COMP 350 Numerical Computing
Assignment #4. Solving a nonlinear equation.

Date given: Tuesday, Oct 16. Date due: Tuesday, October 30, 2018 11:30pm

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1. (4 points) In class, we showed that if Newton's iteration converges to r, a root of $f(x) = 0$, then usually it has quadratic convergence, i.e., $\lim_{n\to\infty} |x_{n+1} - r|/|x_n - r|^2 = c$, where $c \neq 0$ is a constant. From a numerical experiment on $f(x) = x^2 - 2$, we found after $|f(x_n)|$ is small enough, $|f(x_n)|$ is squared every step. In fact it is usually true for a general nonlinear equation. Suppose f, f' and f'' are continuous. Prove that if x_n converges to a root, $f(x_n)$ **usually** converges to 0 with quadratic convergence, i.e.,

$$\lim_{n\to\infty} |f(x_{n+1})|/|f(x_n)|^2 = c$$

for a nonzero constant c.

Note: Use the Taylor series theory in your analysis. The proof is not difficult.

2. (6 points) In class, we derived Newton's method by using the first two terms in the Taylor series. Derive a new method by using the first three terms in the Taylor series in a similar way.

(Bonus 5 points) Show usually the new method has cubic convergence.

3. (10 points) Write a Matlab program `secant.m` for the secant method. Suppose we want to find the largest positive root of $f(x) = x^3 - 5x + 3$. Plot the graph of $y = f(x)$ on an appropriate interval by Matlab (check how to use Matlab build-in function `plot`). Use your `secant.m` to compute the root. Also use the bisection method, the Newton method, and the new method you derived in question 2 to find the root. For the bisection method, use $[1, 3]$ as the initial interval, for the Newton method, use $x_0 = 2$ as the initial point, for the secant method, use $x_0 = 1$ and $x_1 = 2$ as the two initial points, and for the new method, use $x_0 = 2$ as the initial point. You can choose any appropriate initial points and initial interval, Take tolerances `xtol=1.e-12` and `ftol=1.e-12` for Newton's method, the new method, and the secant method, and take `delta=1.e-12` for the bisection method. Set a big number for the maximum number of iterations of the secant method and Newton's method such that the iteration stops only when `xtol=1.e-12` or `ftol=1.e-12` is satisfied. Comment on the speeds of convergence of these four methods. Print out the graph of $y = f(x)$ and the commands you used to plot the graph, your program `secant.m`, and other M-files related to $f(x)$. Also print out the results of each iteration step. You can use M-files `newton.m` and `bisection.m` on the course web site.