University of Idaho

ECE 351 - Section #53

Lab #11 Report

Z-Transform Operations

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Contents

1   Introduction

The goal of this weeks lab was to analyze a discrete system using Python. This lab was completed using *Python* through the *Spyder-IDE*. The packages used in the completion of this lab were `numpy` for definitions of mathematical functions, `matplotlib.pyplot` to plot outputs of functions, `scipy.signal` to compute the frequency response of a digital filter & verify PFE, and `zplane` to create a pole-zero plot for the found transfer function.

All code for this lab, including this report, can be found on my Github.

2   Equations

The equations used within this lab are shown in this section. The equations will be referenced by number throughout the rest of the report.

Given causal function:

$$y[k] = 2x[k] - 40x[k-1] + 10y[k-1] - 16y[k-2] \tag{1}$$

Transfer function calculations (Tasks 1 & 2):

$$H(z) = \frac{-40z^{-1} + 2}{(-8z^{-1} + 1)(-2z^{-1} + 1)} \tag{2}$$

$$h[k] = (6 \cdot 2^k - 4 \cdot 8^k)u[k] \tag{3}$$

3   Methodology

3.1   Lab: Part 1

This lab started out by calculating the transfer function of the given difference equation (Equation (1)). The Z-domain transfer function, H(z), was first found followed by h[k], which can be seen in Equations (2) & (2). Work to find these functions can be seen in the Attachments section of this report. The partial fraction expansion used to find h[k] was verified using the `scipy.singal.residuez()` function (output seen in Figure 1).

Next, a pole-zero plot of H(z) was generated using the provided `zplane()` function written by Christopher Felton. The plot can be seen in Figure 2 and it is able to tell us a lot about the system. Further discussion in the Questions section of this report.

Lastly, a magnitude and phase frequency response plot was generated for H(z) using the `scipy.signal.freqz()` function. Included in this report are two plots of this frequency response Figure 3 with a base 10 scale and Figure 4 with a logarithmic scale. These plots were plotted using the Normalized Nyquist Frequency making the x-axis units π radians per sample. This frequency is commonly used when analyzing discrete-time functions and Fourier Transforms to analyze the stability of a system.

Code implementing Part 1 of this lab can be seen in the below listing.

```
1  ## PART 1 ##
2  # 3
3  num = [2, -40]
4  den = [1, -10, 16]
5  r, p, k = spsig.residuez(num, den)
6  print("Partial Fraction Results")
7  print("Residues: " + str(r))
8  print("Poles: " + str(p))
9  print("Coefficents: " + str(k))
10
11 # 4
12 zplane.zplane(num, den)
13
14 #5
15 w, h = spsig.freqz(num, den, whole=True)
16
17 plt.figure(figsize = (10, 11))
18 plt.subplot(2, 1, 1)
19 plt.plot(w/np.pi, 20*np.log10(abs(h)), "b-")
20 plt.grid(True, which='both', ls='-')
21 plt.ylabel('Magnitude [dB]')
22 plt.title('Magnitude and Phase of H(z)')
23 plt.subplot(2, 1, 2)
24 plt.grid(True, which='both', ls='-')
25 plt.plot(w/np.pi, np.angle(h, deg=True), "b-")
26 plt.ylabel('angle(H(S)) [deg]')
27 plt.xlabel('Frequency [pi/sample]')
28
29 plt.figure(figsize = (10, 11))
30 plt.subplot(2, 1, 1)
31 plt.semilogx(w/2/np.pi, 20*np.log10(abs(h)), "b-")
32 plt.grid(True, which='both', ls='-')
33 plt.ylabel('Magnitude [dB]')
34 plt.title('Magnitude and Phase of H(z)')
35 plt.subplot(2, 1, 2)
36 plt.grid(True, which='both', ls='-')
37 plt.semilogx(w/2/np.pi, np.angle(h, deg=True), "b-")
38 plt.ylabel('angle(H(S)) [deg]')
39 plt.xlabel('Frequency [pi/sample]')
```

4   Results

The results of this lab are very straightforward. The implementation of all functions worked as expected and the results are as expected. More analysis of theory and results is discussed in the Questions section of this report.

The deliverables for Part 1 of this lab are seen in all figures given below.

```
In [1]: runfile('/home/puffballjack/docs/ECE351/ECE351_Code/lab11/lab11.py',
wdir='/home/puffballjack/docs/ECE351/ECE351_Code/lab11')
Partial Fraction Results
Residues: [ 6. -4.]
Poles: [2. 8.]
Coefficents: []
```

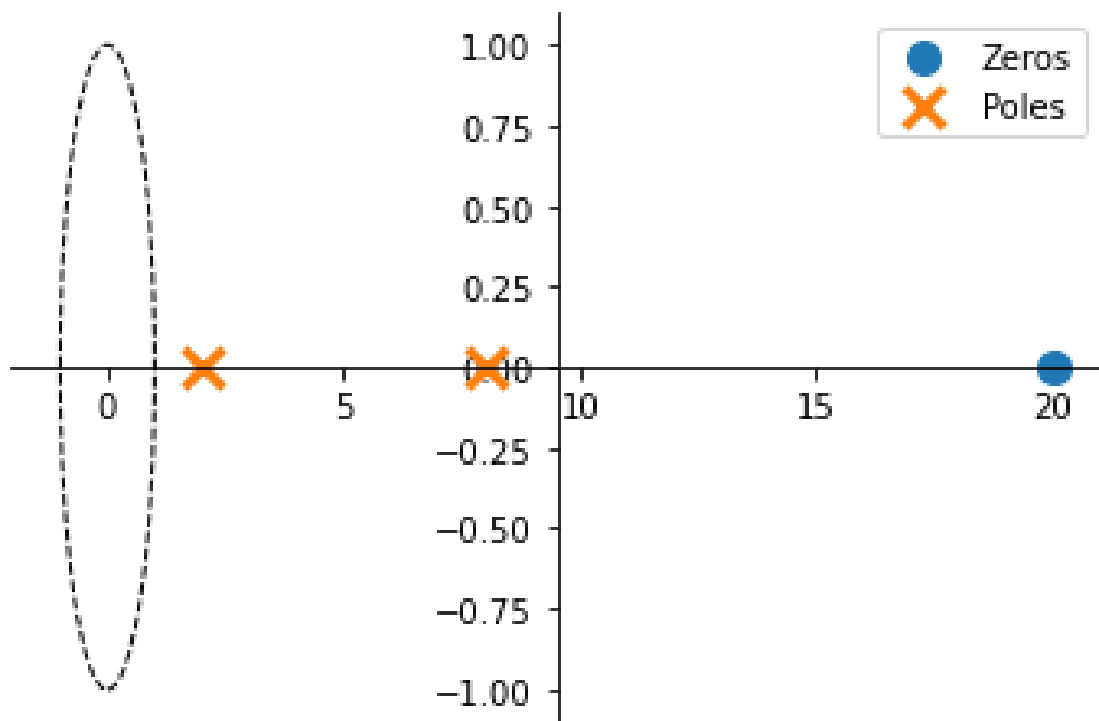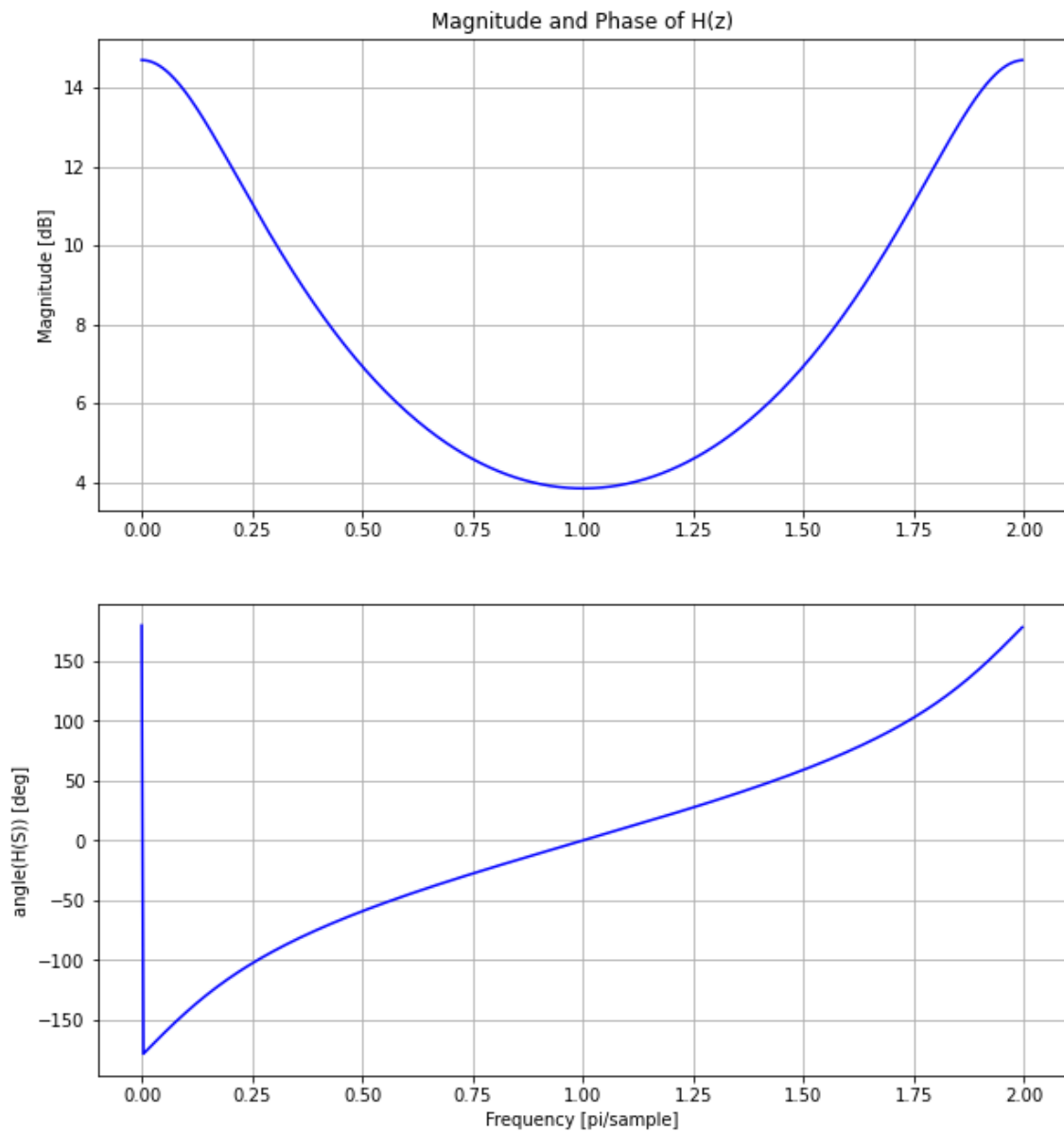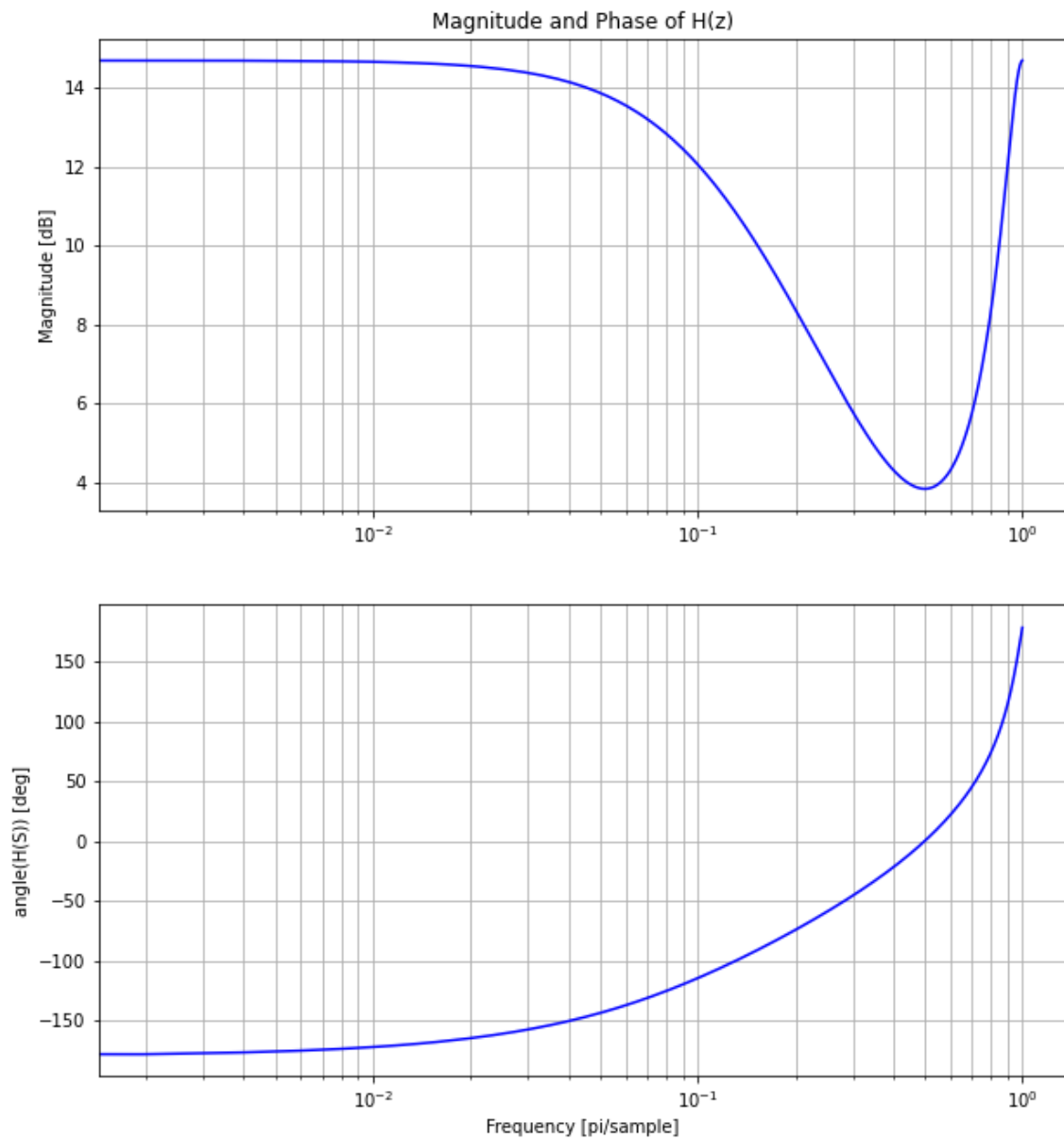Figure 1: Verification of PFE using `scipy.signal.residuez()`

Figure 2: H(z) Pole-Zero Plot using `zplane()`

Figure 3: Magnitude and Phase Response of H(z) [Base 10 Scale]

Figure 4: Magnitude and Phase Response of H(z) [Logarithmic Scale]

5   Error Analysis

No sources of error were seen throughout this lab. Verifying the PFE expansion performed by hand using the `scipy.signal.residuez()` function actually removed, or at least reduced, a possible source of error.

6   Questions

1. Looking at the plot generated in Task 4, is H(z) stable? Explain why or why not.

   - As seen in the plot generated in Task 4, Figure 2, all poles are in the right half plane. As is known, as z approaches those poles accumulation of energy will not end and causes instability. Thus, H(z) is unstable.

2. Leave any feedback on the clarity of lab tasks, expectations, and deliverables.

   - This lab was very clear for all instructions, expectations and deliverables.

7   Conclusion

In conclusion, I feel this lab was very successful. After learning the z-transform content of this course, I almost feel it would have been more useful to know this content first as many concepts seen throughout previous labs seem to rely heavily on these concepts. All in all, I am very satisfied with what this lab has taught me and feel it was an excellent use of time.

8 Attachments

1. H(z) Hand Calculations

1. $y[n] = \left(2x[n] - 40x[n-1] + 10y[n-1] - 16y[n-2]\right)u[n]$

No initial conditions

$Y(z) = 2X(z) - 40\left(z^{-1}X(z) + \cancel{X[-1]}\right)$
$\quad + 10\left(z^{-1}Y(z) + \cancel{Y[-1]}\right) - 16\left(z^{-2}Y(z) + \cancel{z^{-1}Y[-1]} + \cancel{Y[-2]}\right)$

$Y(z) = 2X(z) - 40z^{-1}X(z)$
$\quad + 10z^{-1}Y(z) - 16z^{-2}Y(z)$

$Y(z)\left(1 - 10z^{-1} + 16z^{-2}\right) = X(z)\left(2 - 40z^{-1}\right)$

$\Rightarrow H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{\left(-40z^{-1} + 2\right)}{\left(16z^{-2} - 10z^{-1} + 1\right)} = \dfrac{\left(-40z^{-1} + 2\right)}{\left(-8z^{-1} + 1\right)\left(-2z^{-1} + 1\right)}$

2. $H(z) = \dfrac{A}{\left(-8z^{-1} + 1\right)} + \dfrac{B}{\left(-2z^{-1} + 1\right)}$

$\Rightarrow A = \dfrac{-40z^{-1} + 2}{\left(-2z^{-1} + 1\right)}$ for $z = 8$
$\quad = \dfrac{-40/8 + 2}{\left(-1/4 + 1\right)} = \dfrac{-3}{.75} = -4$

$B = \dfrac{-40z^{-1} + 2}{\left(-8z^{-1} + 1\right)}$ for $z = 2$
$\quad = \dfrac{-20 + 2}{\left(-4 + 1\right)} = \dfrac{-18}{-3} = 6$

$\Rightarrow H(z) = \dfrac{6}{\left(-2z^{-1} + 1\right)} - \dfrac{4}{\left(-8z^{-1} + 1\right)}$

$$\boxed{H(z) = \dfrac{6z}{(z-2)} - \dfrac{4z}{(z-8)}}$$

$\Rightarrow H[n] = \left(6(\cdot 2^n) - 4(\cdot 8^n)\right)u[n]$