Advanced Macro I

Homework DSGE

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Instructions: Students may work together, but they must turn in their assignment individually. Use TeX. Prepare an appendix where you should present the codes in Dynare and Matlab/Octave. Do not forget to plot the data when doing the estimation part. You will have to be explicit about the data, the filtering method, etc.

1 Basic Model

Consider the following model. The economy is populated by a large number of identical households with preferences described by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \tag{1}$$

Budget constraint:

$$d_t + y_t = c_t + i_t + \varphi(k_{t+1} - k_t) + (1 + r_{t-1})d_{t-1} \tag{2}$$

where d_t foreign debt, c_t real consumption, h_t hours worked, r_t real interest rate. Households can borrow and lend at the risk free interest rate in international markets. Finally, $\varphi(.)$ denotes capital adjustment costs and satisfies the property $\varphi(0) = \varphi'(0) = 0$. The capital stock evolves according to: $i_t = k_{t+1} - (1-\delta)k_t$, where δ is the depreciation rate.

Households choose processes $\{c_t, h_t, k_{t+1}, d_t\}_{t=0}^{\infty}$ so as to maximize the utility function subject to the budget constraint and to a no-Ponzi game condition:

$$\lim_{j\to\infty} E_t \frac{d_{t+j}}{\prod_{s=1}^{j}(1+r_s)} \leq 0 \tag{3}$$

Output is produced according to the technology:

$$y_t = A_t F(k_t, h_t) \tag{4}$$

where A_t denotes the TFP and $lnA_t = \rho \ln A_{t-1} + e_t$, where $e_t \sim NIID(0, \sigma^2)$. The interest rate evolves according to

$$r_t = \bar{r} + \psi(e^{d_t - \bar{d}} - 1)$$

where $\bar{r}$ is a constant international interest rate, $\psi > 0$ is a parameter, d_t is the aggregate debt level, and $\bar{d}$ is the steady-state debt level. The last term denotes the country risk premium.

1.1 Functional Forms

Utility takes the form:

$$U(c,h) = \frac{\left(c_t - \frac{1}{\omega}h_t^{\omega}\right)^{1-\gamma} - 1}{1-\gamma}$$

Production function:

$$F(K_t, h_t) = k_t^{\alpha} h_t^{1-\alpha}$$

Capital adjustment cost:

$$\varphi(k_{t+1} - k_t) = \frac{\phi}{2}(k_{t+1} - k_t)^2$$

1.2 Aggregation and definitions

The trade balance is defined as:

$$tb_t \equiv y_t - c_t - i_t - \varphi(k_{t+1} - k_t)$$

Trade-balance-to-output ratio:

$$tby = tb_t/y_t$$

and the current account to output ratio:

$$cay_t = (d_t - d_{t-1})/y_t$$

1.3 Calibration

$\gamma = 2$, $\omega = 1.455$, $\alpha = 0.32$, $\phi = 0.028$, $r = 0.04$, $\delta = 0.1$, $\rho = 0.42$ $\sigma = 0.0103$, $\bar{d} = 0.7442$, $\psi = 0.000742$.

1.4 Task 1: Putting the model in Dynare

- Solve the model (i.e. obtain the first order conditions by setting up the lagrangian), define the equilibrium, find the steady-state and put the model in Dynare (see the lectures slides).
- You have to write a separate m-file with the model steady state. Follow the example given in classroom.
- Simulate the model for 1,000 periods, discarding the first 950 periods, and obtain the IRFs to a TFP shock.
- Compute the same moments as shown in the table below. Present a similar table presenting the moments computed from the model side-by-side with the moments from the data.

1.5 Task 2: After we finish the estimation part of the course.

Pick data on real GDP from some country (annual data). You will have to filter the data to obtain the cyclical component. Estimate the process for TFP. Do all the tasks in estimation: priors check, identification, convergence, and compute the Bayesian IRFs.

1.6 Task 3: Adding Fiscal Policy

Adding government to the model. Suppose the government follows a balanced budget at all times. In this case, it levies a lump sum tax, T_t on households to finance an exogenous stream of expenditure, g_t, where $\ln(g_t/\bar{g}) = \rho_g \ln(g_{t-1}/\bar{g}) + e_t^g$, $\bar{g}$ is government spending in SS and $e_t^g \sim NIID(0, \sigma^g)$. You will have to estimate the parameters of the government expenditure process.

- Do all the tasks in estimation: priors check, identification, convergence, and compute the Bayesian IRFs.

- Compute the IRFs to a TFP and government spending shocks.
- Compute FEVD and historical decomposition (plot using Matlab/Octave).

	Data	M
Volatilities:		
$\text{std}(y_t)$	2.8	
$\text{std}(c_t)$	2.5	
$\text{std}(i_t)$	9.8	
$\text{std}(h_t)$	2	
$\text{std}(\frac{tb_t}{y_t})$	1.9	
$\text{std}(\frac{ca_t}{y_t})$		
Serial Correlations:		
$\text{corr}(y_t, y_{t-1})$	0.61	
$\text{corr}(c_t, c_{t-1})$	0.7	
$\text{corr}(i_t, i_{t-1})$	0.31	
$\text{corr}(h_t, h_{t-1})$	0.54	
$\text{corr}(\frac{tb_t}{y_t}, \frac{tb_{t-1}}{y_{t-1}})$	0.66	
$\text{corr}(\frac{ca_t}{y_t}, \frac{ca_{t-1}}{y_{t-1}})$		
Correlations with Output:		
$\text{corr}(c_t, y_t)$	0.59	
$\text{corr}(i_t, y_t)$	0.64	
$\text{corr}(h_t, y_t)$	0.8	
$\text{corr}(\frac{tb_t}{y_t}, y_t)$	-0.13	
$\text{corr}(\frac{ca_t}{y_t}, y_t)$		