Algorithm
Due Date: 9:20 AM, October 25
Autumn, 2012
The following problem sets are all from CLRS.

Homework 3

1. Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of Θ-notation, prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.

2. Guess a good asymptotic upper bound on the recurrence $T(n) = 3T(n/3) + n$, where $T(1) = 1$. Use the substitution method to verify your answer.

3. Answer the following questions according to the definitions of the asymptotic notations, O, Ω, and Θ.

   (a) Asymptotically upper bound the function $f(n) = n^2 - 2n + 1$ by the O notation. Justify your answer by demonstrating the constants.

   (b) Asymptotically lower bound the function $f(n) = n^2 - 2n + 1$ by the Ω notation. Justify your answer by demonstrating the constants.

   (c) Show that "$f(n) = \Theta(n)$" *if and only if* "$f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$".

   (d) What's wrong with the following argument?

   $$\sum_{1 \le k \le n} k \cdot n = \sum_{1 \le k \le n} O(n) = n \cdot O(n) = O(n^2)$$

   (In fact, we have $\sum_{1 \le k \le n} k \cdot n = n \sum_{1 \le k \le n} k = n \cdot \frac{n}{2}(n+1) = O(n^3)$.)