$$\begin{pmatrix} 4 & 2 & 4 \\ 0 & -5 & 3 \\ 1 & 1 & 1 \end{pmatrix} \longleftrightarrow \quad \boxed{\begin{array}{rcl} 4x + 2y + 4z &=& a \\ -5y + 3z &=& b \\ x + y + z &=& c \end{array}}$$

$\Rightarrow$ ① - 4 × ③

$$-2y = a - 4c$$

$$y = -\tfrac{1}{2}a + 2c$$

$\Rightarrow$ subs

$$3z = b + 5y$$

$$= b - \tfrac{5}{2}a + 10c$$

$$= -\tfrac{5}{2}a + b + 10c$$

Finalement

$$4x = a - 2y - 4z$$

$$= a + (a - 4c) - \tfrac{4}{3}\left(-\tfrac{5}{2}a + b + 10c\right)$$

$$= \tfrac{16}{3}a - \tfrac{4}{3}b - \tfrac{52}{3}c$$

Forme matricielle de la solution

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 16/3 & -4/3 & -52/3 \\ -1/2 & 0 & 2 \\ -5/2 & 1 & 10 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Inversion par operations sur les lignes

$$A \quad \begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ -1 & 1 & -2 & 0 & 1 & 0 \\ -1 & 3 & 1 & 0 & 0 & 1 \end{array} \quad I_3$$

$$\begin{array}{l} \\ L_2 \to L_2 + \tfrac{1}{2}L_1 \\ L_3 \to L_3 + \tfrac{1}{2}L_3 \end{array} \quad \begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -3/2 & 1/2 & 1 & 0 \\ 0 & 3 & 3/2 & 1/2 & 0 & 1 \end{array}$$

$$\begin{array}{l} \\ \\ L_3 \to L_3 - 3L_2 \end{array} \quad \begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -3/2 & 1/2 & 1 & 0 \\ 0 & 0 & 6 & -1 & -3 & 1 \end{array}$$

$$\begin{array}{l} \\ \\ L_3 \to L_3/6 \end{array} \quad \begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -3/2 & 1/2 & 1 & 0 \\ 0 & 0 & 1 & -1/6 & -1/2 & 1/6 \end{array}$$

$$\begin{array}{l} L_1 \to L_1 - L_3 \\ L_2 \to L_2 + \tfrac{3}{2}L_3 \\ \\ \end{array} \quad \begin{array}{ccc|ccc} 2 & 0 & 0 & 7/6 & 1/2 & -1/6 \\ 0 & 1 & 0 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1 & -1/6 & -1/2 & 1/6 \end{array}$$

$$I_3 \quad \begin{array}{ccc|ccc} 1 & 0 & 0 & 7/12 & 1/4 & -1/12 \\ 0 & 1 & 0 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1 & 1/12 & -1/2 & 1/6 \end{array} \quad A^{-1}$$