Fibonacci Heap

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Outline

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    2.3. Delete
    2.4. Consolidate
    2.5. Merge
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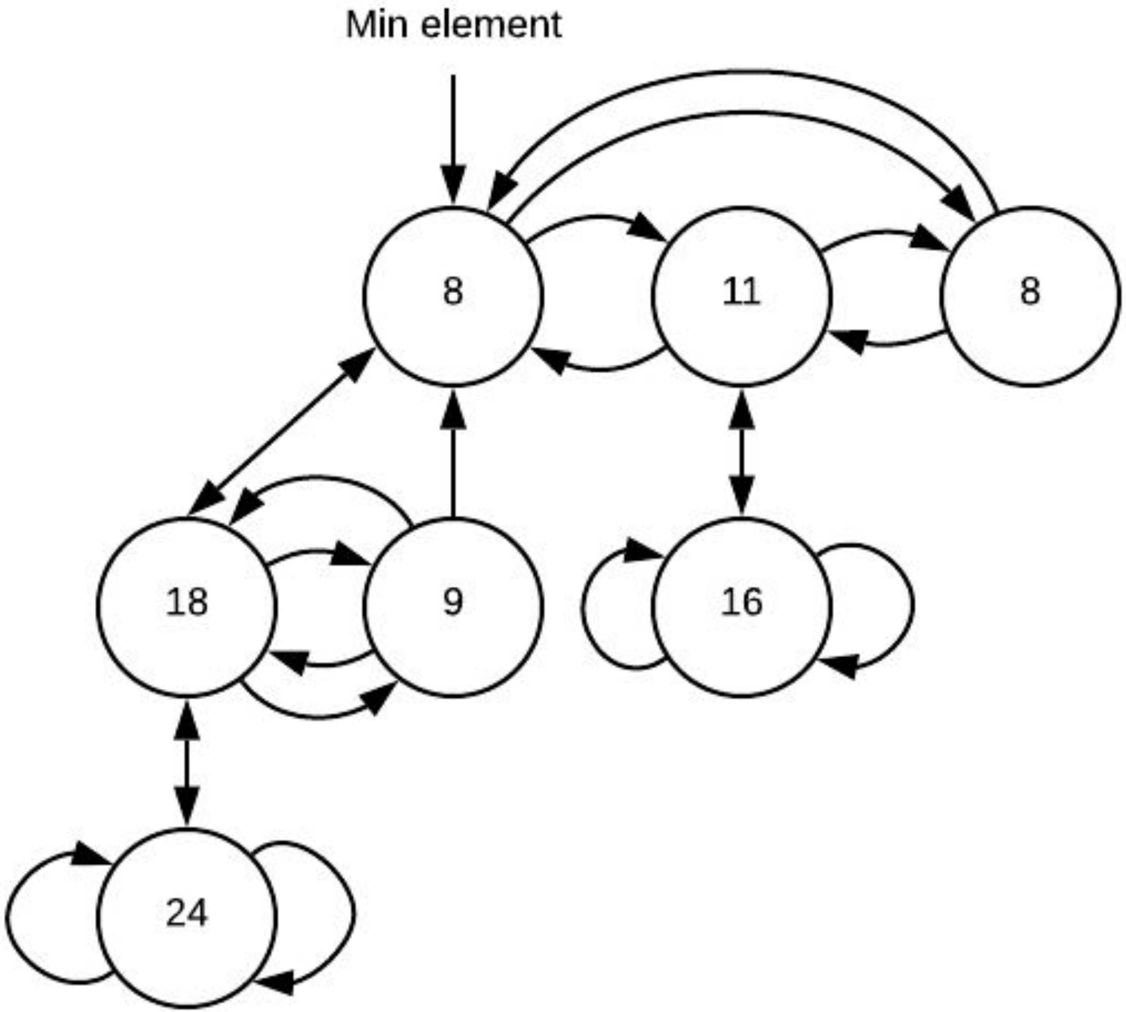
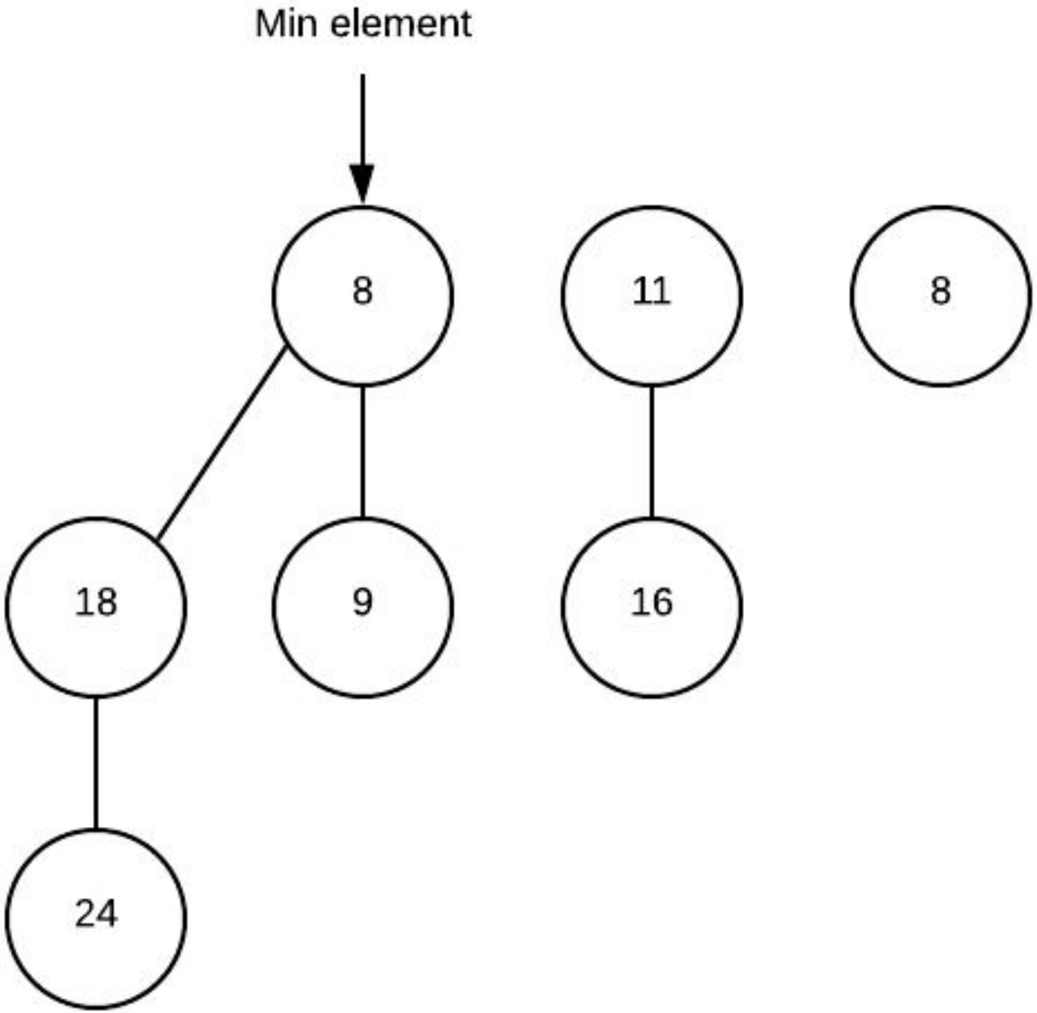
Structure

Structure

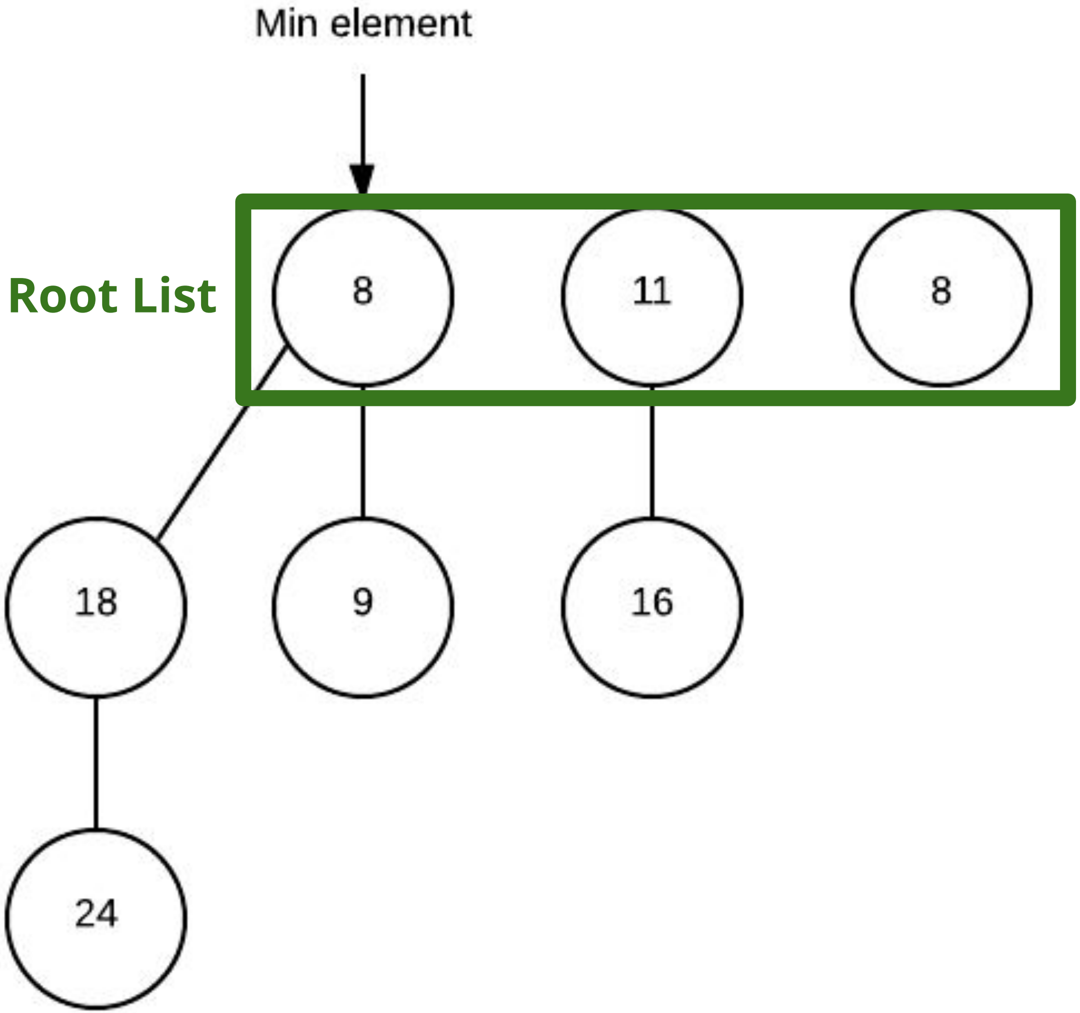
Node:
Left
Right
Child
Parent



Structure



Structure

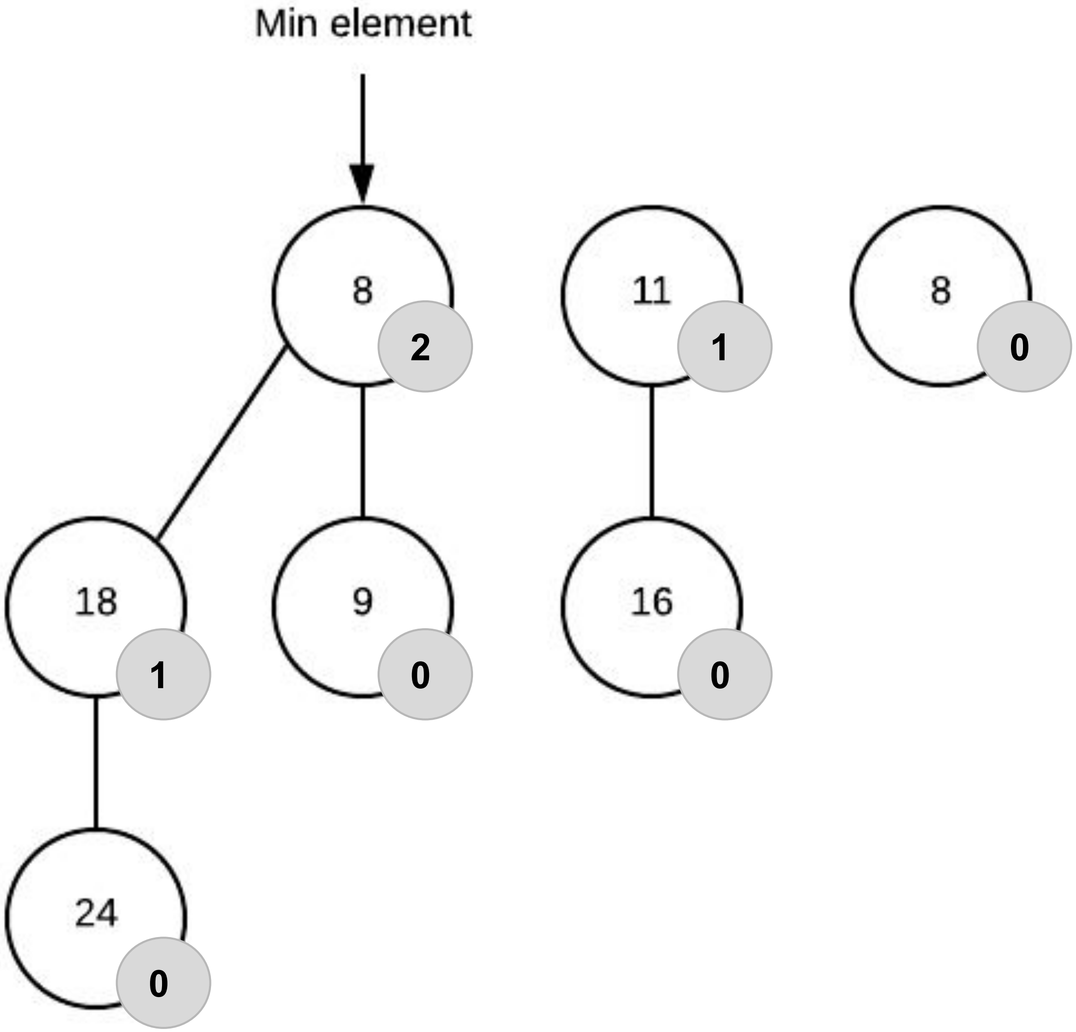


Structure

Rank/Degree

n

n = number of children



Methods

Function	Time complexity
Find Min	O(1)
Delete Min/Consolidate	O(log n)
Insert	O(1)
Decrease-Key	O(1)
Merge	O(1)

Table 1. Fibonacci Heap functions time complexity in big O notation [1]

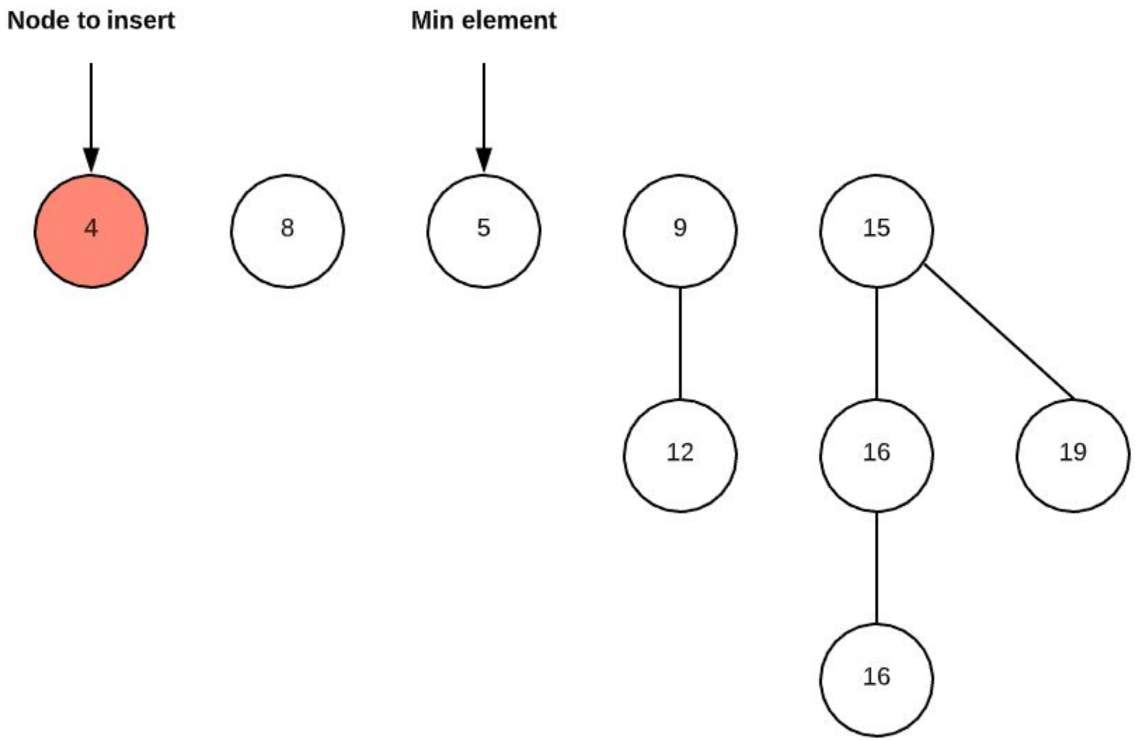
Insert

Algorithm 1: Insert
Source: Adapted from [3]

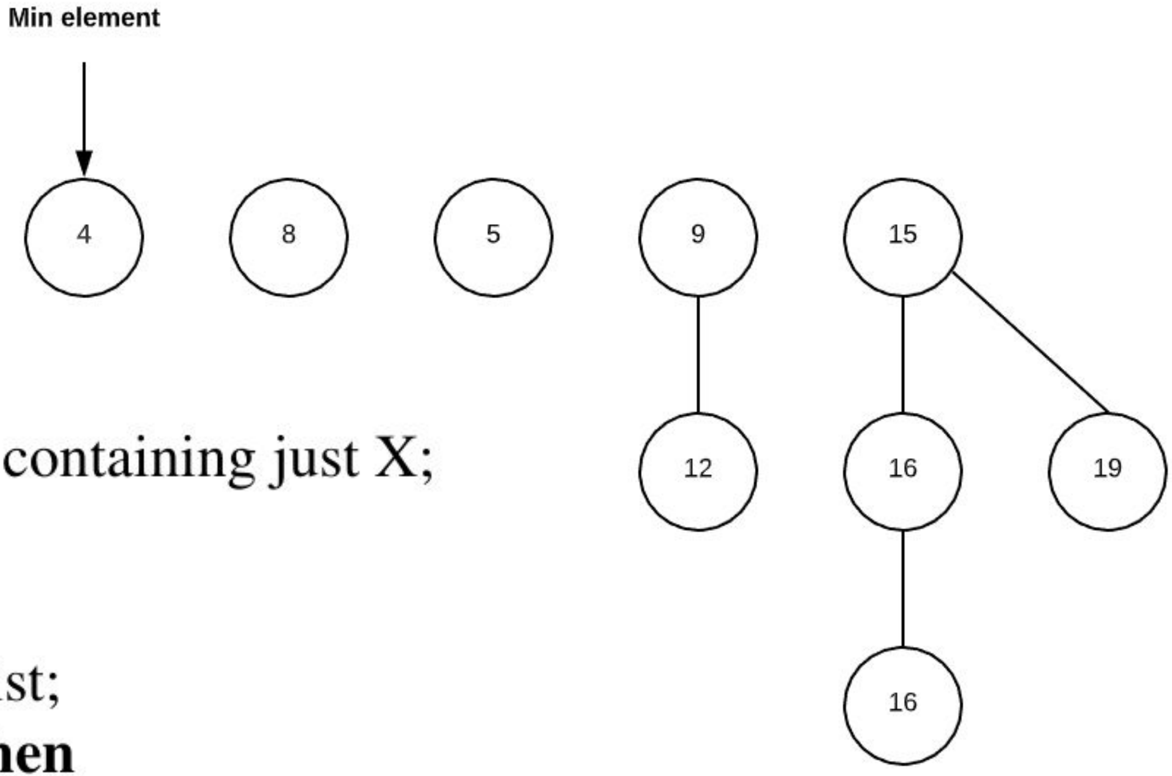
```
x.degree = 0;
x.p = NULL;
x.child = NULL;
x.mark = FALSE;
if H.min == NULL then
    create a root list for H containing just X;
    H.min = x;
else
    insert x into H's root list;
    if x.key < H.min.key then
        H.min = x;
H.n = H.n + 1
```

Insert 4

```
x.degree = 0;
x.p = NULL;
x.child = NULL;
x.mark = FALSE;
```



Insert 4

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    insert x into H's root list;
    if x.key < H.min.key then
        H.min = x;
H.n = H.n + 1
```

Decrease Key

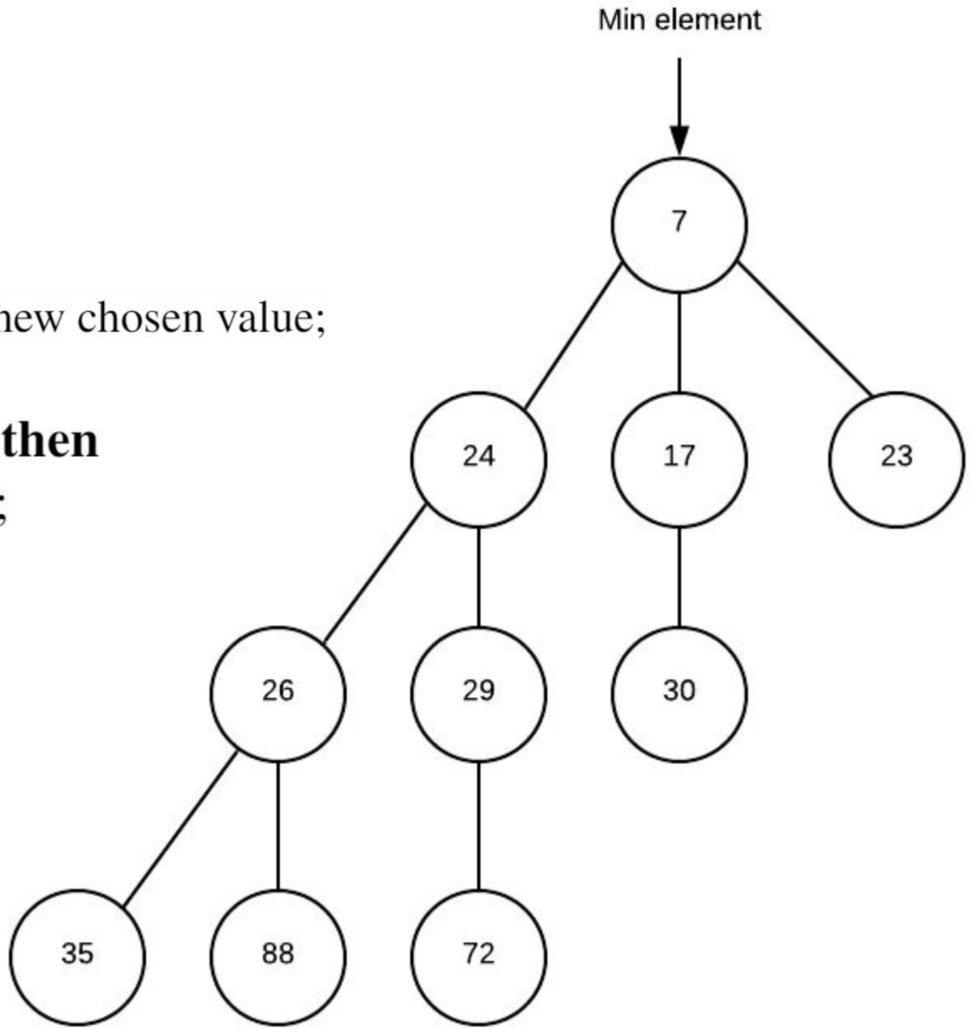
Algorithm 2: Decrease Key

Source: Adapted from [2]

```
1  Decrease the value of the node 'x' to the new chosen value;
2  p[x] = Parent of node 'x';
3  if min heap property is not violated then
4      Update min pointer if necessary;
5  else if min heap property is violated and p[x] is unmarked then
6      Cut off the link between 'x' and p[x];
7      Mark p[x];
8      Add 'x' and its children to the root list and update min pointer if necessary;
9  else if min heap property is violated and p[x] is marked then
10     Cut off the link between 'x' and p[x];
11     Add 'x' to the root list and update min pointer if necessary;
12     Cut off link between p[x] and p[p[x]];
13     Add p[x] to the root list and update min pointer if necessary;
14     if p[p[x]] is unmarked then
15         Mark p[p[x]];
16     else
17         cut off p[p[x]];
18         x = p[p[x]];
19         Jump to line 11;
```

Decrease Key: 17->10

Decrease the value of the node ‘x’ to the new chosen value;
p[x] = Parent of node ’x’;
if *min heap property is not violated* **then**
    Update min pointer if necessary;



Decrease Key: 17->10

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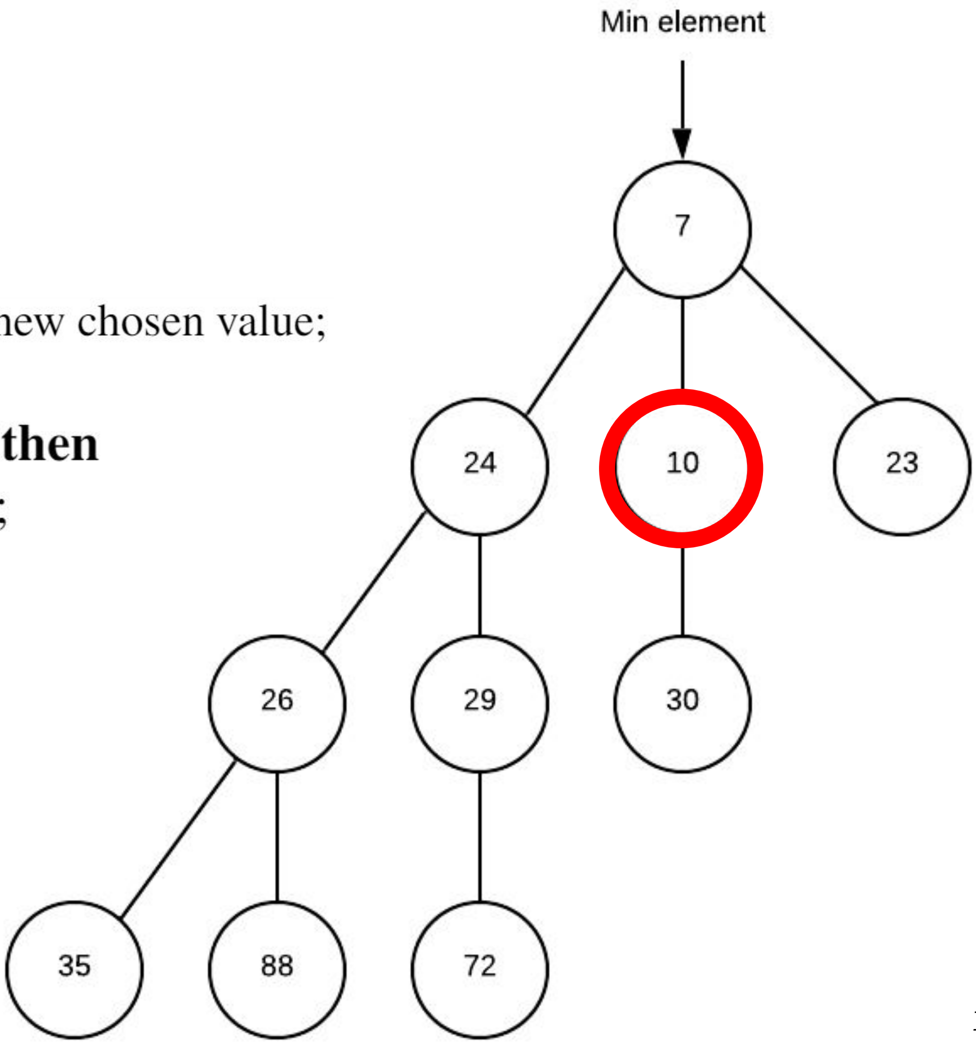
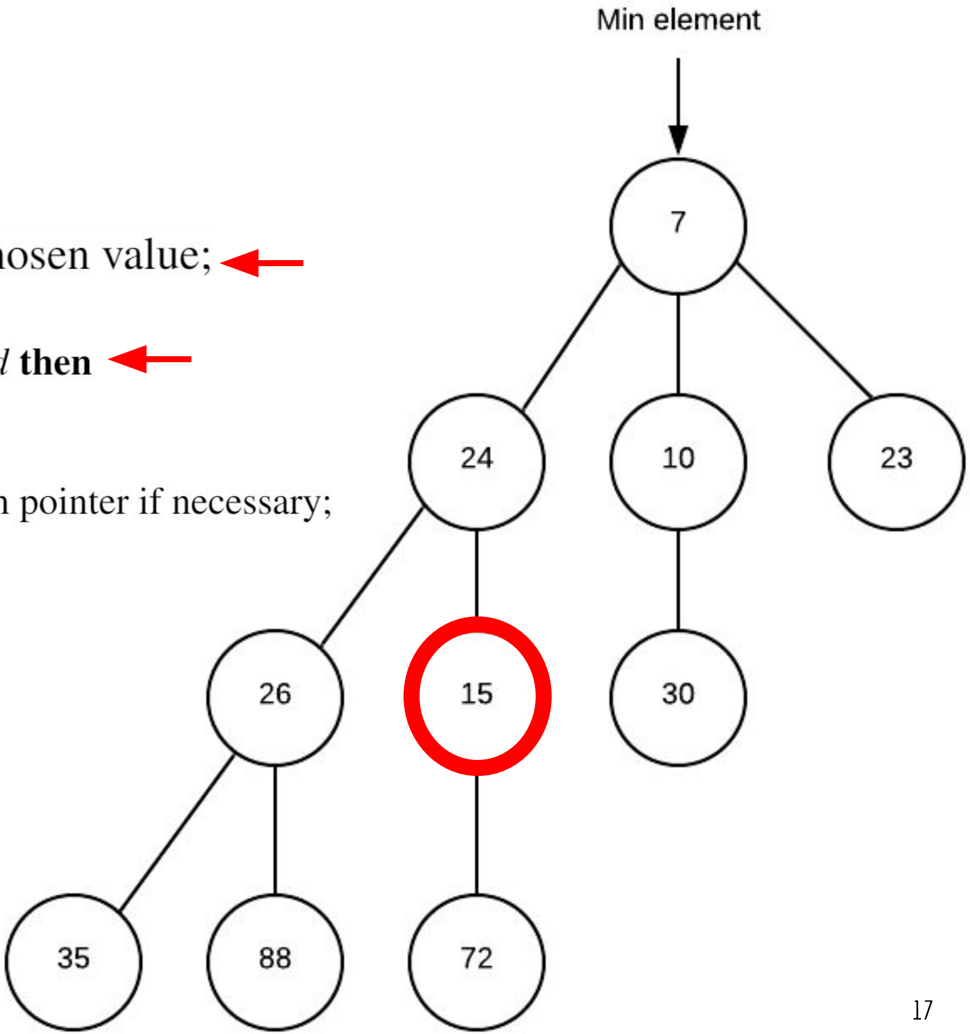


Decrease Key: 29->15

Decrease the value of the node ‘x’ to the new chosen value;
p[x] = Parent of node ’x’;
else if *min heap property is violated and p[x] is unmarked* **then**
    Cut off the link between ‘x’ and p[x];
    Mark p[x];
    Add ‘x’ and its children to the root list and update min pointer if necessary;



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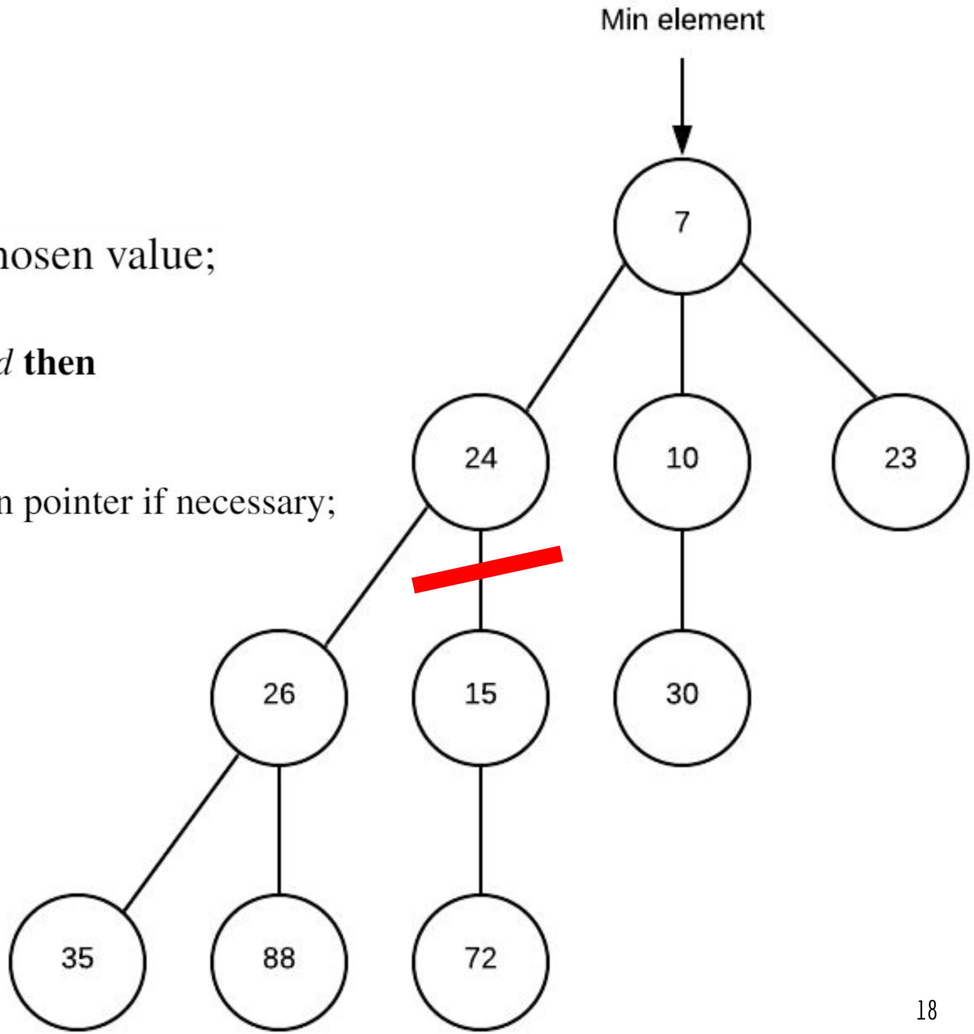

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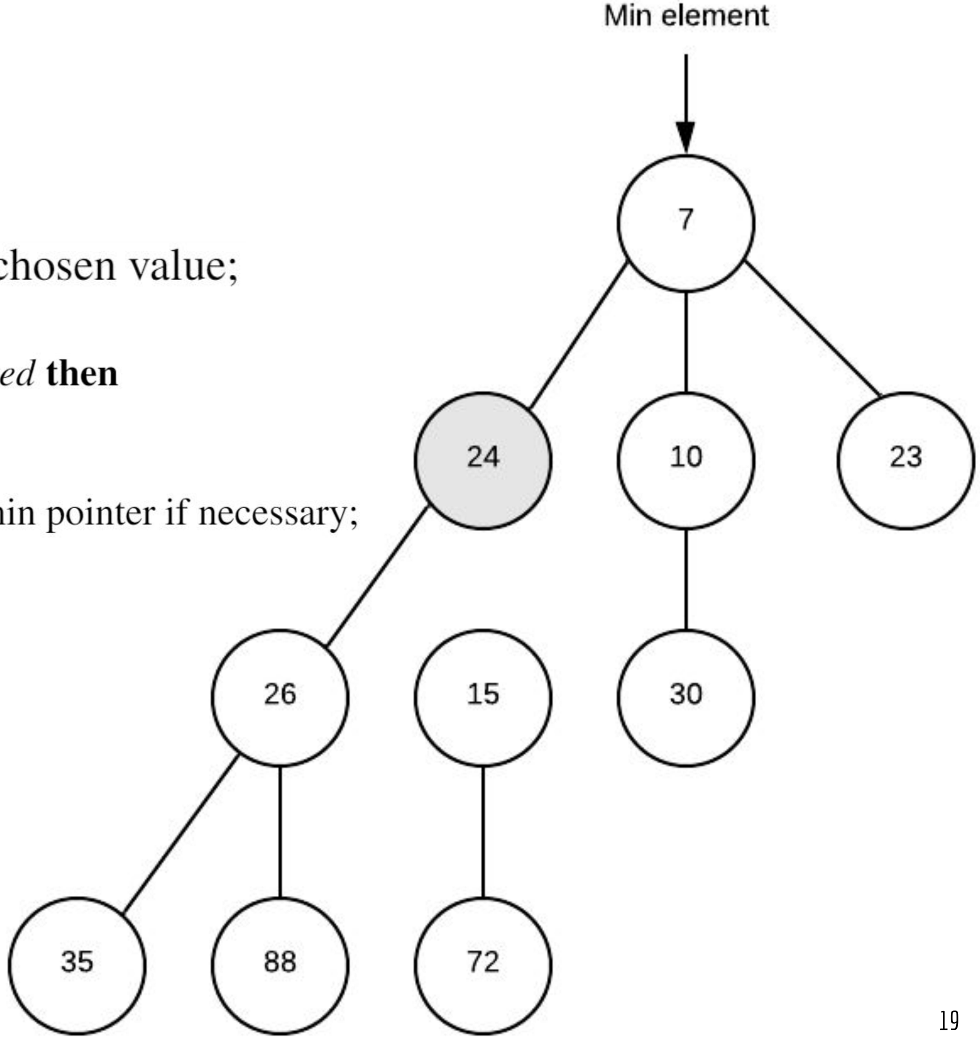

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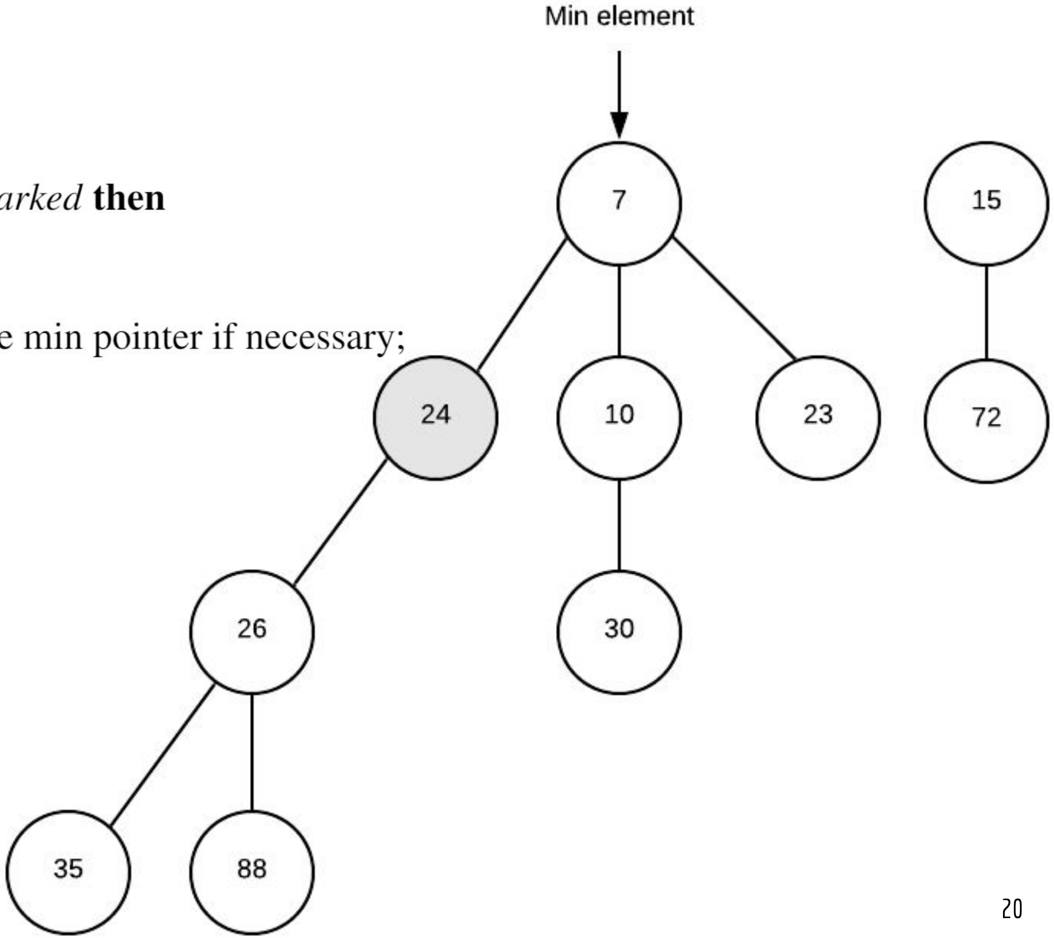

Decrease Key: 26->22

```
1  Decrease the value of the node 'x' to the new chosen value;
2  p[x] = Parent of node 'x';
9  else if min heap property is violated and p[x] is marked then
10 |   Cut off the link between 'x' and p[x];
11 |   Add 'x' to the root list and update min pointer if necessary;
12 |   Cut off link between p[x] and p[p[x]];
13 |   Add p[x] to the root list and update min pointer if necessary;
14 |   if p[p[x]] is unmarked then
15 |   |   Mark p[p[x]];
16 |   else
17 |   |   cut off p[p[x]];
18 |   |   x = p[p[x]];
19 |   |   Jump to line 11;
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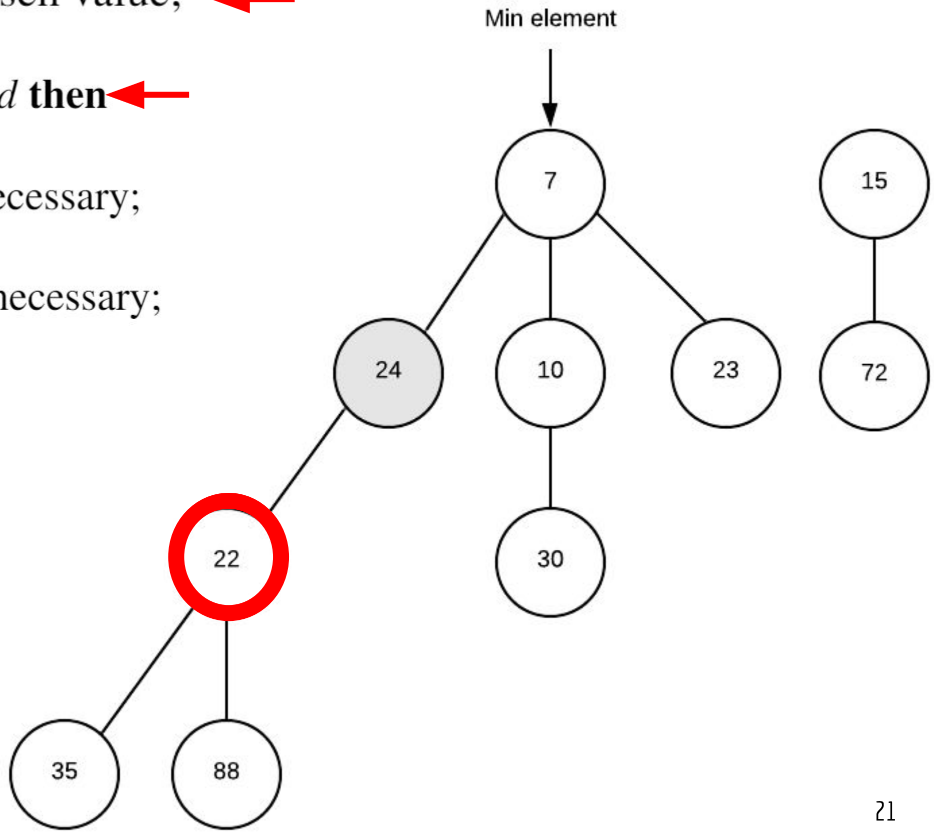

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16 |   else
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18 |   |   x = p[p[x]];
19 |   |   Jump to line 11;
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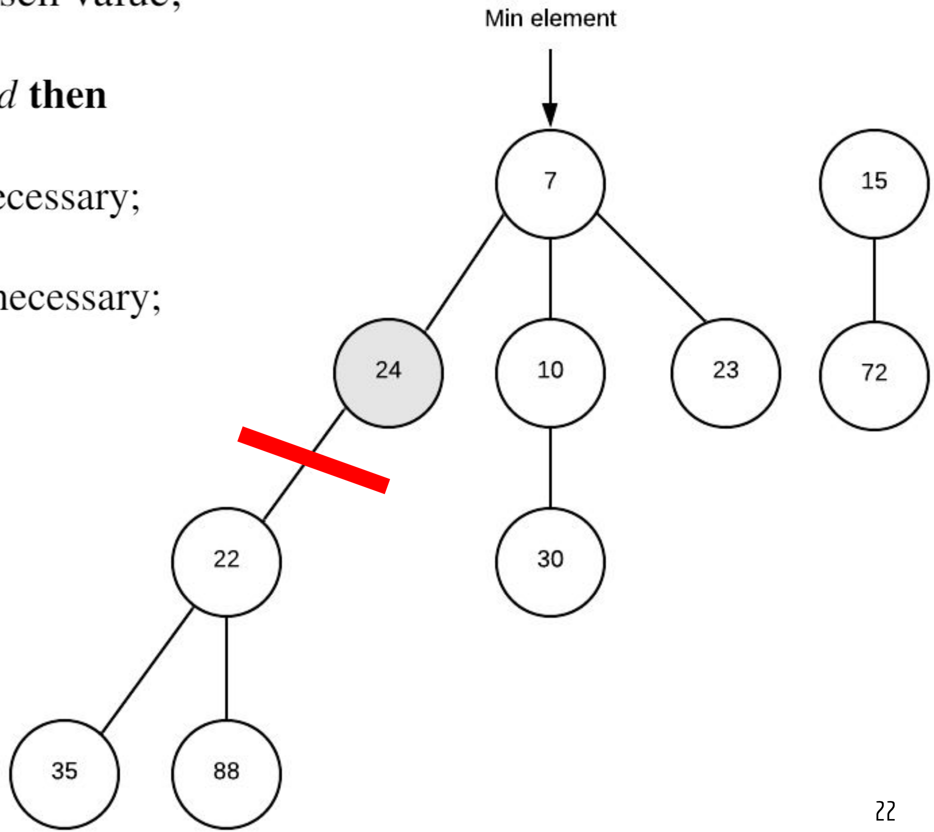
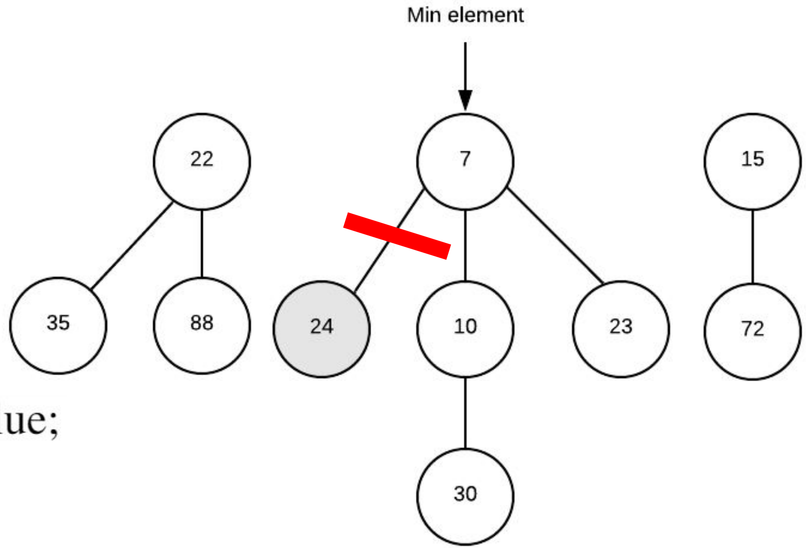


Decrease Key: 26->22

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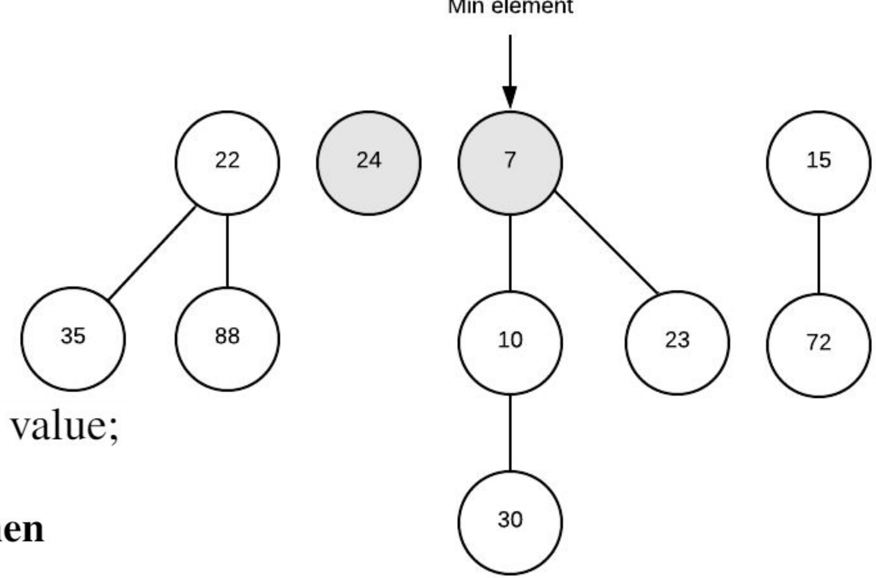
18 |    |    x = p[p[x]];
19 |    |    Jump to line 11;
```

Decrease Key: 26->22

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15         Mark p[p[x]];
16     else
17         cut off p[p[x]];
18         x = p[p[x]];
19         Jump to line 11;
```

Delete Min

Algorithm 3: Delete
Source: Adapted from [4]

```
z = H.min;
if z != NULL then
    for each child x of z do
        add x to root list of H;
        x.p = NULL;
    end
    remove z from the root list of H;
    if z == z.right then
        H.min = NULL;
    else
        H.min = z.right;
        CONSOLIDATE(H);
    H.n = H.n-1;
return z;
```

Consolidate and Merge

Algorithm 4: Consolidate

Source: Adapted from [4]

```
let A[0...D(H.n)] = NULL, .., NULL;
for each node w in the root list of H do
    x = w;
    d = x.degree;
    while A[d] do
        y = A[d];
        if x.key > y.key then
            swap(x.key, y.key);
        MERGE(H,y,x);
        A[d] = NULL;
        d = d + 1;
    end
    A[d] = x;
end
H.min = NULL;
for i = 0 to D(H.n) do
    if A[i] then
        if !H.min then
            create a root list for H containing just A[i];
            H.min = A[i];
        else
            insert A[i] into H's root list;
            if A[i].key < H.min.key then
                H.min = A[i]
end
```

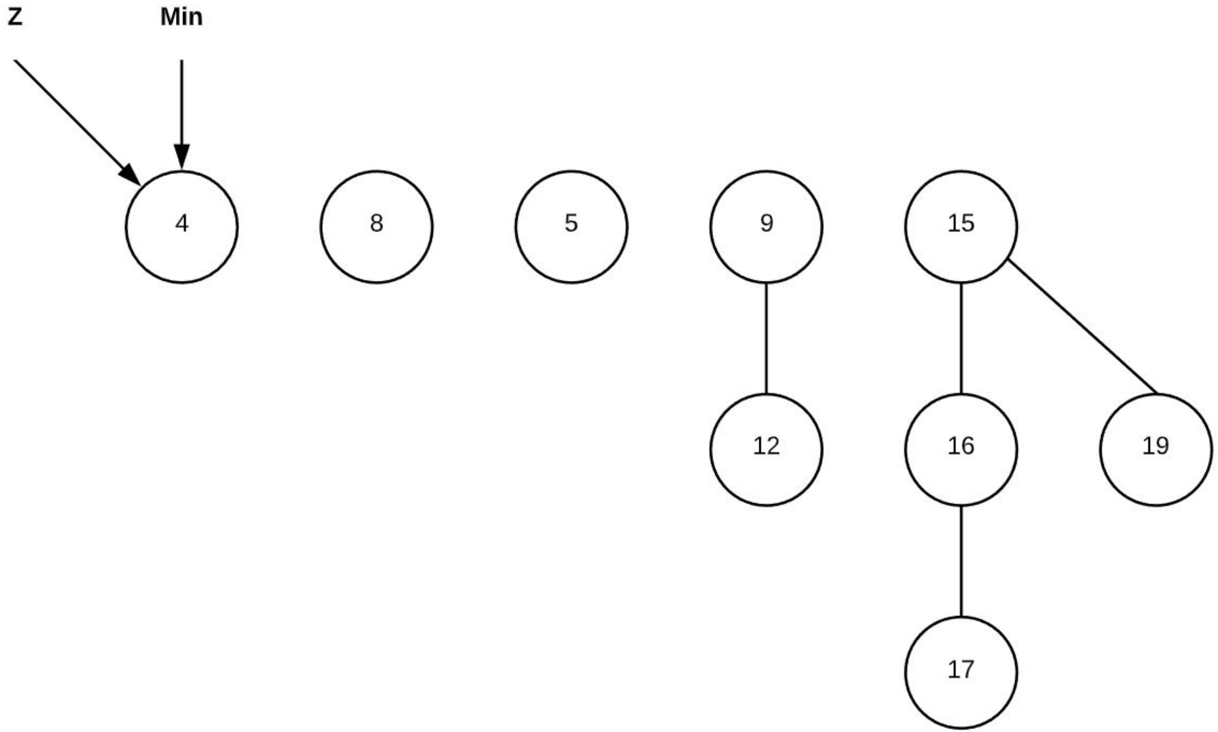
Merge

Algorithm 5: Merge
Source: Adapted from [4]

```
remove y from the root of list H;
make y a child of x, incrementing x.degree;
y.mark = FALSE;
```

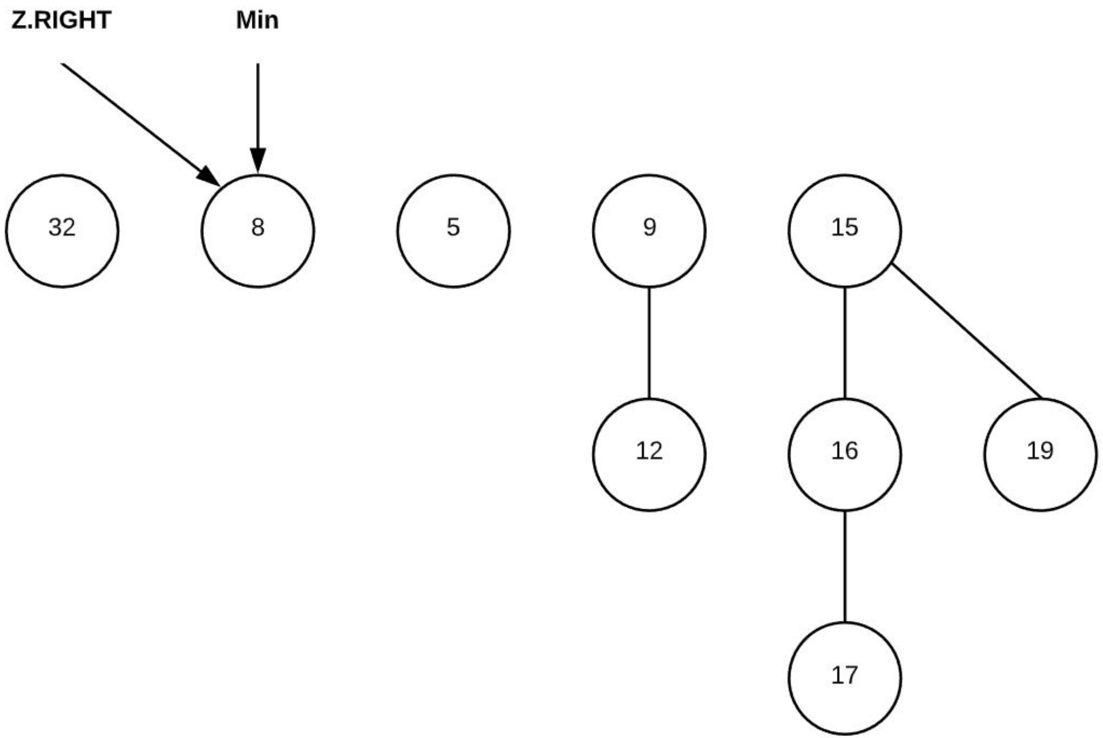
Delete Min

1 z = H.min;



Delete Min

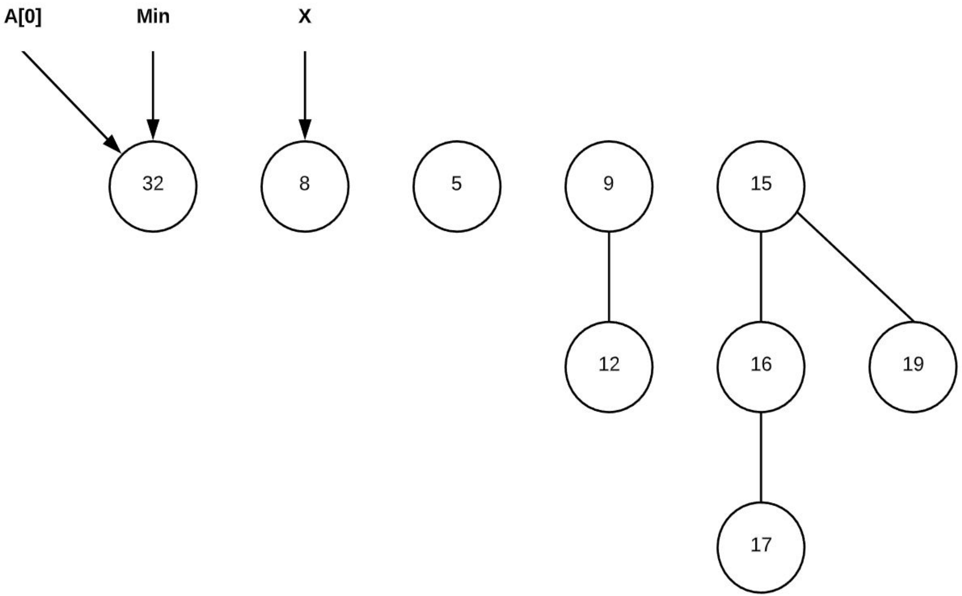
```
if z != NULL then
    for each child x of z do
        add x to root list of H;
        x.p = NULL;
    end
    remove z from the root list of H;
    if z == z.right then
        H.min = NULL;
    else
        H.min = z.right;
        CONSOLIDATE(H);
    H.n = H.n-1;
return z;
```



Consolidate / Merge

```
let A[0...D(H.n)] = NULL, .., NULL;
for each node w in the root list of H do
    x = w;
    d = x.degree;
    while A[d] do
        y = A[d];
        if x.key > y.key then
            swap(x.key, y.key);
        MERGE(H,y,x);
        A[d] = NULL;
        d = d + 1;
    end
    A[d] = x;
```

Consolidate/Merge



Root List = [32, 8, 5, 9, 15]

First Iteration:

d = 0

A[0] = 32

Second iteration:

x = 8

d = 0

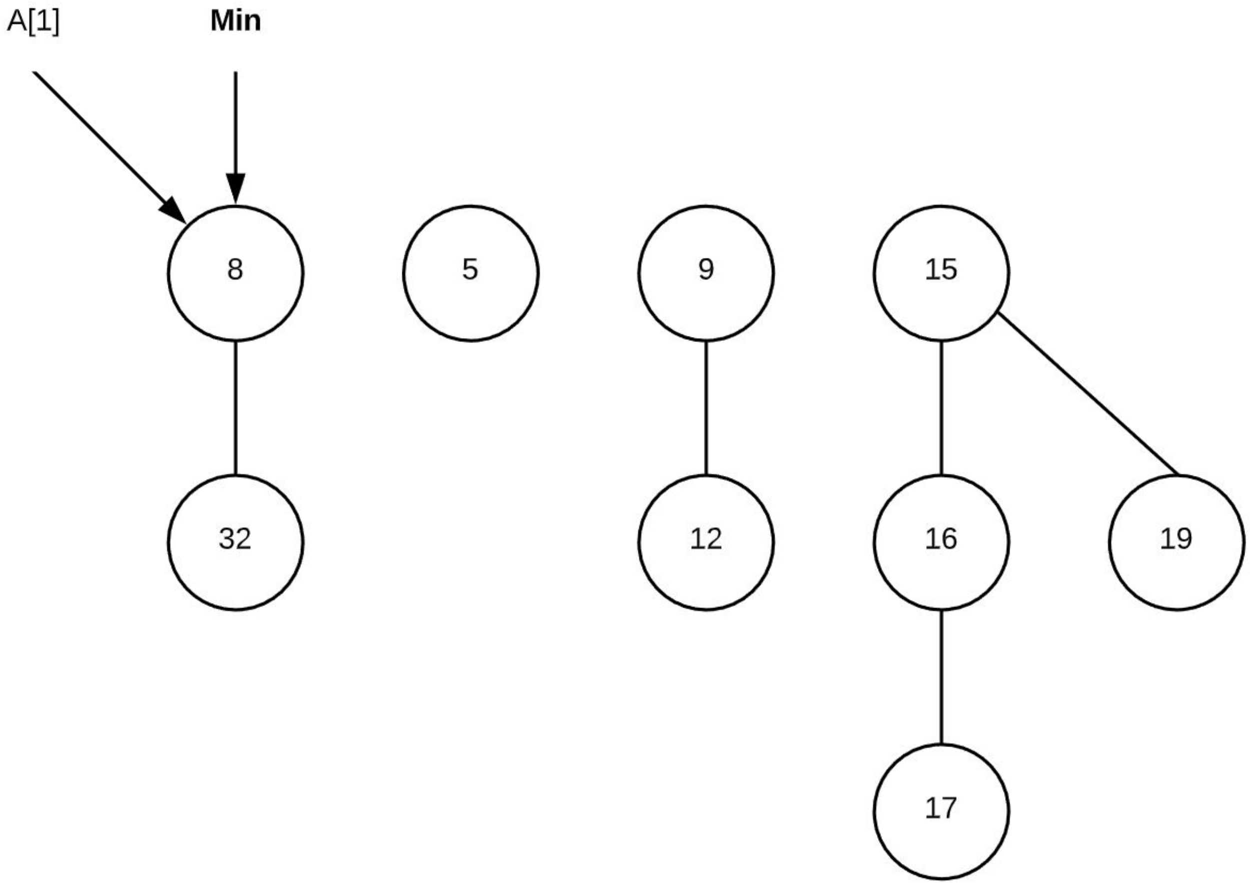
A[0] exists -> while loop:

Y = A[0] / Swap if necessary

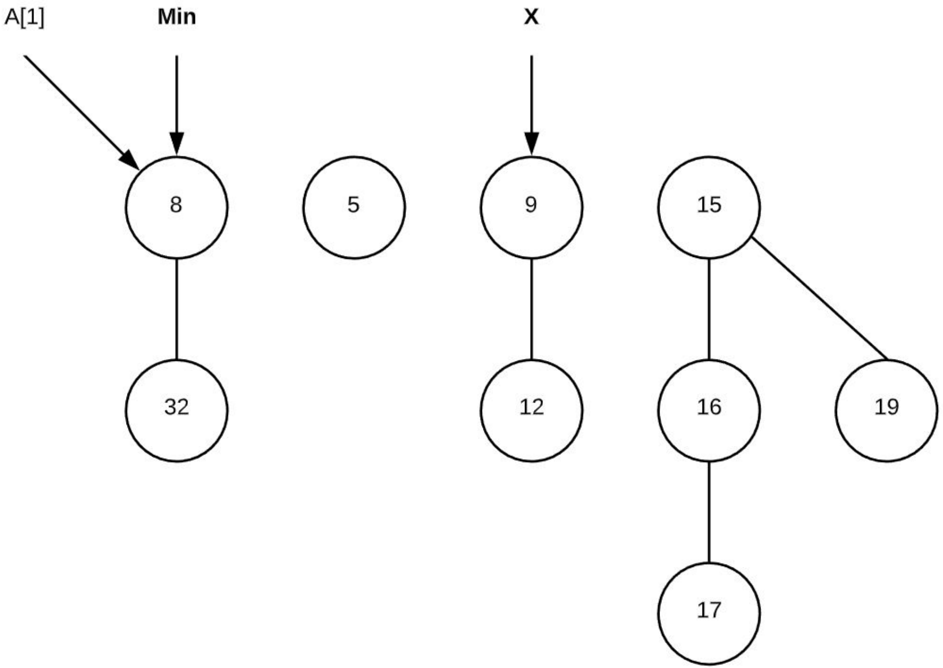
Call MERGE

A[1] = 8 and A[0] = NULL

Consolidate/Merge



Consolidate/Merge



Root List = [8, 5, 9, 15]

Third iteration:

A[0] = 5

Fourth iteration:

x = 9

d = 1

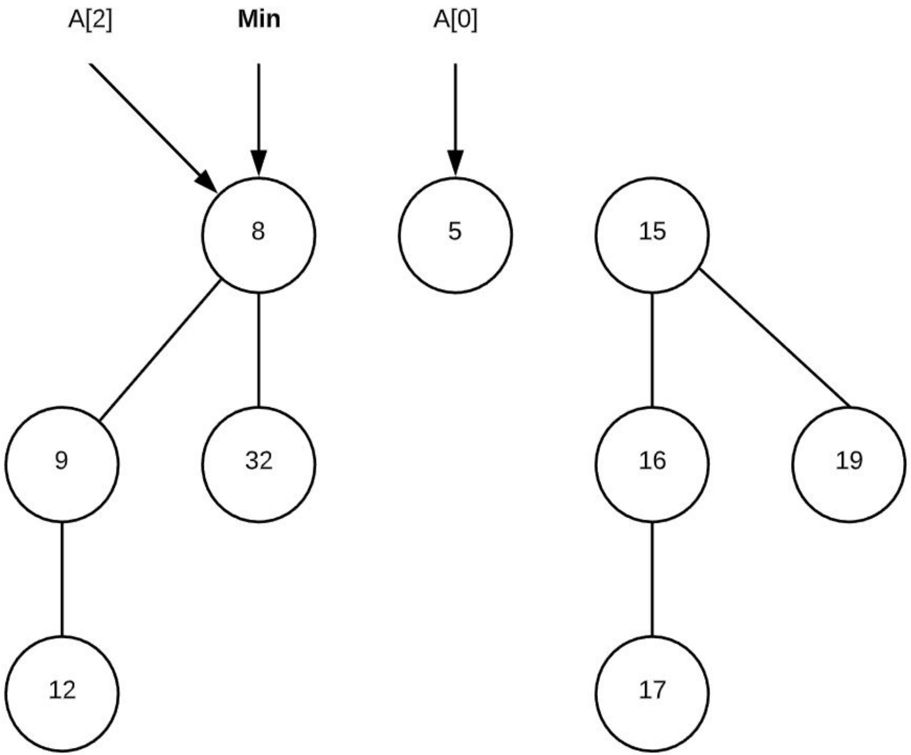
A[1] exists -> while loop:

Y = A[1] / Swap if necessary

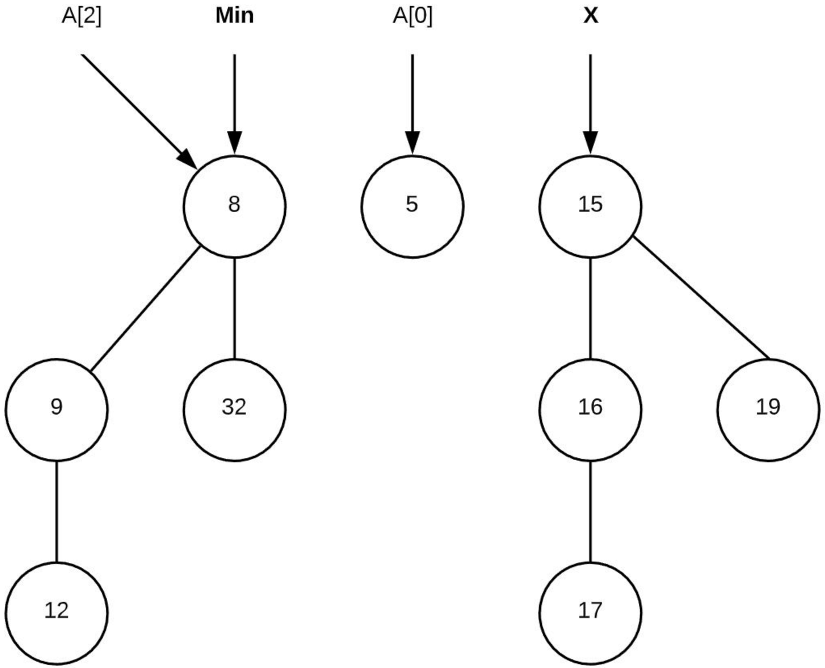
Call MERGE

A[2] = 8 and A[1] = NULL

Consolidate/Merge



Consolidate/Merge



Root List = [8, 5, 15]

Fifth iteration:

x = 15

d = 2

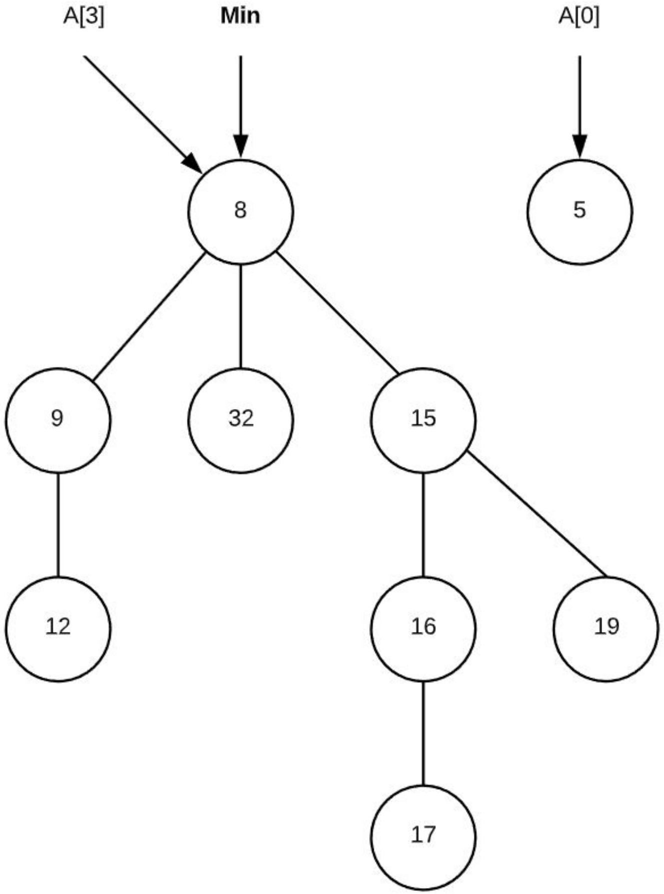
A[2] exists -> while loop:

Y = A[2] / Swap if necessary

CALL MERGE

A[3] = 8 and A[2] = NULL

Consolidate/Merge



Consolidate

```
H.min = NULL;
for i = 0 to D(H.n) do
    if A[i] then
        if !H.min then
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            H.min = A[i];
        else
            insert A[i] into H's root list;
            if A[i].key < H.min.key then
                H.min = A[i]
end
```

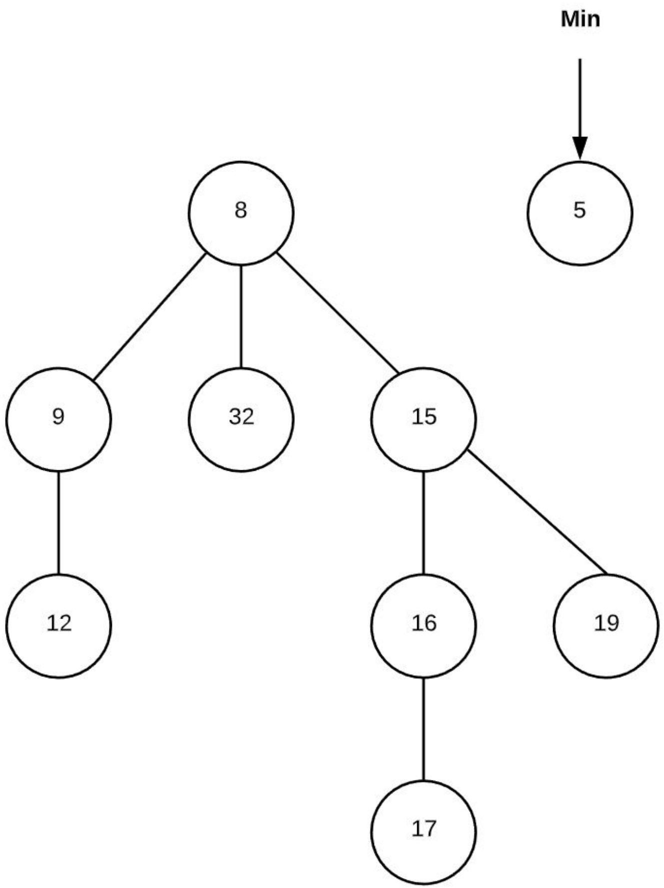

Advantages and Disadvantages

Advantages

- “It is theoretically optimal” [5] (In theory, it has the best performance compared to other trees and heaps)
- Improves asymptotic running time for algorithms that require minimum values

Disadvantages

- Delete worst case has a time complexity of O(n)
- Difficult to implement (4 pointers per node must be manipulated)
- In practice, it is outperformed by simpler heaps and trees, for some applications [5]

Applications

Applications

- Used as the priority queue of Dijkstra Algorithm [1]

- Speed up the scaling algorithm of Edmonds and Karp, for minimum-cost network flow [6]

- Find shortest pairs of disjoint paths [6]

Thanks!
Questions?

Sources

[1] GeeksforGeeks. 2020. Fibonacci Heap | Set 1 (Introduction) - Geeksforgeeks. [online] Available at: <https://www.geeksforgeeks.org/fibonacci-heap-set-1-introduction/> [Accessed 5 July 2020].

[2] GeeksforGeeks. 2020. Fibonacci Heap - Deletion, Extract Min And Decrease Key - Geeksforgeeks. [online] Available at: <https://www.geeksforgeeks.org/fibonacci-heap-deletion-extract-min-and-decrease-key/?ref=rp> [Accessed 5 July 2020].

[3] CORMEN, T., LEISERSON, C., RIVEST, R., &STEIN, C.. (2009). Advanced data structures. En Introduction to Algorithms(pp.510-511). England: THE MIT PRESS.

[4] CORMEN, T., LEISERSON, C., RIVEST, R., &STEIN, C.. (2009). Advanced data structures. En Introduction to Algorithms(pp.512-517). England: THE MIT PRESS.

[5] People.ksp.sk. 2020. Fibonacci Heap | Gnarley Trees. [online] Available at: <https://people.ksp.sk/~kuko/gnarley-trees/?page_id=320> [Accessed 13 July 2020].

[6] Fredman, Michael L., and Robert Endre Tarjan. "Fibonacci heaps and their uses in improved network optimization algorithms." Journal of the ACM (JACM) 34.3 (1987): 596-615.