Evaluation On the Markoff equation

by Norbert Riedel

Conclusion

This manuscript is not in a form adequate to be sent for a detailed review at present. Its length and the many intricate calculations make it difficult to assimilate and verify properly. The interesting ideas should be packaged into a more manageable form and the technical work published in some other review beforehand. I believe I am justified my opinion by the criticisms of prior versions of this approach.

Context

In this mauscript the author proposes an approach to a conjecture de Frobenius, namely that for $z > 2$ there is a unique (normalised) triple of integers $z > y > x > 0$ such that

$$x^2 + y^2 + z^2 - 3xyz = 0.$$

There are have been numerous attempts to solve this conjecture (many false) see for example :

Rosenberger, G., The Uniqueness of the Markoff Numbers, but see MR 53 #280, Math. Comp. 30(1976)

and more recently partial results have given some hope to a definitive solution:

- Jack Button for z prime
- Zhang An elementary proof of Button
- Baragar z, 3z - 2, 3z+ 2 prime
- Bugeaud, Reutenauer, Siksek

The latter work (A Sturmian sequence related to the uniqueness conjecture for Markoff numbers by Yann Bugeaud , Christophe Reutenauer, Samir Siksek), which merely considers Fibonacci and Pell numbers, demonstrates exactly how hard even partial results are to prove.

Indeed some consider the question of whether a Markoff number is prime or not to be of considerable interest:

- Markoff triples and strong approximation, Jean Bourgain, Alexander Gamburd, Peter Sarnak, Comptes Rendus Mathematique.

The manuscript under consideration would appear to be a revised version of Markoff Equation and Nilpotent Matrices which was submitted on 10 Sep 2007 (v1). Subsequently the manuscript was transformed in some manner and became On the Markoff Equation. While mostly the author does not comment the

incremental versions this one has the remark " Comments: A flaw in the proof of the Theorem is being fixed". I would hope that this was a reaction to the work of S. Perrine towit:

Sur la conjecture de Frobenius relative aux solutions de l'équation de Markoff available here

It seems quite incongruous that the authr cites another work of Perrine without thanking him for hs meticulous reading of other versions of this text.

[Pe] S. Perrine, L'interpretation matricielle de la theory de Marko classique, Int. J. Math. Math. Sci. 32 (2002),no.4, 193-262

Diverse comments

The Markoff cubic is not strictly speaking

$$x^2 + y^2 + z^2 - xyz = 0,$$

but

$$x^2 + y^2 + z^2 - 3xyz = 0.$$

Of course there is a simple change of variable to go from one to the other and the Vieta flips between solutions have a nicer form for the latter but this is at the price of every "Markoff number" being divisible by 3.

There are numerous typos and misprints for example :

- In the references I believe it is Jim Propp: J.Popp, "The combinatorics of frieze patterns and Markoff numbers", arXiv:math/0511633
- pp73 the denominator of b is v and not u