

CSE 280 Challenge Set 05 - Solutions

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Question 1

For each relation below, determine which property they satisfy. Each relation has the domain of \mathbf{R} . Remember to use a digraph if you need help.

Relation	Reflexive or Anti-Reflexive	Symmetric or Anti-Symmetric	Transitive	Equivalence
aR_b , if $a^2 = b^2$	Reflexive	Symmetric	Yes	Yes
aR_b , if $\lfloor a \rfloor = b$	Neither	Anti-Symmetric	Yes	No
aR_b , if $a - b \geq 0$	Reflexive	Anti-Symmetric	Yes	No
aR_b , if $-a - b = 1$	Neither	Symmetric	No	No
aR_b , if $a - b = 5$	Anti-Reflexive	Anti-Symmetric	No	No

Question 2

Determine if each of the following relations are equivalences and why. The domain for these relations is $\{A, B, C, D\}$?

- $\{(A, B), (B, A), (C, D), (D, C)\}$
- $\{(A, A), (B, B), (C, C), (D, D), (A, B), (B, A), (C, D), (D, C)\}$
- $\{(A, B), (B, C), (C, B), (B, A), (A, A), (B, B), (C, C), (D, D)\}$
- $\{(A, A), (B, B), (C, C), (D, D), (A, B), (B, C), (A, D), (A, C), (D, C)\}$

Answer:

- No, Not reflexive or transitive
- Yes
- No, Not transitive
- No, Not symmetric

Question 3

For the following parts, assume that the domain of each relation is $\{A, B, C\}$. There may be more than one correct answer for each part.

Part 1

Draw a digraph of a relation that satisfies only the reflexive property.

Part 2

Draw a digraph of a relation that satisfies only the symmetric property.

Part 3

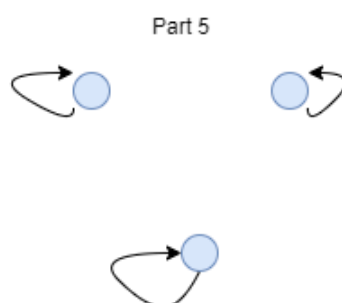
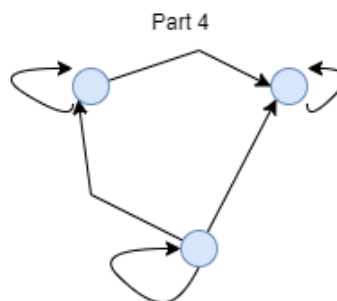
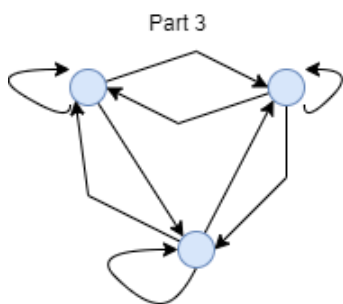
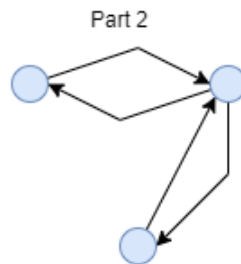
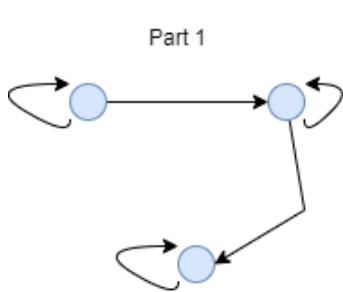
Draw a digraph of a relation that is an equivalence.

Part 4

Draw a digraph of a relation that satisfies both reflexive and transitive properties only.

Part 5

Draw a digraph of a relation that is both an equivalence and anti-symmetric (this is not a trick question ... it is possible).



Question 4

Part 1

Define the relationship $_aR_b$ if a and b have the same remainder when divided by 5. The domain of a and b is \mathbf{Z} . Identify all of the equivalence classes and their contents.

Answer:

- Remainder 0: $[0] = \{5x \mid x \in \mathbf{Z}\} = \{\dots, -10, -5, 0, 5, 10, \dots\}$
- Remainder 1: $[1] = \{5x + 1 \mid x \in \mathbf{Z}\} = \{\dots, -9, -4, 1, 6, 11, \dots\}$
- Remainder 2: $[2] = \{5x + 2 \mid x \in \mathbf{Z}\} = \{\dots, -8, -3, 2, 7, 12, \dots\}$
- Remainder 3: $[3] = \{5x + 3 \mid x \in \mathbf{Z}\} = \{\dots, -7, -2, 3, 8, 13, \dots\}$
- Remainder 4: $[4] = \{5x + 4 \mid x \in \mathbf{Z}\} = \{\dots, -6, -1, 4, 9, 14, \dots\}$

Part 2

Identify the equivalence classes for all of the equivalence relations from Question 1 above.

Answer:

$$_aR_b, \text{ if } a^2 = b^2$$

- $[0] = \{0\}$
- $[a] = \{a, -a\}$ where $a \in \mathbf{R}, a \neq 0$