

CSE 280 Challenge Set 03 - Solutions

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Question 1

Compute the following cartesian products:

Question	Solution
$\{A, B, C\} \times \{H, J\}$	$(A, H), (A, J), (B, H), (B, J), (C, H), (C, J)$
$\{A, B, C\} \times \{H, J\} \times \{H, J\}$	$(A, H, H), (A, J, H), (B, H, H), (B, J, H), (C, H, H), (C, J, H), (A, H, J), (A, J, J), (B, H, J), (B, J, J), (C, H, J), (C, J, J)$

Question 2

Identify four values that are in each of the following sets:

Set	Four Values
$\{x \mid x \in \mathbf{Z}^-, x + 3 \text{ is a multiple of } 5\}$	$-3, -8, -13, -18$
$\{\frac{1}{x^2} \mid x \in \mathbf{Z}, x \geq 2, x \text{ is not prime}\}$	$1/16, 1/36, 1/64, 1/81$

Question 3

Let $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$, $B = \{2, 4, 6, 8, 10, 12, 14\}$, $C = \{1, 3, 6, 9, 12, 15, 17, 20\}$ and the universal set is $U = \{x \mid x \in \mathbf{Z}, 0 \leq x \leq 20\}$. Determine the contents of these resulting sets:

Set Expression	Resulting Set
$(A \cup B) \cap C$	$\{1, 3, 6, 9, 12, 15, 17\}$
$C - B$	$\{1, 3, 9, 15, 17, 20\}$
\overline{B}	$\{0, 1, 3, 5, 7, 9, 11, 13, 15, 16, 17, 18, 19, 20\}$
$C \oplus B$	$\{1, 2, 3, 4, 8, 9, 10, 14, 15, 17, 20\}$

Question 4

Define the sets $A = \{1, 2, 6\}$, $B = \{2, 3, 4\}$, $C = \{5\}$, $D = \{x \mid x \in \mathbf{Z}, 1 \leq x \leq 6\}$, and $E = \{x \mid x \in \mathbf{Z}, 1 < x < 6\}$.

Answer the following questions:

- Can set D be divided into the partitions A , B , and C ? If not, which condition of a partitioning is not satisfied?
- Can set D be divided into the partitions B and C ? If not, which condition of a partitioning is not satisfied?
- Can set E be divided into the partitions B and C ? If not, which condition of a partition is not satisfied?
- What are the different partitionings that can be created from the set $(A \cap B) \cup C$.

Answers:

- No - Not pairwise disjoint

- No - Union does include all of D
- Yes

$$(A \cap B) \cup C = \{2, 5\}$$

Partitions:

- $\{2\}, \{5\}$
- $\{2, 5\}$

Question 5

The definition of set partitioning is a general definition. In many applied cases, additional constraints are imposed. In computing, an operating system will need to partition a collection of tasks to describe the order that they are executed. Here are a list of tasks (where priority 1 is high, 2 is medium, and 3 is low)

- Task A: Requires 3 CPU cores, 8 GB Memory, Priority 1
- Task B: Requires 4 CPU cores, 10 GB Memory, Priority 3
- Task C: Requires 2 CPU cores, 6 GB Memory, Priority 2
- Task D: Requires 3 CPU cores, 6 GB Memory, Priority 2
- Task E: Requires 1 CPU core, 4 GB Memory, Priority 1

Assume the computer has 8 cores and 32 GB memory available. If we divide the tasks into n partitions, then the tasks in each partition would run concurrently and each partition would run sequentially.

- Why would this be a poor partitioning given the computer constraints: $\{A\}, \{B\}, \{C\}, \{D\}, \{E\}$

Answer: Higher priority items will not be completed first. There are unused resources.

- Why would this be a poor partitioning given the computer constraints: $\{A, B, C, D, E\}$

Answer: There are not enough resources on the computer to run them at the same time

- Propose a valid partitioning that would be more ideal given the computer constraints. Explain why it is more ideal.

Answer: $\{A, E, C\}, \{D, B\}$

The first partition will use 6 cores and 18 GB of memory and completes both high priority tasks plus one more. The second partition finishes the remaining tasks using 7 cores and 16 GB of memory. All the work will be done in two frames.

Question 6

Consider the following sets where the universal set for this problem is \mathbb{N} (does not include 0):

- A = The set of all even numbers.
- B = The set of all prime numbers.
- C = The set of all perfect squares.
- D = The set of all multiples of 10.

Using these sets and various set operators (e.g. union, intersection, complement, subset, etc...), represent the following statements in set notation:

- All multiples of 10 are even numbers.
- None of the perfect squares are prime numbers.
- If you take all the prime numbers, all the even numbers, all the perfect squares, and all the multiples of 10, you still won't have all the natural numbers. Can you write this without using \mathbb{N} in your answer?

Answers:

- $D \subseteq A$
- $B \cap C = \emptyset$ (disjoint)
- $\overline{(A \cup B \cup C \cup D)} \neq \emptyset$