Problem Sets 3

IE509/AI533 Advanced Quality Control

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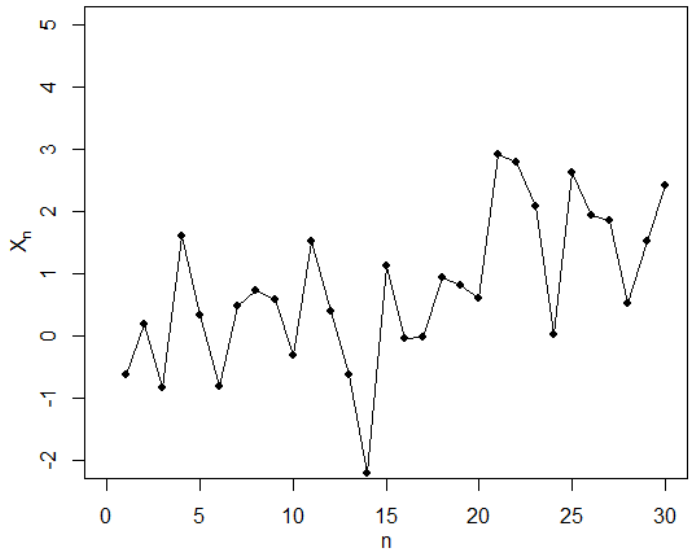
November 16, 2022

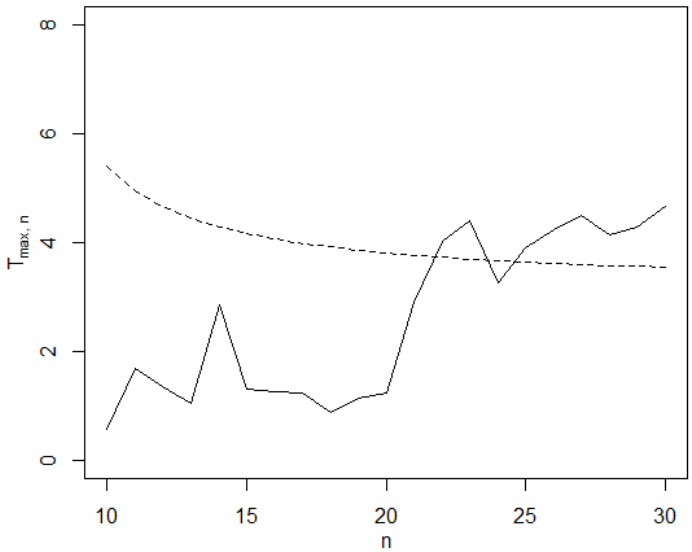
Problem 1

Observations and test statistics.

	x	Wn	Sn2	Tmax	hn
1	-0.63	0.00	0.00	0.00	0.00
2	0.18	0.00	0.00	0.00	0.00
3	-0.84	0.00	0.00	0.00	0.00
4	1.59	0.00	0.00	0.00	0.00
5	0.33	0.00	0.00	0.00	0.00
6	-0.82	0.00	0.00	0.00	0.00
7	0.49	0.00	0.00	0.00	0.00
8	0.74	0.00	0.00	0.00	0.00
9	0.58	1.63	5.27	0.00	0.00
10	-0.30	1.32	5.48	0.57	5.39
11	1.51	2.83	7.21	1.68	4.95
12	0.39	3.22	7.23	1.35	4.65
13	-0.62	2.60	7.96	1.05	4.44
14	-2.21	0.39	13.38	2.86	4.29
15	1.12	1.51	14.50	1.31	4.16
16	-0.04	1.47	14.52	1.27	4.06
17	-0.02	1.45	14.53	1.25	3.98
18	0.94	2.40	15.23	0.88	3.92
19	0.82	3.22	15.68	1.15	3.86
20	0.59	3.81	15.85	1.24	3.81
21	2.92	6.73	22.94	2.92	3.77
22	2.78	9.51	28.72	4.03	3.73
23	2.08	11.59	31.30	4.41	3.70
24	0.01	11.60	31.53	3.26	3.67
25	2.62	14.22	35.92	3.90	3.64
26	1.94	16.16	37.73	4.23	3.62
27	1.84	18.00	39.17	4.49	3.60
28	0.53	18.53	39.19	4.14	3.58
29	1.52	20.06	39.91	4.29	3.56
30	2.42	22.47	42.79	4.67	3.55

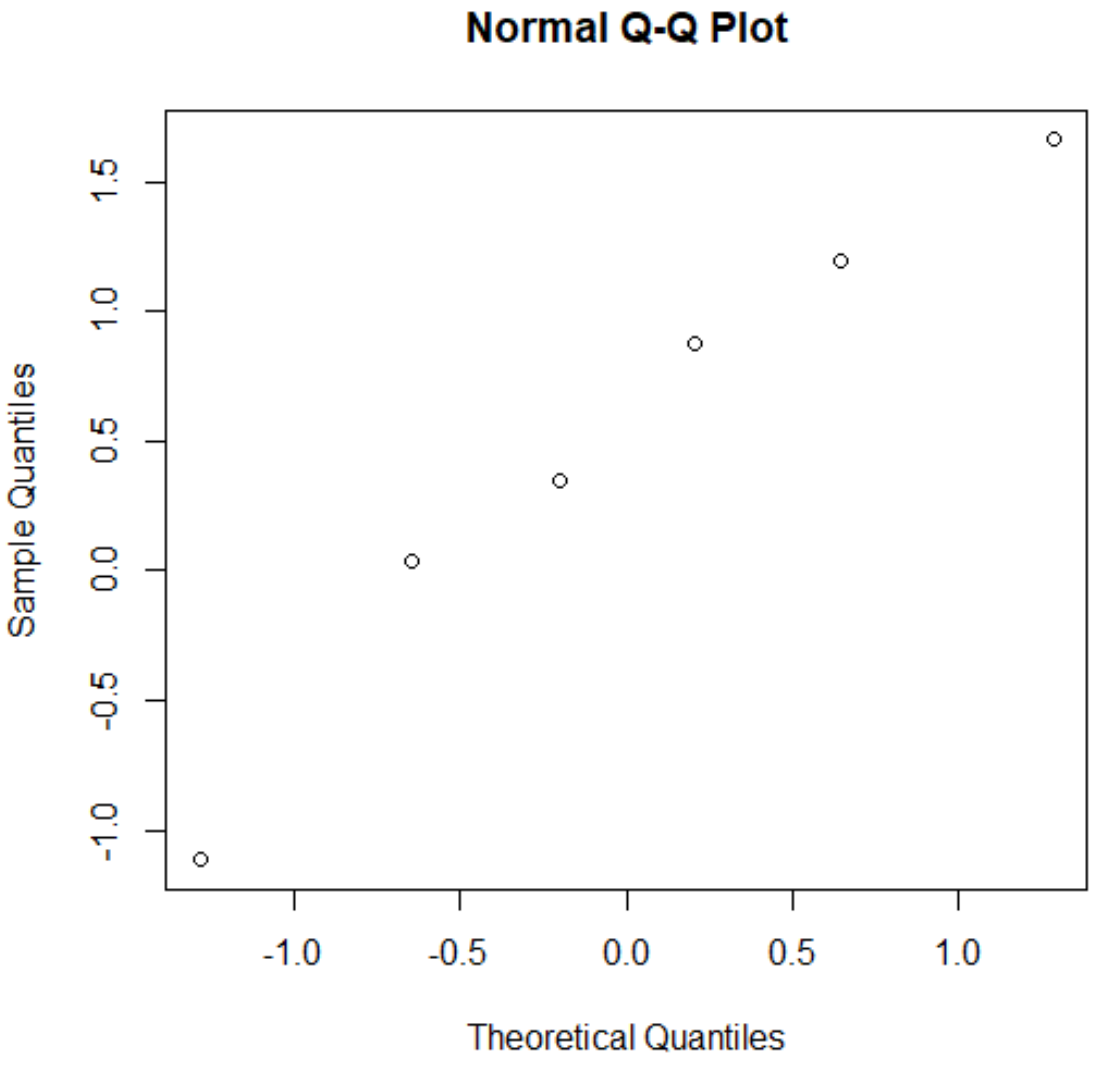


(a) Generated observations



(b) The CPD control chart and the control limit

Problem 2


Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
-1.0
-0.5
0.0
0.5
1.0
1.5
-1.0
-0.5
0.0
0.5
1.0

Problem 3

We will apply multivariate SPC methods(Shewhart, CUSUM, COT) to each scenario. The values of the multivariate Shewhart charting statistic

$$T^2_{0,n} = (X_n - \mu_0)' \Sigma_0^{-1} (X_n - \mu_0), \qquad for \quad n = 1, 2, ..., 30$$

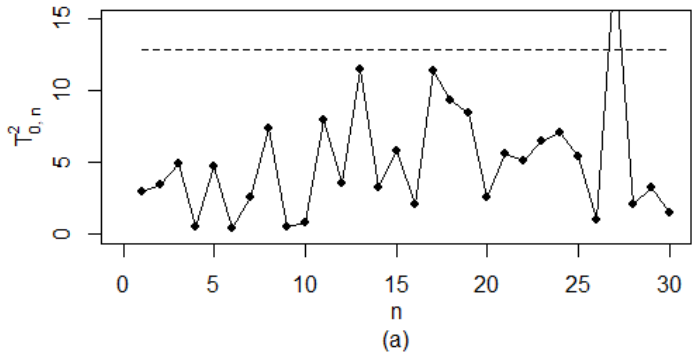
By the general formula of the CUSUM charting statistic, Healy derived the following charting statistic for monitoring the process covariance matrix

$$C_n = max[0, C_{n-1} + T_n^2 - k], \qquad for \quad n \geq 1$$

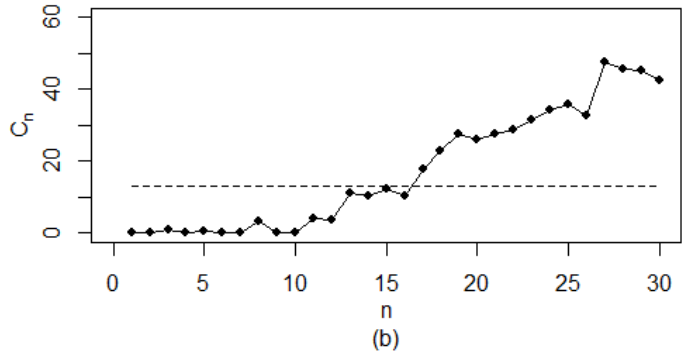
COT charting statistics

$$C_n = max(0, C_{n-1} + T_n - k$$

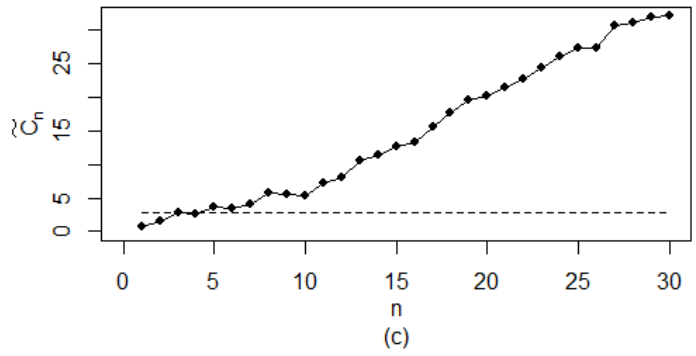
From this example, it can be seen that the Shewhart chart is not sensitive to either the process mean shift or the process covariance matrix shift in cases when such shifts are small, the multivariate CUSUM chart and the COT chart are sensitive to both the process mean shift and the process covariance matrix shift, and it seems that the COT chart is more effective than the CUSUM for detecting either the process mean shift or the process covariance matrix shift or both.



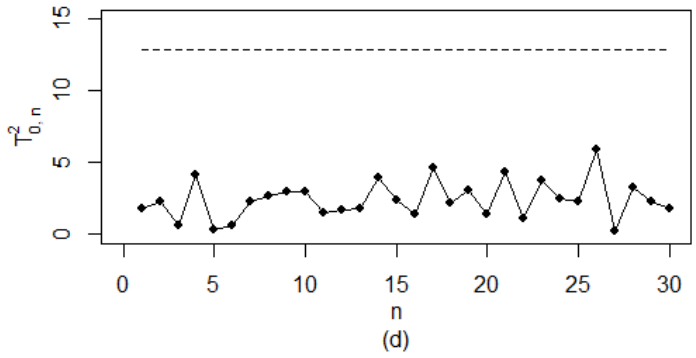
(a) Scenario (i), Shewart chart



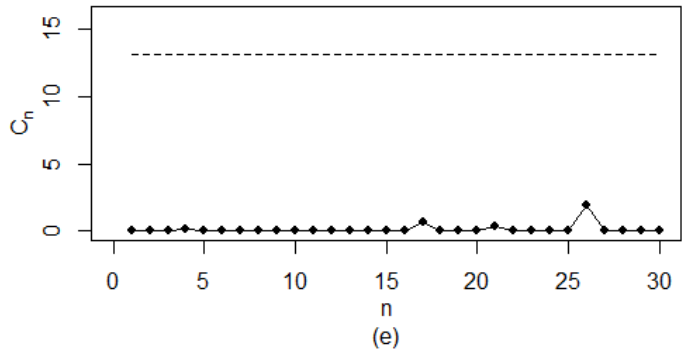
(b) Scenario (i), CUSUM chart



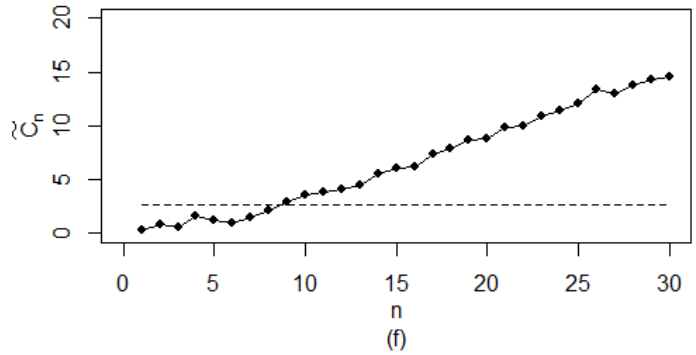
(c) Scenario (i), COT chart



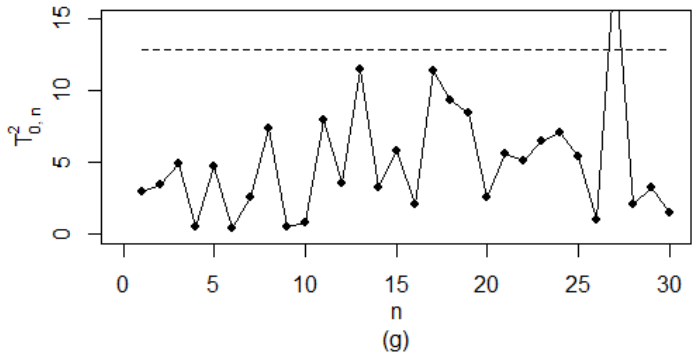
(a) Scenario (ii), Shewart chart



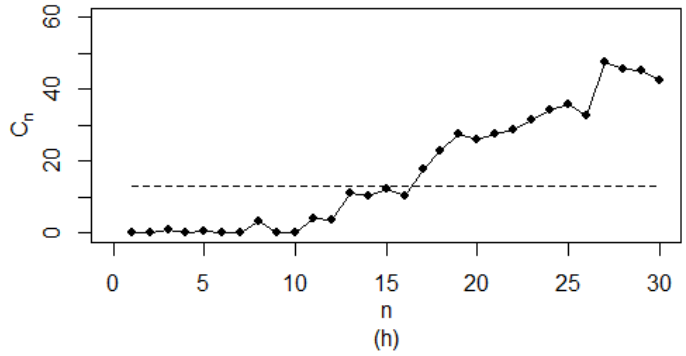
(b) Scenario (ii), CUSUM chart



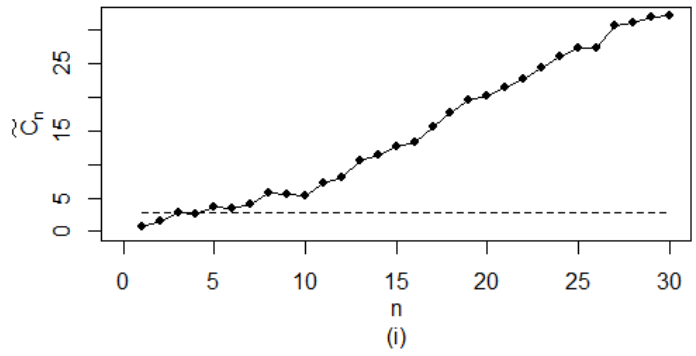
(c) Scenario (ii), COT chart



(a) Scenario (iii), Shewart chart



(b) Scenario (iii), CUSUM chart



(c) Scenario (iii), COT chart