 5a_3[cos(2\pi(230)t) + cos(2\pi(170)t)] + 5b_1[sin(2\pi(210)t) + sin(2\pi(190)t)] \\ &+ 5b_2[sin(2\pi(220)t) + sin(2\pi(180)t)] + 5b_3[sin(2\pi(230)t) + sin(2\pi(170)t)] \end{aligned} \tag{7}$$

After applying the band-pass filter any frequency not in range of [200, 230] Hz we can safely delete.

$$\begin{aligned} s(t) &= 10a_0 cos(2\pi(200t) + 5a_1[cos(2\pi(210)t)] \\ &+ 5a_3[cos(2\pi(230)t)] + 5b_1[sin(2\pi(210)t)] \\ &+ 5b_2[sin(2\pi(220)t)] + 5b_3[sin(2\pi(230)t)] \end{aligned} \tag{8}$$

Taking the Fourier transform to show the spectrum equation.

$$\begin{aligned} S(f) &= 5a_0[\delta(f-200) + \delta(f+200)] + 2.5a_1[\delta(f-210) + \delta(f+210)] \\ &+ 2.5a_3[\delta(f-230) + \delta(f+230)] - 2.5ib_1[\delta(f-210) + \delta(f+210)] \\ &- 2.5ib_2[\delta(f-220) + \delta(f+220)] - 2.5ib_3[\delta(f-230) + \delta(f+230)] \end{aligned} \tag{9}$$