CSE 280 Challenge Set 02 - Solutions

Question 1

Part 1

The NAND operator $\uparrow$ is defined by the following truth table. Use truth tables to show that $p \uparrow q$ is logically equivalent to $\neg(p \wedge q)$.

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

Answer:

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Part 2

Use the result from Part 1 to simplify: $(p \uparrow p) \uparrow (q \uparrow q)$.

Answer:

$\neg(p \wedge p) \uparrow \neg(q \wedge q)$ - NAND Equivalence (above)

$\neg p \uparrow \neg q$ - Indempotent Law

$\neg(\neg p \wedge \neg q)$ - NAND Equivalence

$p \vee q$ - De Morgan

Note: You can create all logical operators using just NAND.

Question 2

The domain of the following predicates is the set of all plants.

$P(x) =$ "x is poisonous."

$Q(x) =$ "Jeff has eaten x."

Translate the following statements into prediate logic:

- Some plants are poisonous.
- Jeff has never eaten any poisonous plant.
- There are some non-poisonous plants that Jeff has not eaten.
- All plants are poisonous and Jeff hasn't eaten any of them

Answer:

- $(\exists x)P(x)$
- $(\forall x)(P(x) \rightarrow \neg Q(x))$ - Note: It does not say anything about non-poisonous therefore we use the conditional ($\rightarrow$)
- $(\exists x)(\neg P(x) \wedge \neg Q(x))$
- $(\forall x)(P(x) \wedge \neg Q(x))$ - Note: This says that there are no non-poisonous plants in existance therefore we use the $\wedge$

Question 3

In the domain of all nonzero integers, let $I(x, y)$ be the predicate "x/y is an integer." Determine whether the following statements are true or false, and explain why. Hint: Find values of x and y for which $I(x, y)$ is true.

- $(\forall y)(\exists x)I(x, y)$
- $(\exists x)(\forall y)I(x, y)$

Answer:

- True - For all integers y, there exists an integer x for which x/y is an integer. This happes when $x = y$
- False - There exists at least one integer x such that all integers y cause x/y to be an integer. There is no integer that is divisible by all other integers. (Not all y will divide any x)

Question 4

The domain of the following predicates is all integers greater than 1:

- $P(a) =$ "a is prime."
- $Q(a, b) =$ "a divides b."

Consider the following statement:

For every x that is not prime, there is some prime y that divides it. This is a true statement.

Part 1

Write the statement in predicate logic. Hint: There is a conditional ($\rightarrow$) in this statement.

Answer: $(\forall x)(\neg P(x) \rightarrow (\exists y)(P(y) \wedge Q(y,x)))$

Part 2

Formally negate the statement (moving the $\neg$ as far to the right as possible).

Answer:

$(\exists x)\neg(P(x) \vee (\exists y)(P(y) \wedge Q(y,x)))$

$(\exists x)(\neg P(x) \wedge \neg(\exists y)(P(y) \wedge Q(y,x)))$

$(\exists x)(\neg P(x) \wedge (\forall y)\neg(P(y) \wedge Q(y,x)))$

$(\exists x)(\neg P(x) \wedge (\forall y)(\neg P(y) \vee \neg Q(y,x)))$

Part 3

Write the english translation of your negated sentance. Is the negated sentance a True statement? If you are having difficulty writing this in english, consider modifying the result in Part 2 to change any $\vee$ to a $\rightarrow$.

Answer:

We can rewrite this to use the implies:

$(\exists x)(\neg P(x) \wedge (\forall y)(P(y) \rightarrow \neg Q(x,y)))$

There is a nonprime x such that all primes y do not divide it.

Or restated, There is a nonprime that is not divisible by a prime.

This is not a true statement.