

# CSE 280 Challenge Set 01 - Solutions

(c) BYU-Idaho

## Question 1

Let the following statements be given:

- $a$  = "You used the pool in the afternoon."
- $b$  = "You cleaned up after lunch."
- $c$  = "You must clean up after dinner."

### Part 1

Use connectives (operators) to translate the following statement into a compound proposition:

"If you used the pool in the afternoon and you didn't clean up after lunch, then you must clean up after dinner."

Answer:  $(a \wedge \neg b) \rightarrow c$

### Part 2

Construct a truth table for the compound proposition you found in Part 1. The eight rows of your table should correspond to the eight different possibilities for  $a$ ,  $b$ , and  $c$ .

Answer:

$a$	$b$	$c$	$a \wedge \neg b$	ANS
T	T	T	F	T
T	T	F	F	T
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

### Part 3

Suppose that the statement given in part 1 is false. What must be true about your pool usage and cleanup duties? Explain how to justify your answer using the truth table.

Answer: The pool was used and neither the lunch nor the dinner was cleaned up.

## Question 2

Use a truth table to prove that the statement  $[(p \vee r) \wedge (\neg p)] \rightarrow r$  is always true or always false, no matter what  $p$  and  $r$  are. What do we call this type of statement?

Answer: A tautology

$p$	$r$	$p \vee r$	$\neg p$	$(p \vee r) \wedge (\neg p)$	ANS
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

## Question 3

Let  $s$  be the following statement: "If it's raining, then the ground is wet."

### Part 1

Give the inverse of  $s$ . Is this the same as  $s$ ?

Answer: If its not raining, then the ground is not wet. -- No

### Part 2

Give the converse of  $s$ . Is this the same as  $s$ ?

Answer: If the ground is wet, then it is raining. -- No

### Part 3

Give the contrapositive of  $s$ . Is this the same as  $s$ ?

Answer: If the ground is not wet, then its not raining. -- Yes

## Question 4

Determine if the following propositions written in English are True or False:

- If 2 is even, then 5 is prime. - True
- If 3 is even, then 6 is prime. - True
- If 5 is odd, then 8 is prime. - False
- If 8 is odd, then 11 is prime. - True

- 10 is even if and only if 4 is prime - False
- 11 is even if and only if 6 is prime - True

## Question 5

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Simplify the following using a truth table:

$$\neg(A \rightarrow B) \vee \neg(A \vee B)$$

Answer:  $\neg B$

## Question 6

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The following "ugly-looking" multi-line compound proposition is a Tautology. Can you explain why without using a truth table. Hint: Refer back to Question 3 above.

$$((A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A)) \wedge ((B \rightarrow A) \leftrightarrow (\neg A \rightarrow \neg B)) \wedge$$

$$((B \rightarrow C) \leftrightarrow (\neg C \rightarrow \neg B)) \wedge ((C \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg C)) \wedge$$

$$((A \rightarrow C) \leftrightarrow (\neg C \rightarrow \neg A)) \wedge ((C \rightarrow A) \leftrightarrow (\neg A \rightarrow \neg C))$$

Answer: Either both the implication and contra-positive are True or either both the implication and the contra-positive are False. This is because the implication and contra-positive are always equal to each other.