CSE 280 Challenge Set 04

Question 1

For each of these functions, determine if they are well-defined, one-to-one, onto and/or a bijection. For these problems, let $A = \{x \mid x \in \mathbf{Z}, 0 < x < 5\}$ and let $B = \{x \mid x \in \mathbf{Z}, 0 \le x \le 5\}$. You may want to draw a digraph of each function (with dots and arrows). Recall that if not well-defined, then it is neither 1-1, onto, nor a bijection.

Function	Well Defined	1-1	Onto	Bijection
$f : A \to A, \text{ where } f = \{(1,2),(2,2),(3,3),(4,3)\}$				
$f : A \to A, \text{ where } f = \{(1,1),(2,2),(3,4),(4,3)\}$				
$f : A \to A, \text{ where } f = \{(2,2),(3,4),(4,3)\}$				
$f : A \to B, \text{ where } f = \{(1,2),(2,3),(3,4),(4,5)\}$				
$f : B \to A, \text{ where } f = \{(0,1),(1,1),(2,2),(3,2),(4,4),(5,3)\}$				

Question 2

For each of these functions, determine if they are well-defined, one-to-one, onto, and/or a bijection. Consider using a graphing calculator like https://www.desmos.com.

Function	Well Defined	1-1	Onto	Bijection
$f : \mathbf{Z} \to \mathbf{Z}, \text{ where } f(x) = 5x - 7$				
$f : \mathbf{Z}^+ \to \mathbf{Z}^+, \text{ where } f(x) = \lceil \frac{x}{5} \rceil$				
$f : \mathbf{R} \to \mathbf{R}, \text{ where } f(x) = x^5 - 3x$				
$f : \mathbf{R}^+ \to \mathbf{R}, \text{ where } f(x) = \frac{1}{x}$				
$f : \mathbf{R} \to \mathbf{R}, \text{ where } f(x) = \log_5(x^2)$				
$f : \{x \mid x \in \mathbf{R}, x > 0\} \to \mathbf{R}, \text{ where } f(x) = \log_5(x^2)$				
$f : \{x \mid x \in \mathbf{R}, x \ne 0\} \to \mathbf{R}, \text{ where } f(x) = \log_5(x^2)$				

Question 3

Part 1

Find the inverse $f^{-1}(x)$ (if it exists) of the following functions:

- $f : \mathbf{R} \to \mathbf{R}, \text{ where } f(x) = (x - 32) * 5/9$

- $f : \mathbf{R} \to \{x \mid x \in \mathbf{R}, x > 0\} \text{ where } f(x) = \frac{1}{2^x}$

- $f : \mathbf{R} \to \mathbf{Z}, \text{ where } f(x) = \lfloor x \rfloor$

- $f: \mathbf{R} \rightarrow \mathbf{R}, \text{ where } f(x) = x^2$
- $f: \mathbf{R} \rightarrow \mathbf{R}, \text{ where } f(x) = x^3$

Part 2

Select a value of x (per the domain) and calculate $(f^{-1} \circ f)(x)$ for all functions that had an inverse in Part 1 to demonstrate that the inverse was successfully created.

Question 4

Define the following two functions:

- $f: \mathbf{R} \rightarrow \mathbf{R}, \text{ where } f(x) = \lceil x \rceil - x$
- $g: \mathbf{R} \rightarrow \mathbf{R}, \text{ where } g(x) = x - \lfloor x \rfloor$

Part 1

Graph function f using a graphing calculator. Explain what this so-called "sawtooth" function represents.

Part 2

Based on your answer above, does an inverse to f exist?

Part 3

Determine what $f(x) + g(x)$ is equal to.