Exo 2 Smtc

Vous devez trouver $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

$$\frac{\partial}{\partial x} x \ln(y^2+1) = \ln(y^2+1)$$

$$\frac{\partial}{\partial y} x \ln(y^2+1) = x \frac{\partial}{\partial y} \ln(y^2+1)$$

$$\frac{\partial}{\partial y}\frac{\partial}{\partial x} f_1 = \frac{\partial}{\partial y} \ln y^2+1 = \frac{2y}{y^2+1}$$

$$\frac{\partial}{\partial x}\frac{\partial}{\partial y} f_1 = \frac{\partial}{\partial x} \frac{2xy}{y^2+1} = \frac{2y}{y^2+1} \frac{\partial}{\partial x} x \quad (1)$$

$$\frac{\partial}{\partial x} \frac{x^2}{x+2y} = \frac{x^2+4xy}{(x+2y)^2}$$

$$\frac{\partial}{\partial y} \frac{x^2}{x+2y} = \frac{-2x^2}{(x+2y)^2}$$

$$\frac{\partial}{\partial y} \frac{x^2+4xy}{(x+2y)^2} = \frac{(x+2y)^2 \times 4x - (x^2+4xy) \times 4y(x+2y)}{(x+2y)^4}$$

$$= \frac{(x+2y) \times 4x - (x^2+4xy) \times 4y}{(x+2y)^3}$$

= vous devez finir

$$\frac{\partial}{\partial x} \frac{-2x^2}{(x+2y)^2} = -2 \frac{(x+2y)^2 \times 2x - 2(x+2y)x^2}{(x+2y)^4}$$

$$= -2 \frac{(x+2y) \times 2x - 2x^2}{(x+2y)^3} = \frac{-8xy}{(x+2y)^3}$$

Exo 3

$$\frac{\partial}{\partial x}\left[\frac{xy(x^2-y^2)}{x^2+y^2}\right] = y\frac{\partial}{\partial x}\frac{x^3}{x^2+y^2} + y^3\frac{\partial}{\partial x}\frac{x}{x^2+y^2}$$

$$= y\frac{(x^4+4x^2y^2-y^4)}{(x^2+y^2)^2}$$

$$\frac{\partial}{\partial y}\left[\frac{xy(x^2-y^2)}{x^2+y^2}\right] = x\frac{(x^4-4x^2y^2-y^4)}{(x^2+y^2)^2}$$

Vous pourrez chercher $\frac{\partial^2}{\partial x \partial y}$ et $\frac{\partial^2}{\partial y \partial x}$ comme d'habitude mais il y a une astuce !

$$\frac{\partial f}{\partial x}(0,t) = -\frac{t^4}{t^4} = -1 \qquad \frac{\partial f}{\partial y}(t,0) = \frac{t^4}{t^4} = 1$$

$$\Rightarrow \frac{\partial f}{\partial y \partial x} = \frac{d}{dt}\frac{\partial f}{\partial x}(0,t) = \frac{d}{dt}(-1) = 0$$