ALGORITHM

Autumn 2009, Final Exam, January 14

9:20-12:05 am

1. Please briefly describe the following standard terms and techniques which are commonly used in algorithm designs. (20%)
   (a) Dynamic Programming
   (b) Greedy Method
   (c) Optimal Substructure
   (d) Simple Uniform Hashing
   (e) Decision tree

2. Design an algorithm to find both the minimum and the maximum of a set of n elements with $3\lfloor n/2 \rfloor$ comparisons in the worse case. (10%)

3. A *subsequence* is a sequence that can be derived from another sequence by deleting some elements. Given two sequences $X = \langle x_1, x_2, \ldots, x_m \rangle$ and $Y = \langle y_1, y_2, \ldots, y_n \rangle$, the ***longest common subsequence problem*** is to find a maximum-length common subsequence of X and Y. (20%)
   (a) Find an LCS of $\langle A, B, C, B, D, A, B \rangle$ and $\langle B, D, C, A, B, A \rangle$.
   (b) Describe an algorithm that solves the longest common subsequence problem in $O(mn)$ time.

4. Please describe briefly the following sorting algorithms along with their time complexities. (20%)
   (a) Counting sort
   (b) Radix sort
   (c) Bucket sort

5. Suppose you are given two sets A and B, each containing n positive integers. You can choose to reorder each set however you like. After reordering, let a_i be the ith element of set A, and let b_i be the ith element of set B. You then receive a payoff of $\prod_{i=1}^{n} a_i^{b_i}$. (20%)
   (a) Give an algorithm that will maximize your payoff.
   (b) Prove that your algorithm maximizes the payoff, and state its running time.

6. Demonstrate the insertion of keys $5, 28, 19, 15, 20, 33, 12, 17, 10$ into a hash table with collisions resolved by chaining. Let the table have 9 slots, and let the hash function be $h(k) = k \mod 9$. (10%)

7. Suppose we use a hash function h to hash n distinct keys into an array T of length m. Assuming simple uniform hashing, what is the expected number of collisions? More precisely, what is the expected cardinality of $\{\{k, \ell\} : k \neq \ell \text{ and } h(k) = h(\ell)\}$? (10%)

8. Show that the **selection problem** can be solved in $O(n)$ time. (10%)