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Partial Fraction Expansion

Pre - Lab 6

Spring 2021

1 Purpose

To use Laplace transforms and partial fraction expansion to find the step response of a second-order differential equation.

2 Deliverables

Turn in your solution to the problem at the beginning of lab 6. Significant values and equations must be typed and properly formatted using LaTeX. Please attach any hand calculations to the back of the typed result.

3 Tasks

A system is described by the following differential equation:

$$y''(t) + 10y'(t) + 24y(t) = x''(t) + 6x'(t) + 12x(t) \tag{1}$$

1. By hand, find the transfer function. Assume all initial conditions are zero.
2. For the system $H(s)$, find $y(t)$ for a step input using partial fraction expansion and inverse Laplace transforms. Perform the calculation by hand and type your final answer and other significant results.

1. $y''(t)+10y'(t)+24y(t)=x''(t)+6x'(t)+12x(t)$

$$s^2Y(s)+10sY(s)+24Y(s) = s^2X(s)+6sX(s)+12X(s)$$

$$\Rightarrow Y(s)(s^2+10s+24)=X(s)(s^2+6s+12)$$

$$\Rightarrow H(s)=\frac{Y(s)}{X(s)}=\frac{s^2+6s+12}{s^2+10s+24}$$

$$\Rightarrow \boxed{H(s)=\frac{s^2+6s+12}{s^2+10s+24}}$$

2. Step-Response:

$$H(s)\cdot U(s)=H(s)\cdot\frac{1}{s}$$

$$=\frac{s^2+6s+12}{s(s+4)(s+6)}=\frac{A}{s}+\frac{B}{s+4}+\frac{C}{s+6}$$

$$\Rightarrow A=\left.\frac{s^2+6s+12}{(s+4)(s+6)}\right|_{s=0}\Rightarrow A=\frac{12}{24}=\boxed{\frac{1}{2}=A}$$

$$\Rightarrow B=\left.\frac{s^2+6s+12}{s(s+6)}\right|_{s=-4}\Rightarrow B=\frac{16+12-24}{-8}=\frac{4}{-8}$$

$$\Rightarrow \boxed{B=-\frac{1}{2}}$$

$$\Rightarrow C=\left.\frac{s^2+6s+12}{s(s+4)}\right|_{s=-6}\Rightarrow C=\frac{12}{-2\cdot-6}=\frac{12}{12}=1$$

$$\boxed{C=1}$$

$$\Rightarrow H(s)\cdot U(s)=\frac{1/2}{s}-\frac{1/2}{s+4}+\frac{1}{s+6}$$

$$\Rightarrow \boxed{H(t) * u(t) = \left(\frac{1}{2} - \frac{1}{2}e^{-4t} + e^{-6t}\right)u(t)}$$