$$\underset{\{c_t, K_t, h_t, d_t\}_{t=0}^{\infty}}{\text{Max}} E_0 \sum_{t=0}^{\infty} B^t U(c_t, h_t) \quad \text{onde} \quad U(c_t, h_t) = \frac{\left[c_t - \frac{1}{w} \cdot h_t^w\right]^{1-\gamma} - 1}{1-\gamma}$$

$$\text{s.a} \quad d_t + y_t = c_t + i_t + \varphi(K_{t+1} - K_t) + (1 + r_{t-1}) \cdot d_{t-1}$$

$$y_t = A_t F(K_t, h_t) = A_t K_t^a h_t^{1-a}$$

$$i_t = K_{t+1} - (1-\delta) K_t$$

$$\varphi(K_{t+1} - K_t) = \frac{\phi}{2} \cdot (K_{t+1} - K_t)^2$$

$$\lim_{j \to \infty} E_t \frac{d_{t+j}}{\prod_{s=1}^{j}(1+r_s)} \leq 0$$

Portanto, $d_t + A \cdot F(K_t, h_t) = c_t + K_{t+1} - (1-\delta) K_t + \frac{\phi}{2} \cdot (K_{t+1} - K_t)^2 + (1 + r_{t-1}) \cdot d_{t-1}$

$$L = E_0 \sum_{t=0}^{\infty} B^t \frac{\left[c_t - \frac{1}{w} \cdot h_t^w\right]^{1-\gamma} - 1}{1-\gamma} + B^t \lambda_t \left[d_t + A_t F(K_t, h_t) - c_t - K_{t+1} + (1-\delta) K_t - \frac{\phi}{2}(K_{t+1} - K_t)^2 - (1 + r_{t-1}) \cdot d_{t-1}\right]$$

$$\frac{dL}{dc_t} = B^t \cdot \frac{(1-\gamma)}{(1-\gamma)} \cdot \left[c_t - \frac{1}{w} \cdot h_t^w\right]^{-\gamma} - B^t \cdot \lambda_t$$

$$\hookrightarrow \lambda_t = \left[c_t - \frac{1}{w} \cdot h_t^w\right]^{-\gamma} \quad (0)$$

$$\frac{dL}{dK_{t+1}} = -B^t \cdot \lambda_t - B^t \lambda_t \cdot \phi (K_{t+1} - K_t) + B^{t+1} \cdot \lambda_{t+1} \left\{A_{t+1} \cdot F'(K_{t+1}, h_{t+1}) + (1-\delta) + \phi \cdot (K_{t+2} - K_{t+1})\right\}$$

$$\lambda_t \left[1 + \phi \cdot (K_{t+1} - K_t)\right] = B \cdot \lambda_{t+1} \left\{A_{t+1} \cdot F'(K_{t+1}, h_{t+1}) + (1-\delta) + \phi \cdot (K_{t+2} - K_{t+1})\right\}$$

$$\left[c_t - \frac{1}{w} \cdot h_t^w\right]^{-\gamma} \cdot \left[1 + \phi(K_{t+1} - K_t)\right] = B \cdot \left[c_{t+1} - \frac{1}{w} h_{t+1}^w\right]^{-\gamma} \cdot \left\{A_{t+1} \cdot a \cdot K_{t+1}^{a-1} \cdot h_{t+1}^{1-a} + (1-\delta) + \phi(K_{t+2} - K_{t+1})\right\} \quad (1)$$

$$\frac{dL}{dh_t} = B^t \cdot \left[C_t - \frac{1}{w} \cdot h_t^w\right]^{-\gamma} \cdot \left(-h_t^{w-1}\right) + B^t \lambda_t \cdot A_t \cdot (1-a) \cdot K_t^a \cdot h_t^{-a}$$

$$\underbrace{\left[C_t - \frac{1}{w} \cdot h_t^w\right]^{-\gamma}}_{\lambda_t} \cdot h_t^{w-1} = \lambda_t \cdot A_t \cdot (1-a) \cdot K_t^a \cdot h_t^{-a}$$

$$\hookrightarrow h_t^{w-1} = A_t(1-a) \cdot K_t^a \cdot h_t^{-a} \qquad (2)$$

Agora vamos para $\frac{dL}{dd_t}$, lembre $r_t = \bar{r} + \psi\left(e_t^{d_t - \bar{d}} - 1\right)$

$$r_{t-1} = \bar{r} + \psi\left(e_{t-1}^{d_{t-1} - \bar{d}} - 1\right)$$

$$L = E_0 \sum_{t=0}^{\infty} B^t \frac{\left[C_t - \frac{1}{w} \cdot h_t^w\right]^{1-\gamma} - 1}{1-\gamma} + B^t \lambda_t \left[d_t + A_t F(K_t, h_t) - C_t - K_{t+1} + (1-\delta) K_t - \frac{\phi}{2}\left(K_{t+1} - K_t\right)^2 - (1 + r_{t-1}) \cdot d_{t-1}\right]$$

$$B^t \cdot \lambda_t \left\{d_t + A_t \cdot F(K_t, h_t) - C_t - K_{t+1} + (1-\delta) K_t - \frac{\phi}{2}\left(K_{t+1} - K_t\right)^2 - d_{t-1} - d_{t-1}\left[\bar{r} + \psi\left(e_{t-1}^{d_{t-1} - \bar{d}} - 1\right)\right]\right\}$$

$$\frac{dL}{dd_t} = B^t \cdot \lambda_t - B^{t+1} \cdot \lambda_{t+1} - B^{t+1} \cdot \lambda_{t+1} \cdot \bar{r} - B^{t+1} \cdot \lambda_{t+1} \cdot \psi\left[e_t^{d_t - \bar{d}} \cdot (1 + d_t) - 1\right]$$

$$\lambda_t = B \cdot \lambda_{t+1} \cdot \left\{1 + \bar{r} + \psi\left[e_t^{d_t - \bar{d}} \cdot (1 + d_t) - 1\right]\right\}$$

$$\left[C_t - \frac{1}{w} \cdot h_t^w\right]^{-\gamma} = B \cdot \left[C_{t+1} - \frac{1}{w} \cdot h_{t+1}^w\right]^{-\gamma} \cdot \left\{1 + \bar{r} + \psi\left[e_t^{d_t - \bar{d}} \cdot (1 + d_t) - 1\right]\right\}$$

$$(3)$$

Relembrar a restrição:

$$d_t + y_t = c_t + i_t + \varphi(K_{t+1} - K_t) + (1 + r_{t-1})\cdot d_{t-1}$$

$$d_t + A_t \cdot K_t^{a} \cdot h_t^{1-a} = c_t + K_{t+1} - (1-\delta)K_t + \frac{\phi}{2}(K_{t+1} - K_t)^2 + (1 + r_{t-1})\cdot d_{t-1}$$

$$r_t = \bar{r} + \psi\left(e^{d_t - \bar{d}} - 1\right) \quad e \quad \ln A_t = \rho \cdot \ln A_{t-1} + e_t$$