Problem Sets2

IE509/AI533 Advanced Quality Control

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Problem 1

$$UCL : \mu + 3\frac{\sigma}{\sqrt{n}}$$
$$CL : \mu$$
$$LCL : \mu - 3\frac{\sigma}{\sqrt{n}}$$

Now we are given β values at $k = 0$ and 2. a) First we consider β values at $k = 0$. These values are This β values signifies:

Color	β
Red	0.95
Black	0.9
Green	0.8
Blue	0.8

$$P(LCL < \bar{X} < UCL|\mu = \mu_1 = \mu_0 + 0 * \sigma) = \beta$$
$$\alpha = P(\bar{X} < LCLor\bar{X} > UCL|\mu = \mu_0) = 1 - \beta$$
$$ARL_0 = \frac{1}{\alpha}$$

This gives the average number of samples required to get a point outside the control limits when the process is in control. Thus based on ARL_0, We can say that we must minimize α. Thus for which process ARL_0 is maximum, we select that process. Thus, we get Hence based on ARL_0 we select the

Color	$\alpha = 1 - \beta$	ARL_0
Red	.05	20
Black	.10	10
Green	.20	5
Blue	.20	5

process defined by RED curve i.e. the curve with $beta = .95$ at $k = 0$ b) Now let us consider β values at $k = 2$ This β values signifies:

Color	β
Red	0.2
Black	0.23
Green	0.16
Blue	0.11

$$P(LCL < \bar{X} < UCL|\mu = \mu_1 = \mu_0 + 2 * \sigma) = \beta$$

Color	$\alpha = 1 - \beta$	ARL_1
Red	.8	1.25
Black	.77	1.29
Green	.84	1.19
Blue	.89	1.12

Out of control ARL : $\frac{1}{1-\beta}$, this gives the average number of samples required to get a point outside the control limits when the process is out of control. Thus based on ARL_1 : We can say that we must minimize β. Thus for which process ARL_1 is minimum we select that process. Hence based on ARL_1 we select the process defined by BLUE curve i.e. the curve with $\beta = .11$ at $k = 2$

Problem 2

a) We will use a np-chart given that its purpose is to detect defectives and the sample size of the observations are the same.

$$n\bar{p} = \frac{\sum np}{k}$$

$$\bar{p} = \frac{\sum np}{\sum n}$$

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

Where np = total number of defectives in the samples, k = number of lots, n = sample size

Based on example,

$$k = 15$$

$$\sum np = 69$$

$$\sum n = 225$$

So,

$$\bar{p} = 69/225 = 0.307$$

$$n\bar{p} = 69/15 = 4.6 = CL$$

Calculate UCL and LCL

$$UCL_{np} = 4.6 + 3\sqrt{4.6 * (1 - 0.307)} = 4.6 + 3\sqrt{3.187} = 4.6 + 5.3 = 9.9$$

$$LCL_{np} = 4.6 - 3\sqrt{3.187} = 4.6 - 5.35 = -0.75 = 0$$

b)

Problem 3

a) LCL and UCL for the shewhart chart:

$$LCL = \bar{\bar{x}} - 3 * \frac{\bar{S}}{a_n\sqrt{n}}$$

$$UCL = \bar{\bar{x}} + 3 * \frac{\bar{S}}{a_n\sqrt{n}}$$

$$\bar{S} = \frac{1}{K}\sum_{k=1}^{K} s_k$$

Calculate the average of the $n = 5$ samples from each bale.

$$\bar{x} = [245, 239, 239, 241, 241, 241, 238, 238, 236, 248, 233, 236, 246, 253, 227, 231, 237, 228, 239, 240]$$

So, $\bar{\bar{x}} = 238.8$ and $\bar{S} = 9.28$

Calculate the lower and upper control limits.

$$LCL = \bar{\bar{x}} - 3 * \frac{\bar{S}}{a_n\sqrt{n}} = 238.8 - 3 * \frac{9.28}{(0.94)(\sqrt{5})} = 225.6$$

$$UCL = \bar{\bar{x}} + 3 * \frac{\bar{S}}{a_n\sqrt{n}} = 238.8 + 3 * \frac{9.28}{(0.94)(\sqrt{5})} = 252.0$$

Problem 5

a) In given two processes, first process is performing equally on both USL and LSL but second process is having asymmetric USL and LSL around mean. Hence, process capability of first process will be better.

b)

$$PCI = min(USL - \sigma)/3 * SD, (\sigma - LSL)/(3 * SD)$$

c) C_p does not take into account the centering of mean. On the other hand, C_pk takes both process variability and centering of mean into account which is more practically accurate and comprehensive approach. In the given problem, we are having two processes A and B. A is centered between USL and LSL and having high variability. Process B is having low variability but its mean is not centered. In this case, relying on C_p only will tell that process B is more capable as it is having low variability. But C_pk, along with variability, will take centering of mean also into account where process B is not doing well. So, considering C_p and C_pk together will present a clearer picture of situation.