Frequency Response

Prelab 10

Spring 2021

1 Purpose

Find the analytical expression for magnitude and phase of a transfer function.

2 Deliverables Overview

- Typed and properly formatted derivation for both the magnitude and phase of the transfer function. *Note: All steps must be shown.*

3 Part 1

3.1 Tasks

Consider the RLC circuit in figure 1.

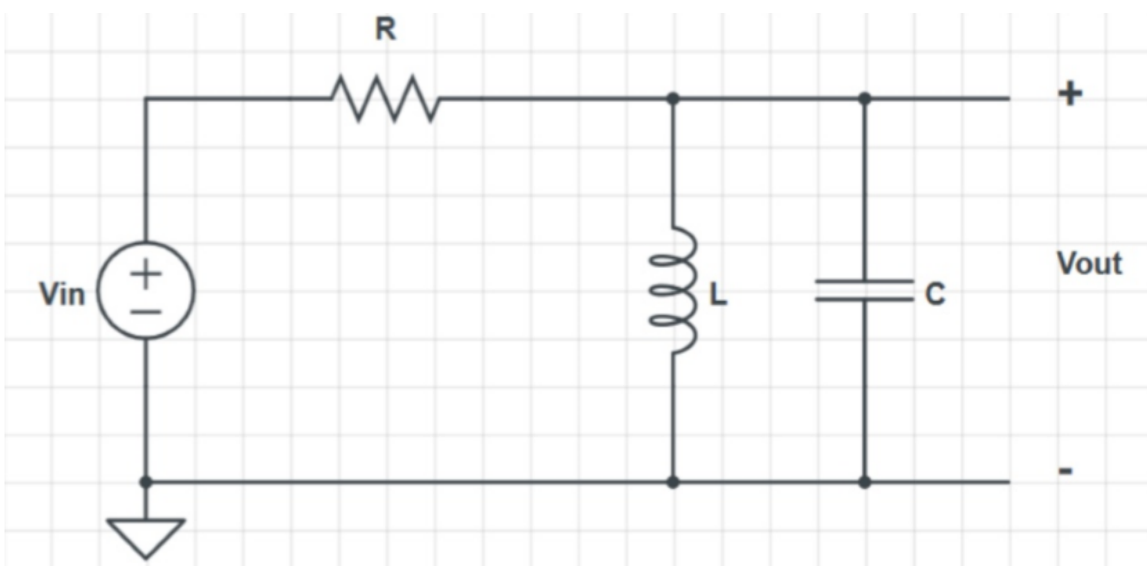

Figure 1: R = 1 kΩ, L = 27 mH, C = 100 nF

Which has the transfer function,

$$H(s) = \frac{\frac{1}{RC}s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}.$$

1. By hand, find the magnitude $|H(j\omega)|$ and the phase $\angle H(j\omega)$ for the RLC transfer function $H(s)$. Type the analytical derivation symbolically in terms of R, L, and C. Do not use your calculator or Python on this step, show all your work.

$$H(j\omega) = \frac{\frac{1}{RC}j\omega}{(j\omega)^2 + \frac{1}{RC}j\omega + \frac{1}{LC}} = \frac{\frac{1}{RC}j\omega}{\frac{1}{RC}j\omega + \left(\frac{1}{LC} - \omega^2\right)} = \frac{j\omega}{RC\left(LCj\omega + RC - RLC^2\omega^2\right)} = \frac{j\omega}{RC^2\left(Lj\omega + R - RLC\omega^2\right)}$$

$$|H(j\omega)| = \frac{\omega}{RC^2\sqrt{(R - RLC\omega^2)^2 + (L\omega)^2}} = \frac{\omega}{RC^2\sqrt{R^2 - 2R^2LC\omega^2 + R^2L^2C^2\omega^4}} = \frac{\omega}{R^2C^2\sqrt{1 - 2LC\omega^2 + L^2C^2\omega^4}}$$

$$\boxed{|H(j\omega)| = \frac{\omega}{R^2C^2\sqrt{1 - 2LC\omega^2 + L^2C^2\omega^4}}}$$

$$\angle H(j\omega) = \frac{\pi/2}{\tan^{-1}\left[\frac{RC^2}{RC^2}\,\frac{L\omega}{R - RLC\omega^2}\right]}$$

$$\boxed{\angle H(j\omega) = \frac{\pi}{2\cdot\tan^{-1}\left(\frac{L\omega}{R(1 - LC\omega^2)}\right)}}$$