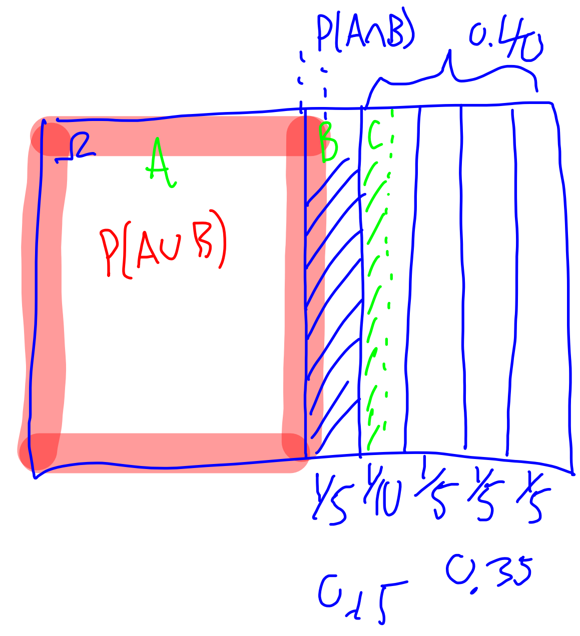
Maclean Whitworth

Homework 1: Part 1

Problem 1) Let $(\Omega, \mathcal{F}, P)$ be a probability space w/ $A \in \mathcal{F}$, $B \in \mathcal{F}$, $C \in \mathcal{F}$ such that $P(A)=0.5$, $P(B)=0.2$, $P(C)=0.1$ and $P(\overline{A \cup B})=0.45$. Compute the following probabilities:



1) Either A or B occurs

$P(\overline{A \cup B}) = A' \cap B'$

$P(A \cup B) = 1 - P(\overline{A \cup B}) = \boxed{0.55}$

2) Both A and B occurs

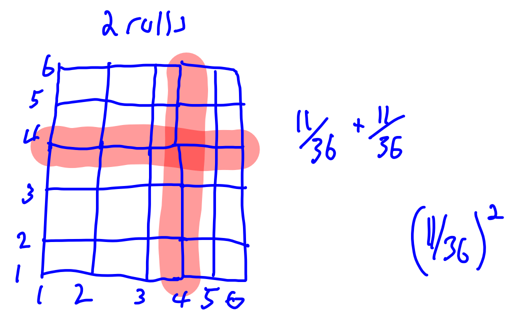
$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.5 + 0.2 - 0.55 = \boxed{0.15}$

3) A occurs but B does not occur

$P(A) - P(A \cap B) = P(A \cap B^c) = 0.5 - 0.15 = \boxed{0.35}$

4) Are A and B mutually exclusive?

No because $P(A \cup B) \neq P(A) + P(B)$ so they are NOT mutually exclusive.



$\frac{11}{36} + \frac{11}{36}$

$\left(\frac{11}{36}\right)^2$

Problem 2

Consider repeatedly rolling a fair 6-sided die.

$N = 6^4 = 1296$ 1 dice $= 1/6$ for a 4 additivity axiom?

1. What is the probability that the top face will be 4 at least once on four rolls of the die? $P(E) = 1 - P(E^c) \rightarrow 1 - (5/6)^4 = \boxed{0.518}$

2. What is the probability that the top face will be 4 at least once on 20 rolls of the die? $(5/6)^{20} \rightarrow 1 - \text{ans} = \boxed{0.974}$

3. How many rolls of the die would you have to do to be 90% confident that you would see at least one 4? In other words, how many rolls of the die would you have to be able to say, with a probability at least 90%, you will see at least one 4 appears.

$0.90 = 1 - (5/6)^n$ $n = 12.62 = \boxed{13 \text{ rolls}}$

hint: For 3, use the answer of 1 and 2 to find a function that takes the number of die rolls to the probability of seeing at least one 4. Then, solve an equation when the probability is 90%, what is the number of die rolls.

Problem 3

A six-sided die is loaded in a way that each even face is twice as likely as each odd face. All even faces are equally likely, as are all odd faces. Construct a probabilistic model for a single roll of this die and find the probability that the outcome is less than 4.

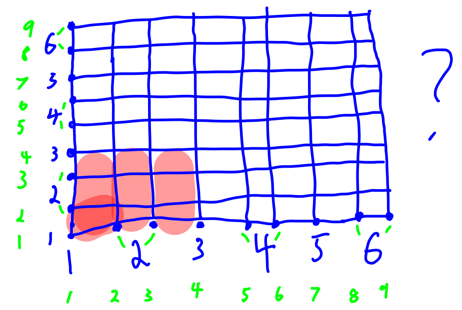
Hint: First, determine the probability of each face on the die. Then, use the addivity axiom.

$1/6$ Even $= 1/3 \rightarrow 2/9, 2/9, 2/9$

odd $= 1/3 \rightarrow 1/9, 1/9, 1/9$

roll < 4

$\{1,2,3\} \rightarrow 1/9 + 2/9 + 1/9 = \boxed{4/9}$



Problem 4

Consider two roommates, Charlie and David, living in the same apartment, where they share the living room. They decide to combine all of their Fall 2023 textbooks. There are 3 books on Probability and Statistics, 2 books on Linear Algebra, 2 books on Machine Learning and 3 culinary books.

Their friend will select 3 books at random (with eyes closed). They decide that if among these 3 books there are: 1 on Probability and Statistics, 1 on Linear Algebra and 1 on Culinary, then Charlie will have to clean the shared living room this week.

How many ways are there to select 3 books? $\binom{10}{3} = \frac{10!}{(10-3)!\,3!} = \boxed{120}$

What is the probability that Charlie will clean the living room this week?

LA: $\binom{2}{1} = \frac{2!}{(2-1)!\,1!} = 2$ $\quad$ PS = C = 3 $\quad$ $P(E) = \frac{18}{120} = \boxed{0.15}$

C: $\binom{3}{1} = \frac{3!}{(3-1)!\,1!} = 3$ $\quad$ $2 \times 3 \times 3 = 18$

Problem 5

A computer manufacturer uses chips from three sources. Chips from sources A, B, and C are defective with probabilities 0.002, 0.02, and 0.001, respectively. Let, Pa, Pb, Pc denote the probabilities that a chip is from source A, B, or C respectively.

1. If the computer manufacturer uses and equal number of chips from each source, what is the probability that a randomly chosen chip is defective? (I.e., if you do not know what manufacturer it came from.) $P(D) = \frac{1}{3}\cdot 0.002 + \frac{1}{3}\cdot 0.02 + \frac{1}{3}\cdot 0.001 = \boxed{0.00767}$

2. If the computer manufacturer uses and equal number of chips from each source and a randomly chosen chip is found to be defective, find the (conditional) probability that it came from each source. $\boxed{P(A|D) = 0.0869}$ $\boxed{P(B|D) = 0.869}$ $\boxed{P(C|D) = 0.0435}$

3. If the computer manufacturer gets 50% of its chips from source A, 10% of its chips from source B, and 40% of its chips from source C, what is the probability that a randomly chosen chip is defective? $P(D) = 0.5\cdot 0.002 + 0.1\cdot 0.02 + 0.4\cdot 0.001 = \boxed{0.0034}$

4. If the computer manufacturer gets 50% of its chips from source A, 10% of its chips from source B, and 40% of its chips from source C, and a randomly chosen chip is found to be defective, find the (conditional) probability that it came from each source.

$\boxed{P(A|D) = 0.294}$ $\boxed{P(B|D) = 0.588}$ $\boxed{P(C|D) = 0.118}$

A = chip from A
B = chip from B
C = chip from C
D = chip defective

$P(A,B,C) = \frac{1}{3}$
$P(D|A) = 0.002$
$P(D|B) = 0.02$
$P(D|C) = 0.001$

$P(A|D) = \frac{\frac{1}{3}\cdot 0.002}{0.00767} = 0.0869$ $\quad$ $P(C|D) = \frac{\frac{1}{3}\cdot 0.001}{0.00767} = 0.0435$

$P(B|D) = \frac{\frac{1}{3}\cdot 0.02}{0.00767} = 0.869$ $\quad$ $P(B|D) = \frac{0.1\cdot 0.002}{0.0034} = 0.588$

$P(C|D) = \frac{0.4\cdot 0.001}{0.0034} = 0.118$

$P(A|D) = \frac{0.5\cdot 0.002}{0.0034} = 0.294$

Problem 6

A card is drawn at random from an ordinary deck of 52 playing cards. Find the probability that it is

(a) an ace, $4/52 = \boxed{1/13}$

(b) a jack of spade, $\boxed{1/52}$

(c) a jack of spade or a six of diamonds, $2/52 = \boxed{1/26}$

(d) any suit except spades or hearts. $\boxed{1/2}$

Problem 7

Consider the experiment where you select one card at a time, at random and without replacement, from a playing 52-card deck (13 cards per suit).

Let Hi be the event that a heart is the ith draw from the deck.

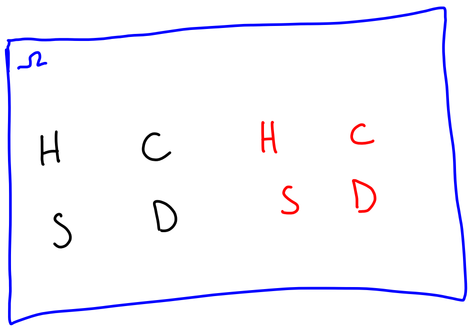
Compute P(H1) $\boxed{P(H_1) = 1/4}$

Compute P(H2) (hint. conditioning on the first card drawn from the deck. and use the law of total probability)

$\boxed{P(H_2) = 4/17}$

Hint: find a set of events that partitions the sample space, be clear in naming these events. For instance, {H,S,D,C}

can be the set of events that indicate hearts, spade, diamond, club. {1, 2, ..., 13} can be ace, two, ..., king. {B,R} can be the set of events that indicate Red or Black.



$\frac{12}{51}$ $\frac{13}{51}$, $\frac{13}{51}$, $\frac{13}{51}$

$P(H_2) = 1 - \frac{39}{51} = \boxed{4/17}$