Exo 1

$$f_1 \quad \begin{pmatrix} x \\ y \end{pmatrix} \mapsto x \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + y \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$$

$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ forment une famille libre

= base de $\operatorname{Im} f_1$

$\Rightarrow \dim \operatorname{Im} f_1 = 2$ $\Rightarrow$ $\operatorname{Im} f_1$ est plan $\subset \mathbb{R}^3$

$\Rightarrow \dim \ker f_1 = 0$

car $\dim \mathbb{R}^2 = \dim \operatorname{Im} f_1 + \dim \ker f_1$

$$\operatorname{Im} f_1 = \{ 9x - 8y - 7z = 0 \} \subset \mathbb{R}^3$$

$$= \{ (9; -8; -7)(x, y, z) = 0 \}$$

<- produit scalaire ->

Pourquoi $(9; -8; -7)$?

On calcule le produit vectoriel

$$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} = \det \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 2 & -1 \\ 3 & -1 & 5 \end{vmatrix} = 9e_1 - 8e_2 - 7e_3 = \begin{pmatrix} 9 \\ -8 \\ -7 \end{pmatrix}$$

le produit vectoriel est perpendiculaire aux facteurs

3/ $\quad \begin{cases} -x - 2y + z = a \\ 2x - y = b \\ x - 3y + z = c \end{cases}$

$y = 2x - b$

$z = c - x + 3y$

$= c - x + 6x - 3b$

$= 5x - 3b + c$

$2x - y = c - a$

$2x - y = b$

Calcul de determinant

$$\begin{pmatrix} -1 & -2 & 1 \\ 2 & -1 & 0 \\ 1 & -3 & 1 \end{pmatrix} = -1 \begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix} + +2 \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix}$$

$$= 1 + 4 + (-6 + 1)$$

$$= 0$$

resolution du systēme pour trouver le noyau

$$\boxed{\begin{matrix} -x - 2y + z = 0 \\ 2x - y = 0 \\ x - 3y + z = 0 \end{matrix}}$$

$\Rightarrow \quad 2x - y = 0$

$\Rightarrow -5x + z = 0$

subs

$$\ker f = \left\{ \begin{pmatrix} x \\ 2x \\ 5x \end{pmatrix}, x \in \mathbb{R} \right\}$$

$$\operatorname{Im} f = \left\{ x \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + z \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \; x, z \in \mathbb{R} \right\}$$

calculer le produit vectoriel pour trouver l'eqn du plan

Exo2

1/ $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ inversible $\Leftrightarrow$ $\det A = ad - bc \neq 0$

$\det \begin{pmatrix} 2 & -2 \\ 3 & -5 \end{pmatrix} = -10+6 = -4$, $A^{-1} = -1/4 \begin{pmatrix} -5 & 2 \\ -3 & 2 \end{pmatrix}$

$\det \begin{pmatrix} 4 & 1 \\ 12 & 3 \end{pmatrix} = 12-12=0$, $\ker A = \left\langle \begin{pmatrix} -1 \\ 4 \end{pmatrix} \right\rangle$

$\det \begin{pmatrix} 2 & 1 \\ -5 & -1 \end{pmatrix} = -2+5=3$, $A^{-1} = \frac{1}{3} \begin{pmatrix} -1 & -1 \\ 5 & 2 \end{pmatrix}$

2/ $\det AB = \det BC = 0$

$\det CA = \det AC = \det A \ \det C = -4 \times 3 = -12$

Exo 3

$$\det\begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix} = 0\begin{vmatrix}0&1\\1&0\end{vmatrix} + (-1)\times\begin{vmatrix}1&1\\1&0\end{vmatrix} + 1\times\begin{vmatrix}1&0\\1&1\end{vmatrix}$$

$$= 0 \quad -1\times -1 \quad + 1 \quad = 2$$

inverser l'application lin

$$\begin{pmatrix}x\\y\\z\end{pmatrix} \mapsto \begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix}$$

$$\begin{cases} y+z = a \\ x+y = b \\ x+z = c \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2}(b+c-a) \\ y = \frac{1}{2}(a+c-b) \\ z = \frac{1}{2}(a+b-c) \end{cases}$$

forme matricielle $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}-\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&-\frac{1}{2}\end{pmatrix}$

$$\det\begin{pmatrix}0&1&1\\1&0&1\\0&1&1\end{pmatrix} = 0\times\begin{vmatrix}0&1\\1&1\end{vmatrix} - 1\begin{vmatrix}1&1\\0&1\end{vmatrix} + 1\begin{vmatrix}1&0\\0&1\end{vmatrix}$$

$$= 0 - 1\times 1 + 1\times 1 = 0$$

système lin

$$\begin{cases} y+z = 0 \\ x+z = 0 \\ y+z = 0 \end{cases} \quad \begin{matrix}\Rightarrow y = -z \\ \Rightarrow x = -z\end{matrix}$$

$\Rightarrow$ $\ker A = \left\{\begin{pmatrix}-z\\-z\\z\end{pmatrix} z\in\mathbb{R}\right\}$

$\ker A = \left\langle\begin{pmatrix}1\\1\\-1\end{pmatrix}\right\rangle$