Ex7

$$I_1 = \int_0^{\frac{\pi}{2}} \sin^3 x \cos x \, dx = \int_0^{\pi/2} \sin^3 x \, d\sin x = \| c.v.\ \sin x = t \| =$$

$$0 \to 0, \quad \frac{\pi}{2} \to 1$$

$$= \int_0^1 t^3 dt = \frac{t^4}{4}\Big|_0^1 = \frac{1}{4}$$

$$I_2 = \int_0^1 e^u \cos(e^u) \, du = \int_0^1 \cos(e^u) \, de^u = \| c.v.\ e^u = t \| =$$

$$0 \to 1, \quad 1 \to e$$

$$= \int_1^e \cos t \, dt = \sin t \Big|_1^e = \sin e - \sin 1$$

$$I_3 = \int_0^{\pi/4} \tan^3 x \, dx = \int_0^{\pi/4} \tan x \left(\frac{1}{\cos^2 x} - 1\right) dx = \int_0^{\pi/4} \tan x \frac{1}{\cos^2 x} dx - \int_0^{\pi/4} \tan x \, dx = \|$$

c.v. 1: $u = \tan x$, $du = \frac{1}{\cos^2 x} dx$, $0 \to 0$, $\frac{\pi}{4} \to 1$, c.v. 2: $t = \cos x$, $dt = -\sin x \, dx$, $0 \to 1$, $\frac{\pi}{4} \to \frac{\sqrt{2}}{2}$ $\| =$

$$= \int_0^1 u \, du + \int_1^{\frac{\sqrt{2}}{2}} \frac{dt}{t} = \frac{u^2}{2}\Big|_0^1 + \ln|t| \Big|_1^{\frac{\sqrt{2}}{2}} = \frac{1}{2} + \ln\frac{\sqrt{2}}{2}$$

$$I_4 = \int_1^2 (3t-1)^{-2/3} dt = \| c.v.\ 3t - 1 = u \| = \int_2^5 u^{-2/3} \frac{du}{3} =$$

$$du = d(3t-1) = 3dt \Rightarrow dt = \frac{du}{3}$$

$$1 \to 2, \quad 2 \to 5$$

$$= \frac{1}{3} \frac{u^{1/3}}{1/3}\Big|_2^5 = 5^{\frac{1}{3}} - 2^{\frac{1}{3}}$$

$$I_5 = \int_1^2 \frac{u+1}{\sqrt{u^2+2u}} du = \int_1^2 \frac{\frac{1}{2} d(u^2+2u)}{\sqrt{u^2+2u}} = \| c.v.\ u^2 + 2u = t \| =$$

$$1 \to 3, \quad 2 \to 8$$

$$= \frac{1}{2}\int_3^8 \frac{dt}{\sqrt{t}} = \sqrt{t}\Big|_3^8 = \sqrt{8} - \sqrt{3}$$

$$I_6 = \int_0^{\sqrt{2}/2} \sqrt{\frac{1+x}{1-x}} dx = \int_0^{\sqrt{2}/2} \sqrt{\frac{(1+x)(1+x)}{(1-x)(1+x)}} dx = \int_0^{\sqrt{2}/2} \frac{1+x}{\sqrt{1-x^2}} dx = \int_0^{\sqrt{2}/2} \frac{1}{\sqrt{1-x^2}} dx +$$

$$+ \int_0^{\sqrt{2}/2} \frac{x}{\sqrt{1-x^2}} dx = \arcsin x \Big|_0^{\sqrt{2}/2} + \int_0^{\sqrt{2}/2} \frac{-\frac{1}{2} d(1-x^2)}{\sqrt{1-x^2}} \| c.v.\ 1 - x^2 = t = \frac{\pi}{4} +$$

$$0 \to 1, \quad \frac{\sqrt{2}}{2} \to \frac{1}{2}$$

$$+ \frac{1}{2}\int_1^{1/2} \frac{-dt}{\sqrt{t}} = \frac{\pi}{4} - \sqrt{t}\Big|_1^{1/2} = \frac{\pi}{4} - \sqrt{\frac{1}{2}} + 1$$

$$I_7 = \int_0^1 \frac{e^x}{e^x+1}\,dx = \int_0^1 \frac{de^x}{e^x+1} = \left\| \text{c.v. } e^x = t \quad \begin{matrix} 0 \to 1 \\ 1 \to e \end{matrix} \right\| = \int_1^e \frac{dt}{t+1} =$$

$$= \ln|t+1|\Big|_1^e = \ln(e+1) - \ln 2$$