Problem Sets 3

IE509/AI533 Advanced Quality Control

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November 6, 2022

Problem 1

(a) Summarize and compare the robustness against non-normality for three types of control chars(the Shewhart charts, the CUSUM charts, and the EWMA charts) from [3].

1. Traditional CUSUM chart.
   [Charting statistics]
$$u^+_{n,N} = max(0, u^+_{n-1,N} + \bar{X}(n) - k_N)$$
$$u^-_{n,N} = max(0, u^-_{n-1,N} + \bar{X}(n) + k_N)$$
   A mean shift in $X(n)$ is signaled if
$$[u^+_{n,N} > h^+_N$$
   or
$$[u^-_{n,N} < -h^-_N$$

For skewed IC process distributions, Different h^+_N and h^-_N can be chosen. When n is large enough, the distributions of the charting statistics $u^+_{n,N}$ and $u^-_{n,N}$ become stable. When the IC process distribution is known, we can determine the steady-state distributions of $u^+_{n,N}$ and $u^-_{n,N}$ by simulation and choose h^+_N and h^-_N such that $P(u^+_{n,N} > h^+_N) = P(u^-_{n,N} < -h^-_N)$ and the prespecified IC ARL value is achiueved.

2. EWMA chart.
   [Charting statistics]
$$v_n = \lambda\bar{X}(n) + (1-\lambda)v_{n-1},$$
   Then, a mean shift in $X(n)$ is signaled if
$$|v_n| \geq h_R$$

Where $h_R > 0$ is a control limit chosen to achieve an prespecified IC ARL level under the normality assumption. It is known that when $\lambda = 0.05$ this chart performed reasonably well in various cases when the IC process distribution was actually nonnormal. This EWMA chart with $\lambda = 0.05$ is called R-EWMA chart hereafter.

3. Shewhart-type chart [Statistic]
$$\psi(n) = 2W^+_n - \frac{m(m+1)}{22} \quad \text{for} \quad n \geq 1$$

It is known that CUSUM charts are more favorable for detecting persistent shifts, compared to Shewhart-type charts. For that reason, we construct a CUSUM chart based on $\psi(n)$ as follows. Let $u^+_{0,S} = u^-_{0,S} = 0$, and for $n \geq 1$
$$u^+_{n,S} = max(0, u^+_{n-1,S} + (\psi(n) - \psi_0) - k_S)$$
$$u^-_{n,S} = min(0, u^-_{n-1,S} + (\psi(n) - \psi_0) + k_S)$$

where k_S is an allowance constantm, and ψ_0 is the IC mean of ψ_n which can be estimated from the IC data. Signals a mean shift in $X(n)$ if

$$u^+_{n,S} > h_S \quad \text{or} \quad u^-_{n,S} > -h_S$$

Where the control limit h_S is chosen to achieve a given IC ARL level. This chart is called S-CUSUM chart hereafter.

4. Comparision on N-CUSUM, R-EWMA, S-CUSUM

Table 1. The actual IC ARL values and their standard errors (in parentheses) of the eight control charts when the nominal IC ARL values are fixed at 500 and the actual IC process distribution is the standardized version of N(0, 1), $t(4)$, $\chi^2(1)$, and $\chi^2(4)$

	N(0, 1)	$t(4)$	$\chi^2(1)$	$\chi^2(4)$
P-CUSUM	501.9 (5.51)	503.3 (5.55)	504.8 (5.46)	501.1 (5.45)
L-CUSUM	498.1 (4.96)	495.3 (5.02)	504.2 (5.13)	497.4 (5.06)
K-CUSUM	499.0 (4.48)	499.7 (4.47)	504.4 (4.61)	496.4 (4.46)
N-CUSUM	498.9 (4.35)	156.0 (1.13)	321.4 (2.63)	371.5 (3.27)
R-EWMA	502.2 (6.24)	578.2 (20.87)	605.5 (8.65)	533.7 (6.57)
S-CUSUM	497.3 (5.22)	532.2 (5.57)	544.5 (5.88)	518.3 (5.63)
St-CUSUM	499.5 (4.87)	3037.7 (27.33)	9316.6 (20.46)	1862.2 (18.08)
I-CUSUM	499.8 (4.84)	499.8 (4.92)	–	–

As you can see in the above table, The actual IC ARL values of the chars N-CUSUM and R-EWMA are quit different from 500 when the actual IC process distribution is nonnormal. For the chart S-CUSUM, its actual IC ARL values are moderately different from 500 due to the discreteness of its charting ststistic. For all nonnormal distributions listed above, S-CUSUM is more robust than R-EWMA and N-CUSUM.

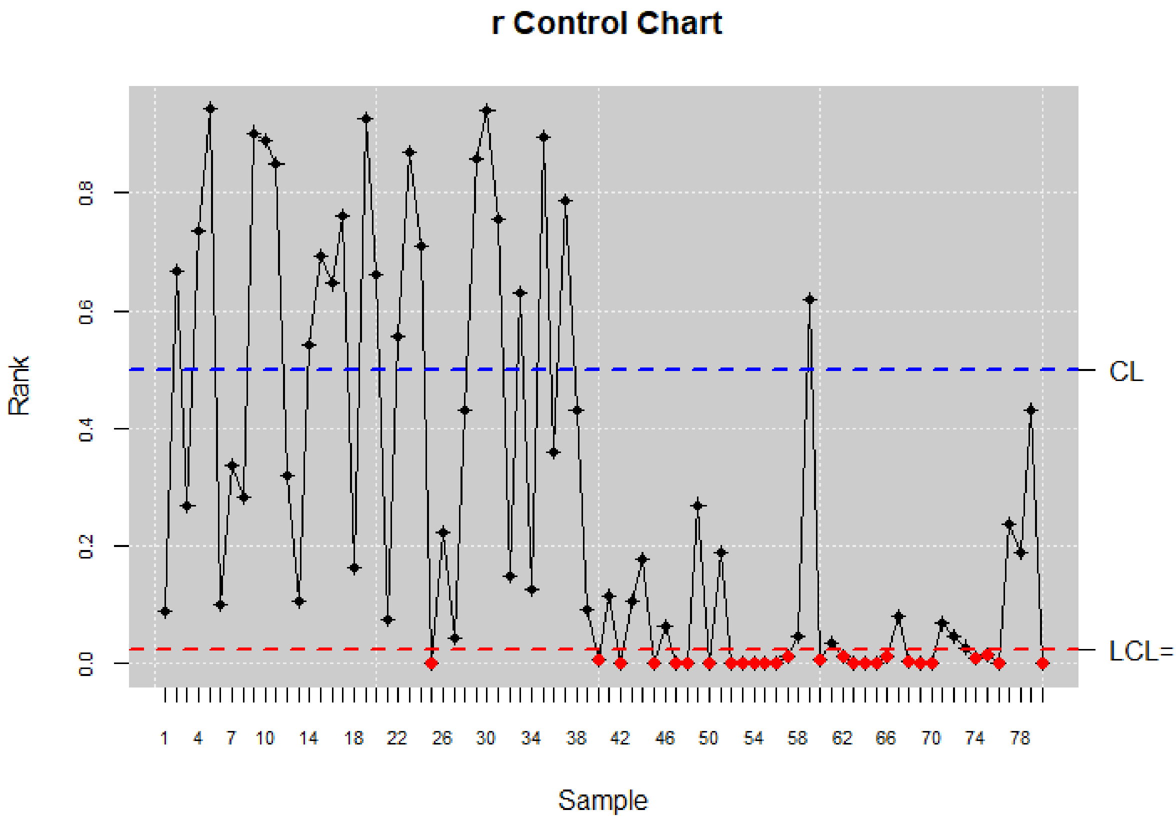
(c) To handle cases when the normality assumption is invalid, what kind of control charts have been developed in [1, 2]? Please specify the type of control chart and explain its main idea.

First we discuss the Shewhart-type control charts. Park and Reynolds develop nonparametric Shewhart-tpye procedures for monitoring the location parameter of a continuous process when the in-control value for the parameter is not specified. These procedures are based on the "linear placement" statistics, introduced by Orban and Wolfe for comparing current samples with a standard sample taken when the process is operating properly. Asymptotic approximations to the run length distributions are obtained.

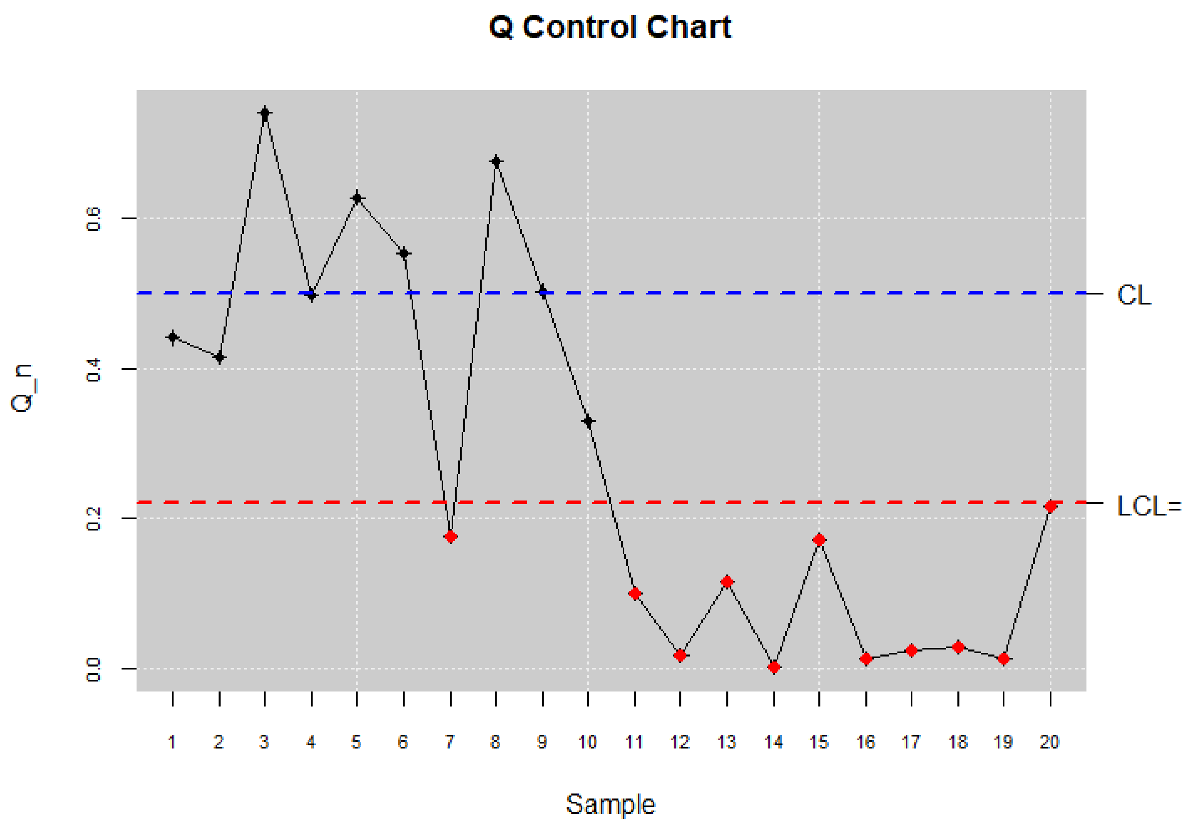
Alloway and Raghavachari consider a Shewhart-type chart for the median θ of a continuous symmetric population, based on a distribution free confidence interval for θ, calculated using the Hodges-Lehmann estimator. The advantage with the Hodeges-Lehmann estimator is that the normality assumption is not required and it is robust against outliers.

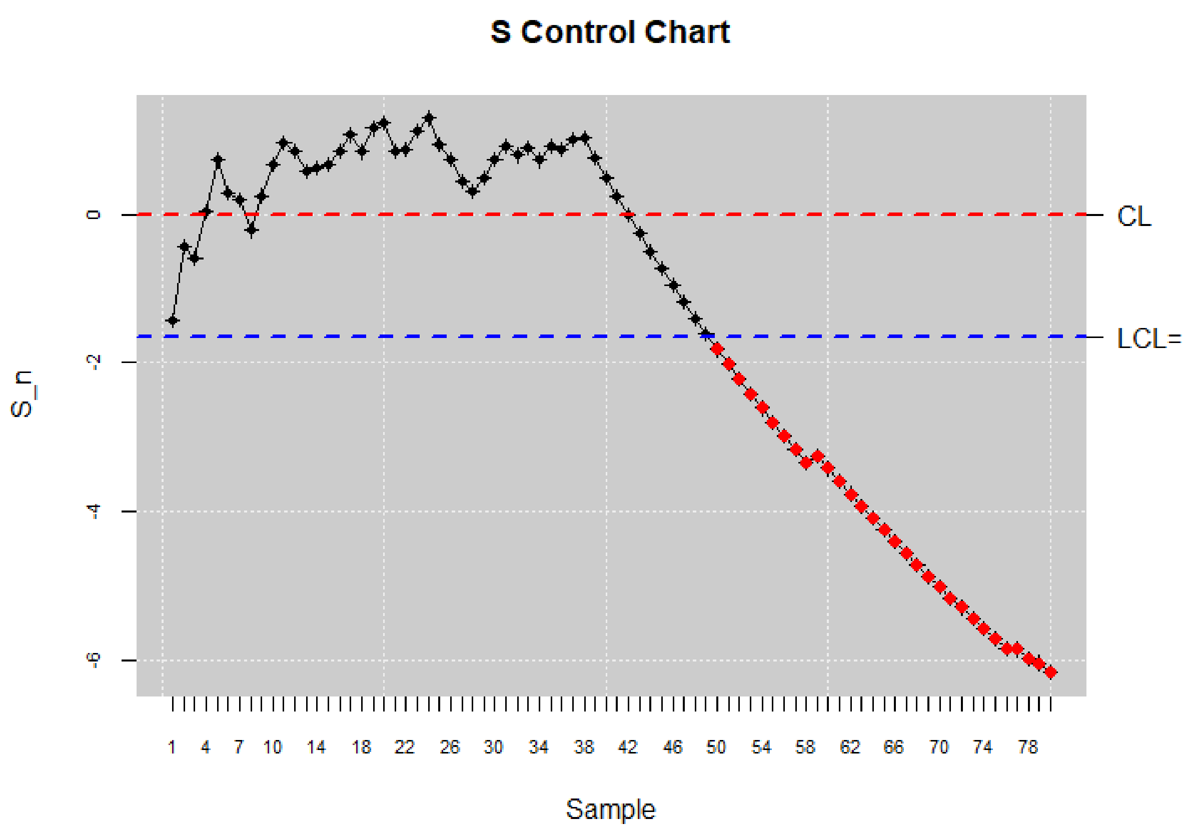
While the Shewhart-type charts are probably the most widely used because of their simplicity, CUSUM procedures are quite appropriate in view of the sequential nature of the process control problem. In the nonparametric setting, Reynolds studies charts based on "signed sequential ranks" of observations. McGilchrist and Woodyer consider a CUSUM technique that allow for distribution-free tests and apply it to the problem of detecting a change in the median of rainfall distribution. Park and Reynolds consider CDUSUM charts for the median of a process based on linear placemnet statistics introduced by Orban and Wolfe. Linear placement statistics are used.

Problem 2



r Control Chart
Rank
0.0
0.2
0.4
0.6
0.8
CL
LCL=
1
4
7
10
14
18
22
26
30
34
38
42
46
50
54
58
62
66
70
74
78
Sample


Q Control Chart
Q_n
CL
LCL=
Sample


S Control Chart
S_n
CL
LCL=
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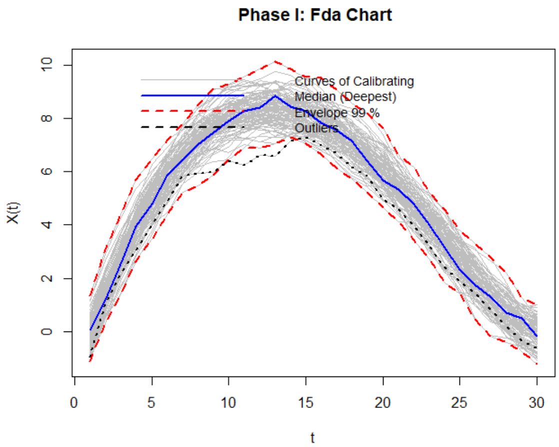
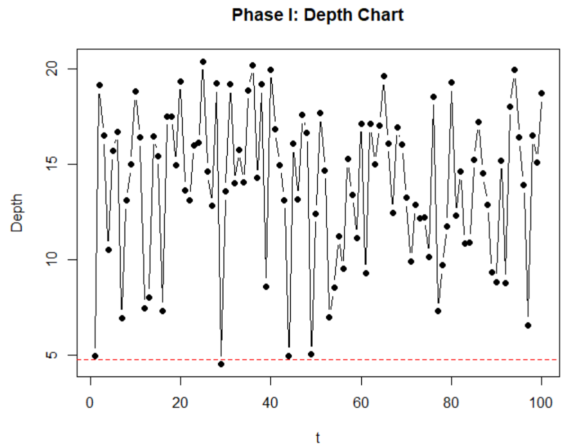

Phase I: Fda Chart
Curves of Calibrating
Median (Deepest)
Envelope 99 %
Outliers
X(t)
t


Phase I: Depth Chart
Depth
t