CSE 280 Challenge Set 04 - Solutions

Question 1

For each of these functions, determine if they are well-defined, one-to-one, onto, and/or a bijection. For these problems, let $A = \{x \mid x \in \mathbf{Z}, 0 < x < 5\}$ and let $B = \{x \mid x \in \mathbf{Z}, 0 \leq x \leq 5\}$. You may want to draw a digraph of each function (with dots and arrows). Recall that if not well-defined, then it is neither 1-1, onto, nor a bijection.

Function	Well Defined	1-1	Onto	Bijection
$f : A \to A,\ \text{where } f = \{(1,2),(2,2),(3,3),(4,3)\}$	x			
$f : A \to A,\ \text{where } f = \{(1,1),(2,2),(3,4),(4,3)\}$	x	x	x	x
$f : A \to A,\ \text{where } f = \{(2,2),(3,4),(4,3)\}$				
$f : A \to B,\ \text{where } f = \{(1,2),(2,3),(3,4),(4,5)\}$	x	x		
$f : B \to A,\ \text{where } f = \{(0,1),(1,1),(2,2),(3,2),(4,4),(5,3)\}$	x		x	

Question 2

For each of these functions, determine if they are well-defined, one-to-one, onto, and/or a bijection. Consider using a graphing calculator like https://www.desmos.com.

Function	Well Defined	1-1	Onto	Bijection
$f : \mathbf{Z} \to \mathbf{Z},\ \text{where } f(x) = 5x - 7$	x	x		
$f : \mathbf{Z}^+ \to \mathbf{Z}^+,\ \text{where } f(x) = \lceil \frac{x}{5} \rceil$	x		x	
$f : \mathbf{R} \to \mathbf{R},\ \text{where } f(x) = x^5 - 3x$	x		x	
$f : \mathbf{R}^+ \to \mathbf{R},\ \text{where } f(x) = \frac{1}{x}$	x	x		
$f : \mathbf{R} \to \mathbf{R},\ \text{where } f(x) = \log_5(x^2)$				
$f : \{x \mid x \in \mathbf{R}, x > 0\} \to \mathbf{R},\ \text{where } f(x) = \log_5(x^2)$	x	x	x	x
$f : \{x \mid x \in \mathbf{R}, x \neq 0\} \to \mathbf{R},\ \text{where } f(x) = \log_5(x^2)$	x		x	

Answer:

- Not onto because you can't get ever integer from an integer.
- Not 1-1 because multiple x map to the same value
- Not 1-1 because 3 different values of x map to the same $y = 0)$

- Not onto because missing 0 and negative
- Not well defined at x=0
- All
- Not 1-1 because 2 x map to the same value

Question 3

Part 1

Find the inverse $f^{-1}(x)$ of the following functions:

- $f : \mathbf{R} \to \mathbf{R}, \text{ where } f(x) = (x - 32) * 5/9$
- $f : \mathbf{R} \to \{x \mid x \in \mathbf{R}, x > 0\} \text{ where } f(x) = \frac{1}{2^x}$
- $f : \mathbf{R} \to \mathbf{Z}, \text{ where } f(x) = \lfloor x \rfloor$
- $f : \mathbf{R} \to \mathbf{R}, \text{ where } f(x) = x^2$
- $f : \mathbf{R} \to \mathbf{R}, \text{ where } f(x) = x^3$

Answer:

- $f^{-1}(x) = (\frac{9}{5} * x) + 32$
- $f^{-1}(x) = \log_2 \frac{1}{x}$
- Not a bijection (not 1-1). No inverse exists.
- Not a bijection (not 1-1 or onto). No inverse exists.
- $f^{-1} = \sqrt[3]{x}$

Part 2

Select a value of x (per the domain) and calculate $(f^{-1} \circ f)(x)$ for both functions in Part 1 to demonstrate that the inverse was successfully created.

- $f(41) = 5; f^{-1}(5) = 41$
- $f(2) = \frac{1}{4}; f^{-1}(\frac{1}{4}) = log_2 4 = 2$
- N/A
- N/A
- $f(-5) = -125; f^{-1}(-125) = -5$

Question 4

Define the following two functions:

- $f : \mathbf{R} \to \mathbf{R}, \text{ where } f(x) = \lceil x \rceil - x$
- $g : \mathbf{R} \to \mathbf{R}, \text{ where } g(x) = x - \lfloor x \rfloor$

Part 1

Graph function f using a graphing calculator. Explain what this so-called "sawtooth" function represents.

Answer:

f reprsents the positive fractional part of x

Part 2

Based on your answer above, does an inverse to f exist?

Answer:

No, an inverse does not exist because we can't determine the original real number from the fractional part.

Part 3

Determine what $f(x) + g(x)$ is equal to.

Answer:

$f(x) + g(x) = \lceil x \rceil - \lfloor x \rfloor$

- If $x \in \mathbf{Z}$ then this is equal to 0
- Otherwise, this is equal to 1