

Stage M2R

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my webpage

Context

The moduli space of Riemann surfaces is a very old and rich subject. The connexion with hyperbolic geometry was recognised by Klein as being of great importance and through the work of Thurston, Penner and many others at the end of the last century has improved our understanding of the geometry of Teichmueller space, the mapping class group and the moduli space itself. One important development has been the discovery of Mirzakhani's volume polynomials, the recursion they satisfy and the resulting formulation of so-called geometric recursion.

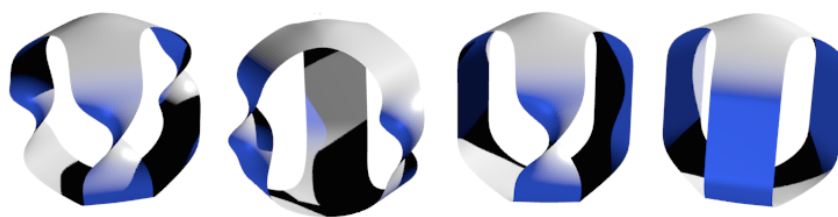


Figure 1: Orientable and non orientable surfaces

The Figure show the one holed torus, one holed Klein bottle, two holed projective plane and the three holed sphere.

A Klein surface is a non orientable topological surface together with a hyperbolic structure. The moduli space of Klein surfaces is much less well understood than that of Riemann surfaces. In particular there seems to be no notion of Mirzakhani's polynomials for Klein surfaces nor asymptotic formula for the number of closed simple geodesics.

In a recent manuscript Gendulpe hakes some calculations and proposes an explanation as to why there can be no such polynomials they should not exist. In another work Magee proves a counting formula for counting one sided simple closed geodesics on Fuchsian thrice punctured projective planes showing surprisingly that the growth rate is not polynomial.

Details of the stage

We will review the basic geometric constructions used in the oriented case and see how they have to be modified in the article of PAPADOPOULOS and PENNER. Next we will study the character variety for small surfaces as in Goldman, et al. Finally we will go through Gendulphe and compare his results with that of Magee to see why there are no polynomials.

References

1. Lengths of geodesics on non-orientable hyperbolic surfaces **Paul Norbury** Preprint
2. What's wrong with the growth of simple closed geodesics on nonorientable hyperbolic surfaces **Matthieu Gendulphe** Preprint arxiv.org/abs/1705.09377
3. Automorphisms of two-generator free groups and spaces of isometric actions on the hyperbolic plane **William Goldman, et al.** Memoirs of the American Mathematical Society 2019.
4. Counting one sided simple closed geodesics on Fuchsian thrice punctured projective planes **Michael Magee** Preprint [arXiv:1705.09377](https://arxiv.org/abs/1705.09377)
5. HYPERBOLIC METRICS, MEASURED FOLIATIONS AND PANTS DECOMPOSITIONS FOR NON-ORIENTABLE SURFACES **A. PAPADOPOULOS† AND R. C. PENNER** ASIAN J. MATH. c 2016 International Press Vol. 20, No. 1, pp. 157–182, January 2016

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