System Step Response Using Convolution

Lab 4

Spring 2021

1 Purpose

Become familiar with using convolution to compute a system's step response.

2 Deliverables Overview

2.1 Part 1

- Plots from **Task 2**.

2.2 Part 2

- Plots for **Task 1**.
- Hand calculated convolution integrals from **Task 2**.
- Plots from **Task 2**.

As usual, plots and equations need to be thoroughly discussed in your report.

3 Part 1

3.1 Purpose

Use the step function you coded previously to write transfer functions to use in this lab.

3.2 Deliverables

1. Plots from **Task 1** in a single figure, to be included in the **Results** section of your report.

3.3 Tasks

1. Create the following signals as user-defined functions:

$$h_1(t) = e^{-2t}[u(t) - u(t-3)]$$

$$h_2(t) = u(t-2) - u(t-6)$$

$$h_3(t) = cos(\omega_0 t)u(t)$$

$$\text{for, } f_0 = 0.25 \text{ Hz.}$$

2. Plot the three functions in a single figure (separate subplots) from $-10 \leq t \leq 10$ with time steps small enough to achieve appropriate resolution.

4 Part 2

4.1 Purpose

Find and plot the step response of the three transfer functions defined in **Part 1** using Python and hand calculations.

4.2 Deliverables

1. Plots from **Task 1** to be included in the **Results** section of your report.
2. Typed equations and solutions from hand calculations in **Task 2** to be included in the **Equations** and **Results** sections of your reports, respectively.
3. Plots from **Task 2** to be included in the **Results** section of your report. *Note: These plots should match the plots from **Part 1**.*

4.3 Tasks

Perform the following tasks for each of the three transfer functions defined in **Part 1**. Plot each response from $-10 \leq t \leq 10$ with an appropriate time step.

1. Plot the step response using the convolution function you created in**Lab 3**.
2. By hand, calculate the step response of each transfer function by solving the convolution integral. Plot the results and ensure they match the plots from **Task 1**.

5 Questions

1. Leave any feedback on the clarity of lab tasks, expectations, and deliverables.

$$h_1 * u(t) = \int_0^t e^{-2\tau}\left(u(t-\tau) - u(t-3-\tau)\right) d\tau$$

$$\int_0^t e^{-2\tau} d\tau = -\frac{1}{2}\left[e^{-2t} - 1\right]$$

$$\Rightarrow \boxed{h_1 * u(t) = \frac{1}{2}\left[e^{-2(t-3)} - 1\right] u(t-3) - \frac{1}{2}\left[e^{-2t} - 1\right] u(t)}$$

$$h_2 * u(t) = \int_{-\infty}^{\infty} \left[u(\tau-2) - u(\tau-6)\right] \cdot u(t-\tau)\, d\tau$$

$$= \int_2^t u(\tau-2)\, d\tau - \int_6^t u(\tau-6)\, d\tau$$

$$\boxed{h_2 * u(t) = (t-2)\cdot u(t-2) - (t-6)\cdot u(t-6)}$$

$$h_3 * u(t) = \int_{-\infty}^{\infty} \cos(\omega_0 \tau) \cdot u(\tau) \cdot u(t-\tau)\, d\tau$$

$$= \int_0^t \cos(\omega_0 \tau)\, d\tau = \boxed{h_3 * u(t) = \frac{1}{\omega_0} \cdot \sin(\omega_0 t) \cdot u(t)}$$