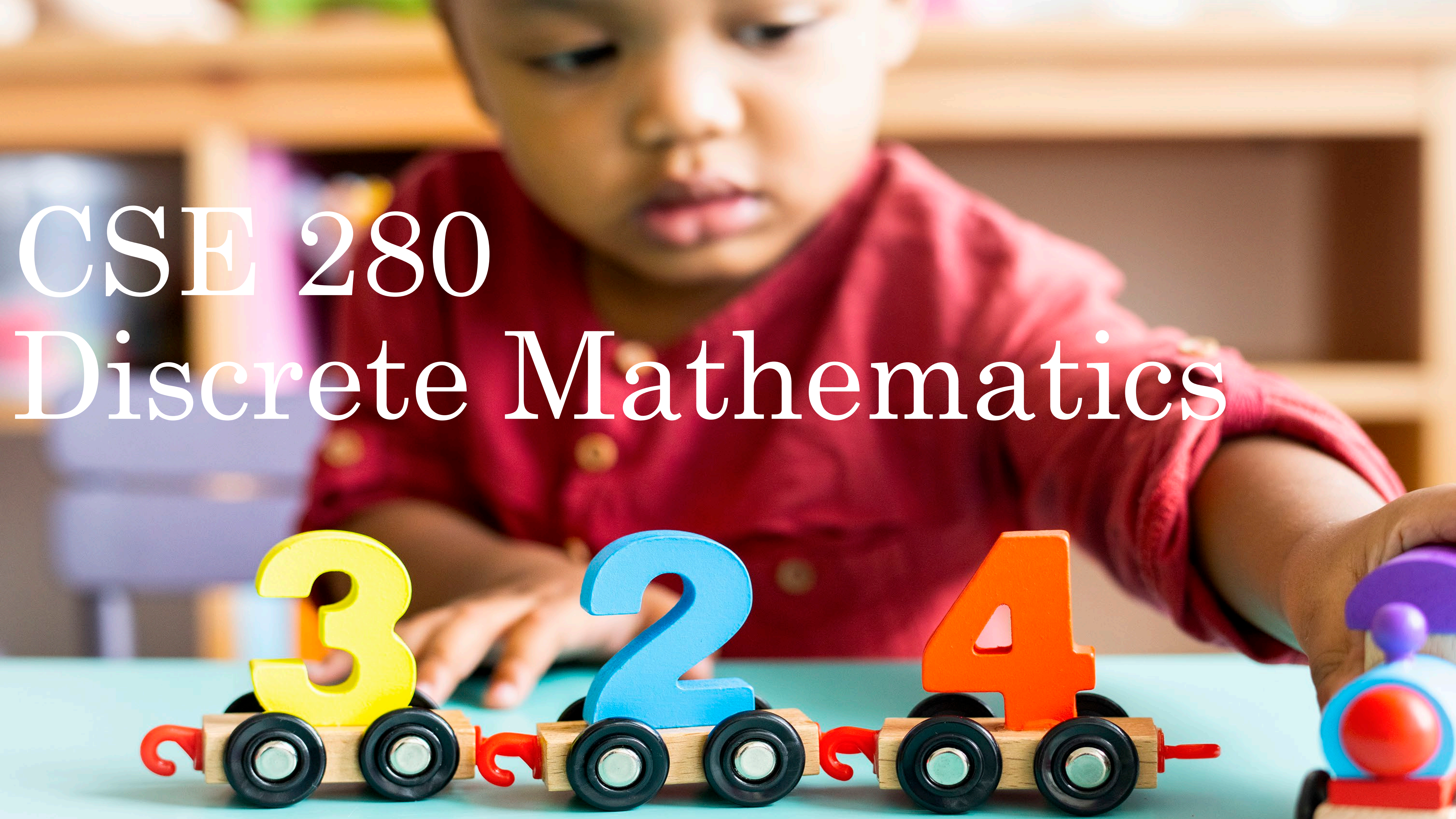
CSE 280
Discrete Mathematics



What is Discrete Mathematics?

"the branch of mathematics dealing with objects that can assume only distinct, separated values" – Wolfram Mathworld

In contrast with continuous mathematics, which deals with things that vary smoothly / continuous numbers (e.g., calculus)

https://mathworld.wolfram.com/DiscreteMathematics.html

Topics

Propositions and Logic

Predicates and Quantifiers

Sets

Functions

Relations

Number Theory
(Integers, prime numbers)

Combinatorics

Sequences and Summations

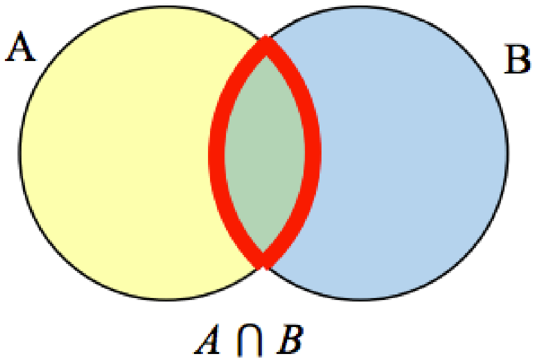
Probability

Graphs

Trees

$p \wedge q$ $\quad p \vee q$

$\forall x\, P(x)$ $\quad \exists x\, P(x)$



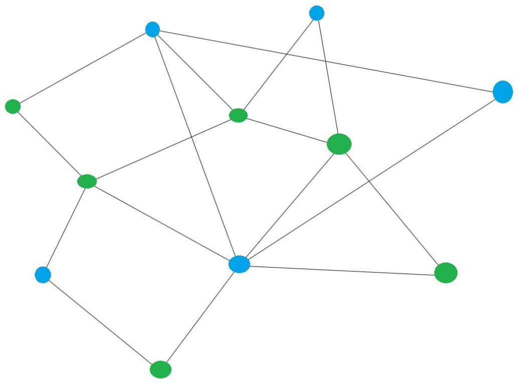
$F(x) = x^2, x \in \mathbb{R}$

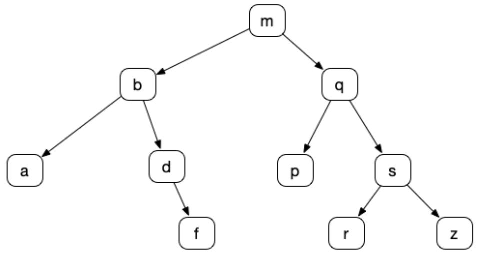
$R = \{(a,b) \mid a + b = 6\}$

2,3,5,7,11,13,17,19…

$\binom{11}{3}$

$$\sum_{i=1}^{4} i = 1 + 2 + 3 + 4 = 10$$





Propositions & Logic

$$p \land q \to r$$

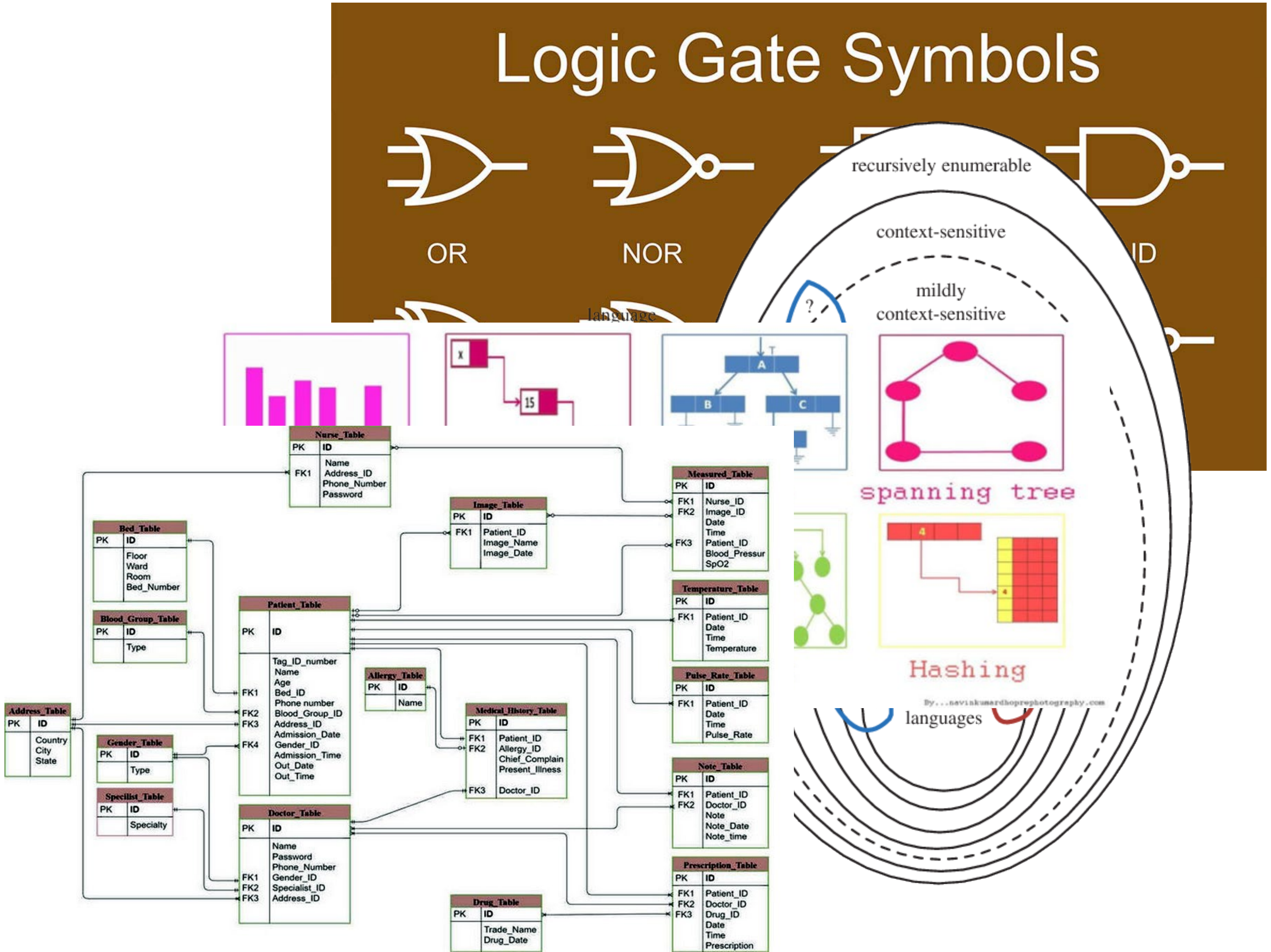
$\neg p \lor \neg q$

$$\neg(p \land r) \lor (q \land s)$$

$$\big((p \lor q) \land (\neg p \lor r)\big) \to (q \lor r)$$

Why do I need this?

- Rules of Logic are everywhere:
  - Mathematical reasoning
  - Computer circuits (Logic gates)
  - Formal Languages (computer languages)
  - Data Structures
  - Algorithm Design
  - Compiler Design
  - Relational Database Theory
  - Computability Theory
  - …
- All built on a foundation of formal logic



Why do I need this?

- Why is it important to think logically?
  - Logic sharpens your thinking, allowing it to “cut” more powerfully.

- Why know logic (formal or informal)?
  - Logic gives you a means to mechanize reasoning.
  - Mechanizing reasoning means programming a computer to do it.

Propositions

A proposition is...

A declarative sentence.

In other words, a sentence that is either ***True*** or ***False***

The sky is blue

Today is Tuesday

We are in a discrete math class

2 + 2 = 5

Propositions

We represent propositions using a variable, such as p, q, r, s

Examples:

p = The sky is blue

q = It is Tuesday

r = My dog's name is Eevee

Propositions

We can build compound propositions by following these rules:

A proposition is...

A ***variable*** representing a ***declarative sentence***	p
proposition preceded by ***not***	$\neg p$
proposition *connective* ***proposition***	$p \wedge q$ $p \vee q$ $p \oplus q$

$p =$ We are in a discrete math class

$q =$ It is Tuesday

$p \wedge q =$ We are in a discrete math class ***and*** it is Tuesday.

Connective	Name	Meaning
$\wedge$	conjunction	and
$\vee$	disjunction	inclusive or "either or"
$\oplus$	exclusive or	exclusive or "one or the other but not both"

Translating to/from English

p = It is raining

q = It is sunny

r = We will go outside

Translate the following into English:

$p \lor q$	It is raining or it is sunny.
$(p \land \neg r) \lor (q \land r)$	It is raining and we will not go outside, or it is sunny and we will go outside.
$p \land q \land r$	It is raining and it is sunny and we will go outside.

Logic symbols in Python

Name	Logic	Example	Python	Python Example
conjunction	$\wedge$	$p \wedge q$	`and`	`p and q`
disjunction	$\vee$	$p \vee q$	`or`	`p or q`
exclusive or	$\oplus$	$p \oplus q$	`^`	`p ^ q`
negation	$\neg$	$\neg p$	`not`	`not p`

Examples

Truth Tables

How many rows will be in the truth table for the following propositions?

$p \land q$ 4 rows

There are 2^n rows in a truth table with n variables

$p \lor q \land r$ 8 rows

$(p \land \neg q) \lor (r \land q)$ 8 rows

$p \land q \land r \land s$ 16 rows!

Truth Tables

Let's create the following truth table.

$$(p \land \neg q) \lor (r \land q)$$

Practical Uses of Logical Operators

- Search Engines
- Conditionals in programming
- Can you think of any others?

Group Activity

With a group of 2-3 people, work through the following Additional exercises at the bottom of reading 1.2

- 1.2.1

Conditionals or Implications

$$p \to q$$

p = You pass the final exam

q = You will get an A in the class

Conditional statement: If you pass the final exam, then you will get an A in the class.

(If you don't pass the final exam, then we don't know if you'll get an A or not. It is not implied by this statement)

Conditionals or Implications

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \to q$

p: You are in this class
q: You are a BYU-I student

if p, then q

if p, q

p only if q

p implies q

p is sufficient for q

q if p

q whenever p

q is necessary for p

You being in this class is sufficient *for me to conclude that* you are a BYU-I student.

Python example

Conditionals: $p \to q$

$p \to q$

if p, then q

if p, q

p only if q

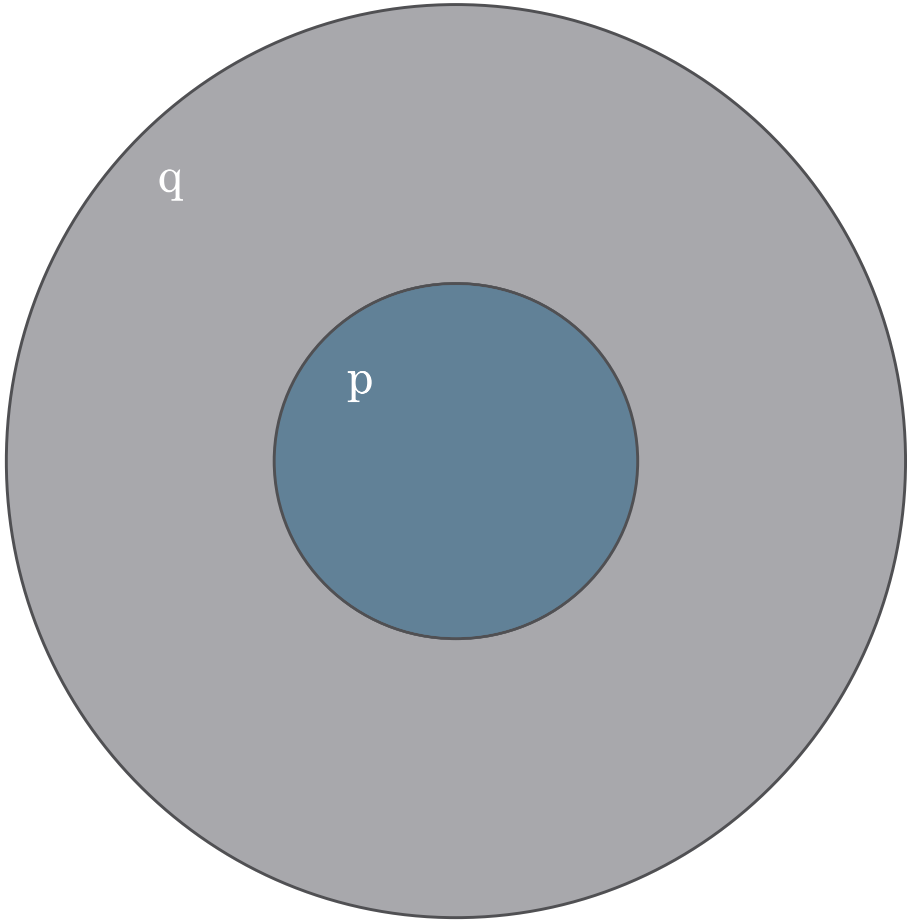
p implies q

p is sufficient for q

q if p

q whenever p

q is necessary for p



p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditionals: $p \rightarrow q$

$p = It\ is\ sunny$

$q = We\ will\ go\ outside$

Conditional:	$p \rightarrow q$	If it is sunny, then we'll go outside.
Converse:	$q \rightarrow p$	If we go outside, then it is sunny.
Inverse:	$\neg p \rightarrow \neg q$	If it is not sunny, then we will not go outside.
Contrapositive:	$\neg q \rightarrow \neg p$	If we do not go outside, then it is not sunny.

Conditionals: $p \rightarrow q$

$p = \textit{You pass the exam}$

$q = \textit{You get an A in the class}$

Conditional:	$p \rightarrow q$	If you pass the exam, then you get an A in the class.
Converse:	$q \rightarrow p$	If you get an A in the class, then you passed the exam.
Inverse:	$\neg p \rightarrow \neg q$	If you didn't pass the exam, then you won't get an A in the class.
Contrapositive:	$\neg q \rightarrow \neg p$	If you don't get an A in the class, then you didn't pass the exam.

Biconditional

$$p \longleftrightarrow q$$

p: You get an A in the class
q: You pass the final exam

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

You will get an A in the class if and only if you pass the final exam.

p if and only if q
p iff q
p is necessary and sufficient for q
if p then q and if q then p
if p then q and conversely

Group Activity

With a group of 2-3 people, work through the following Additional exercises at the bottom of reading 1.3

- 1.3.1