Step and Impulse Response of an RLC Bandpass Filter

Prelab 5

Spring 2021

1 Purpose

Use Laplace transforms to find the time-domain step- and impulse-response of an RLC bandpass filter.

2 Deliverables Overview

Typed solutions for **Task 1** and **Task 2**. *Note: Be sure to show all work.*

3 Tasks

Consider the RLC circuit in figure 1.

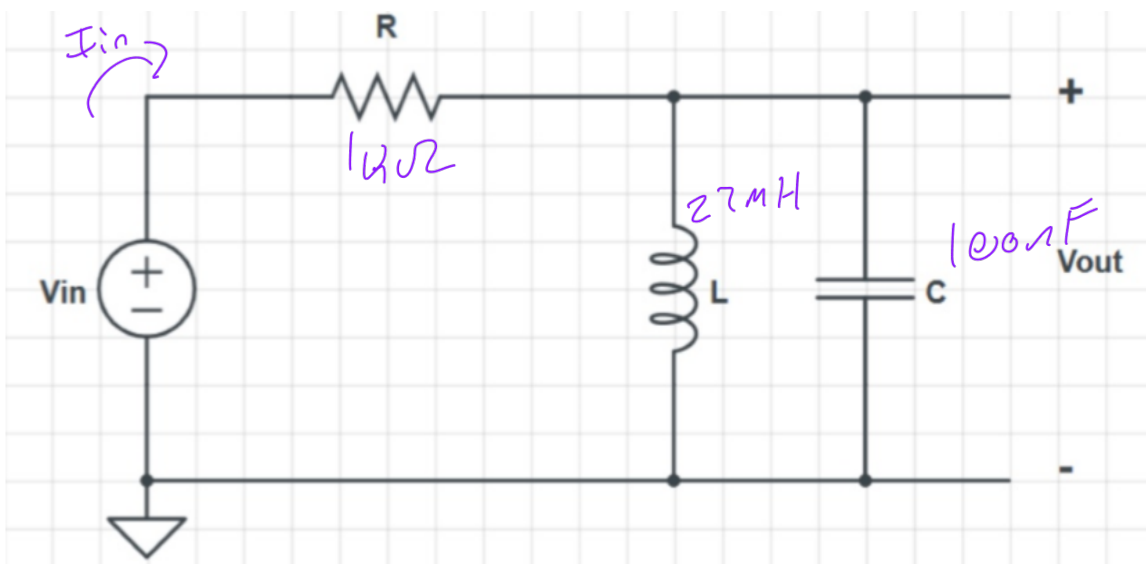

Figure 1: R = 1kΩ, L = 27 mH, C = 100 nF

1. Find the transfer function $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$ symbolically in terms of R, L and C.
2. Find the impulse response $h(t)$.

Assume: $V_L(t)$

$$V_C(t) = V_L(t) = V_{out}(t)$$

$$i_C(t) = C \cdot \frac{d\, V_C(t)}{dt} \Rightarrow \frac{1}{C}\int_0^t i_C(\tau)\, d\tau = V_C(t)$$

$$V_L(t) = L \cdot \frac{d\, i_L(t)}{dt} \Rightarrow \frac{1}{L}\int_0^t V_L(\tau)\, d\tau = i_L(t)$$

KVL:

$$V_{in} = V_R + V_L$$

$$V_{in} = i_R \cdot R + V_C(t)$$

$$i_R = i_L + i_C = \frac{1}{L}\int_0^\tau V_C(\tau)\, d\tau + C \cdot \frac{dV_C(t)}{dt}$$

$$V_{in} = \left(\frac{1}{L}\int_0^\tau V_C(\tau)\, d\tau + C \cdot \frac{dV_C(t)}{dt}\right) \cdot R + V_C(t)$$

$$V_C = V_{out}$$

$$\Rightarrow V_{in}(t) = \left(\frac{1}{L}\int_0^\tau V_{out}(\tau)\, d\tau + C \cdot \frac{d\,V_{out}(t)}{dt}\right) \cdot R + V_{out}(t)$$

$$\overset{\mathcal{L}}{\Rightarrow} V_{in}(s) = \frac{R}{L} \cdot \frac{1}{s} \cdot V_{out}(s) + RC \cdot s\, V_{out}(s) + V_{out}(s)$$

$$\Rightarrow H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \left(\frac{R}{L} \cdot \frac{1}{s} + RC \cdot s + 1\right)^{-1}$$

$$= \left(R \cdot \left(\frac{1}{Ls} + Cs\right) + 1\right)^{-1} \cdot \frac{Ls}{Ls}$$

$$\boxed{= \frac{L \cdot s}{Ls + R(1 + LCs^2)}}$$

$$= \frac{L \cdot s}{R \cdot L \cdot C \cdot s^2 + L \cdot s + R}$$

2. $h(t) = 10{,}000 \cdot e^{-5000t}\left(\cos(18584.1t) - 0.269\sin(18584.1t)\right)$