

Title: An applied analysis to prove Goldbach's strong conjecture.

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**Abstract:**

Goldbach's strong conjecture is one of the oldest number theoretic thought in mathematics. This conjecture was posted in 1742 and, despite being obviously true, has remained unproven. Within this paper, we apply a method to prove its correctness. We are very attracted to this approach because the statement is based on the basic concept of mathematics. In school level mathematics, we explain the clear idea about Christian Goldbach's sir, first one who thought the connection between even numbers and prime numbers. In reality we are going to describe a useful logical method to find out that connections between even and prime numbers. We think that this is the process more practical than theoretical about the Goldbach's strong conjecture. And we, the professionals can give lectures and demonstrates logically to our students in their simple understanding way, without breaking its continuity. In this paper, the method is fully derived depending on teaching techniques. Through this method, we are able to place the conjecture in accurate and exact position in the world of mathematics. In addition, a proof of the conjecture would be an interesting event for the students and readers.

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## Introduction:

The Goldbach's strong conjecture is an unsolved problem in the history of mathematics. It was listed as one of Landau's problems (1912) on prime numbers. In 1742, Prussian mathematician Christian Goldbach wrote Leonard Euler a letter in which Goldbach proposed the following conjecture. It seems that every even number larger than 2 can be written as the sum of three primes. Goldbach considered 1 to be a prime number; this convention has since been abandoned. Euler replied to Goldbach that "every positive even integer<sup>4</sup> can be expressed as the sum of two primes, although I cannot prove it". Today, Goldbach's conjecture is written as follows: "every even positive number greater than 2 can be written as the sum of two primes". Goldbach's conjecture. In a letter to Euler in 1742, Christian Goldbach hazarded the guess that every even integer is the sum of two numbers that are either primes or 1. A somewhat more general formulation is that every even integer greater than 4 can be written as a sum of two odd numbers.

$4=2+2=1+3$ ;  
 $6=3+3=1+5$ ;  
 $8=1+7=3+5$ ;  
 $10=3+7=5+5$ ;  
 $12=5+7=1+11$ ;  
 $14=3+11=7+7=1+13$ ;  
 $16=3+13=5+11$ ;  
 $18=5+13=7+11=1+17$ ;  
 $20=3+17=7+13=1+19$ ;  
 .....

## Methodology:

We are starting to check the correctness of Goldbach's strong conjecture by creating a pair of expression using the basic concept of number theory. Christian Goldbach's original thought is very simple; that is, there is a connection between two prime numbers that have been added together and even numbers. The whole creation can be intuited simply by using number theory in its truest sense. Statement: "All positive even numbers greater than 2 can be represented as the sum of two primes". We know that the sum of any two positive odd numbers is always a positive even number; that is, if  $a$  and  $b$  are two positive odd numbers, then their sum is always a positive even number.

Therefore, we can write  $a+b=2n$ , where  $2n$  is an even number. We must limit the choice of numbers on the left side of the equation to only the set of all positive odd numbers. We know that the set of prime numbers belongs to the set of positive odd numbers. The set of positive odd numbers  $\{1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,\dots\}$  consists of

1. Prime numbers: 2. Composite odd numbers: 3. The positive odd number 1, which is neither prime nor composite.

Now,  $2n=a+b$ ;  $a$  and  $b$  will be odd positive numbers such that

If  $a=1$ , then  $b=\text{any composite odd number}$  and vice versa.

If  $a=1$ , then  $b=\text{any prime number}$ .

If  $a=\text{any composite odd number}$ , then  $b=\text{any prime number}$ .

If  $a=\text{any prime number}$ , then  $b=\text{any composite odd number}$ .

If  $a=\text{any prime number}$ , then  $b=\text{any prime number}$ .

From the above conditions-

we can accept only the last one,  $2n=a$  (any prime number)  $+b$  (any prime number) ..... (1)

Therefore, we accept the values of  $a$  and  $b$  so that they can only be two primes after fulfillment of the condition below.

\*The product of  $a$  and  $b$  is greater than their sum,  $a.b > (a+b)$ .

\*\* The greatest common divisor of  $a$  and  $b$  is equal to 1 i.e.,  $\gcd(a, b) = 1$ .

\*\*\*The least common multiple, i.e.,  $\text{LCM}(a,b)$ , which is equal to  $a.b$ , is divisible by  $a$  or divisible by  $b$  or divisible by their product  $a.b$ . The LCM cannot be divisible by any other odd positive integers.

## Results:

Now, from (1), we can try to establish types of relations that consist of two parts:

$2n=n+n$ , where  $n$  is prime number.

Additionally, for  $k$  is a positive integer,  $(2k-1)$  is always a positive odd integer. Therefore, we can write  $2n$  as

$2n=2(2k-1)+2n-2(2k-1)$ .

$2n=(2k-1)+(2k-1)+n-2(2k-1)+n$ .

$\Rightarrow 2n=[(2k-1)]+[(2k-1)+\{n-2(2k-1)\}+n]$ .

$\Rightarrow 2n = (2k-1) + \{(2k-1) + m + n\}$ , where,  $m = n - 2(2k-1)$ , and  $(2k-1) < n$ ,  $k$  is any positive integer, and  $m$  is any positive or negative integer or the integer 0, which is the main calculating factor.

Now, the positive odd numbers  $a = \{(2k-1)\}$  and  $b = \{m + n + (2k-1)\}$

The product of  $(2k-1)$  and  $\{(2k-1) + m + n\} > (2k-1) + \{m + n + (2k-1)\}$

(\*\*)  $\gcd\{(2k-1), m + n + (2k-1)\} = 1$

(\*\*\*) LCM  $[(2k-1), \{(2k-1) + m + n\}]$  is divisible by  $(2k-1)$ ,  $\{(2k-1) + m + n\}$ , or LCM itself. The LCM is not divisible by any other positive odd numbers. That is,  $2n = a + b$ , where  $a = (2k-1)$  and  $b = \{(2k-1) + m + n\}$ .

Proof: Necessary condition

RHS,  $\{(2k-1)\} + \{(2k-1) + m + n\} = (2k-1) + (2k-1) + (n - 2(2k-1)) + n = 4k - 2 + 2n - 4k + 2 = 2n = \text{LHS}$ .

Sufficient condition: Let  $p$  and  $q$  be two positive odd numbers that are primes. Then,  $(p + q)$  must be a positive even number, therefore,  $2n = p + q$ , where  $p = (2k-1)$  and  $q = \{(2k-1) + m + n\}$ . Thus, it is sufficient that any positive even number  $2n$  can be expressed as the sum of two primes.

Discussion:

Now, to verify the above relation, we will explain by using some examples:

Example 1: Let  $2n = 4$ ; then,  $n = 2$ , which is a prime number.

By using  $2n = n + n$ , we can write  $4 = 2 + 2$ .

Now applying our additional relation, we have

$(2k-1) < n \Rightarrow 2k-1 < 2$  here,  $n = 2$

$\Rightarrow 2k < 3$

$\Rightarrow k = 1$  [because  $k$  is a positive integer], so that by using the additional relation,

$2n = a + b = \{(2k-1)\} + \{(2k-1) + m + n\}$ .

we have for  $k = 1$ ,  $a = (2k-1) = 1$ . Now, we have to find the value of  $b = \{(2k-1) + m + n\}$

We have,  $m = n - 2(2k-1) = 2 - 2(1-1) = 2 - 2 = 0$

Therefore,  $b = \{(2k-1) + m + n\} = 1 + 0 + 2 = 3$ .

Clearly,  $1 \cdot 3 < 1 + 3$ . Therefore, it does not meet our conditions (\*), and we cannot accept values of  $a = 1$  and  $b = 3$ .

Thus, we can write  $4 = 2 + 2$ .

Example 2: Let  $2n = 6$ ; then,  $n = 3$ , which is a prime number.

By using  $2n = n + n$ , we can write  $6 = 3 + 3$ .

Now applying our additional relation, we have

$(2k-1) < n$

$\Rightarrow (2k-1) < 3$

$\Rightarrow 2k < 4 \Rightarrow k < 2$

$\Rightarrow k = 1$  [because  $k$  is a positive integer],

By using our additional relation  $2n = a + b = \{(2k-1)\} + \{(2k-1) + m + n\}$ ,

we have, for  $k = 1$ ,  $a = 2k-1 = 1$ . Now, we have to find the value of  $b = \{(2k-1) + m + n\}$

We have,  $m = n - 2(2k-1) = 3 - 2 \cdot 1 = 1$

Therefore,  $b = \{(2k-1) + m + n\} = 1 + 1 + 3 = 5$

Clearly,  $1 \cdot 5 < 1 + 5$

This does not meet our conditions (\*), and we cannot accept values of  $a = 1$  and  $b = 5$ . Thus, we can write  $6 = 3 + 3$

Example 3: Let  $2n = 8$ ; then,  $n = 4$ , which is not a prime number, so we have

$(2k-1) < n$

$\Rightarrow (2k-1) < 4$

$\Rightarrow 2k < 5$

$\Rightarrow k = 1 \& 2$ . [ $k$  is a positive integer]

By using the relation  $2n = \{(2k-1)\} + \{(2k-1) + m + n\}$ ,

We have  $k = 1$ ; then,  $a = (2k-1) = 1$ . Now, we have to find the value of  $b = \{(2k-1) + m + n\}$

We have,  $m = n - 2(2k-1) = 4 - 2 \cdot 1 = 2$ . Therefore,  $b = \{(2k-1) + m + n\} = 1 + 2 + 4 = 7$

Clearly,  $1 \cdot 7 < 1 + 7$ . It does not meet our conditions (\*), and we cannot accept values of  $a = 1$  and  $b = 7$ . We can take the value  $k = 2$ ; then,  $a = (2k-1) = 3$

We have to find the value of  $b$ :

We have  $m = n - 2(2k-1) = 4 - 2 \cdot 3 = (-2)$ ; Therefore,  $b = (2k-1) + m + n = 3 + (-2) + 4 = 5$ .

Now,  $3 \cdot 5 > 3 + 5$  so that it meets our condition (\*). In addition,  $\gcd(3, 5) = 1$ , so that it meets our condition (\*\*).

Additionally, LCM  $(3, 5) = 3 \cdot 5 = 15$  is divisible by 3, 5 and 15 (LCM itself). The LCM is not divisible by any other positive odd integers; it meets our condition (\*\*\*).

Therefore, by using our relation,  $2n = a + b = (2k-1) + \{(2k-1) + m + n\}$ ,

We can write  $8=3+5$ .

Example 4: Let  $2n=14$ ; then,  $n=7$ , which is a prime number  $\Rightarrow 14=7+7$ .

Additionally,  $(2k-1) < n$

$\Rightarrow (2k-1) < 7, \Rightarrow k < 4$

$\Rightarrow k=1,2,3$ . If  $k=1$ , then  $a=(2k-1)=1$ . Now, we have to find the value of  $b = \{(2k-1) + m + n\}$ .

We have,  $m=n-2(2k-1)=7-2 \cdot 1=5$

Therefore,  $b = \{(2k-1) + m + n\} = 1+5+7=13$ . Clearly,  $1 \cdot 13 < 1+13$

It does not meet our conditions (\*), and we cannot accept values of  $a=1$  and  $b=13$ .

Now, for  $k=2$ , we have  $a=(2k-1)=3$ ; moreover, we have  $m=n-2(2k-1)=7-2 \cdot 3=1$ .

Therefore,  $b=(2k-1) + m + n = 3+1+7=11$ .

Now,  $3 \cdot 11 > 3+11$  so that it meets our condition (\*). In addition,  $\gcd(3,11)=1$ ; so that it meets our condition (\*\*).

Additionally,  $\text{LCM}(3,11)=3 \cdot 11=33$ , which is divisible by 3, 11 and 33 (LCM itself). The LCM cannot be divisible by any other positive odd integers; thus, it satisfies our condition (\*\*\*). We can write  $14=3+11$

When  $k=3$ , then  $a=(2k-1)=5$ ; moreover, we have,  $m=n-2(2k-1)=7-2 \cdot 5=7-10=(-3)$ .

Then,  $b=\{(2k-1) + m + n\} = 5+(-3)+7=9$ .

Now,  $5 \cdot 9 > 5+9$ , so that it meets our condition (\*).

In addition,  $\gcd(5,9)=1$ ; so that it meets our condition (\*\*).

Additionally,  $\text{LCM}(5,9)=5 \cdot 9=45$  is divisible by 5, 9 and 45 (LCM itself). However, the LCM is also divisible by 3, so that it does not meet our condition (\*\*\*), and we cannot accept values of  $a=5$  and  $b=9$ . Thus,  $14=7+7=3+11$ .

Example 5: Let  $2n=20$ ; then,  $n=10$ , which is not a prime number. Therefore, by using our additional relation,

$2n=a+b=\{(2k-1)\} + \{(2k-1) + m + n\}$ ,

We have  $(2k-1) < n$

$\Rightarrow (2k-1) < 10 \Rightarrow k=1,2,3,4, \text{ and } 5$ .

For,  $k=1$ ,  $a=(2k-1)=1$ ; moreover, we have,  $m=n-2(2k-1)=10-2 \cdot 1=8$ . Then,  $b=(2k-1) + m + n = 1+8+10=19$ . Clearly,  $1 \cdot (19) < 1+19$ . It does not meet our conditions (\*), and we cannot accept values of  $a=1$  and  $b=19$ .

For,  $k=2$ , we have  $a=(2k-1)=3$ , so we can try to find the value of  $b$ .

We have  $m=n-2(2k-1)=10-2 \cdot 3=4$ ; thus,  $b=(2k-1) + m + n = 3+4+10=17$ .

Now,  $3 \cdot (17) > 3+17$ , so that it meets our condition (\*).

And,  $\gcd(3,17)=1$ ; so that it meet our condition (\*\*).

Additionally,  $\text{LCM}(3,17)=3 \cdot 17=51$ , which is divisible by 3, 17 and 51 (LCM itself). The LCM 7 and 51 (LCM itself). The LCM cannot be divisible by any other positive odd integers; thus, it satisfies our condition (\*\*\*). We can write  $20=3+17$ .

For  $k=3$ , we have  $a=(2k-1)=5$ , so we can try to find the value of  $b$ . Here,  $m=n-2(2k-1)=10-2 \cdot 5=0$ ;

Thus,  $b=(2k-1) + m + n = 5+0+10=15$ ,

Now,  $5 \cdot (15) > 5+15$ , so that it meets our condition (\*).

In addition,  $\gcd(5,15)=5$ ; so that it does not meet our condition (\*\*).

We cannot accept the values of  $a=5$  and  $b=15$ .

For  $k=4$ , we have  $a=(2k-1)=7$ , so we can try to find the value of  $b$ .

Here,  $m=n-2(2k-1)=10-2 \cdot 7=(-4)$ ; therefore,  $b=(2k-1) + m + n = 7+(-4)+10=13$ ,

Now,  $7 \cdot (13) > 7+13$ , so that it meets our condition (\*).

In addition,  $\gcd(7,13)=1$ ; so that it meets our condition (\*\*).

Additionally,  $\text{LCM}(7,13)=7 \cdot 13=91$ , which is divisible by 7, 13 and 91 (LCM itself). The LCM cannot be divisible by any other positive odd integers; thus, it satisfies our condition (\*\*\*).

We can write  $20=7+13$ .

For  $k=5$ , we have  $a=(2k-1)=9$ . Therefore, we have to find the value of  $b$ . Moreover, we have  $m=n-2(2k-1)=10-2 \cdot 9=-8$ . Thus,  $b=(2k-1) + m + n = 9+(-8)+10=11$ . Now,  $9 \cdot (11) > 9+11$ , so that it meets our condition (\*).

In addition,  $\gcd(9,11)=1$ ; so that it meets our condition (\*\*).

Additionally,  $\text{LCM}(9,11)=9 \cdot (11)=99$  is divisible by 9, 11 and 99 (LCM itself). However, the LCM is also divisible by 3, so that it does not meet our condition (\*\*\*), and we cannot accept values of  $a=9$  and  $b=9$ . We can write  $20=3+17=7+13$ .

Example 6: Let  $2n=28$ ; then,  $n=14$ , which is not a prime number.

Now, we can use our additional relation,  $2n=a+b=\{(2k-1)\} + \{(2k-1) + m + n\}$ .

Now,  $(2k-1) < n \Rightarrow (2k-1) < 14$

$\Rightarrow k=1,2,3,4,5,6 \text{ and } 7$ .

For  $k=1$ , we have  $a=(2k-1)=1$ , so we have to find the value of  $b$ .

Moreover, we have  $m=n-2(2k-1)=14-2.1=12$

Thus,  $b=(2k-1)+m+n=1+12+14=27$ . Clearly,  $1.(27)<1+27$ .

It does not meet our conditions (\*), and we cannot accept values of  $a=1$  and  $b=27$ . For  $k=2$ , we have  $a=(2k-1)=3$ .

We have  $m=n-2(2k-1)=14-2.3=8$ .

Therefore,  $b=(2k-1)+m+n=3+8+14=25$ .

Now,  $3.(25)>3+25$ , so that it meets our condition (\*).

In addition,  $\gcd(3,25)=1$ ; so that it meets our condition (\*\*).

Additionally,  $\text{LCM}(3,25)=3.(25)=75$  is divisible by 3, 25 and 75 (LCM itself). However, the LCM is also divisible by 5, so that it does not meet our condition (\*\*\*), and we cannot accept values of  $a=3$  and  $b=25$ . For  $k=3$ , we have  $a=(2k-1)=5$ , so we can try to determine the value of  $b$ . Here,  $m=n-2(2k-1)=14-2.5=4$ . Therefore,  $b=(2k-1)+m+n=5+4+14=23$ .

Now,  $5.(23)>5+23$ , so that it meets our condition (\*).

In addition,  $\gcd(5,23)=1$ ; so that it meets our condition (\*\*). Additionally,  $\text{LCM}(5,23)=5.23=115$ , which is divisible by 5, 23 and 115 (LCM itself). The LCM cannot be divisible by any other positive odd integers; thus, it satisfies our condition (\*\*\*). We can write  $28=5+23$ .

For,  $k=4$ , we have  $a=(2k-1)=7$ . Now, we have to find the value of  $b$ .

Here,  $m=n-2(2k-1)=14-2.7=0$ . Therefore,  $b=(2k-1)+m+n=7+0+14=21$ .

Now,  $7.(21)>7+21$ , so that it meets our condition (\*). In addition,  $\gcd(7,21)=7$ ; so that it does not meet our condition (\*\*). We cannot accept values of  $a=7$  and  $b=21$ .

For  $k=5$ , we have  $a=(2k-1)=9$ , and we have to find the value of  $b$ . Here,  $m=n-2(2k-1)=14-2.9=(-4)$ ; therefore,  $b=(2k-1)+m+n=9+(-4)+14=19$

Now,  $9.(19)>9+19$ , so that it meets our condition (\*).

In addition,  $\gcd(9,19)=1$ ; so that it meets our condition (\*\*).

Additionally,  $\text{LCM}(9,19)=9.(19)=171$  is divisible by 9, 19 and 171 (LCM itself). However, the LCM is also divisible by 3 so that it does not meet our condition (\*\*\*), and we cannot accept values of  $a=9$  and  $b=19$ .

For,  $k=6$ , we have  $a=(2k-1)=11$ , and we can try to find the value of  $b$ .

Here,  $m=n-2(2k-1)=14-2.11=(-8)$ ; therefore,  $b=(2k-1)+m+n=11+(-8)+14=17$ . Now,  $11.(17)>11+17$ , so that it meets our condition (\*). In addition,  $\gcd(11,17)=1$ ; so that it meets our condition (\*\*). Additionally,  $\text{LCM}(11,17)=11.17$ , which is divisible by the 11, 17 and  $(11.17)$  LCM itself. The LCM cannot be divisible by any other positive odd integers; thus, it satisfies our condition (\*\*\*). We can write  $28=11+17$ .

For  $k=7$ , we have  $a=(2k-1)=13$ , and we have to find the value of  $b$ .

Here,  $m=n-2(2k-1)=14-2.13=(-12)$ ; therefore,  $b=(2k-1)+m+n=13+(-12)+14=15$ .

Now,  $13.(15)>13+15$ , so that it meets our condition (\*).

In addition,  $\gcd(13,15)=1$ ; so that it does not meet our condition (\*\*).

Additionally,  $\text{LCM}(13,15)=13.(15)$ , is divisible by the 13, 15 and  $(13.15)$  LCM itself. However, the LCM is also divisible by 3 and 5 so that it does not meet our condition (\*\*\*), and we cannot accept values of  $a=13$  and  $b=15$ . Thus,  $28=5+23=11+17$ .

Conclusion: Every even integer greater than 2 can be expressed as the sum of two prime numbers. This statement is simple and quietly intuitive. It attracts everyone who loves mathematics. The statement compels us to think simply about the relation between even integers and prime numbers. By the above pair of formulae, Goldbach's strong conjecture can easily be validated and accorded its proper position in number theory. Therefore, we must conclude that Goldbach's strong conjecture is fully true.

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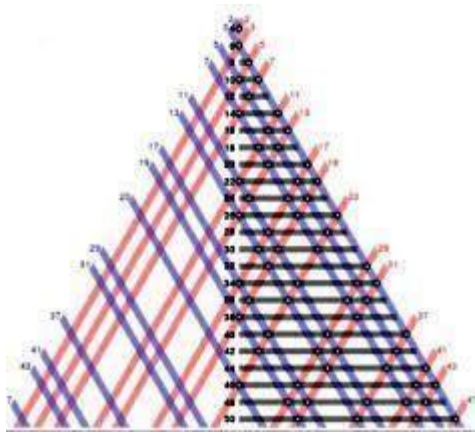


Figure 1: Python.

Figure legend title: Even integers are the sum of two prime numbers.

Calculations in python show that the blue and red lines both indicate the prime numbers 2,3,5,7, 11, ....respectively. The blue line represents prime number 2 at the top position of Python after intersection with the red line representing prime number 2. Therefore, we have  $4=2+2$ . The blue line representing prime number 3 and the red line representing prime number 3 intersect only one point. Therefore, we have  $6=3+3$ . Again, the blue line representing the prime number 3 and the red line representing the prime number intersect each other. Therefore, we obtain  $8=3+5$ . Next, the blue line representing the prime number 3 and the red line representing the prime number 7 intersect at a point, the blue line representing the prime number 5 and the red line representing the prime number 5 intersect at a point, but these intersecting points lie in a horizontal line. Therefore, we have  $10=3+7=5+5$ . From Python, the positive even numbers are represented as the points of intersections of the blue lines and the red lines. The relation of even numbers and prime numbers is displayed in the real line  $\mathbb{R}$  in the positive direction, which means positive integers or simply natural numbers  $\mathbb{N}$ . From the figure, this statement is intuitively easy to understand.