Notes on network and generation

Suppose we have one infected individual and one susceptible individual. Infected individual makes effective contact at a constant rate β and has a constant recovery rate of λ. Then, we can write the probability that the susceptible remains uninfected at time t as follows:

$$s(\tau) = \exp(-\beta \int_{t=0}^{\tau} \exp(-\lambda t) dt)$$

Solving the integral, we have

$$s(\tau) = \exp\left(-\frac{\beta}{\lambda}(1 - \exp(-\lambda\tau))\right)$$

Define $K(\tau) = F(\tau)\beta$ as the rate of secondary case, where $F(\tau) = \exp(-\lambda\tau)$ is the probability that an infectious individual has not recovered t time units after infection.

We define network generation interval as follows:

$$g(\tau) = \frac{K(\tau)s(\tau)}{\int_0^\infty K(\tau)s(\tau)d\tau}$$

which can be simplified into

$$g(\tau) = \frac{F(\tau)s(\tau)}{\int_0^\infty F(\tau)s(\tau)d\tau}$$

Let's try to solve the integral... First, let's do substitution:

$$\begin{aligned} dF &= -\lambda \exp(-\lambda\tau)d\tau \\ &= -\lambda F d\tau \end{aligned}$$

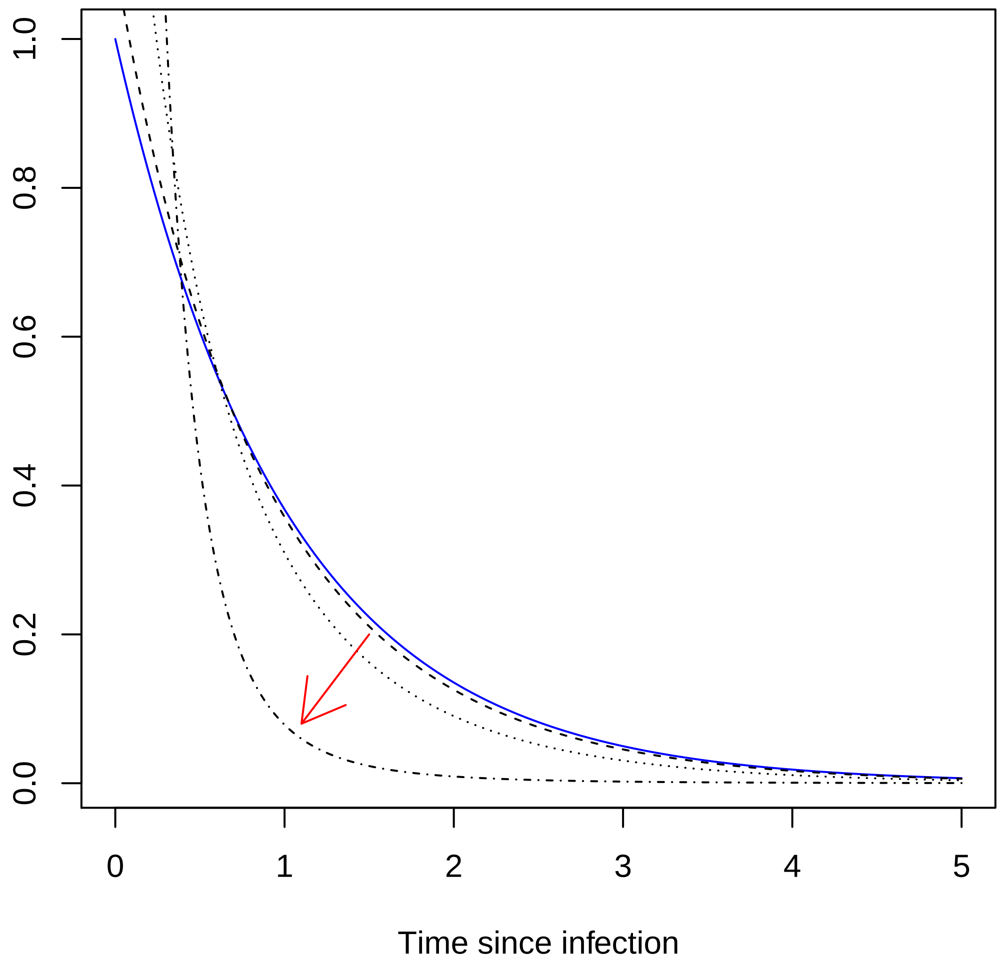
Substitute it:

$$\begin{aligned} &\int_0^\infty F(\tau)s(\tau)d\tau \\ &= -\frac{1}{\lambda}\int_1^0 \exp(-\frac{\beta}{\lambda}(1-F))dF \\ &= \frac{1}{\beta}\left[\exp(-\frac{\beta}{\lambda}(1-F))\right]_0^1 \\ &= \frac{1}{\beta}\left(1 - \exp(-\frac{\beta}{\lambda})\right) \end{aligned}$$

Therefore, the generation interval is

$$g(\tau) = \beta\left(1 - \exp(-\frac{\beta}{\lambda})\right)^{-1} \exp(-\lambda\tau)\exp\left(-\frac{\beta}{\lambda}(1-\exp(-\lambda\tau))\right)$$

Comparison of intrinsic generation with network generation distribution for SIR model:



As we increase effective transmission rate, network generation follows the red arrow. Let's try to write a general equation. . .