20213359 이효정 AI Toolkits Assignment 3

1. (Dual Ascent) we solve fallowing problem.

minimize $f(x,y) = \frac{1}{2}(x^2+y^2)$
Subject to $2x-y=5$

(a) Update rules for $x_{t+1}, y_{t+1}, \lambda_{t+1}$

$f(x,y) = f(x)+g(y) \quad f(x) = \frac{1}{2}x^2 \quad g(y) = \frac{1}{2}y^2$

minimize $f(x)+g(y)$
Subjeat to $2x-y=5$

Lagrangian is

$Lp(x,y,\lambda) = f(x)+g(y)+\lambda^T(2x-y-5)$

Update rule

$$\begin{cases} x_{t+1} = \underset{x}{\operatorname{argmin}}\, Lp(x, y_t, \lambda_t) \\ y_{t+1} = \underset{y}{\operatorname{argmin}}\, Lp(x_{t+1}, y, \lambda_t) \\ \lambda_{t+1} = \lambda_t + p(2x_{t+1} - y_{t+1} - 5). \end{cases}$$

(b). Solve the problem and provide the exact solution.

$$\frac{\partial Lp(x,y_t,\lambda_t)}{\partial x} = f'(x) + \frac{\partial \lambda^T(2x-y-5)}{\partial x}$$
$$= x + 2\lambda = 0$$
$$\therefore x = -2\lambda$$

$$\frac{\partial Lp(x_{t+1},y,\lambda_t)}{\partial y} = g'(x) + \frac{\partial \lambda^T(2x-y-5)}{\partial y}$$
$$= y - \lambda = 0$$
$$\therefore y = \lambda$$

$[x=2,\ y=-1,\ \lambda=-1]$

Subsbitute

$$2(-2\lambda) - \lambda = 5$$
$$-4\lambda - \lambda = 5$$
$$-5\lambda = 5$$
$$\lambda = -1$$

2. (Method of Multipliers) We solve the following problem.

minimize $f(x,y) = 2\beta xy$
subject to $2x-y=0$.

(a) Show that $f(x,y)$ is not convex for both $\forall x$ and $\forall y$.

· check the hessian matrix.

$f_x(x,y) = \frac{\partial}{\partial x}(2\beta xy) = 2\beta y$

$f_y(x,y) = \frac{\partial}{\partial y}(2\beta xy) = 2\beta x$

Hessian.
$$\begin{bmatrix} 0 & 2\beta \\ 2\beta & 0 \end{bmatrix}$$

$f_{xx}(x,y) = \frac{\partial}{\partial x}(2\beta y) = 0$

$f_{xy}(x,y) = \frac{\partial}{\partial y}(2\beta y) = 2\beta$

$f_{yx}(x,y) = \frac{\partial}{\partial x}(2\beta x) = 2\beta$

$f_{yy}(x,y) = \frac{\partial}{\partial y}(2\beta x) = 0$

check the eigenvalues of hessian matrix

$\begin{bmatrix} 0 & 2\beta \\ 2\beta & 0 \end{bmatrix}$ determinant $-4b^2+\lambda^2$

$\therefore \lambda_1 = -2b \quad \lambda_2 = 2b.$

there is negative eigenvalues in the hessian matrix
$\Rightarrow$ non-convex.

(b) Show that the augmented Lagrangian is convex for both $\forall x$ and $\forall y$ for some condition on p

augmented Lagrangian.

$$f(x,y) = 2\beta xy + \frac{p}{2}\| 2x-y \|^2$$

Check the hessian matrix for augmented Lagrangian.

$f_x(x,y) = 2\beta y + \rho(2x-y)\cdot(2) = 2\beta y + 2\rho(2x-y)$

$f_y(x,y) = 2\beta x + \rho(2x-y)\cdot(-1) = 2\beta x - \rho(2x-y)$

$f_{xx}(x,y) = \dfrac{\partial(2\beta y + 2\rho(2x-y))}{\partial x} = 2\rho\cdot 2 = 4\rho$

$f_{xy}(x,y) = \dfrac{\partial(2\beta y + 2\rho(2x-y))}{\partial y} = 2\beta + 2\rho(-1) = 2\beta - 2\rho$

$f_{yx}(x,y) = \dfrac{\partial(2\beta x - \rho(2x-y))}{\partial x} = 2\beta - \rho(2) = 2\beta - 2\rho$

$f_{yy}(x,y) = \dfrac{\partial(2\beta x - \rho(2x-y))}{\partial y} = -\rho(-1) = \rho$

$$\begin{bmatrix} 4\rho & -2\beta\rho + 2\rho \\ 4\beta\rho - 2\rho & \rho \end{bmatrix}$$

(c) update steps.

$L_\rho(x,y,\lambda) = f(x,y) + \frac{\rho}{2}\|2x-y\|_2^2$

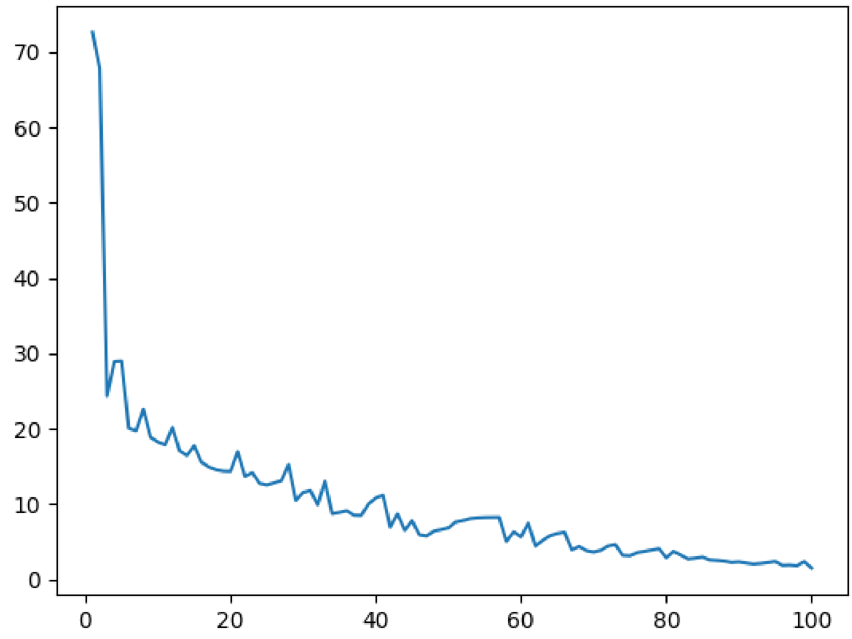
$x^{t+1} = \underset{x}{\operatorname{argmin}}\, L_\rho(x, y^t, \lambda^t)$

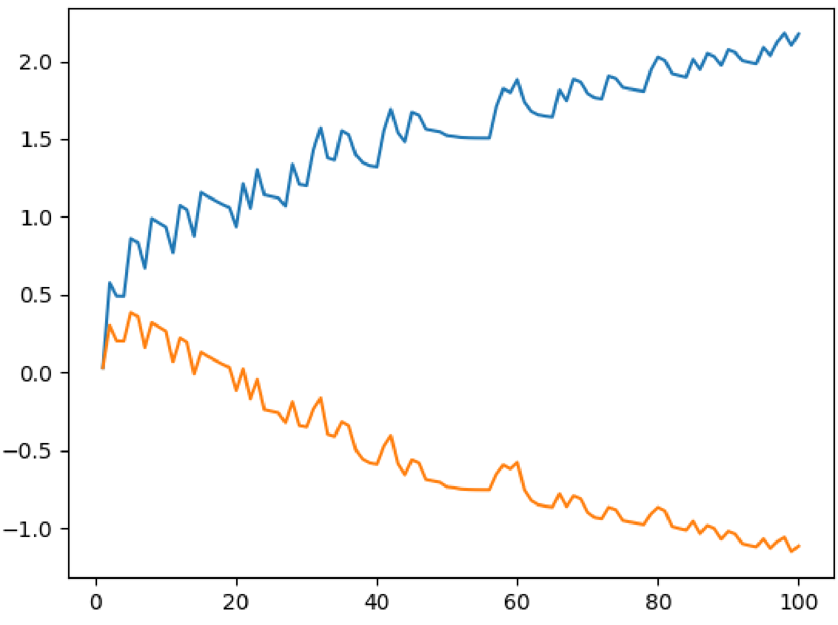
$y^{t+1} = \underset{y}{\operatorname{argmin}}\, L_\rho(x^{t+1}, y, \lambda^t)$

$y^{t+1} = \lambda^t + \rho(2x^{t+1} - y^{t+1})$

Q3 (SGD) Implement Stochastic Gradient Descent

Plot 1, 2 : loss and weight during 100 iterations





[FOR THE CODE, PLEASE REFER TO ATTACHED PYTHON FILE]

[Results]

weight of t = 5 : [0.6802595280000001, 0.16886861759999997]

weight of t = 10 : [1.0765977771050919, 0.22137610963655294]

weight of t = 50 : [1.9412157473559197, -0.5630068619554149]

weight of t = 100 : [2.2590301472623806, -1.1839525932057553]

Prediction : [3.2252326621698755, 6.668215402638012, 4.191435177077372]

Loss : 1.5935182383297608

Final weight : [2.2590301472623806, -1.1839525932057553]

Q4. The task is to determine if a given 9 x 9 sudoku board is valid

[FOR THE CODE, PLEASE REFER TO ATTACHED PYTHON FILE]

[Results]

For invalid inputs

column-wise duplicate 8 in (3, 0)

column-wise duplicate 8 in (0, 0)

checking 0th 3x3 box

duplicates in 3x3 box

checking 1th 3x3 box

No duplicates

checking 2th 3x3 box

No duplicates

checking 3th 3x3 box

No duplicates

checking 4th 3x3 box

No duplicates

checking 5th 3x3 box

No duplicates

checking 6th 3x3 box

No duplicates

checking 7th 3x3 box

No duplicates

checking 8th 3x3 box

No duplicates

Process finished with exit code 0

For valid inputs

checking 0th 3x3 box

No duplicates

checking 1th 3x3 box

No duplicates

checking 2th 3x3 box

No duplicates

checking 3th 3x3 box

No duplicates

checking 4th 3x3 box

No duplicates

checking 5th 3x3 box

No duplicates

checking 6th 3x3 box

No duplicates

checking 7th 3x3 box

No duplicates

checking 8th 3x3 box

No duplicates

Process finished with exit code 0