Algorithm
Autumn 2011, Midterm Exam, November 10
9:20-12:05am

1. (20 %) Please explain the following terms:

   a. Lower median
   b. Priority queue
   c. Recurrence
   d. Decision tree
   e. Min-heap property

2. (20 %) Please describe briefly the following sorting algorithms along with their time complexities. Which of them are stable sorting algorithms? Which of them are in-place sorting algorithms?

   a. Counting sort
   b. Radix sort
   c. Bucket sort
   d. Merge sort
   e. Insertion sort

3. (20 %)

   a. What are the two major concerns in algorithm design?
   b. Describe the "divide and conquer method" in algorithm design.
   c. Describe the three types of arguments in mathematical proofs.
   d. Show that $n^2 + 2n = \Theta(n^2)$.
   e. Show that if $f(n) = a_m n^m + \ldots + a_1 n + a_0$, then $f(n) = O(n^m)$.

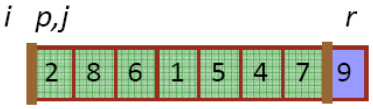
4. (15 %) Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 3T(n/3) + n$. Use the substitution method to verify your answer.

5. (10 %) Use the master method to give tight asymptotic bounds for the following recurrences.

   a. $T(n) = 8T(n/2) + n^3$.
   b. $T(n) = 8T(n/2) + n^4$.
   c. $T(n) = 8T(n/2) + n^5$.

6. (15 %) Show that any decision tree that sorts n elements has height $\Omega(n \lg n)$.

7. (15 %) Analysis the time complexity of the RANDOMIZED-QUICKSORT algorithm. Illustrate the operation of PARTITION on the following array.

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1.  RANDOMIZED-QUICKSORT(A, p, r)
2.  if p < r
3.    then q ← RANDOMIZED-PARTITION(A, p, r)
4.         RANDOMIZED-QUICKSORT(A, p, q−1)
5.         RANDOMIZED-QUICKSORT(A, q + 1, r)
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```
RANDOMIZED-PARTITION(A, p, r)
1.  i ← RANDOM(p, r)
2.  exchange A[r] ↔ A[i]
3.  return PARTITION(A, p, r)
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```
PARTITION(A, p, r)
1.  x ← A[r]
2.  i ← p − 1
3.  for j ← p to r − 1
4.      do if A[ j ] ≤ x
5.           then i ← i + 1
6.                exchange A[i ] ↔ A[j ]
7.  exchange A[i+1] ↔ A[r]
8.  return i + 1
```

尊重自己就是尊重別人