Muc Edmundson

Fourier Series Approximation of a Square Wave

Prelab 8

Spring 2021

1 Purpose

Find a general expression for the coefficients of a Fourier series.

2 Deliverables

Typed and properly formatted derivation of the k-th Fourier series coefficients a_k and b_k.

3 Tasks

Consider the square wave in figure 1. Assume this is a real-valued function and can be explained by the Fourier series:

$$x(t) = \frac{1}{2}a_0 + \sum_{k=1}^{\infty} a_k cos(k\omega_0 t) + b_k sin(k\omega_0 t) \tag{1}$$

Where,

$$a_k = \frac{2}{T}\int_0^T x(t)cos(k\omega_0 t)dt \tag{2}$$

$$b_k = \frac{2}{T}\int_0^T x(t)sin(k\omega_0 t)dt \tag{3}$$

$$\omega_0 = \frac{2\pi}{T} \tag{4}$$

1. By hand, find the expressions for a_k, b_k, and $x(t)$.

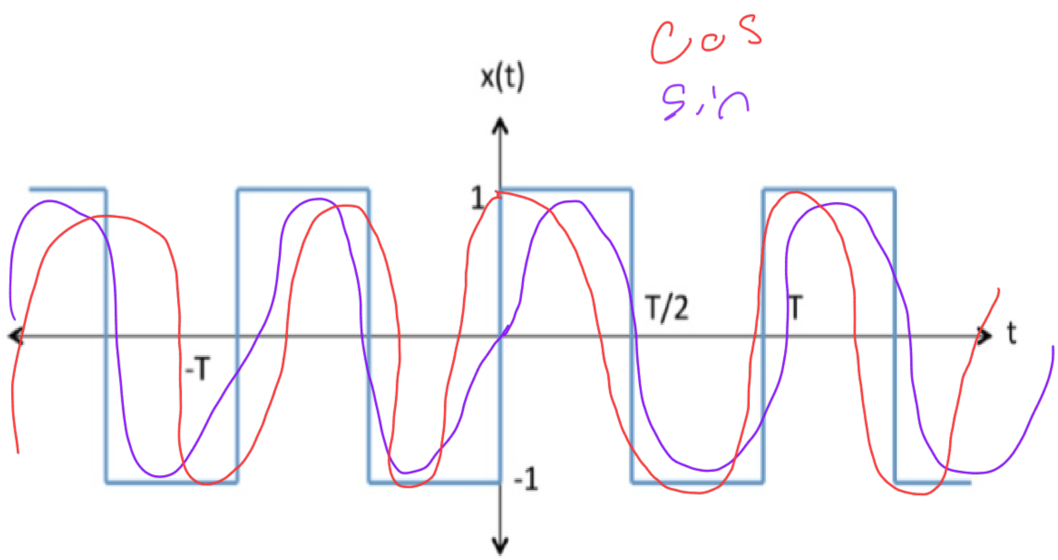
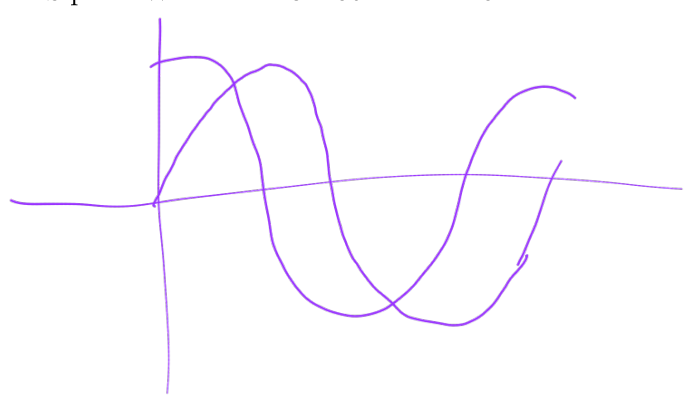


Figure 1: Square Wave for ECE 351 Prelab 8



As seen on graph,

$a_0 = \emptyset$. (No DC component)

$$a_k = \frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)\,dt$$

$$= \frac{2}{T}\left(\int_0^{T/2} \cos(k\omega_0 t)\,dt - \int_{T/2}^{T} \cos(k\omega_0 t)\,dt\right)$$

$$= \frac{2}{T}\left(\frac{\sin(kT\omega_0/2)}{k\omega_0} - \frac{\sin(kT\omega_0) - \sin(kT\omega_0/2)}{k\omega_0}\right)$$

$$= \frac{2}{\cancel{T}}\left(\frac{\cancel{T}\sin(k\pi)}{k2\pi} - \frac{\cancel{T}(\sin(2\pi k) - \sin(k\pi))}{k\cancel{2}\pi}\right)$$

$$= \frac{\sin(k\pi) - \sin(2\pi k) + \sin(k\pi)}{k\pi}$$

$$= \frac{2\sin(k\pi) - \sin(k2\pi)}{k\pi}$$

$$b_k = \frac{2}{T}\int_0^T x(t)\sin(k\omega_0 t)\,dt = \frac{2}{T}\left(\int_0^{T/2}\sin(k\omega_0 t)\,dt - \int_{T/2}^{T}\sin(k\omega_0 t)\,dt\right)$$

$$= \frac{2}{T}\left(\left[-\frac{\cos(k\omega_0 t)}{k\omega_0}\right]_0^{T/2} + \left[\frac{\cos(k\omega_0 t)}{k\omega_0}\right]_{T/2}^{T}\right)$$

$$= \frac{2}{T}\left[\left(\cancel{\frac{-\cos(k\omega_0\frac{T}{2})}{k\omega_0}} + \frac{1}{k\omega_0}\right) + \left(\frac{\cos(k\omega_0 T)}{k\omega_0} - \cancel{\frac{\cos(k\omega_0\frac{T}{2})}{k\omega_0}}\right)\right]$$

$$= \frac{2}{T}\left(\frac{\cos(k\omega_0 T)}{k\omega_0} + \frac{1}{k\omega_0}\right) = \frac{2}{\cancel{T}}\left(\frac{\cancel{T}\cos(k2\pi)}{k2\pi} + \frac{\cancel{T}}{k2\pi}\right)$$

$$= \frac{\cos(k2\pi) + 1}{k\pi}$$

$$\Rightarrow x(t) = \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

In-Lab: (gone-over with Kate)

$$\cancel{a_k = \frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)\,dt}$$

we know this goes to $\emptyset$.

$$b_k = \frac{2}{T}\int_0^T x(t)\sin(k\omega_0 t)\,dt$$

$$b_k = 2\cdot\frac{2}{T}\int_0^{T/2}(1)\sin(k\omega_0 t)\,dt$$

$$= \frac{4}{T} - \frac{1}{k\omega_0}\left[\cos\left(\frac{k\omega_0 t}{2}\right) - 0\right]$$

$$= \frac{2}{k\pi}\left(1 - \cos(k\pi)\right)$$