Second order bias correction

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2021 Aug 11 (Mon)

Introduction

We consider Taylor series approximation approach for bias correction. Suppose $f(.)$ represents a back-transformation function, μ the mean and σ_μ^2 on the linear predictor scale (**univariate case // what happens in multivariate case?**) predictor. Then using Taylor expansion around μ, the approximation on the original scale is given by

$$f(\mu) + \frac{1}{2}\sigma_\mu^2 f''(\mu)$$

To start with, we consider logistic function

$$g(\mu) = \frac{1}{1+\exp(-\mu)},$$

with the first and second derivatives give by

$$g'(\mu) = g(\mu)(1 - g(\mu))$$

and

$$g''(\mu) = g(\mu)g(-\mu)(1 - 2g(\mu))$$

respectively.

```
taylorapprox <- function(fun, mu, sigma, ...) {
    untrans <- fun(mu)
    der2 <- fun(mu)*fun(-mu)*(1 - 2*fun(mu))
    corrected <- untrans + der2*sigma^2/2
    return(corrected)
}
```

Let us consider a simple univariate case

```
b0 <- 6
b1 <- 0.1
N <- 100
x <- scale(rnorm(N))
eta <- b0 + b1*x

corrected <- taylorapprox(plogis, mean(eta), sd(eta))
out <- c(NULL
    , mean = mean(plogis(eta))
    , uncorrected = plogis(mean(eta))
```

```r
    , corrected = corrected
)
print(out)
```

```
##        mean uncorrected   corrected
##   0.9975152   0.9975274   0.9975151
```

??

- How can we verify this?
- Does anything change in multivariate case?
- How does it work in *focal–non-focal* variables case?