ALGORITHM

Autumn 2011, Final Exam, January 12

9:20-12:05 am

1. Please briefly describe the following standard terms and techniques which are commonly used in algorithm designs. (20%)

   (a) Binary Search Tree Property

   (b) Red Black Tree Property

   (c) Greedy Choice Property

   (d) Dictionary Operation

   (e) Prefix Code

2. Design an algorithm to find both the minimum and the maximum of a set of n elements with $3\lfloor n/2 \rfloor$ comparisons in the worse case. (10%)

3. Given a sequence $X = \langle x_1, x_2, \ldots, x_m \rangle$, the longest increasing subsequence problem is to find an increasing subsequence of X with the longest length. (15%)

   (a) Find a longest increasing subsequence of $\langle 1, 8, 4, 12, 2, 10, 6 \rangle$.

   (b) Describe an algorithm that solves the longest increasing subsequence problem in $O(n^2)$ time.

4. Please describe briefly the following sorting algorithms along with their time complexities. (10%)

   (a) Insertion sort

   (b) Quick sort

5. Suppose we assign n persons to n jobs. Let C_{ij} be the cost of assigning the ith person to the jth job. (15%)

   (a) Give an algorithm that will minimize the total cost of assigning all n persons to all n jobs.

   (b) Prove that your algorithm maximizes minimize the total cost, and state its running time.

6. Suppose that $bh(x)$ is the number of black nodes on the path from x to leaf, but not counting x. Show that the subtree rooted at any node x contains at least $2^{bh(x)} - 1$ internal nodes. (10%)

7. Suppose we use a hash function h to hash n distinct keys into an array T of length m. Assuming simple uniform hashing, what is the expected number of collisions? More precisely, what is the expected cardinality of $\{\{k, \ell\} : k \neq \ell \text{ and } h(k) = h(\ell)\}$? (10%)

8. Suppose we use RANDOMIZED-SELECT to select the minimum element of the following array $A = \langle 3, 2, 9, 0, 7, 5, 4, 8, 6, 1 \rangle$. Describe a sequence of partitions that results in a worst-case performance of RANDOMIZED-SELECT. (10%)